Analysis of residual CFO impact on downlink massive MISO systems

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Massive antenna technologies provide a good power focusing gain for emerging communication systems. They can easily be integrated into an orthogonal frequency-division multiplexing (OFDM) system. However, OFDM is known to be prone to carrier frequency offset (CFO) due to the loss of the orthogonality among OFDM subcarriers. In this Letter, the authors investigate the impact of residual CFO (RCFO) on the downlink performance of massive multiple-input single-output (MISO) OFDM systems using matched-filter (MF) and maximum-ratio-transmission (MRT) precoders. Particularly, the exact mean-square-error (MSE) expressions of the equalised received signal of both MF and MRT systems are derived. Numerical simulations with Rayleigh fading channels are carried out to validate the analysis. The results show that the RCFO causes a MSE plateau compared to the ideal case of no CFO.

Introduction: Massive antenna systems have been considered as an interesting technology for next-generation communication systems [1]. In combination with precoding techniques such as matched-filter (MF) [2] or maximum-ratio-transmission (MRT) [3], a high power gain can be achieved thanks to the signal focusing on particular points in space. Moreover, precoding techniques can easily be integrated into orthogonal frequency-division multiplexing (OFDM) systems widely used in cellular systems.

In communication systems, carrier frequency offset (CFO) comes from the frequency mismatch between local oscillators used for frequency up- and down-conversion at the transmitter and receiver, respectively. It results in a linearly increasing phase rotation in the time domain, destroying the orthogonality between OFDM subcarriers, and thus leads to system performance degradation. Many recent works have studied the CFO impact and its compensation on such large-scale antennas OFDM systems (see [4, 5] and references therein). However, these investigations have only considered a frequency-flat channel. Furthermore, the derivation of the mean-square-error (MSE) of equalised received symbols to evaluate such system performance under the residual CFO (RCFO) impact is not available to the best of our knowledge. In this Letter, we derive the exact MSE expressions of the RCFO impact on the downlink performance of precoded multiple-input single-output (MISO) OFDM systems over Rayleigh fading channels.

Notation: Bold italic values of lower-case and upper-case letters denote column vectors and matrices, respectively. $\boldsymbol{I}_Q$, $\boldsymbol{F}_Q$ are the $Q \times Q$ identity and Fourier matrices, respectively; $[A]_{mn}$ is the mnth element of the matrix $\boldsymbol{A}$; $\boldsymbol{\Lambda}_X$ is the diagonal matrix whose diagonal entries are the elements of the vector $\boldsymbol{X}$; $\mathrm{tr}\{\boldsymbol{A}\}$ is the trace of a square matrix $\boldsymbol{A}$; $\|\cdot\|$, $(\cdot)^*$, $(\cdot)^{\mathrm{H}}$, $(\cdot)^{\mathrm{T}}$, $\odot$, $\mathrm{Re}(\cdot)$ and $\mathbb{E}[\cdot]$ are the Euclidean norm, complex conjugate, Hermitian transpose, transpose, element-wise product, real-part and expectation operators, respectively.

