Outage Probability Analysis of the Relay Network with Correlated Relaying Channels

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***Abstract*—A relay network can provide the extended link capacity and coverage thanks to its good spatial diversity. In order to analyze the relay network performance, most of the works in literature assume the independent relaying channels. In practice, the received signals experience the common scatterers, hence the correlated relaying channels should be considered. In this paper, we first derive a novel and accurate probability density function (PDF) of a modulus of the sum of products of two correlated zero-mean complex Gaussian random variables (RVs), which is useful for the statistical analysis of the cascaded relaying fading channels. Based on this result, we secondly investigate the received signal-to-noise ratio (SNR) outage probability of a relay network in the presence of correlated relaying channels. It is found that the correlation magnitude is more important than the correlation phase for the analysis of power-based signal-detection of relay networks. Numerical simulations have been carried out to confirm the correctness of the derivation.**

***Index terms*—** Probability density function, correlated complex Gaussian RVs, relay network.

I. Introduction

Relay networks have been considered as a promising solution in emerging wireless communication systems thanks to their ability to increase the link capacity and coverage [1]. Similar to the multiple antenna systems, a relay network can benefit from the spatial diversity, created by the different relay links [2]. The mobile broadband communication systems, such as the long-term evolution-advanced (LTE-A), IEEE 802.16j, IEEE 802.16m, have already integrated the relay-aided transmission [3]. A relay network, composed of a source node, several relay nodes and a destination node, generally has two main transmission phases. The source node broadcasts the signal to all relay nodes in the first phase. After that the relay nodes choose to either purely amplify and retransmit the signal to the destination (amplify-and-forward (AF) scheme) or to decode the signal and then transmit the reconstructed signal to the destination (decode-and-forward (DF) scheme). In this work, we focus on the AF scheme because of its low complexity and high efficiency [2].

In relay networks, the cascaded Rayleigh fading channel has been reported theoretically and experimentally to be an appropriate channel model, i.e., multi-hop mobile-to-mobile channels [4]. Most of the works in the literature consider the statistically independent cascaded channels [5]. In practice, because the signals experience common scatterers, the channels can be correlated [6], [7], especially in the cooperative sensor networks, where low-complexity nodes are densely deployed. There are various kinds of channel correlations existing in the relay network and the review of such channel correlations has been thoroughly documented in [8]. The authors then analyzed the impact on the performance of the best-relay-selection-scheme based relay network, when there is a correlation between source-relay and relay-destination channels for each relay. The spatial diversity hence may not be fully exploited.

In this paper, we investigate the received signal-to-noise ratio (SNR) outage probability of the relay network in the presence of the correlated relaying channels. On the contrary to [8], we assume the system uses all the signals coming from the relay nodes (no relay selection). More specifically, our contributions are summarized as (1) the derivation of the exact probability density function (PDF) of the modulus of sum of products of two correlated Gaussian random variables (RVs), which is useful to characterize the cascaded relaying channels; (2) the derivations of the PDF and cumulative distribution function (CDF) of received SNR; (3) the received SNR outage probability analysis of the relay nework with correlated relaying channels and numerical validations.

Notation: $\mathbb{E}[\cdot]$ and $|\cdot|$ are the expectation and absolute operators, respectively; X_R and X_I denote the real and imaginary parts of X, respectively; $x!$ is the factorial of a positive integer x; $\Gamma(\cdot)$ is the Gamma function and $\Gamma(x) = (x-1)!$ for a positive integer number x; $\mathbb{J}_L(\cdot)$ is the L-th order Bessel function of the first kind; $\mathbb{I}_L(\cdot)$ and $\mathbb{K}_L(\cdot)$ are the L-th order modified Bessel functions of the first and second kind, respectively.

II. Pre-requisite: PDF of the Modulus of Sum of Products of two correlated Gaussian RVs

We consider that (X_l, Y_l) are complex-valued, zero-mean, Gaussian RVs, i.e., $X_l \sim \mathcal{CN}\left(0, \sigma_X^2\right)$, $Y_l \sim \mathcal{CN}\left(0, \sigma_Y^2\right)$ of cross-correlation $\mu = \mathbb{E}\left[X_l \cdot Y_l\right] \Big/ \sqrt{\mathbb{E}\left[|X_l|^2\right] \cdot \mathbb{E}\left[|Y_l|^2\right]} = |\mu| \cdot \exp(j\varepsilon)$; and such that (X_l, Y_l) are statistically independent and identically distributed (i.i.d.) with any other pairs (X_k, Y_k) for $\forall l \neq k$.

