Computational Design of Bistable Deployable Scissor Structures: Trends and Challenges

Liesbeth I.W. Arnouts1,2, Thierry J. Massart1, Niels De Temmerman2 and Péter Z. Berke1

1 BATir Department, Université libre de Bruxelles (ULB), Avenue Franklin Roosevelt 50, 1050 Brussels, Belgium. liarnouts@ulb.ac.be, thmassar@ulb.ac.be, pberke@ulb.ac.be
2 Department of Architectural Engineering, Vrije Universiteit Brussel (VUB), Pleinlaan 2, 1050 Brussels, Belgium. Niels.De.Temmerman@vub.be

Editor’s Note: This space reserved for the Editor to give such information as date of receipt of manuscript, date of receipt of revisions (if any), and date of acceptance of paper. In addition, a statement about possible written discussion is appended.

DOI: Digital Object Identifier to be provided by Editor when assigned upon publication

ABSTRACT

Mobile deployable scissor structures are transportable and can be transformed rapidly from a compact folded state offering a huge volume expansion. Intended geometrical incompatibilities during transformation can be introduced as a design strategy to obtain bistability, which allows instantaneously achieving some structural stability in the deployed state. In such bistable deployable structures, these incompatibilities result in the elastic bending of some specific members that are under compression with a controlled snap-through behaviour. Attempts to optimally design deployable bistable structures remain scarce, since the underlying structural-mechanical concepts are complex. Furthermore, the requirements of flexibility during deployment while ensuring some structural stability in the deployed state prevents the use of simple design methodologies relying on the structural behaviour under service loads only. In this contribution, the trends and challenges of using computational tools in the structural analysis and design process of deployable bistable structures are discussed. Computational tools are crucial for the geometrical and structural design, for the definition of a rigorous design methodology and for a deeper understanding of the complex transformation behaviour of these structures.

Keywords: deployable structures, transformable structures, scissor structures, bistability, snap-through, computational tools, parametric design, nonlinear computational mechanics, structural analysis, finite elements

1. INTRODUCTION

Nature itself is constantly evolving through transformations. Populations adapt through evolution and individuals transform during their lifetime to adapt to their environment. In contrast to biological systems, buildings are often designed to fulfill a single set of functions throughout their lifetime. By consequence, it is a challenge for them to adapt to changing circumstances. In our dynamic and rapidly evolving society there is a need for flexible spaces. Convertible, temporary and lightweight structures can adapt their shape or function to meet changing needs and circumstances on either the short or on the long term. They are sustainable because of their modular design, reusability and easy adaptivity [1]. The design of transformable structures is inherently a design for change, since it includes explicitly the concept of time and different stable structural states.

Transformable structures can be either kit-of-parts systems, having reversible connections allowing for disassembly, or deployable structures, having the ability to transform from a compact state to a deployed configuration [2]. This contribution focuses on the latter having the advantage of fast and easy transformation without requiring on-site (dis)assembly operations. Applications of deployable structures within the field of architecture and civil engineering include mobile or temporary structures for disaster relief, for construction and maintenance sites, for military operations or for recreational purposes. Examples can be found for bridges [3], towers [4] and shelters [5]. Alternatively, they form transformable extensions to static constructions, such as retractable roof structures [6] for halls or sports stadiums [7] or adaptable façades [8]. It is worth mentioning that deployable systems are also heavily used in the space industry for technologies such as deployable antennas [9], masts and satellites [10], however produced with very differ-
ent manufacturing tolerances and cost compared to civil engineering structures, and intended for use in a zero-gravity environment, with often a one-time deployment. Transformable structures also have everyday applications for smaller devices, such as umbrellas and foldable chairs, as medical devices [11] and in the toy industry [12].

The main common characteristics of all deployable structures is that they are both mechanisms and structural systems. As mechanisms, they transform between multiple predetermined configurations. As structural systems, they can bear loads in their stable configuration(s). Research on deployable structures is therefore located at the intersection of different disciplines. Besides the geometrical and kinematical aspects (the kinematic design of the transformation is usually based on rigid wireframe models), there are also the structural and mechanical aspects (taking the applied loads and structural compliance into account), which makes the design problem complex. Furthermore, because deployable structures are mechanisms, they require tolerances that are stricter than for usual civil engineering constructions [13]. In the past, most design approaches were heavily simplified for the ease of application. Today, the continuously improving computer technology and advanced numerical methods allow for dealing with highly complex problems [14], making the use of computational tools efficient and indispensable as part of the design process.

The aim of this contribution is to describe the trends and challenges in the use of computational tools as part of the design of deployable structures with a special focus on bistable structures. Also, an illustration is given of the use of computational tools in the design of a large bistable deployable dome.

