Advancing towards the correct modeling and subsequent understanding of laminar-to-turbulent transition during atmospheric reentry is paramount for the future of aerospace technology. The coexistence of multiple physical phenomena and the grand amount of conditioning factors require the progressive extension of the applicability capabilities of the theoretical models. Past efforts have been mostly dedicated to investigate high-temperature and non-equilibrium effects using parallel stability theories. However, the implications of coupling these thermochemical phenomena with non-parallelism remains uncertain. Advanced state of the art thermodynamic and transport models are employed both in parallel and weakly non-parallel stability theories (LST and LPSE). A parametric study about the influence of non-local effects under different re-entry conditions and flow assumptions (i.e. CPG, TPG, CNE and LTE) showed that non-parallel effects stabilize/destabilize the boundary-layer, depending on the altitude and independently from the gas model employed. Particularly, they lead to a stronger destabilization of the 2nd Mack mode at the earliest points of the atmospheric re-entry flight envelope, reducing their effect until being weakly stabilizing at the lowest altitudes. Drastic N factor increments occurred assuming LTE, due to the presence of unstable supersonic modes, promoted by the boundary-layer cooling, caused by the intense chemical activity.

Nomenclature

**Acronyms**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEW</td>
<td>Blottner-Eucken-Wilke transport model</td>
</tr>
<tr>
<td>CE</td>
<td>Chapman &amp; Enskog’s transport model</td>
</tr>
<tr>
<td>CNE</td>
<td>Chemical Non-Equilibrium</td>
</tr>
<tr>
<td>CPG</td>
<td>Calorically Perfect Gas</td>
</tr>
<tr>
<td>DNS</td>
<td>Direct Numerical Simulations</td>
</tr>
<tr>
<td>EBD</td>
<td>Effective binary diffusion</td>
</tr>
<tr>
<td>LPSE</td>
<td>Linear Parabolized Stability Equations</td>
</tr>
<tr>
<td>McB</td>
<td>McBride’s NASA-7 and NASA-9 thermal model</td>
</tr>
<tr>
<td>NPSE</td>
<td>Non-linear Parabolized Stability Equations</td>
</tr>
<tr>
<td>RRHO</td>
<td>Rigid rotor and harmonic oscillator thermal model</td>
</tr>
<tr>
<td>SCEBD</td>
<td>Self-consistent effective binary diffusion</td>
</tr>
<tr>
<td>TCNE</td>
<td>Thermo-Chemical Non-Equilibrium</td>
</tr>
<tr>
<td>TPG</td>
<td>Thermally Perfect Gas</td>
</tr>
<tr>
<td>TPS</td>
<td>Thermal Protection Systems</td>
</tr>
<tr>
<td>LST</td>
<td>Linear Stability Theory</td>
</tr>
<tr>
<td>LTE</td>
<td>Local Thermodynamic Equilibrium</td>
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</tbody>
</table>

**Roman Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>Reduced spanwise wavenumber [-]</td>
</tr>
<tr>
<td>$b$</td>
<td>Dimensional spanwise wavenumber [m$^{-1}$]</td>
</tr>
<tr>
<td>$F$</td>
<td>Reduced frequency [-]</td>
</tr>
<tr>
<td>$f$</td>
<td>Dimensional frequency [s$^{-1}$]</td>
</tr>
<tr>
<td>$g_{ij}$</td>
<td>Metric tensor [-]</td>
</tr>
<tr>
<td>$H$</td>
<td>Semi-total enthalpy (= $h + u^2/2$) [J/kg]</td>
</tr>
<tr>
<td>$h$</td>
<td>Static enthalpy [J/kg]</td>
</tr>
<tr>
<td>$h$</td>
<td>Altitude [m]</td>
</tr>
<tr>
<td>$\Im$</td>
<td>Imaginary component of complex number</td>
</tr>
<tr>
<td>$\mathcal{J}$</td>
<td>Energy diffusion flux in tensorial notation [J/m$^2$-s]</td>
</tr>
<tr>
<td>$j$</td>
<td>Mass diffusion flux in tensorial notation [kg/m$^2$-s]</td>
</tr>
<tr>
<td>$\ell$</td>
<td>Blasius length based on the boundary-layer edge quantities [m]</td>
</tr>
<tr>
<td>$L$</td>
<td>Wedge length [m]</td>
</tr>
</tbody>
</table>

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Greek Symbols

- \( \alpha \): Streamwise wavenumber [1/m]
- \( \beta \): Spanwise wavenumber [1/m]
- \( \gamma_q \): Perturbation growth rate of quantity \( q \) [m\(^{-1}\)]
- \( \epsilon \): Perturbation order of magnitude [-]
- \( \kappa \): Thermal conductivity [W/K-m]
- \( \lambda \): Second coefficient of dynamic viscosity [kg/m-s]
- \( \mu \): First coefficient of dynamic viscosity [kg/m-s]
- \( \rho \): Density [kg/m\(^3\)]
- \( \mu u \): Mass flux [kg /m\(^2\)-s]
- \( \tau_c \): Chemical time scale
- \( \omega \): Perturbation frequency [1/s]
- \( \Omega \): Collision integral of order \( (i, j) \) between species \( s \) and \( \ell \) [m\(^2\)]

Subscripts

- \( q_e \): Boundary-layer edge value
- \( q_\infty \): Freestream pre-shock value
- \( q_{\text{max}} \): Maximum value of a quantity
- \( \hat{q} \): Vector quantity
- \( \underline{q} \): Fixed position quantity
- \( q_w \): Wall quantity
- \( \underline{q} \): Matrix quantity
- \( q_i \): Covariant variable
- \( q_j \): Covariant derivative of a variable
- \( q_{\text{Mod}} \): Quantity referred to the different energy modes
- \( q_r \): Reaction quantity
- \( q_s \): Species quantity
- \( q^b \): Quantity referred to \( k_{br} \)
- \( q^{eq} \): Quantity referred to \( K_{eqr} \)
- \( q^f \): Quantity referred to \( k_{fr} \)

Superscripts

- \( q^\prime \): Perturbation variable
- \( q^\nu \): Root-Mean-Square of physical perturbation
- \( \tilde{q} \): Base-flow variable
- \( \tilde{q} \): Perturbation complex conjugate
- \( q^i \): Contravariant variable
I. Introduction

The accurate prediction of the laminar-to-turbulent transition onset in boundary layers constitutes one of the major requirements for the development of optimized and reliable atmospheric-entry and hypersonic cruise vehicles [1]. The regime change from laminar to turbulent drastically increases the surface heating (by a factor of 8 or more [2]), thus presenting a potential mission killer, and a major design concern for Thermal Protection Systems (TPS).