System model and precoding schemes: We consider a $N_T \times 1$ MISO OFDM system (Fig. 1). We assume that the CFO and channel between each UT and BS are estimated in the uplink and the estimates are used to compensate for the CFO and to precode the signal for the downlink communication. Unfortunately, the estimation is usually imperfect. In order to focus on the impact of the RCFO, we assume that the channel estimation error is negligible. Furthermore, we focus on only one Q-subcarrier OFDM symbol, $\boldsymbol{x} = [X_0 \ \dots \ X_{Q-1}]^{\mathrm{T}}$ (X_q is an independent zero-mean random variable with variance $\mathbb{E}[|X_q|^2] = \sigma_X^2$), which is sent over the precoded system. This OFDM symbol is then precoded across frequencies by a matrix $\boldsymbol{\Lambda}_{\boldsymbol{p}^k}$ on each antenna branch k. We define $\boldsymbol{h}^k := [H_0^k \ H_1^k \ \dots \ H_{Q-1}^k]^{\mathrm{T}}$ as the channel frequency response (CFR) associated with the kth antenna. Vector $\boldsymbol{p}^k$ depends on the precoding techniques. Particularly, at the qth subcarrier and the kth antenna, the MF precoding is $P_q^k = (H_q^k)^*/\sqrt{N_T}$ and the MRT precoding is $P_q^k = (H_q^k)^*/\|\bar{\boldsymbol{h}}_q\|$, where $\bar{\boldsymbol{h}}_q = [H_q^0 \ H_q^1 \ \dots \ H_q^{N_T-1}]^{\mathrm{T}}$ requiring the CFR information exchange among antennas [3]. The signals are then transformed to the time domain. The signal is then made cyclic by adding the cyclic prefix and propagated over the channel, which is mathematically equivalent to the left multiplication with the $Q \times Q$ circulant matrix $\tilde{\boldsymbol{H}}^k$ of the kth channel impulse response (CIR). We assume that the CIRs between each transmit antenna and receive antenna are spatially independent from one to another. The matrix $\tilde{\boldsymbol{H}}^k$ can be factorised as $\tilde{\boldsymbol{H}}^k = \boldsymbol{F}_Q^{\mathrm{H}} \cdot \boldsymbol{\Lambda}_{\boldsymbol{h}^k} \cdot \boldsymbol{F}_Q$. At the receiver side, the received signals are corrupted by the RCFO $\boldsymbol{\Lambda}_\theta$ (where $\boldsymbol{\theta} = [1 \ \exp(j2\pi\theta/Q) \ \dots \ \exp(j2\pi\theta(Q-1)/Q)]^{\mathrm{T}}$) and by the additive white Gaussian noise (AWGN) $\tilde{\boldsymbol{v}}$. A common oscillator at the transmitter side is considered. The signals are brought back to the frequency-domain (FD) by a fast Fourier transform (FFT) and can be expressed as

$$\boldsymbol{y} = \boldsymbol{F}_Q \cdot \boldsymbol{\Lambda}_\theta \cdot \left(\sum_{k=0}^{N_T-1} \tilde{\boldsymbol{H}}^k \cdot \boldsymbol{F}_Q^{\mathrm{H}} \cdot \boldsymbol{\Lambda}_{\boldsymbol{p}^k} \right) \cdot \boldsymbol{x} + \boldsymbol{v}, \qquad (1)$$

where $\boldsymbol{v} = \boldsymbol{F}_Q \cdot \tilde{\boldsymbol{v}}$ is the FD AWGN of variance σ_V^2. Defining $\boldsymbol{\Theta} := \boldsymbol{F}_Q \cdot \boldsymbol{\Lambda}_\theta \cdot \boldsymbol{F}_Q^{\mathrm{H}}$ as the $Q \times Q$ circulant matrix of the RCFO, whose first column is $[\psi_0 \ \psi_1 \ \dots \ \psi_{Q-1}]^{\mathrm{T}}$ with $\psi_q = 1/Q \sum_{l=0}^{Q-1} \exp(j2\pi l\theta/Q) \cdot \exp(-j2\pi lq/Q)$ and defining $\boldsymbol{\Lambda}_\kappa := \sum_{k=0}^{N_T-1} \boldsymbol{\Lambda}_{\boldsymbol{h}^k} \cdot \boldsymbol{\Lambda}_{\boldsymbol{p}^k}$, in which $\boldsymbol{\kappa} = 1/\sqrt{N_T}[\sum_{k=0}^{N_T-1}|H_0^k|^2 \ \dots \ \sum_{k=0}^{N_T-1}|H_{Q-1}^k|^2]^{\mathrm{T}}$ for MF precoding and $\boldsymbol{\kappa} = [\|\bar{\boldsymbol{h}}_0\| \ \dots \ \|\bar{\boldsymbol{h}}_{Q-1}\|]^{\mathrm{T}}$ for MRT precoding, then (1) can be rewritten as $\boldsymbol{y} = \boldsymbol{\Theta} \cdot \boldsymbol{\Lambda}_\kappa \cdot \boldsymbol{x} + \boldsymbol{v}$.