2. DESIGN OF DEPLOYABLE STRUCTURES

2.1. Classification of deployable structures

Several authors such as A. Hanaor [15] and C. Gengnagel and N. Burford [16] attempted to classify deployable structures based on their morphological and kinematical properties, the interested reader can refer to G.E. Fenci and N.G.R. Currie [1]. In most cases the distinction is made between skeletal structures and continuous structures. Skeletal structures are e.g. pantographic structures, bars with articulated joints, ruled surfaces, reciprocal grids, tensegrity or bending-active structures. Continuous structures are for instance rigid or curved origami structures, pneumatic structures, tensioned membranes or sliding or retractable plates.

This contribution focuses on pantographic structures, also called scissor structures. They consist of scissor-like elements (SLE’s). An SLE is the assembly of two beams connected at a point along their length by a revolute joint (Fig. 1). This joint allows large rotations around its hinge axis being perpendicular to the common plane of the connected beams. By altering the position of this scissor hinge, three basic unit types are obtained: translational, polar and angulated units. For translational units (Fig. 1a), unit lines that connect the upper and lower nodes of one SLE remain parallel during deployment. For polar (Fig. 1b) and angulated units (Fig. 1c), they intersect at a single point. For angulated units, the angle between the unit lines is constant while for polar units, the angle varies during deployment. Units are linked to each other by the beam extremities. The assembly of different types of units in the modular deployable structure results in different unfolded shapes and transformation kinematics.

Scissor structures can be further classified on the basis of their structural behaviour in foldable, incompatible and bistable structures. Foldable structures are geometrically compatible throughout all stages of deployment, which means that their beams remain straight throughout the transformation process. They therefore behave like pure mechanisms (no internal stresses are generated during transformation). Once deployed, external manipulation is required (e.g. bracing) for them to bear (even small) loads. Incompatible structures are geometrically incompatible in the folded or deployed configuration, i.e. internal stresses are always present. These stresses are undesired especially in the deployed configuration because they make the structure more susceptible to buckling, but they also can be a desired feature in order to obtain

![Figure 1: A translational, polar and angulated unit.](image)
self-deploying structures. Finally, bistable structures are geometrically compatible in the folded and deployed configuration while they are incompatible throughout transformation (Fig. 2). The transformation of bistable structures generates internal stresses that increase and then drop to zero when going from one stable configuration to the other. It takes an extra effort to overcome this incompatibility, after which the structure snaps into the next compatible configuration. When structures are geometrically incompatible throughout the transformation phase, they are self-locking [17], since they will attempt to fix themselves into one of their stable configurations. Other examples of bistable structures are bistable wings [18], morphing structures [19], and tape springs [20].

The geometric incompatibilities in bistable scissor structures during deployment originate from angular distortions or peripheral incompatibilities [21]. Angular distortions occur when the angles between SLE planes at a central node do not add up to $2\pi$ during transformation (Fig. 2). Peripheral incompatibilities occur during transformation when the lengths of the beams at the perimeter are insufficient to encircle the grid that lies within (Fig. 3). During deployment, the surface area of a grid might increase relatively to the available length of the perimeter. The modules on the perimeter deform in order to cope with this relative decrease in perimeter length, leading to an incompatible grid for certain deployment stages. The size of the snap-through effect can be altered by changing the geometry or stiffness of the beams subject to buckling, by adding or removing members or by using flexible joints or telescopic beams [21]. The complex behaviour of bistable deployable structures during deployment makes their design complicated, which is a drawback that is balanced by the benefits of compact storage, transportability and easy (un)folding.

2.2. Trends in the design of scissor structures

The invention of concepts for movable and rapidly deployable structures dates back several millennia, with ancient applications in umbrellas, Mongolian yurts and the Egyptian hunter’s chair [22]. In medieval times, scissor linkages were encountered in Leonardo da Vinci’s drawings [23]. Throughout history, particularly the simple plane translational scissor linkage, called a lazy-tong mechanism, has been well known and applied.

In the early 1960’s, E.P. Piñero was the first to build scissor structures with more complex configurations and of larger scales [24]. He designed mobile theatres, pavilions and exhibition buildings. Since Piñero’s structures behaved as mechanisms, stability was achieved by adding cables. Later, F. Escrig presented the foldability conditions for scissor structures; demonstrated how structures could be obtained by placing pantographs in a grid; and showed how curvature could be introduced [25]. Together with J.P. Valcárcel [26] and J. Sánchez [27], he proposed several geometric models. Some of his grids exhibited a ‘snap-through’ effect due to geometric incompatibilities. According to Escrig, this was a problem because energy input is required for transformation. T.R. Zeigler was the first to theoretically identify the ‘snap-through’ phenomenon. He proposed a self-supported spherical dome which does not require additional members for stabilization [28]. The design aiming for a limited range of geometric shapes resulted in the existence of bent elements in the deployed configuration which make the structure more prone to buckling. R. Clarke further developed this idea and explained the origin of the geometric incompatibilities in Zeigler’s dome [29].