The early stages of transition in a low disturbance environment [3], where instabilities grow linearly and without interacting can be studied with Linear Stability Theory (LST) [4,5]. More physically-inclusive theories, such as the Linear Parabolized Stability Equations (LPSE) [6], Nonlinear Parabolized Stability Equations (NPSE) [7] or Direct Numerical Simulations (DNS) [8] allow to include successively larger ranges of the transitioning region. Whereas LST assumes the flow to be locally parallel, LPSE accounts for weak non-parallel effects. Both of them, however, are linear theories, and therefore neglect perturbation interactions. These are taken in consideration by NPSE, where a series of modes are simultaneously tracked, while letting them interacting. Finally, DNS solve the Navier-Stokes equations without any simplification. On the other hand, the mesh requirements needed to appropriately capture the smaller flow scales makes it computationally very expensive. Hence, only simple flows and geometric configurations are often studied.

During the last two decades there has been a remarkable effort to model and investigate the influence of high-temperature phenomena on transition [9–14]. In order to accurately capture these aspects of the hypersonic flow physics, one must depart from the Calorically Perfect Gas (CPG) assumption and include additional physics. Thermally Perfect Gas (TPG) extends the model to the excitation of particles internal energy modes, while Chemical Non-Equilibrium (CNE) further includes air chemistry. Thermo-Chemical Non-Equilibrium (TCNE) considers the different energy modes to be governed by one of several temperatures, thus providing an accurate description of high-enthalpy flows. In the presence of both thermal and chemical equilibrium, one reaches Local Thermodynamic Equilibrium (LTE). [15 Ch. 10]

The majority of past stability and transition studies in hypersonic regime has been conducted within the LST theory. In particular, Malik et al. [9] studied dissociation effects by investigating self-similar boundary layers in Local Thermodynamic Equilibrium (LTE). Stuckert et al. [10] extended this analysis to boundary layers in chemical non-equilibrium, while Hudson et al. [11] considered also thermal non-equilibrium effects, by modeling the vibrational energy mode with a separate temperature from the translational and rotational. Lyttle et al. [12] and Franko et al. [13] carried out sensitivity studies to compare the effects of using different thermodynamic and transport models, while Klentzman et al. [16] investigated viscosity effects on receptivity and stability in a dissociating binary oxygen mixture. Bitter et al. [17] studied supersonic modes in vibrational non-equilibrium yet chemically frozen, and Knisely et al. [18] extended their analysis to thermo-chemical non-equilibrium. Miró Miró et al. [19] proposed an alternative compatibility boundary condition for the species partial density perturbations, which significantly improved the conditioning of the matrix system. It was then used to study diffusion and strong chemistry effects on adiabatic-wall boundary layers [20], and ionization effects in a Martian return mission [21]. Recently, Miró Miró et al. [13] revisited some of these literature results, analyzing the influence of thermodynamic, transport and chemistry models, in terms of stability and transition predictions. This also allowed to explain disagreements between different reference sources, confirming the high sensitivity of stability results to the gas properties modeling, particularly the transport one.

Transition studies in hypersonic flows by means of more advanced techniques are very limited. Examples of investigations by means of DNS are the works of Marxen et al. [22], Ma et al. [23], Mortensen et al. [24] or Di Giovanni et al. [25]. Concerning LPSE, the first application to chemically-reacting flows was done by Chang et al. [26], where non-parallel effects on a wedge flow were studied for a single frequency, and found to give an N factor increment of 2. Malik [27] and Johnson et al. [28] used LPSE to analyze the 2nd mode instability in experimental data from flight experiments. In particular, they estimated transition on the Reentry-F blunt cone [29] [30], attributing a destabilizing effect both to chemistry and non-parallelism. Recently, Kline et al. [31] extended the capabilities of the NASA code LASTRAc to perform LPSE analysis in TCNE conditions. A crossflow instability over an infinite swept wing was investigated, showing negligible non-parallel effects in terms of N factors. Finite-rate reactions were found to destabilize the 2nd mode disturbance, while they were stabilizing the crossflow waves.

Despite these studies, the interaction between non-parallel effects and flow chemistry remains unclear. In particular, no extended investigations have been performed at variation of the flow conditions and chemistry assumptions. The present work addresses this by jointly performing LST and LPSE analyses, in order to clarify whether the consequences of non-parallelism observed in CPG and LTE [32,33] are equally present when assuming frozen chemistry (TPG) or CNE. A benchmark test case is revisited [26], in order to verify the coupling of the developed LPSE solver with the chemistry and high temperature effects. A parametric study on a wedge geometry for different re-entry conditions and flow assumptions is considered to isolate the non-parallel effects. The laminar basic-state flowfields are obtained with
the DEKAF solver \cite{21,34}, and the stability computations with the VESTA toolkit \cite{35}, benefiting from its Automatic Derivation and Implementation Tool \cite{36}.

The paper is structured in the following manner. The problem formulation is described in § II reporting the governing equations for the laminar basic state § II.A, the stability problem § II.B and the different thermodynamic and transport models employed § II.C. Results are displayed in § III. Finally, summary and conclusions are drawn in § IV.

II. Problem formulation

A. Laminar basic state

The laminar basic state is obtained from solving firstly the inviscid flowfield region, and then imposing the inviscid wall conditions at the freestream boundary of the boundary layer \cite{37}. Despite notably neglecting viscous-inviscid interaction, the transition predictions obtained with this approach are valid, as long as the interaction parameter is lower than one: $M_{\infty}/Re_{L,\infty} << 1$ \cite{38}.

1. Inviscid region

The inviscid flow region is characterized by the considered geometry: in the present work only flows over wedges are studied. The wedge shock jumps are obtained from the oblique jump-shock relations, consistently with the employed flow assumption, as detailed in Miró Miró et al. \cite{21}.

When working with the CNE assumption, the non-equilibrium is assumed to be restricted to the boundary layer, letting the inviscid region be frozen. This assumption is based on the observations made for a Mach 45 wedge in CNE \cite{21}. Despite the high post-shock temperatures, the flow high speed renders the chemical activity considerably slower than the flow, and therefore justifies the frozen flow assumption.