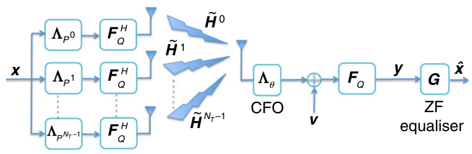

Fig. 1 *Schematic of either MF or MRT precoding MISO OFDM system*

Considering a zero-forcing equaliser $\boldsymbol{G} = \boldsymbol{\Lambda}_\kappa^{-1}$, the received equalised signal can be expressed as $\hat{\boldsymbol{x}} = \boldsymbol{\Lambda}_\kappa^{-1} \cdot \boldsymbol{\Theta} \cdot \boldsymbol{\Lambda}_\kappa \cdot \boldsymbol{x} + \boldsymbol{\Lambda}_\kappa^{-1} \cdot \boldsymbol{v}$.

In the absence of the RCFO, $\boldsymbol{\Theta} = \boldsymbol{I}_Q$ and the received signal becomes $\hat{\boldsymbol{x}} = \boldsymbol{x} + \boldsymbol{\Lambda}_\kappa^{-1} \cdot \boldsymbol{v}$. Note that, thanks to the precoding, in the absence of the RCFO the equaliser coefficients are real-valued, which brings a receiver complexity reduction.

Performance analysis in the presence of RCFO: We define the estimated symbol error as $\boldsymbol{e} := \hat{\boldsymbol{x}} - \boldsymbol{x}$, the MSE is then calculated by $MSE = \mathbb{E}[\boldsymbol{e}^{\mathrm{H}} \cdot \boldsymbol{e}] = \mathbb{E}[\mathrm{tr}\{\boldsymbol{e} \cdot \boldsymbol{e}^{\mathrm{H}}\}]$. Defining the normalised MSE (NMSE) $NMSE := MSE/(Q\sigma_X^2)$ and after some manipulations, we obtain

$$\begin{aligned}\mathrm{NMSE} = {} & \frac{\gamma^{-1}}{Q}\mathbb{E}\left[\mathrm{tr}\left\{\boldsymbol{\Lambda}_\kappa^{-2}\right\}\right] + 1 - \frac{2}{Q}\mathrm{Re}(\mathrm{tr}\{\boldsymbol{\Theta}\}) \\ & + \frac{1}{Q}\mathbb{E}\left[\mathrm{tr}\left\{\boldsymbol{\Lambda}_\kappa^2 \cdot \boldsymbol{\Theta}^{\mathrm{H}} \cdot \boldsymbol{\Lambda}_\kappa^{-2} \cdot \boldsymbol{\Theta}\right\}\right],\end{aligned} \qquad (2)$$

where $\gamma = \sigma_X^2/\sigma_V^2$. We define $(\sigma_l^k)^2 := \mathbb{E}[|h_l^k|^2]$ as the variance of lth tap of the kth CIR. The corresponding CFR components are uniformly spread over the bandwidth and the random variables K_q are identically distributed. Therefore, in the absence of the RCFO, the ideal NMSE, $\mathrm{NMSE_{ideal}}$, is given by

$$\mathrm{NMSE_{ideal}} = \frac{\gamma^{-1}}{Q}\mathbb{E}\left[\mathrm{tr}\left\{\boldsymbol{\Lambda}_\kappa^{-2}\right\}\right] = \gamma^{-1}\mathbb{E}\left[1/K_q^2\right]. \qquad (3)$$

Due to the fact that the random variable $z = \sum_{k=0}^{N_T-1}|H_q^k|^2$ has a probability density function $f_Z(z) = (z^{N_T-1}/(N_T-1)!)\mathrm{e}^{-z}$ [6]. Considering the MF precoding and $N_T > 2$, the expectation in (3) can be calculated by $\mathbb{E}[1/K_q^2] = \int_0^\infty (N_T/z^2)(z^{N_T-1}/(N_T-1)!)\mathrm{e}^{-z}dz = N_T/(N_T-1)(N_T-2)$. A similar derivation is done for the MRT precoding to obtain the following $\mathrm{NMSE_{ideal}}$ expression

$$\mathrm{NMSE_{ideal}} = \begin{cases} \dfrac{\gamma^{-1}N_T}{(N_T-1)(N_T-2)}, & \text{MF precoding\& } N_T > 2 \\ \dfrac{\gamma^{-1}}{N_T-1}, & \text{MRT precoding\& } N_T > 1 \end{cases} \qquad (4)$$