W.P. Zalewski and S. Krishnapillai improved Zeigler’s work and found flat and spherical polygonal modules that were geometrically compatible in the folded and deployed configuration [30], but bending some of the elements during transformation. Krishnapillai constructed simple models following a trial-and-error procedure, verifying the feasibility.
of his ideas. Based on this work, R.D. Logcher and Y. Rosenfeld carried out trial and error approaches with drawings and physical models [31]. In parallel, C.J. Gantes, together with J.J. Connor, developed a geometric design approach and studied the structural response of structures that consist of assembled polygonal modules, using finite element models [32]. Meanwhile, C.H. Hernandez used the scissor principle to construct cylindrical barrel vaults which behave as mechanisms [33]. Next to the employment of scissor systems in civil engineering applications, A. Kwan used scissor systems in space technologies and developed masts and satellite panels [10]. Active cables zig-zagged over small pulleys, located at the top and bottom nodes of his structures.

Using purely geometric approaches, A. Zanardo [34] and T. Langbecker [35] extended the foldability condition of Escrig to determine the foldability of scissor structures and to analyse their kinematics. Also based on the previous studies of Escrig, W. Chen et al. [36] and E. Gutierrez and J. Valcacer [37] developed scissor systems in order to obtain planar or dome-like structures. I. Raskin and J. Roorda focused numerically on the stiffness and stability characteristics of linear structures that behave as mechanisms [38].

A different type of scissor structures was developed and commercialized by C. Hoberman [12]. His designs use angulated units, i.e. units with ‘kinked’ beams, which allow forming closed-loop linkages. For structural analyses, Hoberman uses a ‘snapshot’ method, where the structure is analysed in a particular state of transformation. The various innovative sculptures and architectural projects of Hoberman Associates brought scissor grids to the attention of a broad public. S. Pellegrino investigated pantographic structures as deployable antennas, together with Z. You, and made further progress on the angulated unit by investigating multi-angled elements [39]. W. Gan and Pellegrino explained the geometry of structural mechanisms in analytical and numerical ways [40]. The concept of the multi-angled element was further developed by P.E. Kassabian by providing this type of structure with cover elements which provide, both in the open and closed positions, a weatherproof surface [41]. F.V. Jensen found that it is possible to remove the angulated elements and connect the plates directly to each other [42]. S.D. Guest and S. Pellegrino developed foldable plate cylinders which fold down to flat polygons [43].

More recently, Y. Akgiin and K. Korkmaz, amongst others, developed a new modified SLE [44]. With the use of actuators, these scissor elements can be adapted with changes of the global shape of the scissor system. The polygonal hyperboloids of M. Al Khayer and H. Lalvani [45] are other remarkable examples of innovative designs of deployable scissor structures. Other individual researchers are worth mentioning, among who J. Patel and G.K. Ananthasuresh [46], who have explained the geometry and kinematics of scissor systems based on angulated elements and plates, as well as A. Kaveh and A. Daravan [47], B.P. Nagaraj et al. [48] and J.-S. Zhao and J.-Y. Fengh [49], who made an in-depth geometric and kinematic analysis of planar and spatial scissor structures.

Japan has formed a major source of research on deployable scissor grids with contributions by e.g. K. Kawaguchi, K. Atake, T. Kokawa and M. Saito. One of the innovative proposals is the design of a scissor arch which uses zigzag cables with pulleys to (un)fold [50]. The research in Japan has led to a remarkable amount of prototypes, e.g. a deployable bistable geodesic full sphere [51]. In Belgium, N. De Temmerman proposed various architecturally and structurally viable concepts for mobile applications in architecture [52]. T. Van Mele, combined scissor structures with membrane structures to design covering systems [7]. L. Alegraia Mira developed methods for a structural assessment and optimization of scissor grids in conceptual design stages [5]. She developed the concept of the Universal Scissor Component, which can be reconfigured and reused to generate a wide variety of scissor grids [53]. A. Koumar designed a deployable scissor arch that enables quick sheltering solutions for areas struck by disaster [54]. K. Roovers enlarged the spectrum and application possibilities of scissor structures by revealing the mathematical principles required to convert surface geometries into scissor systems [21].