2. Boundary layer

The continuity, momentum, energy and species continuity boundary-layer equations respectively read:

\begin{align}
\frac{\partial \bar{\rho} \bar{u}}{\partial x} + \frac{\partial \bar{\rho} \bar{v}}{\partial y} &= 0, \\
\bar{\rho} \frac{\partial \bar{u}}{\partial x} + \bar{\rho} \frac{\partial \bar{u}}{\partial y} &= -\frac{\partial p_e}{\partial x} + \frac{\partial}{\partial y} \left( \bar{\mu} \frac{\partial \bar{u}}{\partial y} \right), \\
\frac{\partial \bar{H}}{\partial y} &= 0, \\
\bar{\rho} \frac{\partial \bar{H}}{\partial y} + \bar{\rho} \bar{v} = \bar{\rho} v \left( \bar{u} \frac{\partial \bar{T}}{\partial y} \right) + \bar{\mu} \frac{\partial \bar{u}}{\partial y} \left( \frac{\partial \bar{u}}{\partial y} \right)^2 + \bar{\mu} \frac{\partial \bar{w}}{\partial y} + \frac{\partial}{\partial y} \left( \sum_{s \in S} \bar{h}_s \bar{J}_s \right), \\
\bar{\rho} \frac{\partial \bar{Y}_s}{\partial x} + \bar{\rho} \bar{v} \frac{\partial \bar{Y}_s}{\partial y} &= \bar{\rho} \frac{\partial \bar{J}_s}{\partial y} + \bar{\omega}_s,
\end{align}

where the wall-normal diffusion fluxes have different expressions depending on the diffusion model (see § II.C.3).

For CPG and TPG the composition $Y_s$ is frozen, and for LTE it is a function of pressure and temperature through the equilibrium relations. Therefore the resolution of the species conservation equations (1e) and the concentration condition $\sum_{s \in S} \bar{Y}_s \approx 1$ is not required for these flow assumptions.

The wall boundary is characterized by the no-slip condition ($\bar{u} = \bar{w} = 0$), the impenetrability condition ($\bar{v} = 0$), the non-catalytic condition ($\partial \bar{T}/\partial y = 0$) and the isothermal condition ($\bar{T} = T_w$).

B. Stability analysis

1. Governing equations

Stability equations are derived starting from the multi-species Navier-Stokes equations, here expressed in their dimensional invariant:

\begin{align}
\frac{\partial \rho_s}{\partial t} + \left( u^j \rho_s \right)_j &= -J^j_{s,j} + \bar{\omega}_s, \\
\end{align}

where $J^j_{s,j}$ is the species diffusion flux.
\[
\begin{align*}
\rho \frac{\partial u^i}{\partial t} + \rho u^i u_j = & -g^{ij} p_j + \tau^{ij}, \\
\rho \frac{\partial h}{\partial t} + \rho u^i h_j = & \frac{\partial p}{\partial t} + u^i p_j + (k g^{ij} T_j)_j - \mathcal{J}^i_j + g_k \tau^{kj} u_j,
\end{align*}
\]

where \(g^{ij}\) is the metric tensor \[^{39}\], \(J^i_j\) are the diffusion mass fluxes of each species, \(\mathcal{J}^i\) are the energy diffusion fluxes:

\[
\mathcal{J}^i = \sum_{s \in S} h_s J^i_s,
\]

and \(\tau^{ij}\) is the viscous tensor:

\[
\tau^{ij} = \lambda g^{ij} u_k^s + \mu \left( g^{ik} u_j^s + g^{jk} u_i^s \right).
\]

Equations \[^{2}\] are then linearized, by decomposing the flow variables into their basic state and perturbation components \(q = \bar{q} + \epsilon q'\), subtracting the basic-state equations (Eqs. \[^{2}\] evaluated with \(q = \bar{q}\)), and dropping terms of order \(\epsilon^i, i > 1\). The LST and LPSE equations are ultimately reached by substituting the perturbation quantities for their corresponding ansatz and imposing the appropriate assumptions on the basic-state quantities:

\[
\begin{align*}
\text{LST} & \quad \rightarrow \quad \bar{q} = \bar{q}(y), \quad q' = \bar{q}(y) \exp \left[ -i \left( \alpha x + \beta z - \omega t \right) \right] + \text{cc.}, \\
\text{LPSE} & \quad \rightarrow \quad \bar{q} = \bar{q}(x, y), \quad q' = \bar{q}(x, y) \exp \left[ -i \left( \int_{x_0}^{x} \alpha(\xi) \, d\xi + \beta z - \omega t \right) \right] + \text{cc.},
\end{align*}
\]

where Eq. \[^{5}\] requires an additional orthogonality condition to solve the ambiguity caused by the double presence of the streamwise component, both in the amplitude and in the wave function:

\[
\int_0^{\gamma_e} \bar{q}^i \frac{\partial \bar{q}}{\partial x} \, dy = 0.
\]

In addition, in LPSE equations elliptic terms \(\partial^2 \bar{q} / \partial x^2\) are dropped, making the equations nearly parabolic (an ellipticity residual is still present indeed \[^{40}\]).

The LST and LPSE problems ultimately reduce to a generalized eigenvalue problem and a marching problem respectively:

\[
\begin{align*}
\text{LST} & \quad \rightarrow \quad \left( A \alpha^2 + B \alpha + C \right) \bar{q} = 0, \\
\text{LPSE} & \quad \rightarrow \quad \left( L + M \frac{\partial}{\partial x} + N \frac{\partial}{\partial x} \right) \bar{q} = 0.
\end{align*}
\]

2. Nondimensionalization and scaling considerations

Most variables are nondimensionalized with respect to their boundary-layer edge values, denoted by the subscript \(e\). Some exceptions occur, as described in Table \[^{1}\] reporting the VESTA nondimensionalization convention. In particular, spatial coordinates are normalized with the Blasius length \(\ell\):

\[
\ell = \sqrt{\frac{\mu_e}{u_e \rho_e}}.
\]