Theorem: Let $Z = \sum_{l=1}^{L} X_l \cdot Y_l$, the PDF of $R = \sqrt{Z_R^2 + Z_I^2}$ is provided in (1).

Proof: See the Appendix. It is worth noticing that the derivation steps are suggested in [9], unfortunately, the detailed derivation was omitted. Furthermore, we correct some errors in previously derived joint PDF formulas, i.e., [9, eqs. (11) and (12)]. Inspecting (1), we observe that the correlation phase ε plays no role in this PDF, meaning that in the power-based detection system, only the correlation magnitude $|\mu|$ has an impact on the performance. Note that, the derived PDF has also been applied on the performance analysis of the time-reversal communications [10], when the correlation is equal to zero.

III. Outage Probability of the Relay Network with Correlated Relaying Channels

A. Relay Network Model

We consider a relay network, where two terminals $\mathcal{T}_1$ and $\mathcal{T}_2$ are equipped with a single antenna. The communication between two terminals is carried out via L single-antenna relays $\mathbf{r} = (\mathcal{R}_1, \mathcal{R}_2, ..., \mathcal{R}_L)^T$. We define $\mathbf{h} \triangleq (h_1, h_2, ..., h_L)^T$ and $\mathbf{g} \triangleq (g_1, g_2, ..., g_L)^T$ as the channel vectors from $\mathcal{T}_1$ to each relay $\mathcal{R}_l$ and from each relay $\mathcal{R}_l$ ($\forall l = 1, ..., L$) to $\mathcal{T}_2$, respectively. We assume that any of two relaying channels (between $\mathcal{T}_1$ and $\mathcal{R}_l$ as well as between $\mathcal{R}_l$ and $\mathcal{T}_2$) are reciprocal and subject to equally correlated block Rayleigh fading. According to the Rayleigh assumption, the elements of the two vectors $\mathbf{h}$ and $\mathbf{g}$ follow zero-mean circularly symmetric complex Gaussian RV with variances σ_h^2 and σ_g^2, respectively. The relaying channels h_l and g_l are assumed to be correlated with correlation $\mu = |\mu| \cdot \exp(j\varepsilon)$. This assumption makes sense especially in the case there is no line-of-sight (LoS) communication between $\mathcal{T}_1$ and $\mathcal{T}_2$, i.e., relay network of vehicule-to-vehicule (V2V) communications at the crossing streets and/or critical points [4], [8]. We assume that the transmit power at the source $\mathcal{T}_1$ and each relay $\mathcal{R}_l$ are P_t and P_r, respectively. During the first transmission phase, $\mathcal{T}_1$ transmits the symbol x to each relay $\mathcal{R}_l$, where the value of x can be assigned from *M*-phase shift keying (*M*-PSK) (or *M*-quadrature amplitude modulation (*M*-QAM)) modulation and $\sigma_x^2 \triangleq \mathbb{E}[|x|^2]$. In order to ensure a normalized signal at each relay, $\mathcal{R}_l$ simply scales the received signal by a fixed factor $\alpha \triangleq \sqrt{P_r/(\sigma_h^2 P_t + \sigma_{v1}^2)}$, where σ_{v1}^2 denotes the variance of the additive white Gaussian noise (AWGN) at each relay node. Finally, relay nodes broadcast the signals to $\mathcal{T}_2$ during the second transmission phase. The received signal at $\mathcal{T}_2$ is expressed as

$$\tilde{y} = \mathbf{g}^T \cdot (\alpha \cdot \mathbf{h} \cdot x + \alpha \cdot \mathbf{v}_1) + v_2, \tag{2}$$