2.3. Computational tools in the design of scissor structures

Developing and analysing scissor structures is a contemporary subject in engineering research. Often an experimental approach is used on reduced scale models in which, in the case of bistable scissor structures, researchers try to combine an easy deployment process and adequate stiffness in the deployed state. This is achieved by starting from an initial guess and iteratively fine-tuning it to get a
good balance between having a self-locking effect for stability with a limited force needed for deployment. Trial and error experiments on reduced scale models [21,31,51] give a valuable insight in the structural behaviour, but they are inefficient as a design approach for large civil engineering structures.

Computational tools can contribute significantly to a deeper understanding of the complex transformation behaviour of bistable deployable structures and to the development of a rigorous design methodology [17]. Designing the complete transformation behaviour in terms of kinematics and applied forces is a prerequisite for conceiving practical applications, which are currently rare in civil engineering. The design of bistable deployable structures relies on different computational tools: computer-aided design and nonlinear structural analysis.

Computer-aided drawing and modelling can be used to produce drawings and renderings. Moreover, it simplifies the complicated geometric design process and it allows finding the geometry parametrically. The geometric design approach for scissor structures is based on the requirement for geometric compatibility in the deployed as well as in the folded configuration, which means that the beams are located theoretically on one line in the folded configuration. This can be translated into the deployability constraint [25]:

\[ a + b = c + d \]  

(1)

Figure 4: Deployability constraint.

This constraint (Fig. 4) ensures that all units in the assembly simultaneously reach their most compact state. Using this constraint, together with other geometric constraints [17,21,52], a rigid wireframe model (i.e. neglecting the compliance of the structural elements) can be obtained that is compatible in the folded and deployed configuration. Verifying mathematical expressions indicates if a given assembly is compatible. It is however impractical as a tool to search for new design possibilities or to study a variety of shapes, due to the amount of parameters and equations that arise using irregular units or when creating larger assemblies. A more efficient and intuitive design approach can be provided by integrating graphical design methods in digital design environments in the form of parametric models [21]. The locus of all valid intermediate hinges that comply with the deployability constraint is an ellipse, with the common end nodes of both units as its focal points [52]. This graphic representation of the deployability constraint makes it possible to draw the geometry of scissor structures using ellipses or circles, depending on the desired shape (Fig. 7).

Computational structural analysis (linear for the deployed state and non-linear for the transformation) allows assessing the structural performance of a given design. Since scissor structures usually have a complicated 3D geometry and are subject to large geometric and structural transformations, no efficient analytical alternative exists to numerical modelling. Finite element models can be used to do a linear analysis of scissor structures in the deployed state subjected to service loads (e.g. wind, snow) [5,54]. A nonlinear analysis of bistable or incompatible scissor structures is required during deployment, subjected to large displacements and geometric incompatibilities [32]. Although some of the researchers in the past used the finite element method to perform structural analyses in the deployed state, modelling the complete transformation cycle of scissor structures is very scarce, since the underlying phenomena and modelling tools and concepts are complex. Nevertheless, simulation of the deployment process is a crucial part of the analysis, requiring sophisticated computational modelling techniques.

2.4. Challenges in the design of bistable deployable structures

Analytical or numerical work that investigates the behaviour of bistable scissor structures is inherently complex. Often a trial and error approach is used which is laborious and does not allow to fully understand their behaviour [21,31,51]. Additionally, such approaches are not a general design methodology and limit the number of design alternatives. Other approaches were based on the modelling of structures without discrete joints or imperfections, based on a large number of simplifications.

The difference in the behaviour of bistable scissor structures during deployment and in the deployed configuration prevents the formulation of a straightforward, simple design methodology (Fig. 5). To obtain feasible designs, a bistable deployable struc-
ture should be lightweight, flexible during deployment and should provide sufficient stiffness in the deployed state. These desired requirements are often contradicting, and the challenge is thus to find an acceptable compromise in the computational design. A low mass and a compact structure in the folded state are important for transportation. Flexibility of specific members during deployment is important to ensure a limited force needed for deployment and additionally an elastic material behaviour must be ensured. Not taking the transformation stage into account could lead to inefficient, expensive and potentially unfeasible designs. Sufficient stiffness in the deployed state is needed to bear self-weight and possibly service loads without needing external bracing.

![Figure 5: Conflicting requirement during transformation and in the deployed configuration, as presented by C.J. Gantes [55]. The magnitude of the geometric incompatibilities is given in function of the utilisation ratio, which is the actual performance value divided by the maximum allowable performance value.](image)

Optimisation methods have been developed for structures with contradicting objectives, subject to several constraints. These methods can solve a wide range of problems, but their drawback is that each iteration requires a structural analysis with a considerable computational effort. A number of remedies have been suggested by Gantes to reduce the number of required exact analyses to optimise bistable scissor structures [17]: breaking down the problem into a number of smaller sub-problems, using approximations and simplified models. In the above design approach however, there are several iterations which have to be performed manually, making it difficult to automatize.