The reference diffusion coefficient \(D_e\) is chosen as the one of the most present species pair in the freestream, while the reference chemical time scale \(\tau_{c,e}\) is obtained as the maximum of the \(\rho_e / \dot{\omega}_e\) quantity among all species. The nondimensional reduced frequency is defined as \(F = 2 \pi f \mu_e / (u_e^2 \rho_e) = \omega / Re\), where the Reynolds number is based on \(\ell\), whereas the reduced spanwise wavenumber is \(b = b \mu_e / (u_e \rho_e) = \beta / Re\). LST uses the local edge quantities at every \(x\) location as the reference. PSE however uses constant reference values for all \(x\) locations. These are obtained at a single \(x_0\) location: here always chosen as the first domain position.
3. Initial and boundary conditions

At the wall, the no-slip and impenetrability condition and the thermal inertia of the surface allow to apply homogeneous Dirichlet boundary conditions for the temperature and velocity perturbations. Within the CPG, TPG and LTE flow assumptions, a compatibility condition for pressure or density is applied, by evaluating the y-momentum equations at the bottom boundary. Similarly, in CNE a species y-momentum conservation is used to account for each species densities [19]. Alternatively, a non-catalytic boundary condition can also be imposed for the \( \rho_s \) perturbations.

At the freestream different possibilities can be chosen. Homogeneous Dirichlet or Neumann are commonly applied, assuming disturbances exponentially decay outside the boundary-layer. Exception are made for the pressure or density component, in the case of CPG, TPG or LTE flow assumptions, and the vertical velocity component \( \tilde{v} \), in case of CNE flows. For these perturbations, compatibility conditions are obtained by evaluating respectively the y-momentum and the mixture continuity equations at the top boundary.

Finally, regardless of the flow assumption, initial conditions for LPSE computations are obtained from LST solutions.

4. Measures of growth

Contrary to LST disturbances, within the PSE theory, perturbations grow and decay with different rates. In general the growth rate of a quantity \( \tilde{q} \) can be written as:

\[
\gamma_{\tilde{q}} = -\Im(\alpha) + \Re\left(\frac{\tilde{q}_x}{\tilde{q}}\right).
\]

According to the quantity \( \tilde{q} \), different definitions are possible. The ones mainly used in this work are based on:

- maximum of a velocity component fluctuation \( u''_{\text{max}} \):
  \[
  \tilde{q} = \tilde{u}^i
  \]

- maximum of temperature fluctuation \( T''_{\text{max}} \):
  \[
  \tilde{q} = \tilde{T}
  \]

- disturbance kinetic energy:
  \[
  \tilde{q} = KE = \left[ \frac{1}{S} \int_0^{\infty} \tilde{\rho} (|\tilde{u}|^2 + |\tilde{v}|^2 + |\tilde{w}|^2) dy \right]^{1/2}
  \]

- maximum of the mass flux fluctuation \( (\rho u')''_{\text{max}} \):
  \[
  \tilde{q} = (\rho u)' = (\tilde{\rho} \tilde{u} + \tilde{\rho} \tilde{u})
  \]

where the symbol \( '' \) denotes what in the experiments is the root-mean-square of the physical perturbation profile, mathematically equal to the module of the relative perturbation \( q' \).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Reference</th>
<th>Variable</th>
<th>Reference</th>
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</thead>
<tbody>
<tr>
<td>( x^i )</td>
<td>( \ell_e )</td>
<td>( \lambda )</td>
<td>( \mu_e )</td>
</tr>
<tr>
<td>( t )</td>
<td>( \ell_e/u_e )</td>
<td>( k )</td>
<td>( k_e )</td>
</tr>
<tr>
<td>( \tilde{u}^i )</td>
<td>( u_e )</td>
<td>( \mathcal{M}_s, \mathcal{M}_e )</td>
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</tr>
<tr>
<td>( T )</td>
<td>( T_e )</td>
<td>( \tilde{\omega}_s )</td>
<td>( \rho_e/\tau_{v,e} )</td>
</tr>
<tr>
<td>( \rho, \rho_s )</td>
<td>( \rho_e u_e^2 )</td>
<td>( h, h_s, h_e )</td>
<td>( h_e )</td>
</tr>
<tr>
<td>( \rho, \rho_s )</td>
<td>( \rho_e )</td>
<td>( \mathcal{D}_{st} )</td>
<td>( \mathcal{D}_e )</td>
</tr>
<tr>
<td>( \mu )</td>
<td>( \mu_e )</td>
<td>( \mathcal{D}_s )</td>
<td>( \mathcal{D}_e )</td>
</tr>
</tbody>
</table>
C. Thermodynamic, transport and chemistry models

Thermodynamic, transport and chemistry models provide the relations required to close the system of equations, between dependent quantities \( (\mu, \kappa, h, \ldots) \) and independent quantities \( (u, T, \rho_s) \). In DEKAF and VESTA several models, among state-of-the-art and most commonly used in the literature, are available. In the following only the ones considered in this work are reported. For a more detailed description of all the models refer to Miró Miró et al. [14].

Thermal equilibrium is always assumed, thus only one temperature is sufficient to describe the different energy modes. Also, for all the models, mixture molar mass and multi-species equation of state are defined respectively as:

\[
\mathcal{M} = \left( \sum_{s \in S} \frac{Y_s}{\mathcal{M}_s} \right)^{-1},
\]

\[
p = \sum_{s \in S} \rho_s \frac{\mathcal{R}}{\mathcal{M}_s} T = n k_B T.
\]

1. Thermal models

Enthalpy in CPG conditions can be simply obtained as a linear function of temperature (i.e. \( h = C_p T \)), assuming a constant specific heat capacity \( C_p \), in this study set equal to 1004.5 J/Kg-K. Within other flow hypothesis, enthalpies and species heat capacities can be obtained from the McBride’s NASA-5 or NASA-9 curve fits [41,42] (McB), whose expressions are summarized in Miró Miró et al. [14].