where $\mathbf{v}_1 = [v_{11}, v_{12}, ..., v_{1L}]^T$ is the AWGN added at each relay after the first phase of the communication, v_2 is the AWGN of variance σ_{v2}^2 at the receiver $\mathcal{T}_2$. Exploiting the fact that $\mathbf{g}$ and $\mathbf{v_1}$ are independent of each other, we define $\mathbf{n} \triangleq \mathbf{g}^T \cdot \mathbf{v}_1 + v_2/\alpha$ as the equivalent additive noise such that (2) can be rewritten as $y = \mathbf{g}^T \cdot \mathbf{h} \cdot x + \mathbf{n}$, where $y \triangleq \tilde{y}/\alpha$ is the scaled received signal. It can be observed that the statiscal characterization of equivalent cascaded channel $\mathbf{h}^T \cdot \mathbf{g}$ is important for the analysis of the received signal in the presence of the correlated relaying channels. Note that, $\mathbf{g}^T \cdot \mathbf{v}_1$ also follows the distribution of cascaded channels, with $\mu = 0$ (no correlation). It is easy to show that $\mathbf{n}$ has zero-mean and variance of $\sigma_n^2 = L\sigma_g^2\sigma_{v1}^2 + \sigma_{v2}^2/\alpha^2$. In order to focus on the impact of the correlated relaying channels on the relay network performance, in what follows, we assume that $\mathbf{n}$ is the AWGN.

B. Relay Network Statistical Properties

The equivalent baseband received signal of a relay network is written as $y = R \cdot x + n$, where the fading amplitude R is a RV following the PDF (1), whose shape is specified by the variances of relaying channels σ_h^2, σ_g^2, the number of relay nodes L and the correlation μ, otherwise R is talked of without knowing what it means. The AWGN n has the variance of σ_n^2. The instantaneous signal-to-noise ratio (SNR) per received symbol is $\gamma = R^2\sigma_x^2/\sigma_n^2$ and the corresponding average SNR is $\overline{\gamma} = \Omega\sigma_x^2/\sigma_n^2$, where $\Omega = \mathbb{E}[R^2]$.

By applying a simple variable change, Ω is derived as

$$\Omega = \int_0^\infty r^2 \cdot f_R(r)\, dr = \frac{\sigma_h^2\sigma_g^2\left(1-|\mu|^2\right)^{L+2}}{2^{L+1}\Gamma(L)} \int_0^\infty t^{L+2}\mathbb{K}_{L-1}(t)\,\mathbb{I}_0(|\mu|\,t)\,dt.$$

Applying [11, eq. (6.576.5)], we achieve $\Omega = \sigma_h^2\sigma_g^2\left(1-|\mu|^2\right)^{L+2} {}_2\mathbb{F}_1\left(L+1, 2; 1; |\mu|^2\right)$, where ${}_2\mathbb{F}_1(\cdot, \cdot; \cdot; \cdot)$ is Gauss hypergeometric function. Applying [12, eq. (07.23.03.0142.01)] and after some manipulations, we obtain Ω as follows

$$\Omega = \sigma_h^2\sigma_g^2 L\left(1 + L|\mu|^2\right). \tag{3}$$

By changing the variables [13] and defining $A \triangleq 2\Big/\left(1-|\mu|^2\right)$ and $B \triangleq \sqrt{L\left(1+L|\mu|^2\right)\Big/\overline{\gamma}}$, the PDF of γ can be obtained as

$$f_\gamma(\gamma) = \frac{A \cdot B^{L+1}}{\Gamma(L)}\gamma^{(L-1)/2}\mathbb{I}_0\left(|\mu|\,AB \cdot \gamma^{1/2}\right) \cdot \mathbb{K}_{L-1}\left(AB \cdot \gamma^{1/2}\right). \tag{4}$$

$$f_R(r) = \frac{4 \cdot r^L}{\Gamma(L) \cdot (\sigma_X \sigma_Y)^{L+1} \left(1 - |\mu|^2\right)} \cdot \mathbb{I}_0\left(\frac{2|\mu|}{\sigma_X \sigma_Y \left(1 - |\mu|^2\right)} r\right) \cdot \mathbb{K}_{L-1}\left(\frac{2}{\sigma_X \sigma_Y \left(1 - |\mu|^2\right)} r\right) \tag{1}$$

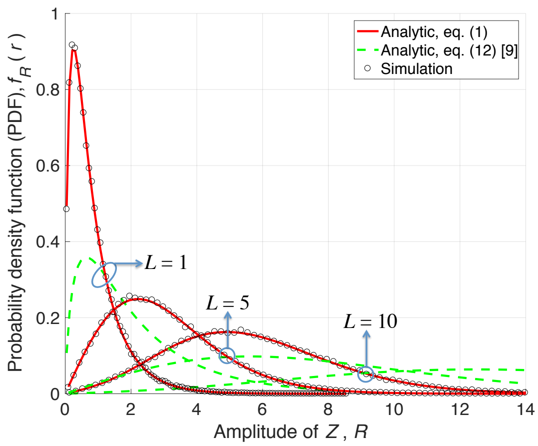

Fig. 1. PDFs of the amplitude of RV Z, $f_R(r)$ when variances $\sigma_g = 0.7$, $\sigma_h = 1.5$ and correlation $\mu = 0.5 \cdot \exp(j\pi/6)$.