Because of the progress in computational efficiency over the recent years, several researchers in the field of pantographic structures did an effort to use optimisation methods to solve a variety of problems. Foldable scissor structures were optimised by Z. You to minimise the weight or maximise the stiffness [56]. An initial effort optimising bistable scissor structures was done by C. Gantes [57]. Genetic algorithms were used to minimise the weight by solving a non-linear problem that simulates the deployment and a linear analysis. A. Kaveh minimised the weight of scissors structures which behave as mechanisms [58]. For movable bridges composed of linkages, A.P. Thrall employed shape and size optimisation to minimise the self-weight and the power required for operation. She also used size and shape optimisation for pantographic structures to minimise the weight [59]. L. Alegria Mira did a topology, shape and size optimisation of the Universal Scissor Component to minimise its weight [53]. A. Koumar performed for the first time a multi-objective optimisation of a barrel vault, leading to a pareto front with several optimal solutions. The mass and the compactness were minimised [54].

Structural optimisation can guide the designer in choosing a feasible design or can suggest improvements, without requiring a trial and error approach. One of the main challenges in the field of bistable deployable structures today is to efficiently perform a multi-objective size and shape optimisation to obtain lightweight bistable scissor structures that provide stiffness in the deployed state and flexibility during deployment by choosing the best combinations of the geometry, the cross-sections of the beams and the material properties.

3. EXAMPLE OF THE USE OF COMPUTATIONAL TOOLS IN THE DESIGN OF A BISTABLE SCISSOR STRUCTURE

To illustrate how computational tools are used in the particular case of bistable deployable structures, a large structural example is presented in the following paragraphs. A bistable deployable dome with a radius of 5 m is designed which consists of a triangulated double layer grid of polar units with concurrent unit lines (final design shown on Fig. 12). The structure is bistable due to angular incompatibilities. By consequence, the structure is a self-locking doubly curved dome with a triangulated grid that can be reduced to a compact bundle of bars.
This structure could potentially be used as a mobile and temporary structure for disaster relief, military operations or for exhibition or recreational purposes.

The complete design process is given in Fig. 6. The first part is the geometric design, in which the rigid wireframe design, i.e. the beam lengths and the position of the hinges, is defined. The second part is the structural design, in which the bistable scissor structure is structurally analysed during transformation and in the service state using non-linear finite element modelling.

3.1. Geometry and kinematics

Digital parametric design environments present a powerful tool to handle the high degree of complexity of the design process of scissor structures in an interactive and efficient manner. General design methods for the rigid wireframe design of certain types of scissor units can be translated into algorithms, which translate user-defined input parameters into a deployable scissor grid in a step-wise and interactive manner. It can provide direct visual and numerical feedback on the solution throughout the process. By changing the input parameters, a myriad of solutions can quickly be generated and explored. The various alternatives can efficiently be evaluated and compared, leading to better informed designs. These tools can also be extended e.g. to simulate the deployment or to perform linear elastic analyses [5].

The computational tool used in this contribution is the 3D modelling software Rhinoceros® and its parametric design plug-in Grasshopper® [60]. Grasshopper’s visual interface, relying on a visual programming approach, knows various plug-ins such as the live-physics plug-in Kangaroo which can be used for form finding, for the optimisation of meshes and scissor grids, and for deployment simulations that are not based on finite elements [61]. By using Kangaroo, the transformation of scissor structures can be approximated by allowing rotations between the beams, however the structural stiffness and the deformability of the elements is not properly considered at this stage. Rigorous finite element modelling is required to capture the correct transformation behaviour.

In the geometric design, two stable states are considered i.e. the folded and the deployed state. The movement between those two configurations (the kinematics) is of course implicitly present and is properly incorporated only in the non-linear finite element analysis in the structural design stage.

For three-dimensional scissor grids, the deployability constraint is translated into networks of tangent rotational ellipsoids placed along a surface. However, in some specific cases, such as for flat structures or the designed dome, this graphical representation can be simplified to a circle packing (Fig. 8a).
which is a configuration of circles with specified patterns of tangency [21]. Each pair of tangent circles \((c_i; c_j)\) holds an SLE with unit lines normal to the circles and going through the respective centres \((C_i; C_j)\) of the circles. The intermediate hinge (revolute joint) \(I_i\) of the SLE is located at the point where the circles touch (Fig. 8b). In the case of this dome, all of the intermediate hinge points will lie on the surface of the sphere and all of the unit lines intersect at the centre \(O\) of the sphere.

