Experimental Demonstration of AoA Estimation Uncertainty for IoT Sensor Networks

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***Abstract*—In practice, the subspace-based algorithms such as Multiple Signal Classification (MUSIC) suffer from sensitivity to antenna-array response errors and therefore they require the assessment of the calibration gain and phase perturbations. This paper evaluates experimentally the accuracy of Angle-of-Arrival (AoA) estimation based on the MUSIC algorithm only coming from these perturbations in the context of Internet-of-Thing (IoT) applications. First of all, a new Over-the-Air (OTA) calibration method is proposed and gain and phase uncertainties are investigated. The impact of these uncertainties on the accuracy of AoA estimation is then studied and compared with the theoretical analysis. The experimental results show that the calibration errors coming from hardware imperfections can cause some degrees of uncertainty in AoA estimation.**

I. Introduction

The subspace-based algorithms for narrowband Angle-of-Arrival (AoA) estimation have received more attention due to their high accuracy. These algorithms are suitable for Internet-of-Thing (IoT) applications [1]. Bluetooth Low Energy (BLE) is the most popular wireless connection technology for IoT sensor networks thanks to its low power consumption, low cost, and high availability [2]. The direction of the received signal is determined by processing the signal impinging on an antenna-array [3]. Multiple Signal Classification (MUSIC) algorithm is one of the well-known subspace-based algorithms [4] in terms of practical use and ease of implementation.

MUSIC algorithm provides a high AoA estimation accuracy for narrowband antenna array-based systems under far-field conditions [5]. However, in practice, finite sample sets, a perturbed antenna-array response (gain and phase perturbations) and an imprecisely known noise covariance are sources of error in AoA estimation using the MUSIC algorithm. The performance of the MUSIC algorithm based on a finite number of the samples has been analyzed in [6]. This effect is negligible when the number of the samples is large or the Signal-to-Noise Ratio (SNR) is high. In [7], the uncertainty of the MUSIC algorithm is evaluated under the perturbed noise covariance and antenna-array response. Moreover, the effects of calibration gain and phase perturbations on the AoA estimation performance of the MUSIC algorithm have been investigated in [8]. The gain and phase errors are caused due to measurement errors, environment change, and sensor misplacement.

Recently, we experimentally validated the accuracy of AoA estimation based on the MUSIC algorithm when the BLE signal transmitted by a beacon device [9]. The antenna array-based systems have been implemented with Universal Software Radio Peripheral (USRP) platform. The experimental results demonstrated a promising accuracy of the MUSIC algorithm, although the impact of hardware imperfections has not studied carefully. To the best of authors' knowledge, no experimental investigation of the MUSIC algorithm performance is found in the literature under calibration perturbations. Therefore, the aim is to experimentally demonstrate and assess the uncertainty of the MUSIC algorithm under these perturbations. At first, we propose a new Over-the-Air (OTA) calibration method which is easier and more flexible than cable calibration in terms of implementation. Later a detailed experimental demonstration is performed to precisely assess the calibration error and estimate the resulting uncertainty of the MUSIC algorithm. The validation of the experimental measurements is then thoroughly tested in comparison with the theoretical results.

This paper is organized as follows. Section II briefly introduces a theoretical model of the calibration gain and phase perturbations and provide explicit expression to predict the variance of the angle estimation with the MUSIC algorithm originating from these perturbations. Section III describes the experimental demonstration and proposed OTA calibration method implemented with a USRP platform. Accordingly, the experimental results are given to show the uncertainty of the MUSIC algorithm due to calibration perturbations. Finally, the conclusion is presented in Section IV.