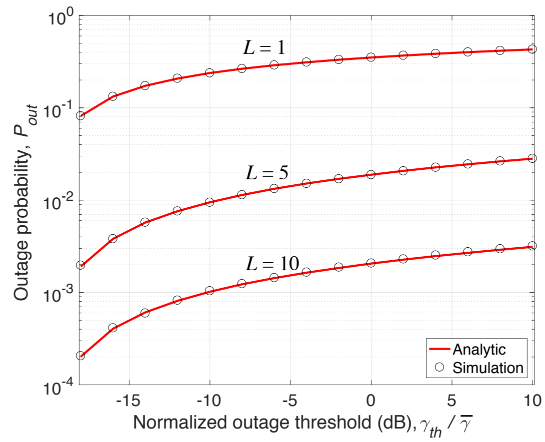

Fig. 2. Outage probability versus the normalized thresholds, when variances $\sigma_g = 0.7$, $\sigma_h = 1.5$ and correlation $\mu = 0.5 \cdot \exp(j\pi/6)$.

The cumulative distribution function (CDF) of γ is calculated by $F_\gamma(\gamma) = \int_0^\gamma f_\gamma(x)\, dx$. This finite integral can be evaluated via numerical integration. In order to get rid of special Bessel functions, we apply the elementary-functions-based series expansion of those functions. Firstly, using some simple variable changes, the CDF can be rewritten as

$$F_\gamma(\gamma) = \frac{2}{A^L \Gamma(L)} \int_0^{AB\gamma^{1/2}} x^L \mathbb{I}_0(|\mu| x)\, \mathbb{K}_{L-1}(x)\, dx.$$

By applying the primary definition of $\mathbb{I}_0(\cdot)$ [12, eq. (03.02.02.0001.01)] $\mathbb{I}_0(|\mu| x) = \sum_{k=0}^{\infty} \left(|\mu|^{2k} \Big/ \left((k!)^2 4^k\right)\right) x^{2k}$ and the series expansion of $\mathbb{K}_L(\cdot)$ [14, eq. (17)] $\mathbb{K}_{L-1}(x) = \sum_{q=0}^{\infty} \sum_{l=q}^{\infty} \Lambda(L-1, l, q)\, \mathrm{e}^{-x} x^{q-L+1}$, after some manipulations, the CDF can be derived in (5), where $\Lambda(L, l, q) \triangleq \frac{(-1)^q \sqrt{\pi} \cdot \Gamma(2L) \cdot \Gamma\left(\frac{1}{2} + l - L\right) \cdot \mathbb{L}(l, q)}{2^{L-q} \cdot \Gamma\left(\frac{1}{2} - L\right) \cdot \Gamma\left(\frac{1}{2} + l + L\right) \cdot l!}$, $\mathbb{L}(l, q) \triangleq \begin{pmatrix} l-1 \\ q-1 \end{pmatrix} \frac{l!}{q!}$ and $\Gamma_{low}(\eta, z) \triangleq \int_0^z t^{\eta-1} \mathrm{e}^{-t} dt$ is the lower incomplete Gamma function. Note that, the derived CDF is valid for $L > 1$. When $L = 1$ (one relay), we use the recurrence identity of $\mathbb{K}_L(\cdot)$, i.e., $\mathbb{K}_0(x) = \mathbb{K}_2(x) - 2x^{-1}\mathbb{K}_1(x)$ [12], and the CDF can be obtained. We skip it here due to the space constraint.

C. Outage Probability (P_{out})

The outage probability is defined as the probability that the received SNR is smaller than a given threshold, γ_{th}. Therefore, the outage probability can be easily deduced by $P_{out}(\gamma_{th}) = F_\gamma(\gamma_{th})$.