![Diagram](image)

**Figure 8:** Populating a spherical circle packing with polar scissor units, based on the work of K. Roovers [21].

For a given spherical circle packing for a bistable deployable dome, there is one free parameter to choose, which is the angle \(\varphi\) between the two beams of an SLE in the deployed configuration (Fig. 8b). The design satisfies the deployability constraint (Eq. 1) because this angle is the same for all of the SLE’s in the structure, as the intermediate hinges are located at the points where the circles touch and as the unit lines go through the centres of the circles as well as through the centre of the sphere. Changing the angle \(\varphi\) changes outer and inner dimensions of the dome (Fig. 9). Varying this parameter also changes the structural behaviour of the bistable deployable dome in the deployed configuration subject to service loads as well as during deployment. In general, the larger this angle, the higher the stiffness of the dome in the deployed state and the higher the force required for deployment.

![Diagram](image)

**Figure 9:** Bistable scissor grids based on the same circle packing with a different angle between their beams in the deployed configuration.

Spheres can be covered by an infinite variety of circle packings and therefore give rise to several scissor grids (Fig. 10). First, an initial grid (i.e. a network of lines) is designed on the spherical surface, which is optimised to hold a circle packing and then populated by scissor units (Fig. 7). The design freedom therefore largely lies in shaping the initial grid. It can result from e.g. an arbitrary point cloud, projecting a planar grid onto a sphere or a geodesic pattern. For this example of a bistable deployable dome, the scissor grid is based on a geodesic pattern (Fig. 7).

![Diagram](image)

**Figure 10:** Bistable scissor grids based on different initial grids.

In general, the denser the circle packing (i.e. the shorter the beams of the scissor structure), the higher are the stiffness of the dome in the deployed state and the force required for deployment. Increasing the density of a circle packing does not change the global motion because the same geometry is enveloped when the structure is deployed. Nevertheless, the number of elements and joints are increased by using a denser circle packing, which makes the structure locally more complex. Moreover, since finite elements are used to analyse the structure during transformation, the number of the degree of freedom of the system increases when the number...
of joints and elements increases. The parameters that can be fine-tuned to obtain a feasible design with a given external shape and dimensions of the structure, are the angle $\phi$ between the two beams of an SLE in the deployed configuration (Fig. 9) and the initial grid to hold a circle packing (Fig. 10).

In the previous line models, the joints were represented by points. Joints connect several SLE's at their end points in a three-dimensional configuration and are of finite size. To represent realistic hubs, joint lines are added to the line model that create space between the different rotation axes of the beams. A relatively simple method is chosen to include in the parametric design algorithm and that maintains the concurrency of the unit lines throughout transformation i.e. the unit lines intersect in a single point during the whole transformation phase [21]. One parameter can be chosen, which is the angle $\lambda$ (Fig. 11). A disadvantage of this method is the resulting amount of different hubs that is needed to build the structure because the lower hubs are smaller than the upper hubs.

![Figure 11: Joints in the line model such that the unit lines and the central axes of the joints are concurrent. Based on the work of K. Roovers [21].](image)

The design of the proposed closed dome is realistic for a roof type of application. For a temporary tent-like structure however, as aimed for here, openings have to be created. The desirable amount of SLE's on the bottom of the dome can be removed to open space for entrance on the dome surface. In this example, five SLE's were removed, creating openings with a height of 2.5 m. The top view, side view and 3D view of the resulting structure is shown in Fig. 12. The dome has an outer diameter of 12.85 m and a total height of 6.05 m. The inner diameter of the dome is 9.6 m and the inner height is 4.9 m.

![Figure 12: Final geometric design of the dome with hinges and openings.](image)

The beams of the structure are made of aluminium with a Young’s modulus of 70 GPa, a poisson’s ratio of 0.35, a density of 2700 kg/m³ and a yield strength of 160 MPa. Hollow rectangular aluminium beam cross-sections of 25x50x5 mm were chosen in the design procedure, which corresponds to realistic beam cross-sections for scissor structures [5,17,52]. The mass of the structure is 360.5 kg.

### 3.2. Structural analysis

The same software used for the kinematic design (Rhinoceros) has a plugin Karamba3D that can be applied for linear structural analysis using the finite element method [62]. The advantage of doing the geometric/kinematic and preliminary linear structural design in the same software is that there is immediately some structural feedback in the early design stages [5]. As it is still impossible to analyse snap-through problems in Rhinoceros and Grasshopper, another finite element software is needed. In this example, Abaqus is used [63] for the detailed analysis of the non-linear transformation phase as well as for the linear analysis of the deployed configuration under service loads. Rhinoceros and Grasshopper were used to design the structure geometrically, to create the input file for the finite element model in Abaqus, to run the simulation in Abaqus and to read the results of the analysis. The
coupling of Rhinoceros/Grasshopper with Abaqus is done by using in house written Python scripts.