II. MUSIC Analysis with Calibration Gain and Phase Perturbations

Consider a Uniform Linear Array (ULA) of M antennas with half-wavelength inter-antenna spacing ($d = \lambda/2$). The waveforms received at each antenna are linear combinations of additive noise and the narrow-band signal from a source, centered at frequency f_c, located in the far-field of the ULA. The signal and the additive noises at each antenna are zero-mean stationary random variables. It is assumed that the noises are independent between antennas and are uncorrelated with the signal. The antenna-array response of the *m*-th antenna $a_m(\theta)$ can be expressed by gain and phase responses of each antenna as $\alpha_m(\theta)$ and $\phi_m(\theta)$, respectively, for the signal impinging from direction θ. In practice, there is some tolerance in gain and phase responses. We assume a zero-mean random gain error $\Delta\alpha_m$ and phase error $\Delta\phi_m$ at the *m*-th antenna with variances $\sigma^2_{\alpha_m}$ and $\sigma^2_{\phi_m}$, respectively. These errors are assumed to be independent of each other and between antennas. The received signal is given by [8]

$$\tilde{r}_m(t) = \tilde{a}_m(\theta)s(t)e^{-j2\pi f_c \tau_m(\theta)} + n_m(t), \qquad (1)$$

where $\tau_m(\theta)$ is propagation delay between the source and the *m*-th antenna, $n_m(t)$ is additive noise at the *m*-th antenna and $\tilde{a}_m(\theta)$ refers to the perturbed antenna-array response, i.e.,

$$\tilde{a}_m(\theta) = \left(\alpha_m(\theta) + \Delta\alpha_m\right) e^{j(\phi_m(\theta)+\Delta\phi_m)}. \qquad (2)$$

Assuming small $\Delta\alpha_m$ and $\Delta\phi_m$,

$$\tilde{a}_m(\theta) \approx a_m(\theta) + (\Delta\alpha_m + j\Delta\phi_m)a_m(\theta). \qquad (3)$$

The antenna-array covariance matrix is perturbed and consequently it affects the AoA estimation performance. Therefore, the MUSIC spectrum is represented by [8]

$$\tilde{\mathbf{P}}(\theta) = \mathbf{v}^H(\theta)\tilde{\mathbf{U}}_n\tilde{\mathbf{U}}_n^H\mathbf{v}(\theta), \qquad (4)$$

where $\mathbf{v}(\theta)$ is normalized direction vector which is defined as

$$\mathbf{v}(\theta) = \frac{1}{\sqrt{M}}\left[1, e^{j\pi\sin(\theta)}, \cdots, e^{j(M-1)\pi\sin(\theta)}\right]^T, \qquad (5)$$

and $\tilde{\mathbf{U}}_n = [\tilde{\mathbf{u}}_2, \cdots, \tilde{\mathbf{u}}_M]$ is known as the perturbation noise subspace, where $\tilde{\mathbf{u}}_m$ is corresponding to *m*-th perturbation eigenvector when $\tilde{\lambda}_1 \geq \tilde{\lambda}_2 = \cdots = \tilde{\lambda}_m$ are the eigenvalues of perturbed covariance.

From [8], the mean-square error (MSE) of AoA estimation is approximated by first order Taylor series expansion of MUSIC spectrum as follows

$$\mathcal{E}[\Delta\theta^2] \approx \mathcal{E}\left[\left(\tilde{\mathbf{P}}'(\theta)\right)^2\right] / \left(\mathbf{P}''(\theta)\right)^2, \qquad (6)$$

where $\Delta\theta = \hat{\theta} - \theta$, and $\mathcal{E}[\cdot]$ denotes the expectation over $\Delta\alpha_m$ and $\Delta\phi_m$. The second derivative of $\mathbf{P}(\theta)$ with respect to θ is computed as

$$\mathbf{P}''(\theta) = 2\mathbf{v}'^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{v}'(\theta), \qquad (7)$$

where $\mathbf{v}'(\theta)$ is the first derivative of $\mathbf{v}(\theta)$. Therefore, the denominator of (6) can simplified as

$$\left(\mathbf{P}''(\theta)\right)^2 = \frac{(M^2-1)^2}{36}(\pi\cos\theta)^4. \qquad (8)$$

Applying the orthogonality condition of eigenvectors and approximation of the perturbed covariance matrix by collecting the second order terms [8], we obtain the numerator of (6) as follows

$$\mathcal{E}\left[\left(\tilde{\mathbf{P}}'(\theta)\right)^2\right] \approx \frac{4}{M}\mathbf{v}'^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\Sigma_\phi\mathbf{v}'(\theta), \qquad (9)$$

where

$$\mathbf{\Sigma}_\phi = \mathrm{diag}\left\{\sigma^2_{\phi_1}, \cdots, \sigma^2_{\phi_M}\right\}. \qquad (10)$$