IV. Numerical Validation

For the sake of simplicity, we set variances $\sigma_g^2 = 0.7$, $\sigma_h^2 = 1.5$ and the correlation $\mu = 0.5 \cdot \exp(j\pi/6)$ in all simulations in order to quickly validate the analysis. The histograms are plotted after 1000 000 realizations. The marker symbols and solid lines denote numerical and analytical results, respectively.

We consider three cases, where the number of relay nodes are set to $L = 1$, $L = 5$ and $L = 10$. Fig. 1 plots the derived PDF (1), $f_R(r)$, and the associated simulated PDF, in comparion to the PDF (12), $f_{\overline{R}}(r)$ [9]. It can be seen that our proposed PDF better fits the simulated ones. We further plot the derived outage probability $P_{out}(\gamma_{th})$ as a function of the normalized outage thresholds, $\gamma_{th}/\overline{\gamma}$ in Fig. 2. Again, it is observed that the analytical curves match the numerical ones, confirming the correctness of our derivations. It is further observed that the outage probability strongly improves with the number of relays.

V. Conclusion

We have investigated the outage probality of the relay network with the correlated relaying channels. To this end, we have derived the exact PDF of the modulus of sum of products of correlated zero-mean Gaussian RVs. The statistics of the cascaded relay channel have also been analyzed. The numerical simulations have been carried out to confirm the correctness of our derivations.

Appendix

In order to derive the PDF of R, we carry out the step-by-step derivation: (A) Derivation of joint charac-

$$F_\gamma(\gamma)=\frac{2}{A^L\Gamma(L)}\sum_{k=0}^{\infty}\sum_{q=0}^{\infty}\sum_{l=q}^{\infty}\frac{|\mu|^{2k}}{(k!)^2 4^k}\Lambda(L-1,l,q)\cdot\Gamma_{low}\left(2k+q+2,AB\gamma^{1/2}\right). \quad (5)$$

teristic function (CF) of Z_R and Z_I; (B) Inverse CF transformation to yield the joint PDF of Z_R and Z_I; (C) Cartesian-polar transformation to yield the joint PDF of the amplitude $R=\sqrt{Z_R^2+Z_I^2}$ and the phase $\Theta=\arctan(Z_I/Z_R)$; (D) Integration of the joint PDF over the RV Θ to yield the PDF of R.

A. Joint CF derivation

We first consider the case $L=1$ and omit the subscripts l of RVs X_l and Y_l for simplicity. Once we find the CF of Z, the generalization with any value of L is straightforward thanks to the properties of the CF. Let us express Y as the contribution of X plus a RV $U\sim\mathcal{CN}\left(0,\sigma_U^2\right)$ independent of X as follows

$$\begin{cases} Y_R &= \dfrac{\sigma_Y|\mu|\cos\varepsilon}{\sigma_X}X_R+\dfrac{\sigma_Y|\mu|\sin\varepsilon}{\sigma_X}X_I+U_R \\ Y_I &= \dfrac{\sigma_Y|\mu|\sin\varepsilon}{\sigma_X}X_R-\dfrac{\sigma_Y|\mu|\cos\varepsilon}{\sigma_X}X_I+U_I \end{cases}$$

where the variance $\sigma_{U_R}^2=\sigma_{U_I}^2=\frac{\sigma_U^2}{2}=\frac{\sigma_Y^2}{2}\left(1-|\mu|^2\right)$. It is easy to check that the correlation between X and Y is μ. From aforementioned assumptions and the distributions of X and Y, it is easy to obtain the distribution of Z_R and Z_I conditioned on X as $Z_R|\,X\sim\mathcal{N}\left(\frac{\sigma_Y|\mu|\cos\varepsilon}{\sigma_X}|X|^2,\frac{\sigma_Y^2}{2}\left(1-|\mu|^2\right)|X|^2\right)$ and $Z_I|\,X\sim\mathcal{N}\left(\frac{\sigma_Y|\mu|\sin\varepsilon}{\sigma_X}|X|^2,\frac{\sigma_Y^2}{2}\left(1-|\mu|^2\right)|X|^2\right)$.