Alternatively, the enthalpies expression can be computed from the differentiation of the energy modes partition functions, assuming molecules behave like a rigid rotor and a harmonic oscillator (RRHO):

\[
h_s = \sum_{\text{Mod}} e_{s,\text{Mod}}^{\text{Mod}} + h_{f,s}^0 + \frac{\mathcal{R}}{\mathcal{M}_s} T,
\]

where the energy associated to a specific mode \( \text{Mod} \) reads

\[
e_{s,\text{Mod}}^{\text{Mod}} = \frac{\mathcal{R}}{\mathcal{M}_s} T^2 \frac{\partial \ln Q_{s,\text{Mod}}}{\partial T},
\]

The species formation enthalpy at 0 K can be obtained using Hess’ law, while the expressions for the different partition functions are reported for example in [14]. The mixture enthalpy is then computed from:

\[
h = \sum_{s \in S} Y_s h_s.
\]

2. Transport models

Transport models were found to highly affect stability results [12,14]. The most common model used in literature to compute viscosity and thermal conductivity is the combination of Blottner’s curve fits [43] with Eucken’s relation [44] and Wilke’s mixing rule [45] (BEW):

\[
\mu_s = 0.1 \exp \left( \left( A_s^\mu \ln T + B_s^\mu \right) \ln T + C_s^\mu \right),
\]

\[
\kappa_s = \left( \frac{5}{2} c_{s,\text{Trans}}^\text{Trans} + c_{s,\text{Rot}}^\text{Rot} + c_{s,\text{Vib}}^\text{Vib} + c_{s,\text{Elec}}^\text{Elec} \right) \mu_s,
\]

\[
\phi_s = \sum_{\ell \in S} \left( 1 + \frac{Q_{s,\ell}}{Q_{s,\ell}} \right)^{1/2} \left( \frac{\mathcal{M}_s}{\mathcal{M}_\ell} \right)^{1/4} \left( \frac{1}{8} \left( 1 + \frac{\mathcal{M}_s}{\mathcal{M}_\ell} \right) \right)^{1/2}.
\]
\[ Q = \sum_{s \in S} X_s Q_s \phi_s, \]  

where \( Q \) is either \( \mu \) or \( \kappa^{Fr} \), and where values for \( A^\mu_s, B^\mu_s \) and \( C^\mu_s \) are given in Blottner \[43\] or in Miró Miro \[20\].

Nevertheless, the most accurate model is the first and second approximation to the Chapman and Enskog’s molecular theory of gases (CE), using Laguerre-Sonine polynomials \[46\]

\[ Q = - G Q_s \ell X_s X_\ell 0 / |G Q_s \ell|, \forall s, \ell \in \mathcal{H}, \]  

where \( Q \) can be the viscosity \( \mu \) or the heavy-particle translational thermal conductivity \( \kappa^{Trans} \). The full expression for the thermal conductivity includes addends for the contribution of the internal energy modes:

\[ \kappa^{Fr} = \kappa^{Trans} + \kappa^{Rot} + \kappa^{Vib} + \kappa^{Elec}. \]

Solving Eq. (22) is computationally expensive, thus a simplified expression was proposed by Brokaw \[47\] (Brokaw):

\[ \mu = \sum_{s \in S} \frac{A^\mu_s}{\sum_{\ell \in S} X_{\ell} \Delta_s^{(2)}}, \]  

\[ \kappa^{Trans} = \frac{15}{4} k_B \sum_{s \in S} X_s \sum_{\ell \in S} \phi_s \Delta_s^{(2)} \Delta_{\ell}^{(2)}. \]

Terms appearing in Eq. (22), Eq. (23) and Eq. (24) can be found in Miró Miro \[20\].

Finally, within the CPG assumption, commonly Sutherland’s law \[48\] is used to compute viscosity, while thermal conductivity is obtained from assuming a constant Prandtl number. Second viscosity coefficient derives from the Stokes’ hypothesis (i.e. \( \lambda = -\frac{2}{3} \mu \)).

3. Diffusion models

Several diffusion models are available, depending if the diffusion of one species into the mixture or into each other species is considered. The former is modeled as:

\[ J_s^j = - \rho D_{eff, s} d_s^j, \forall s, \ell \in S, \]  

where \( D_{eff, s} \) is the effective diffusion coefficient of species \( s \) within the mixture and \( d_s^j \) is the diffusion driving force of species \( s \); while the latter takes the form:

\[ J_s^j = - \sum_{\ell \in S} \rho_s D_{s, \ell} d_s^j, \forall s, \ell \in S, \]  

where \( D_{s, \ell} \) is the diffusion coefficient of the species pair \( s \cdot \ell \). The diffusion driving force is commonly assumed to be either the mass fraction or the mole fraction gradients (\( d_s^j = g^{ij} Y_{s,j} \) or \( d_s^j = g^{ij} X_{s,j} \)).

Yos \[49\] \[50\] proposed an effective binary diffusion (EBD) model, characterized by the an effective diffusion coefficient defined as:

\[ D_{eff, s} = \frac{1 - X_s}{\sum_{\ell \neq s} X_{\ell} / \mathcal{D}_{s, \ell}}, \forall s \in S. \]

where \( \mathcal{D}_{s, \ell} \) is the binary diffusion coefficient of the species pair \( s \cdot \ell \) \[13\]. Nevertheless, this model is not self-consistent, since the null sum of all diffusion fluxes is not granted. Thus, Ramshaw \[51\] presented a correction in order to preserve self-consistency, which leads to the self-consistent effective binary diffusion (SCEBD) coefficients:

\[ D_{s, \ell} = \frac{\delta_{s, \ell} - Y_{s, \ell}}{X_{s, \ell}} \frac{1 - Y_{s, \ell}}{1 - X_{s, \ell}}, \forall s, \ell \in S. \]
4. Collisional models

Species pair’s collisional cross-section integrals are needed in the expressions of some transport and diffusion models. Commonly these quantities are obtained by interpolation between tabled values [52] as a function of the temperature governing the collisions. In stability studies, on the contrary, polynomial fits are preferred, since they have the advantage of having continuous analytical derivatives. Possible fittings are the ones proposed by Gupta et al. [53], (Gupta):

$$\ln \pi^{(i,j)}_{st} = D^{(i,j)}_{st} + C^{(i,j)}_{st} \ln T + B^{(i,j)}_{st}(\ln T)^2 + A^{(i,j)}_{st}(\ln T)^3, \quad \forall s, t \in S,$$

whose coefficients were obtained from fitting the curve to the different collision integrals computed by the NATA (Nonequilibrium Arc Tunnel Analysis) code [54].

Alternatively, an higher-polynomial-order version of this fit was proposed by Miró Miró [14]:

$$\ln \Omega^{(i,j)}_{st} = A^{(i,j)}_{st} + B^{(i,j)}_{st} \ln T + C^{(i,j)}_{st}(\ln T)^2 + D^{(i,j)}_{st}(\ln T)^3 + E^{(i,j)}_{st}(\ln T)^4 + F^{(i,j)}_{st}(\ln T)^5, \quad \forall s, t \in S,$$

where the coefficients were retrieved from fitting the latest collisional data in Wright et al. [52], (Wright).