Using $\mathbf{U}_n\mathbf{U}_n^H = \mathbf{I} - \mathbf{v}\mathbf{v}^H$, where $\mathbf{I}$ is identify matrix, (9) is simplified as

$$\begin{aligned}\mathcal{E}\left[\left(\tilde{\mathbf{P}}'(\theta)\right)^2\right] &\approx \frac{4}{M}\big(\mathbf{v}'^H(\theta)\Sigma_\phi\mathbf{v}'(\theta) \\ &\quad - \mathbf{v}'^H(\theta)\mathbf{v}\mathbf{v}^H\Sigma_\phi\mathbf{v}'(\theta)\big), \qquad (11)\\ &= \frac{4}{M}(\pi\cos\theta)^2\Big(\sum_{m=1}^{M}(m-1) \\ &\quad \cdot \big(m - (\frac{M+1}{2})\big)\sigma^2_{\phi_m}\Big). \qquad (12)\end{aligned}$$

Substituting (8) and (12) into (6), we compute the MSE of the AoA estimation under the calibration uncertainties as follows

$$\begin{aligned}\mathcal{E}[\Delta\theta^2] &\approx \Big(\frac{12}{M(M^2-1)\pi\cos\theta}\Big)^2 \cdot \\ &\quad \sum_{m=2}^{M}\sigma^2_{\phi_m}(m-1)\big(m - (\frac{M+1}{2})\big). \qquad (13)\end{aligned}$$

The variance of the gain does not appear in this equation which means that the gain uncertainty does not impact AoA estimation of the MUSIC algorithm. When the phase calibration errors are identically distributed between antennas ($\mathbf{\Sigma}_\phi = \sigma^2_\phi\mathbf{I}$), the summation in the numerator of (13) can be simplified to

$$\sum_{m=2}^{M}\sigma^2_{\phi_m}(m-1)\big(m - (\frac{M+1}{2})\big) = \sigma^2_\phi\frac{M(M^2-1)}{12}. \qquad (14)$$

Therefore, the variance of AoA estimation is expressed in closed-form as [8]

$$\sigma^2_\theta \approx \frac{12\sigma^2_\phi}{M(M^2-1)(\pi\cos\theta)^2}. \qquad (15)$$

In the following, we assess the theoretical model of AoA uncertainty with experiments.

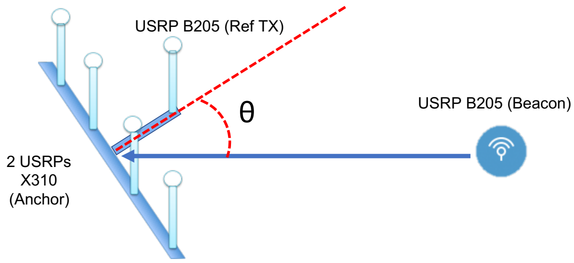

Fig. 1. Experimental setup for AoA measurements.

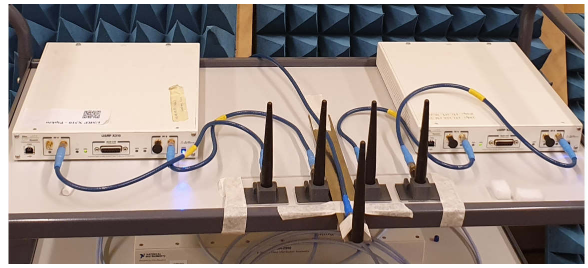

Fig. 2. Anchor setup with the near-field OTA calibration method.