The joint CF of Z_R and Z_I conditioned on X can be expressed by $\Psi_{Z_R,Z_I|X}(j\omega_1,j\omega_2|\,X)=\mathbb{E}\left[\exp\left(j\left(\omega_1 z_R+\omega_2 z_I\right)\right)|X=x\right]=\exp\Big\{j\frac{\sigma_Y|\mu|}{\sigma_X}\left(\omega_1\cos\varepsilon+\omega_2\sin\varepsilon\right)|x|^2 -\frac{\sigma_Y^2}{4}\left(1-|\mu|^2\right)\left(\omega_1^2+\omega_2^2\right)|x|^2\Big\}$.

The joint CF of Z_R and Z_I can now be derived as $\Psi_{Z_R,Z_I}(j\omega_1,j\omega_2)=\int\limits_{-\infty}^{\infty}\Psi_{Z_R,Z_I|X}(j\omega_1,j\omega_2|X)\cdot f_X(x)\,dx$, where the PDF of the RV X is expressed by [15] $f_X(x)=\frac{1}{\pi\sigma_X^2}\mathrm{e}^{-\frac{|x|^2}{\sigma_X^2}}$. By changing variables, i.e., $x_R=t\cos\phi$ and $x_I=t\sin\phi$, after some mathematical manipulations, the joint CF can be rewritten as

$$\Psi_{Z_R,Z_I}(j\omega_1,j\omega_2)=\frac{1}{\pi\sigma_X^2}\int\limits_0^{\infty}\int\limits_0^{2\pi}t\cdot\exp\Bigg\{-\frac{t^2}{\sigma_X^2} +j\frac{\sigma_Y|\mu|}{\sigma_X}\left(\omega_1\cos\varepsilon+\omega_2\sin\varepsilon\right)t^2 -\frac{\sigma_Y^2}{4}\left(1-|\mu|^2\right)\left(\omega_1^2+\omega_2^2\right)t^2\Bigg\}d\phi\;dt \quad (6)$$

By solving the trivial problem (6), the CF corresponding to $L=1$ is obtained. The CF is generalized to any value of L as presented in (7), in which we apply the property that the CF of the summation of independent RVs is equal to the multiplication of all individual CFs w.r.t each RV. It can be observed that we obtain again the CF derivation as in [9].

B. Joint PDF of Z_R and Z_I

The joint PDF of Z_R and Z_I can be derived by performing the inverse CF transformation as $f_{Z_R,Z_I}(z_R,z_I)=\frac{1}{4\pi^2}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\Psi_{Z_R,Z_I}(j\omega_1,j\omega_2)\cdot\mathrm{e}^{-j(\omega_1 z_R+\omega_2 z_I)}d\omega_1 d\omega_2$. By changing the variables $t_1=\omega_1-j\frac{2|\mu|\cos\varepsilon}{\sigma_X\sigma_Y\left(1-|\mu|^2\right)}$ and $t_2=\omega_2-j\frac{2|\mu|\sin\varepsilon}{\sigma_X\sigma_Y\left(1-|\mu|^2\right)}$, $f_{Z_R,Z_I}(z_R,z_I)$ can be rewritten as

$$f_{Z_R,Z_I}(z_R,z_I)=\frac{4^{L-1}\left(1-|\mu|^2\right)^{-L}}{\pi^2(\sigma_X\sigma_Y)^{2L}}\cdot\mathrm{e}^{\frac{2|\mu|(z_R\cos\varepsilon+z_I\sin\varepsilon)}{\sigma_X\sigma_Y(1-|\mu|^2)}}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\left(\frac{4}{\sigma_X^2\sigma_Y^2\left(1-|\mu|^2\right)^2}+t_1^2+t_2^2\right)^{-L}\cdot\mathrm{e}^{-j(z_R t_1+z_I t_2)}dt_1 dt_2\;. \quad (8)$$

Let $C=\frac{4^{L-1}\left(1-|\mu|^2\right)^{-L}}{\pi^2(\sigma_X\sigma_Y)^{2L}}\cdot\exp\left(\frac{2|\mu|(z_R\cos\varepsilon+z_I\sin\varepsilon)}{\sigma_X\sigma_Y\left(1-|\mu|^2\right)}\right)$ and changing the variables $t_1=u\cos\varphi$ and $t_2=u\sin\varphi$, (8) is rewritten as $f_{Z_R,Z_I}(z_R,z_I)=C\int\limits_0^{\infty}u\left(\frac{4}{\sigma_X^2\sigma_Y^2\left(1-|\mu|^2\right)^2}+u^2\right)^{-L}\cdot\int\limits_0^{2\pi}\mathrm{e}^{ju(z_R\cos\varphi+z_I\sin\varphi)}d\varphi du$.