For all of the structural members, 2-noded geometrically nonlinear Timoshenko beam finite elements in Lagrangian formulation are used. Four beam finite elements are used to model the semi-length of each beam, which was verified to be a converged.

The joints, which allow the beams to rotate around a rotation axis, are simulated with the Abaqus connector type ‘hinge’. The rotation axis of the joints between the two beams in an SLE is defined perpendicular to the common plane of the two connected beams and is updated during structural transformation. This hinge axis co-rotates with the beams and remains perpendicular to the common plane of both beams during transformation. 3D hubs, which connect several beams of different SLE’s, are modelled here as a stiff grid composed of short beam elements (Fig. 13) with the Abaqus connector type ‘hinge’ at their extremities.

The lower corner points of the structure are fixed in the vertical direction as if the structure was standing on a surface. To prevent rigid body modes, the lower centre point is allowed to move only in the vertical direction while rotations around the vertical axis are prohibited. In the deployed configuration, the only applied force is gravity. This corresponds to a bistable scissor structure just after deployment, before the structure is fixed to the ground. In real life applications wind-bracing could be added before the structure is subjected to other service loads such as wind or snow. Therefore, these are disregarded in this simplified simulation.

To obtain a feasible structure in the deployed configuration, the following constraints are considered:

1. The maximum stress is less than the yield stress of the material.

2. The maximum deflection is \( L/100 \) and the maximum horizontal displacement at any point of the structure is \( H/100 \) with \( L \) the length and \( H \) the height of the structure. This constraint is less strict than for traditional structures [54].

3. An analytical verification is implemented for the local buckling of each member by taking into account the normal force and the bending moments around the two axes following Eurocode 9, based on [54].

4. The calculation of the global buckling is based on an eigenvalue-based buckling analysis of the model.

The results from the linear analysis of the bistable scissor structure in the deployed configuration show that, as expected, the maximum displacement is the vertical displacement in the lower centre node, which is 3.6 cm (Fig. 14). This is lower than \( H/100 \) since the height of the structure is 6.05 m. The maximum von Mises stress in the beams is 13.7 MPa (Fig. 15), which is lower than the yield strength of Aluminium and local buckling has been checked for all of the structural members.

Figure 13: Dome with finite joint dimensions (left) and detail of the stiff grid of beams that represents the hubs (right). The rotation axes of the hinges are given by the small lines perpendicular to the beams.

Figure 14: The displacement of the beams on the deformed configuration of the dome subjected to gravity.

Figure 15: The von Mises stress in the beams of the dome subjected to gravity.
Buckling load magnitudes and associated buckling modes are calculated for global buckling of the structure. The obtained buckling load factor is the number by which the applied load must be multiplied to obtain the buckling load magnitude and the corresponding buckling mode presents the shape of the structure when it buckles. The first buckling mode (Fig. 16) which has the smallest buckling load magnitude has a buckling load magnitude of 8.84, which means that the structure will buckle when the magnitude of the applied loads is the magnitude of the gravity loads multiplied by 8.84.

The results of the analysis in the deployed configuration show that the structure will be a feasible structure just after deployment since it bears gravity loads without needing further bracing. To be able to use the dome in the deployed configuration as a structure, additional computations should be performed in which realistic fixations to the ground, wind-bracing and a cover (e.g. a membrane attached to the structure) are be considered. Doing so is technically straightforward, requiring the modification of the FE input files only, however it relies on making complex design choices (e.g. membrane attachment) and is therefore out of the scope of this contribution.

To simulate the transformation phase, the deployed configuration is used as the initial configuration, because in the geometrically ideal folded configuration all nodes would lie on a straight line and because scissor structures are designed geometrically in the deployed configuration. The structure is folded to its most compact configuration in force driven simulations using the modified Riks solution strategy to capture the snap-through. Around 60 loading steps are used to compute the load magnitude and its nonlinear evolution during transformation. The finite element model has 1500 elements, 4732 nodes and 10752 variables. The computational time of a single folding simulation is around 1 minute.