III. Experimental Demonstration

We build an experimental setup to study the impact of the calibration phase uncertainty on the AoA measurements. Figure 1 illustrates the experimental setup with one anchor and one beacon in a clear Line of Sight (LoS) channel. The setup is implemented with the USRP platform. The anchor is equipped with a ULA consisting of four vertical dipoles antennas with half-wavelength inter-antenna spacing ($d = 6$ cm). The ULA is created by two USRPs X310 which are synchronized by an OctoClock timing reference [10]. A beacon periodically broadcasts BLE advertising packets with a data rate of 1 Mb/s according to the Bluetooth core specification [2]. The advertising packets are modulated using Gaussian Frequency Shift Keying (GFSK) with $BT = 0.5$ and modulation index $h = 0.5$, so the maximum frequency is $fd = 250$ kHz. A USRP B205mini-i with a vertical dipole antenna is used to build the beacon. The beacon position should fulfill the far-field condition which applies at distances greater than one meter from the ULA. All USRPs are connected to a host computer via the Gigabit Ethernet cables and configured by GNU Radio Companion.

A. OTA calibration method

USRPs introduce a hardware phase difference between antennas during the acquisition of signals, although all USRPs are sharing the same external clock and Pulse-per-Second (PPS) timing signal. The phase drift is caused by hardware manufacturing tolerances in the Radio Frequency (RF) feeding network, thermal effects in the Power Amplifiers (PAs) and Low Noise Amplifiers (LNAs), and group delay variations in the filters. Hence, gain and phase calibration has to be applied on the receiver side of the antenna-array.

We propose a new OTA calibration method which is easy and flexible to implement in comparison with a cable calibration. Since the cable calibration should be implemented by connecting the antenna ports of USRPs with power splitter [10], the phase calibration should be performed every time before starting the experiments and it is highly inconvenient to move cables every time to perform calibration.

In the proposed OTA calibration, another USRP B205mini-i with a vertical dipole antenna is used in near-field distance located with $a = 10$ cm from the middle of the antenna-array as a Reference Transmitter (Ref Tx). The anchor setup with proposed OTA calibration is shown in Figure 2. The periodic pulses of sine wave are transmitted from the single antenna port and receive by antennas with a phase shift which needs to be compensated. The phase calibration defines the reference direction. This method can be performed simultaneously with AoA measurements and allows the accurate estimation of AoA uncertainty under the calibration errors. The gain levels of both calibration and BLE signals are settled by USRP power adjustment in GNU Radio. Hence, BLE TX, Ref TX, RX Gains of USRPs are fixed to 60 dB, 30 dB and 20 dB, respectively. Note that when the Ref Tx is not far away from the antenna-array, the phase shift $\Phi = \frac{2\pi f_c b}{c}$ due to the near-field distance needs to be compensated where c is the free-space propagation velocity and b can be simply calculated by euclidean distance, i.e, $b = \sqrt{a^2 + (\frac{3d}{2})^2} - \sqrt{a^2 + (\frac{d}{2})^2}$.

In this calibration method, the antenna-array can simultaneously be calibrated during the AoA measurements. In order to avoid the conflict between BLE Tx and Ref Tx, BLE and sine waveforms should be generated with an appropriate frequency spacing. Therefore, the whole bandwidth should be monitored by ULA and related filters have to be applied to recover both signals. Figure 3 shows the Welch estimation [11] of Power Spectral Density (PSD) of received signal as a function of frequency with window length of $L = N/2 = 500$ samples.

B. Experimental results

Two kinds of experiments are performed to assess the AoA uncertainty due to the calibration phase error as follows

- First of all, two ULAs are calibrated separately within 24 hours every 1 min and the phase difference between two USRPs in each ULA has been measured periodically. The variance of phase error is obtained from the phase measurements. We can then derive theoretically the variance of AoA from Eq. 13.

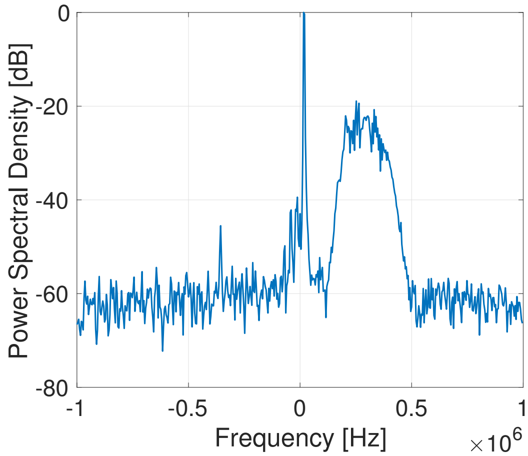

Fig. 3. Estimated PSD with the Welch method.