5. Chemical models

For a set of reactions \( \mathcal{R} \) between a set of species \( S \):

$$\sum_{s \in S} v'_{sr} s \leftrightarrow \sum_{s \in S} v''_{sr} s, \quad \forall r \in \mathcal{R},$$

the mass production rate of each species can be approximated by the law of mass action [55]:

$$\dot{\omega}_s = \mathcal{M}_s \sum_{r \in \mathcal{R}} \left( v''_{sr} - v'_{sr} \right) \left( k_f r \prod_{\ell \in S} \left( \frac{P_{\ell r}}{\mathcal{M}_\ell} \right)^{v'_{\ell r}} - k_b r \prod_{\ell \in S} \left( \frac{P_{\ell r}}{\mathcal{M}_\ell} \right)^{v''_{\ell r}} \right),$$

with the forward and backward reaction rates defined as:

$$k_f r = A_f^r T^{n_f^r} \exp{-\theta_f^r / T},$$

$$k_b r = \frac{k_f r}{K_{eq r}}.$$

Bortner [53, 56] proposed to use an Arrhenius-like expression both for the forward and backward reaction rates, which lead as a consequence to an equivalent expression for the equilibrium constant, (Bortner):

$$K_{eq r} = A_{eq r} T^{n_{eq r}} \exp{-\theta_{eq r} / T},$$

where the coefficients are drawn from Gupta et al [53]. Note that Bortner actually presented an Arrhenius-like expression for the backward reaction rate with \( A_b^r, n_b^r \) and \( \theta_b^r \). It is however trivial to reach Eq. (35) defining \( A_{eq r} = A_f^r / A_b^r \), \( n_{eq r} = n_f^r - n_b^r \) and \( \theta_{eq r} = \theta_f^r - \theta_b^r \).

In case of LTE conditions the mixture composition is computed by solving the equilibrium system of equations, involving \( K_{eq r} \) and the species partial pressures \( p_s \) (see Anderson [15] Ch. 10).

III. Results

In this section results are presented divided in two parts. The first one deals with the verification of the LPSE solver for finite-rate chemically reacting flows; the second one includes a parametric study about the stability of a boundary-layer flow over a wedge, varying the high-envelope position and the gas model. All cases investigate exclusively second mode instabilities (\( \beta = 0 \)), which are well known to be dominant for these velocities at 0-deg angle of attack.
A. Verification of the LPSE solver

The VESTA LPSE solver has been already verified in the past, for flows in CPG and LTE conditions \cite{32,33,35}. In the following, a literature test case studying the stability of finite-rate chemically reacting boundary-layers is reproduced, in order to verify the correct implementation of the LPSE algorithm. The 6-deg wedge firstly studied by Chang et al. \cite{26} and recently revisited by Kline et al. \cite{31} is one of the few literature benchmark problems dealing with LPSE in presence of chemistry. The flow conditions feature a Mach number of 20, a unit Reynolds number of $2.953 \cdot 10^6$ m$^{-1}$, a freestream temperature of 236.67 K, and a composition of 21% $O_2$ and 79% $N_2$ in mass. The wall temperature is fixed to $0.91 T_{adiab}$. Because of the high sensitivity of stability results to the mean flow and gas properties modeling \cite{14}, generally it is important to remove, or at least minimize, all possible sources of discrepancies in both these aspects, when trying to reproduce literature results. To this end, LPSE analyses on two base flow solutions were performed: one computed with DEKAF \cite{34}, and one with Blottner’s chemical nonequilibrium boundary-layer code \cite{43}. For the latter case, the mean flow data were kindly provided by H.L. Kline, and they correspond to the same laminar solution used in Kline et al. \cite{31}.

Table 2 reports the models used in the calculation of the two mean flows and in the stability analyses. It is important to point out that in Kline et al. \cite{31} different models were used to compute the base flow and the LPSE solutions. In order to be consistent, the stability study on the DEKAF mean flow was conducted by choosing the same models both for the stability and the laminar solution. They were selected to match as close as possible the ones used in Blottner’s code, since the effect of gas properties modeling in the base flow is greater than in the perturbations \cite{14}.

Table 2 Models used in the base flow and stability computations of the Mach 20 6-deg wedge case \cite{31}

<table>
<thead>
<tr>
<th>Model</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base flow</td>
<td>Stability</td>
</tr>
<tr>
<td>Thermal</td>
<td>DEKAF RRHO</td>
<td>VESTA RRHO</td>
</tr>
<tr>
<td>Transport</td>
<td>BEW BEW</td>
<td>SCEBD SCEBD</td>
</tr>
<tr>
<td>Diffusion</td>
<td>Gupta Gupta</td>
<td>Yun \cite{58} Gupta Gupta</td>
</tr>
<tr>
<td>Chemical</td>
<td>Bortner Bortner</td>
<td>Bortner Bortner</td>
</tr>
</tbody>
</table>

In the LPSE analysis, non-catalytic boundary conditions were used at the wall, for the species disturbance densities, while homogeneous Neumann were imposed at the freestream to all perturbations variables, except for the vertical velocity component (see II.B.3). On the contrary, Kline et al. \cite{31} used non-reflecting boundary conditions at the far field, for all disturbances.

Figure 1 shows the comparison between the laminar solutions obtained by the two boundary-layer codes, at three different streamwise locations along the wedge. There is generally a satisfactory profiles match. The Oxygen dissociation is very low and it does not significantly affect the temperature profile. These discrepancies could be due to the different diffusion fluxes or collisional data.

Stability results are firstly compared in terms of growth rate and phase speed, for a single frequency of 60 kHz. These quantities are expressed based on the disturbance kinetic energy. Figure 2 displays the comparison between the VESTA LPSE and literature reference results. For this particular frequency, the VESTA solution computed on the DEKAF base flow, surprisingly matches better the overall growth rate reference curve. A slight lower peak is retrieved, while, in the proximity of the leading edge there is an evident disagreement. The results obtained using the Blottner’s code mean flow depict a better agreement in terms of maximum peak value and first neutral point detection, while a minimal shift towards downstream locations occurs. Similarly, phase speed diagrams agree well (Figure 2).