From (2) and (3), in the presence of the RCFO causing the inter-carrier interference, the NMSE is the summation of $\mathrm{NMSE_{ideal}}$ and $\mathrm{NMSE_{CFO}}$, defined as follows:

$$\mathrm{NMSE_{CFO}} = 1 - \frac{2}{Q}\mathrm{Re}(\mathrm{tr}\{\boldsymbol{\Theta}\}) + \frac{1}{Q}\mathbb{E}\left[\mathrm{tr}\left\{\boldsymbol{\Lambda}_\kappa^2 \cdot \boldsymbol{\Theta}^{\mathrm{H}} \cdot \boldsymbol{\Lambda}_\kappa^{-2} \cdot \boldsymbol{\Theta}\right\}\right]. \qquad (5)$$

The second term on the right-hand-side (RHS) of (5) is derived as

$$T_1 = \frac{2}{Q}\mathrm{Re}(\mathrm{tr}\{\mathbf{\Theta}\}) = \frac{2}{Q}\cos\left(\frac{\pi\theta(Q-1)}{Q}\right)\frac{\sin(\pi\theta)}{\sin(\pi\theta/Q)}. \quad (6)$$

In order to derive the last term on the RHS of (5) $T_2 = (1/Q)\mathbb{E}[\mathrm{tr}\{\mathbf{\Lambda}_\kappa^2\mathbf{\Theta}^{\mathrm{H}}\mathbf{\Lambda}_\kappa^{-2}\mathbf{\Theta}\}]$, we first re-write T_2 as $T_2 = (1/Q)\mathbb{E}[\mathrm{tr}\{(\mathbf{\Omega}\odot\mathbf{\Theta}^{\mathrm{H}})\mathbf{\Theta}\}]$, where $\mathbf{\Omega} = \mathbb{E}[\boldsymbol{a}\cdot\tilde{\boldsymbol{a}}^{\mathrm{T}}]$, $\boldsymbol{a} = [K_0^2\ K_1^2\ \ldots\ K_{Q-1}^2]^{\mathrm{T}}$ and $\tilde{\boldsymbol{a}} = [1/K_0^2\ 1/K_1^2\ \ldots\ 1/K_{Q-1}^2]^{\mathrm{T}}$. Applying the results in [7, Equation 4 and Theorem 2], $[\Omega]_{pq}$ can be derived as

$$[\Omega]_{pq} = \begin{cases} \dfrac{24|\rho_{q-p}|^4 - 8(N_T+2)|\rho_{q-p}|^2 + N_T(N_T+2)}{(N_T-2)(N_T-4)}, & \text{MF precoding } \& \ N_T > 4 \\ \dfrac{N_T - 2|\rho_{q-p}|^2}{N_T - 2}, & \text{MRT precoding } \& \ N_T > 2 \end{cases} \quad (7)$$

in which ρ_{q-p} is the CFR correlation coefficient, which, given our assumptions, is equal to $\rho_{q-p} := \mathbb{E}[H_p^k\cdot(H_q^k)^*] = \sum_{l=0}^{L-1}(\sigma_l^k)^2 \exp(j2\pi(q-p)l/Q)$, $\forall k = 0, \ldots, (N_T-1)$.

Based on the fact that $\mathrm{tr}\{\boldsymbol{C}^{\mathrm{T}}\boldsymbol{D}\} = \sum_{p=0}^{Q-1}\sum_{q=0}^{Q-1}[C]_{pq}[D]_{pq}$, T_2 is finally derived as

$$T_2 = \frac{1}{Q}\sum_{p=0}^{Q-1}\sum_{q=0}^{Q-1}[\Omega]_{pq}|\psi_{q-p}|^2. \quad (8)$$