Applying the fact that $\int_0^{2\pi}\mathrm{e}^{x\cos\varphi+y\sin\varphi}d\varphi=2\pi\mathbb{I}_0\left(\sqrt{x^2+y^2}\right)$ [11, eq. 3.338-4] and $\mathbb{I}_\alpha(x)=j^{-\alpha}\mathbb{J}_\alpha(jx)$, f_{Z_R,Z_I} can be rewritten as

$$f_{Z_R,Z_I}(z_R,z_I)=2\pi\cdot C\cdot\int\limits_0^{\infty}\frac{u\cdot\mathbb{J}_0\left(u\sqrt{z_R^2+z_I^2}\right)}{\left(D^2+u^2\right)^L}du \quad (11)$$

where $D=\frac{2}{\sigma_X\sigma_Y\left(1-|\mu|^2\right)}$. The integral in (11) is of Hankel-Nicholson type [18] and due to the fact that $\mathbb{K}_\alpha(x)=\mathbb{K}_{-\alpha}(x)$, (11) can be derived as $f_{Z_R,Z_I}(z_R,z_I)=\frac{2\pi C\left(\sqrt{z_R^2+z_I^2}\right)^{L-1}}{\Gamma(L)\cdot(2D)^{L-1}}\cdot$

$$\Psi_{Z_R,Z_I}(j\omega_1,j\omega_2)=\left[\frac{\frac{4}{\sigma_X^2\sigma_Y^2\left(1-|\mu|^2\right)}}{\left(\omega_1-j\frac{2|\mu|\cos\varepsilon}{\sigma_X\sigma_Y\left(1-|\mu|^2\right)}\right)^2+\left(\omega_2-j\frac{2|\mu|\sin\varepsilon}{\sigma_X\sigma_Y\left(1-|\mu|^2\right)}\right)^2+\frac{4}{\sigma_X^2\sigma_Y^2\left(1-|\mu|^2\right)^2}}\right]^L \quad (7)$$

$$f_{Z_R,Z_I}(z_R,z_I)=\frac{2\left(z_R^2+z_I^2\right)^{\frac{L-1}{2}}}{\pi\cdot\Gamma(L)\cdot(\sigma_X\sigma_Y)^{L+1}\left(1-|\mu|^2\right)}\cdot \mathrm{e}^{\frac{2|\mu|}{\sigma_X\sigma_Y\left(1-|\mu|^2\right)}(z_R\cos\varepsilon+z_I\sin\varepsilon)}\mathbb{K}_{L-1}\left(\frac{2\sqrt{z_R^2+z_I^2}}{\sigma_X\sigma_Y\left(1-|\mu|^2\right)}\right) \quad (9)$$

$$f_{R,\Theta}(r,\theta)=\frac{2\cdot r^L}{\pi\cdot\Gamma(L)\cdot(\sigma_X\sigma_Y)^{L+1}\left(1-|\mu|^2\right)}\cdot \mathrm{e}^{\frac{2|\mu|}{\sigma_X\sigma_Y\left(1-|\mu|^2\right)}r\cos(\theta-\varepsilon)}\mathbb{K}_{L-1}\left(\frac{2}{\sigma_X\sigma_Y\left(1-|\mu|^2\right)}r\right) \quad (10)$$

$\mathbb{K}_{L-1}\left(\frac{\sqrt{z_R^2+z_I^2}}{D}\right)$. Substituting the expressions of C and D, we achieve the full formula of the joint PDF (9), which is different compared to [9, eq. (11)]. Interestingly, when $\mu=0$, in our formula the joint PDF is still a function of the variances of RVs X and Y.

C. Joint PDF of R and Θ

We obtain the polar coordinate form of the joint PDF of Z_R and Z_I by changing the variables in (9) as $z_R=r\cos\theta$ and $z_I=r\sin\theta$. Taking into account the Jacobian matrix determinant of r, we reach the joint PDF of the amplitude R and the phase Θ (10). Again we can observe additional terms related to the variances of X and Y compared to the one derived in [9, eq. (12)].