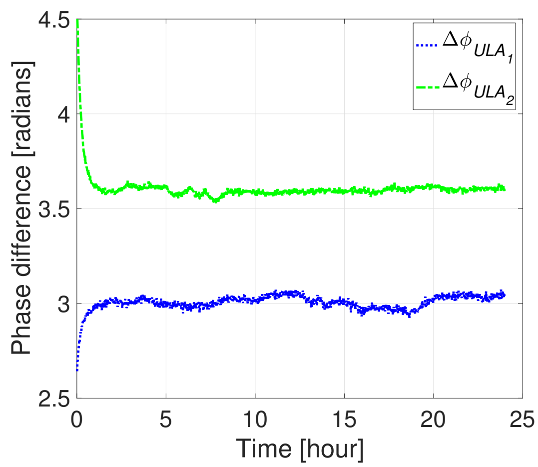

Fig. 4. Phase difference between two USRPs within 24 hours.

- Later on, AoA is directly estimated by evaluating the calibration and received signals from different pre-defined directions within 24 hours every 1 min in the same experimental conditions. We can then compute the variance of AoA from the measurements for each direction and validate this result in comparison with the result of the first experiment.

In order to omit the effect of the finite sample set, the SNR is assumed to be high such that only a limited amount of samples ($N = 100$) are enough to guarantee the hypothesis of the low number of samples.

In Fig. 4, the phase difference between two USRPs ($\Delta\phi$) is shown within 24 hours for two ULAs. Note that the phase difference error between two antennas of one USRP is approximately constant. As can be seen, there is a phase drift over time and significantly during the warming-up period of USRPs. Additionally, a considerable change in phase difference occurs between two ULAs due to the different hardware manufactures.

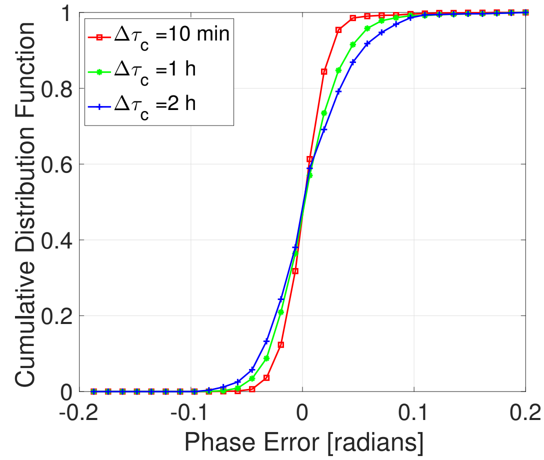

Fig. 5. Cumulative Distribution Function of the phase error for different time delays from calibration for ULA_1.

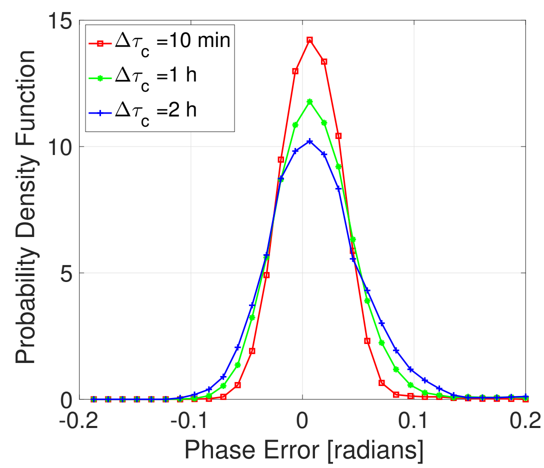

Fig. 6. Probability Density Function of the phase error for different time delays from calibration for ULA_1.