From the derived NMSE expression, the RCFO always causes a performance degradation since $\mathrm{NMSE_{CFO}} > 0$. To gain insight in the expression, we consider a small RCFO, meaning that $\sin(\theta)\approx\theta$ and $\cos(\theta)\approx 1$ so that $T_1\approx 2$ and $T_2 \approx 1 + T_3$, where $T_3 = (1/Q)\sum_{p=0}^{Q-1}\sum_{q=0,\,q\neq p}^{Q-1}[\Omega]_{pq}(\theta/(q-p-\theta))^2$. From (5), $\mathrm{NMSE_{CFO}}$ is approximated by T_3, revealing that at high SNRs (equivalent to $\mathrm{NMSE_{CFO}} \gg \mathrm{NMSE_{ideal}}$), RCFO causes a plateau value for NMSE. Note that when N_T tends to infinity (which implies $[\Omega]_{pq}\to 1$), T_3 depends only on the RCFO, regardless of the CFR correlation and precoding methods.

Simulation results: We consider a 256-subcarrier OFDM system, whose subcarrier spacing is 15 kHz. The carrier frequency is set to be 2 GHz. A multi-path channel of type extended pedestrian A [2], whose power delay profile is normalised to unity, is used in the simulations. We assume that channels remain constant within the transmission time frame of one OFDM symbol. The analytical results (solid lines) are validated by the simulation results (markers).

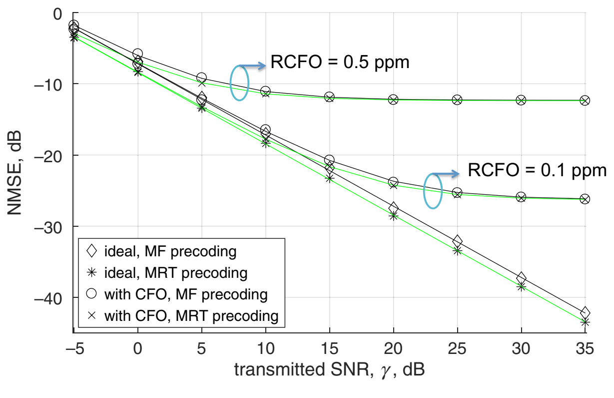

Fig. 2 *NMSE versus SNR, N_T is set to 8*

We first plot NMSE versus SNR for the RCFOs of 0.1 and 0.5 ppm in Fig. 2, when $N_T = 8$. The MRT precoding system provides lower NMSEs at low SNRs than the MF precoding system. The higher RCFO value leads to the higher plateau. The two systems converge to the same plateau at high SNRs. Next, NMSE versus number of antennas is plotted in Fig. 3 when the RCFO is 0.5 ppm and SNR = 5 dB. In the presence of the RCFO, increasing the number of antennas, N_T, barely improves the MSE, confirming again the results predicted by the aforementioned analysis. For all simulations, the numerical results match the analytical ones, which validate our derivation.

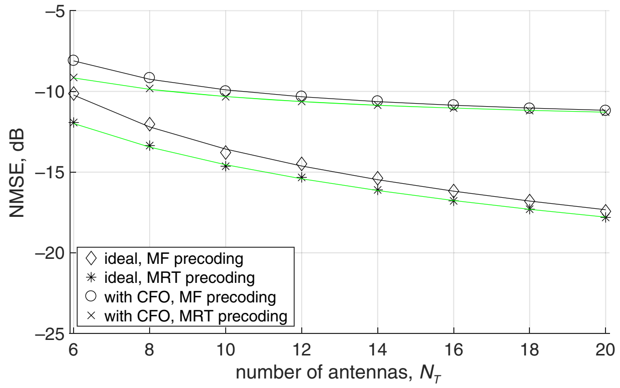

Fig. 3 *NMSE versus number of antennas, SNR = 5 dB*

Conclusion: We have studied the RCFO impact on the MISO OFDM precoder system. Assuming Rayleigh fading channels, we derived the exact MSE expressions of the received symbols for both MF and MRT precoding. The MSE expressions notably depend on the number of antennas. The MSEs of the MF and MRT precoding systems converge to the same plateau in the presence of RCFO. The correctness of the analytical derivation has been numerically confirmed.

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One or more of the Figures in this Letter are available in colour online.

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