The lower corner points of the structure are fixed in the vertical direction. Several sets of forces can be used to transform scissor structures. In this example, horizontal radial forces are applied to the upper corner points of the structure and an additional vertical force is needed on the lower centre point of the structure (Fig 17). The horizontal forces have the same magnitude. The vertical force is 200 times larger than the horizontal forces to be able to fold the structure. Gravity is also taken into account during deployment.
The required total folding force (i.e. the sum of the magnitudes of the forces) is given as a function of the displacement in Fig. 18a and the structural configurations corresponding to points A, B, C, D and E are shown in Fig. 18b. Point A corresponds to the initial deployed configuration with gravity applied. Due to the geometric incompatibilities, some of the beams bend, requiring an increasing force to fold the structure (A-B). The maximum folding load is reached at point B, followed by a snap-through from B to D. In point C is an unstable configuration in which no externally applied force is required. The required load to keep on folding becomes negative (i.e. changes direction) from C to E. This implies that without this restraining force, the structure would ‘snap’ to point E in a dynamic fashion. Between points D and E, the magnitude of the vertical restraining force decreases because the bent beams become straight again and internal stresses relax. Point E can be considered as the final folded configuration i.e. a transformed configuration in equilibrium under gravity load.

To obtain a feasible mechanism during deployment, the following constraints are considered:

1. The maximum stress during deployment is lower than the yield stress of the material.

2. The maximum negative load during the folding process should be lower than zero, so the structure does not unfold under gravity from the folded configuration.

\[ \text{Maximum stress during deployment} = \text{Less than yield stress} \]

\[ \text{Maximum negative load} = \text{Less than zero} \]

![Figure 19: Maximum von Mises stress as a function of the displacement of the upper center node.](image)

The results of the analysis during transformation show that the structure is realizable. The maximum von Mises stress during transformation is 139 MPa. (Fig. 19), which is below the yield strength of Aluminium (depending on the alloy). Figure 19 shows graphically how internal stresses build up and relax during transformation. In this case, the deployment is more critical than the deployed configuration in terms of stress levels reached. The maximum negative load during the folding process (point D in Fig. 18a) is well below zero.

For this example, the mass of the structure is 360.5 kg and the required folding force is 1.52 kN. This means that the transformation cannot be controlled by human power but that machines such as cranes or a set of cables connected to winches will be needed for transportation and transformation. To decrease the mass and the required folding force, the structure can be optimised by changing the geometry, the material or the cross-sections of the beams or by changing the way the loads are applied e.g. by using a cable running over pulleys to transform the structure. Another option is to use two groups of elements with different cross-sections and material properties e.g. aluminium for beams that do not bend during transformation and a material that can easily bend elastically for the beams that bend during transformation. Optimising a bistable scissor structure requires several iterations with the geometry, cross-sections, materials and applied loads. Using computational optimisation algorithms would be an ideal solution for the design of bistable scissor structures in terms of mass, stiffness in the deployed state and flexibility during transformation and it is part of future work.

4. CONCLUSIONS AND OUTLOOK

The aim of this contribution is to investigate trends and challenges in the use of computational tools for the design of bistable deployable structures.

In the past most design aspects had to be heavily simplified in order to be understood and used. Today, the continuously improving computer technology and advanced numerical methods allow for dealing with highly complex problems, making the use of computational tools crucial and efficient as part of the design process. The structural assessment of bistable deployable structures relies on different computational tools: computer-aided design and nonlinear structural analysis. An example was given of the use of such computational tools for the design of a large, real life scale bistable deployable dome. Parametric models were used to geometrically design a dome, which was afterwards structurally analysed in the deployed configuration as well as during transformation using finite element models. The importance to take the nonlinear structural response during transformation into account makes the design of bistable structures a non-trivial problem.
Generally, as for the example of the bistable deployable dome, the geometry, kinematics and structural analysis are developed and investigated as separate entities with different tools or software packages. However, the desired requirements to obtain a lightweight structure which is flexible during deployment while offering sufficient stiffness in the deployed state are often contradicting and the challenge is to find an acceptable compromise between the geometry, cross-sections and materials of the beams. To do this, the geometric design and structural analysis in the deployed state as well as during transformation have to be considered. As future development, an optimisation approach to such structures is envisioned which links the geometry to structural and transformation analyses.

Next to the use of computational tools in the design of bistable scissor structures, a realizable design should include a geometric design and a cover adapted to its function, detailing of the beams, connections and hubs, and the use of physical scale models and prototypes.

ACKNOWLEDGMENTS

This work was supported by a Research Fellow (ASP – Aspirant) fellowship of the Fund for Scientific Research – FNRS (F.R.S.-FNRS) (grant number FC 23469).

REFERENCES


A. Zana rdo, "Two-dimensional articulated systems developable on a single or double curvature surface," *Meccanica*, vol. 21, no. 2, pp. 106-111, 1986. DOI: 10.1007/BF01560628