The correct description of the growth rate in the vicinity of the first neutral point becomes important when comparing the N factors. Results reported in Fig. 3 show that the solution obtained by using Blottner’s mean flow approaches better the reference one, regarding both the N factors peaks and the neutral point estimation. Computing stability using the DEKAF mean flow leads to an underestimation of the N factor maximum values.

All sets of results reveal very different behaviors after the N factors peaks, corresponding to regions downstream of the classical growth rate bell-shape. In these zones, supersonic modes are present \cite{26}, originating from interactions between discrete modes and the acoustic continuous branch of the spectrum. Because of this mode-to-mode interaction and the fact that they do not exponentially decay outside the boundary-layer, growth rate solutions are very sensitive to
Figure 1  Mean flow profiles for the Mach 20 6-deg wedge case [31]: comparison between DEKAF and Blottner’s code solutions

Figure 2  Growth rate (left) and phase speed (right), based on disturbance kinetic energy, for a frequency of 60 kHz in CNE conditions: comparison against Kline et al. [31]

grid discretization and freestream boundary conditions. Huge discrepancies occur not only between the two stability codes, but also within the same code applied to different base flows. In the former case an important role is probably played by the different boundary conditions employed. However, the supersonic mode region does not contribute in this problem to the N envelope definition and the eventual transition estimation. Differences noted in the first neutral-branch-point capturing can be linked to the initialization of the LPSE computations. Depending on the initial conditions origin, a numerical transient usually occurs. Because LST solutions are used in VESTA, stronger transient effects appear as the leading edge is approached.

Moreover, several other aspects affect the simulations, namely: the sensitivity of the stability results to the aerothermodynamic models, the uncertainties associated to basic states quantities and their derivatives, the stability codes numerics. Because of all these sources of discrepancies and the differences reported in similar literature
comparisons, the verification of the LPSE solver can be considered successful.

B. Non-parallel effects investigation

In order to investigate non-parallel effects within different flow assumptions, a 10-deg wedge of length $L = 5$ m at different points within a typical planetary re-entry flight envelope are analyzed (see Table 3). At the freestream, air is assumed to have 23.3% $O_2$ and 76.7% $N_2$ in mass. Different flow assumptions are considered: CPG, both with Sutherland’s law and Chapman-Enskog transport models, TPG, CNE, CNE with perturbations modeled in TPG, and finally LTE. State-of-the-art models are used to describe gas properties both in the base flow and stability computations, as summarized in Table 4. A constant temperature of 1400 K is imposed at the wall. This temperature constitutes a classic operational point of re-entry TPS. Results are displayed in terms of N factors envelopes around the following frequencies: [189.6, 167.3, 146.9, 128.5, 112.0, 97.5, 84.9, 74.2, 65.5, 58.7, 53.9, 51.0, 50.0] kHz.

### Table 3  Freestream flow conditions for the 10-deg wedge studied test cases

<table>
<thead>
<tr>
<th>Mach</th>
<th>Altitude [m]</th>
<th>Temperature [K]</th>
<th>Pressure [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>31655</td>
<td>228.3</td>
<td>914.0</td>
</tr>
<tr>
<td>8</td>
<td>37567</td>
<td>244.2</td>
<td>388.2</td>
</tr>
<tr>
<td>10</td>
<td>42327</td>
<td>257.6</td>
<td>203.0</td>
</tr>
<tr>
<td>15</td>
<td>45904</td>
<td>267.6</td>
<td>127.5</td>
</tr>
<tr>
<td>20</td>
<td>46565</td>
<td>269.4</td>
<td>117.2</td>
</tr>
<tr>
<td>25</td>
<td>46509</td>
<td>269.3</td>
<td>118.0</td>
</tr>
</tbody>
</table>

The streamwise velocity, temperature and O concentration basic state profiles, for different flow assumptions and Mach numbers, are reported in Appendix A Fig. 7. For all envelope points below $M=10 - h=42.3$ Km, vibrational excitation and chemistry effects can be neglected. At $M=15 - h=45.9$ Km, vibrational excitation gains importance and makes the TPG and CNE curves depart from the CPG. Similarly, Sutherland’s law is seen to no longer be accurate, making the CPG-Suth differ from the CPG-CE curve. Chemical activity in this operation point is only meaningful if
Table 4  Models used in the base flow and stability computations of the 10-deg wedge parametric study

<table>
<thead>
<tr>
<th>Model</th>
<th>Flow assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CPG</td>
</tr>
<tr>
<td>Thermal</td>
<td>RRHO</td>
</tr>
<tr>
<td>Transport</td>
<td>Sutherland’s/CE</td>
</tr>
<tr>
<td>Diffusion</td>
<td>–</td>
</tr>
<tr>
<td>Collisional</td>
<td>Wright</td>
</tr>
<tr>
<td>Chemical</td>
<td>–</td>
</tr>
</tbody>
</table>

equilibrium is reached, leading to a ≈ 10% dissociation of O_2 in the middle of the boundary-layer. At M=20 - h=46.5 Km, higher levels of dissociation are reached, not only in equilibrium, but also with the finite-rate reactions occurring along the streamwise span.

When non-parallel effects are taken into account, different perturbation variables are no longer assumed to growth or decay with the same rate, as within the LST framework. Figure 4 displays the N factor comparison between LST and LPSE results, based on different quantities. As an example, two extreme conditions are shown in terms of non-parallel effects influence: a CPG flow with Sutherland’s transport model at Mach 6 and 25. From all test cases run, it has been seen that, at higher altitudes, the variation between LPSE N factors based on different quantities becomes larger. Interestingly, the perturbation that grows the most is the vertical velocity component, showing a difference in the N factor going from 1 to 0.5, while moving towards the end of the domain. Temperature, streamwise velocity and kinetic energy have very close N curves also at high Mach numbers. Non-parallel effects has a stabilizing influence on the mass-flux perturbation variable, whose N curve was found to be lower with respect to the LST one, in all studied conditions. The other quantities behave in relation to the LST solution in the same way, regardless of the re-entry conditions and flow assumptions. Commonly, in supersonic and hypersonic flows PSE growth rates and N factors are reported in terms of mass-flux, because it can directly be compared with experimental data, or temperature or kinetic energy, because they usually experience the greater growth. In the following, it has been chosen to show LPSE results based on kinetic energy, since it is a more representative variable in the context of the considered transition scenario. Even if the \( \tilde{v} \) perturbations exhibits the largest growth, its magnitude was found to be half of the streamwise component. Its contribution to the kinetic energy growth can be considered negligible, as it is evident from the fact that N curves based on kinetic energy and \( \tilde{u} \) velocity are basically always superposed.