D. PDF of R

The PDF of R is derived by integrating the joint PDF over the RV Θ. Applying again [11, eq. (3.338-4)], we reach the PDF of R presented in (1). For comparison, we derive the subsequent erroneous PDF of R (12), $f_{\overline{R}}(r)$, based on [9, eq. (12)] as follows

$$f_{\overline{R}}(r)=\frac{\left(1-|\mu|^2\right)^L\cdot r^L}{2^{L-1}\cdot(L-1)!}\cdot\mathbb{I}_0(|\mu|\,r)\cdot\mathbb{K}_{L-1}(r). \quad (12)$$

ACKNOWLEDGMENT

The authors would like to thank the financial supports of the Copine-IoT Innoviris project, the Icity.Brussels project and the FEDER/EFRO grant.

REFERENCES

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: efficient protocols and outage behavior,” IEEE Trans. Inf. Theory, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[2] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, “Multihop diversity in wireless relaying channels,” IEEE Trans. Commun., vol. 52, no. 10, pp. 1820–1830, Oct. 2004.

[3] Y. Yang, H. Hu, J. Xu, and G. Mao, “Relay technologies for WiMax and LTE-advanced mobile systems,” IEEE Commun. Mag., vol. 47, no. 10, pp. 100–105, Oct. 2009.

[4] H. Ilhan, M. Uysal, and I. Altunbas, “Cooperative diversity for intervehicular communication: Performance analysis and optimization,” IEEE Trans. Veh. Technol., vol. 58, no. 7, pp. 3301–3310, Sep. 2009.

[5] P. Agrawal, and N. Patwari, “Correlated link shadow fading in multi-hop wireless networks,” IEEE Trans. Wirel. Commun., vol. 8, no. 8, pp. 4024–4036, Aug. 2009.

[6] Q.-H. Chu, J.-M. Conrat, and J. Cousin, “Experimental characterization and modeling of shadow-fading correlation for relaying systems,” in Proc. IEEE Vehicular Technology Conference (VTC2011-fall), San Francisco, USA, Sep. 2011.

[7] X. Cheng, and *et al.*, “Cooperative MIMO channel modeling and multi-link spatial correlation properties,” IEEE J. Sel. Areas Commun., vol. 30, no. 2, pp. 388–396, Feb. 2012.

[8] B. V. Nguyen, and K. Kim, “Performance analysis of amplify-and-forward systems with single relay selection in correlated environments,” Sensors, vol. 16, no. 9, article # 1472, Sep. 2016.

[9] J. G. Proakis, “Probabilities of error for adaptive reception of M-phase signals,” IEEE Trans. Commun. Technol., vol. 16, no. 1, pp. 71–81, Feb. 1968.

[10] T.-H. Nguyen, J.-F. Determe, M. Van Eeckhaute, J. Louveaux, P. De Doncker, and F. Horlin, “Frequency-domain time-reversal precoding in wideband MISO OFDM communication systems,” 2019. [Online] Available: https://arxiv.org/abs/1904.10727.

[11] I. S. Gradshteyn, and I. M. Ryzhik, “Tables of integrals, series and products,” San Diego Academic, 7th edition, 2007.

[12] “The Wolfram functions site.” [Online] Available: http://functions.wolfram.com

[13] M. K. Simon, and M.-S. Alouini, “Digital communication over fading channels,” New York: Wiley, 2nd edition, 2005.

[14] M. M. Molu, “Series representation of modified Bessel functions and its application in AF cooperative systems,” arXiv:1506.04283 [cs.IT], Jun. 2015. Link: https://arxiv.org/pdf/1506.04283.pdf.

[15] A. Papoulis, “Probability, random variables and stochastic processes,” McGraw-Hill Companies, 3rd edition, 1991.

[16] I. S. Ansari, S. Al-Ahmadi, F. Yilmaz, M. S. Alouini, and H. Yanikomeroglu, “A new formula for the BER of binary modulations with dual-branch selection over generalized-K composite fading channels,” IEEE Trans. Commun., vol. 59, no. 10, pp. 2654–2658 , Oct. 2011.

[17] H. Chergui, M. Benjillali, and S. Saoudi, “Performance analysis of project-and-forward relaying in mixed MIMO pinhole and Rayleigh dual-hop channel,” IEEE Commun. Lett., vol. 20, no. 3, pp. 610–613, Mar. 2016.

[18] Y. L. Luke, “Integrals of Bessel functions,” McGraw-Hill Book Company, 1st edition, 1962.