The Cumulative Distribution Function (CDF) of the phase error is shown in Fig. 5 for different time delays from calibration ($\Delta\tau_c$). To derive the CDF of the phase error, we assume a sliding window with a size of time delay from the calibration on the phase difference obtained from the previous figure. Fig. 6 illustrates Probability Density Function (PDF) of the phase error. It shows that the calibration phase error has a Gaussian distribution. The mean of the distribution is not null due to the drift with time. By increasing the delay from the calibration time, the accuracy of the calibration method is less reliable since the PDF is wider.

The standard deviation of phase error as a function of the time delay from calibration is depicted in Fig. 7 for two ULAs. It confirms that increasing the time delay from calibration adds more uncertainty on phase calibration. Furthermore, it displays the impact of hardware change on the standard deviation of phase error.

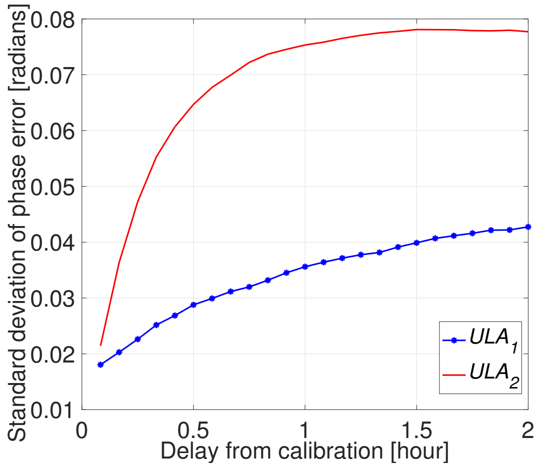

Fig. 7. Standard deviation of phase error as a function of the time delay from calibration for two ULAs.

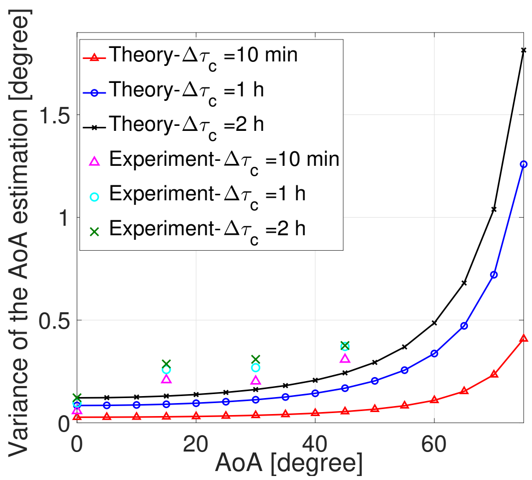

Fig. 8. Variance of AoA as a function of the angle of arrival for ULA_1.

Fig. 8 finally compares the variance of AoA obtained from the experiment to that obtained from theory as a function of the angle-of-arrival for ULA_1. Knowing approximately constant phase difference error between two antennas of one USRP and substituting the extracted phase variance from Fig. 7 into Eq. 13, we can compute the variance of AoA due to the phase error for different directions of arrival. For the next part of the experiment, we conducted the test for pre-defined angles of $0°, 15°, 30°$, and $45°$. The AoA is estimated for each direction within 24 hours every 1 min with different time delays from calibration. Note that the mismatch between the theory and experiment comes from the fact that in theory, the AoA estimation errors are approximated when the phase errors are distributed as zero-mean Gaussian random variables, although, for experimental demonstration, phase errors follow a non-zero-mean Gaussian distribution. In both experiment and theory, the variance of AoA estimation is increased with the increment of the direction-of-arrival and the delay between measurements and calibration time. The phase uncertainty comes from hardware manufactures, as in this case, it introduces up to 2 degrees uncertainty to AoA measurements.

IV. Conclusion

In this paper, an experimental demonstration of the AoA estimation uncertainty under the calibration errors is presented. We propose a new OTA calibration method to investigate precisely the calibration errors and their impact on the AoA estimation uncertainty. We experimentally study the accuracy of the calibration method which is less reliable by increasing the delay from the calibration time. We also show that the phase uncertainty depends on the hardware manufactures and this uncertainty can lead to AoA estimation error which is not negligible for higher angles of the incident signal.

Acknowledgment

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