![Figure 4](image)

**Figure 4** N factor computed with LST and LPSE, based on different variables. CPG flow assumption with Sutherland’s transport model at Mach 6 - h=31.6 Km (left) and Mach 25 - h=46.5 Km (right)

In Fig. 5 the LST N factor curves for different altitudes and flow assumptions are displayed. The well known
destabilization effects on the second Mack mode, due to the boundary layer cooling is visible. Assuming the hypersonic flow in CPG conditions leads, in general, to an underestimation of the disturbances growth, with Sutherland’s law transport model giving slightly more unstable results than the Chapman-Enskog model. Chemical reactions increase the N factor, with a drastic N increment observed in the high Mach number LTE cases. For this particular problem the maximum difference between the finite-rate and no-reacting N curves appears at the end of the domain, at Mach 25. It is about \( \Delta N = 1 \), with respect to the TPG flow assumptions and \( \Delta N = 2 \) compared to the CPG with Chapman-Enskog model. At the same locations the N factor predicted in LTE conditions is about 12 (not shown in the graph). The cause of
this phenomenon in LTE is the presence of supersonic modes, which are promoted by the cooling of the boundary-layer, as a consequence of the endothermic recombination reactions. This confirms the fact that supersonic modes can not only be found in flows with highly-cooled walls (e.g. $T_w/T_e \approx [0.1, 0.4]$) [17,26], but also in hot flows cooled by chemical reactions [21,33]. Figure 6 displays the perturbation growth rates and their relative nondimensional phase speeds for the whole set of analyzed frequencies, for the Mach 25 flow in LTE conditions. Disturbances become supersonic right after the growth rate peak location, resulting in a considerably enlarged downstream unstable region. From Fig. 5 it is also evident the importance of properly accounting not only for the transport model, but also the thermodynamic one. Indeed, CPG with Chapman-Enskog is still far from the more accurate solutions. The effect of the perturbations field model is minor compared to the mean flow one: the difference between the CNE curves computed by assuming the perturbations frozen or reacting is generally not appreciable, except at Mach 25, when it shows a small effect. Such differences were seen to continue increasing for successfully larger Mach numbers [21].

Figure 6 Growth rate (left) and phase speed (right), based on disturbance kinetic energy, for a Mach 25 flow in LTE conditions. The arrow indicates decreasing frequencies

It is evident that nonparallelism has an effect which depends on the re-entry trajectory flight envelope. It is slightly stabilizing at Mach 6 and it becomes more and more destabilizing as the Mach increases. This behavior has been already seen in Zanus et al. [57], for adiabatic and isothermal flat plates in CPG and LTE conditions. However, in that case, at variation of the single freestream Mach number, non-parallel effects were stabilizing or destabilizing depending on the flow assumption. The current study shows instead that, at fixed Mach conditions, non-parallel effects have the same influence regardless the flow model. In other words, N factor curves for different flow assumptions evolve relative to each other in the same manner within the parallel or non-parallel flow assumption.

For this particular problem, the maximum difference in terms of N values is around 0.3, obtained at Mach 25, at the end of the domain. Nonparallelism is here associated only to the boundary-layer natural growth. For this reason its effect is more visible, even at low Mach conditions, closer to the leading edge. Limited N factors are reached, except within the LTE flow hypothesis. Non-parallel effects do not have a strong destabilizing influence and the predicted transition-onset locations are not expected to significantly change. Nevertheless, it is important to note the stabilizing/destabilizing qualitative trend, since in problems with stronger non-local effects, such as for example in presence of highly curved surfaces, the differences in results computed within the parallel or non-parallel theories could be remarkable.

**IV. Conclusions**

In this work the influence on stability of weak non-parallel effects in hypersonic flows, with different flow assumptions was investigated. The VESTA LPSE solver for stability studies in presence of chemistry and high-temperature effects was successfully verified against the results from Kline et al. [31]. Despite stability computations being done on the exact same reference mean flow, some minor discrepancies occur in the growth rate and in the ascending curve of the N factors. Differences appeared in the regions where supersonic modes were present. This can be explained by the different stability numerical algorithms, and it is most probably due to the freestream boundary conditions for the
supersonic modes growth rate detection. Despite this, the CNE LPSE solver can be considered verified. VESTA's tools were used within the parallel and non-parallel flow theories to perform a parametric study on a 10-deg hypersonic wedge, at flow conditions corresponding to selected points along a planetary re-entry trajectory. For each of them, different aerothermodynamic and chemistry models were used, and the following conclusions were drawn. Comparison between VESTA results obtained by the LST solver (verified multiple times [14]) and the LPSE one increases the confidence in the correct derivation and implementation of the non-parallel solver for finite-rate chemically reacting boundary-layers.

Assuming a CPG flow, always leads to an underestimation of the N factor, even at the lowest Mach number. The excitation of the internal energy modes destabilizes the flow and leads to higher N factors, even in situations of minor base flow cooling. The dissociation of air’s molecular species further cools the boundary-layer, destabilizing the second-Mack-mode, and leading to higher growth rates and N factors. The correct modeling of the base-flow confirms to have a stronger impact on the N factor computation, than the perturbation field one.

Within the non-parallel theory different perturbations variables experience different growth rates, with discrepancies increasing towards lower re-entry altitudes. In all test cases the vertical velocity disturbance showed the highest growth rate, while temperature and kinetic energy reached similar values. Mass-flux was the only variable to be stabilized by non-local effects. Non-parallel effects influence the N factor depending on the re-entry flow conditions. They are slightly stabilizing at the lowest re-entry altitudes, and destabilizing at the other locations, with an effect increasing with the altitude. Non-local effects and gas-modeling showed no mutual interaction, with respect to the N factor computation.

Drastic N factor increments occur for the high Mach LTE flow cases. This was attributed to the presence of wide regions of unstable supersonic modes, confirming the fact that they can be promoted also in hot highly-reacting boundary-layers.

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References


A. 10-deg wedge flow: basic state profiles

Figure 7  Streamwise velocity, temperature and O concentration basic state profiles, for different flow assumptions and re-entry trajectory points: 10-deg wedge test case with 1400 K isothermal wall