

Contents

1 Symmetric spaces	1
1.1 General features of symmetric spaces	2
1.2 Rank of symmetric Lie algebras	6
1.3 Pseudo-Riemannian symmetric spaces	7
1.3.1 Pseudo-Riemannian symmetric spaces with constant sectional curvature	11
1.3.2 Semisimple pseudo-Riemannian symmetric spaces	17
1.4 Classification of indecomposable Riemannian symmetric spaces	20
1.4.1 Isotropic Riemannian symmetric spaces	23
1.5 Classification of indecomposable Lorentzian symmetric spaces	24
1.5.1 Cahen-Wallach spaces	27
1.6 Symplectic symmetric spaces with Ricci-type curvature	29
1.6.1 Classification of Ricci-type symplectic symmetric spaces	32
1.7 Totally geodesic submanifolds	45
1.7.1 Totally geodesic submanifolds in pseudo-Riemannian symmetric spaces	47
1.7.2 Totally geodesic submanifolds in symplectic symmetric spaces	48
2 Integral geometry on Riemannian symmetric spaces	53
2.1 Framework of homogeneous spaces in duality	55
2.2 Radon transforms on Riemannian homogeneous spaces	58
2.2.1 Main dual Radon transform	58
2.2.2 Shifted dual Radon transform	61
2.3 Totally geodesic Radon transforms on isotropic Riemannian symmetric spaces	61
2.3.1 Spherical integrals	64
2.3.2 Inversion formulas on Riemannian symmetric spaces with constant sectional curvature	68
2.3.3 Inversion formulas on isotropic Riemannian symmetric spaces of compact type	77
2.3.4 Inversion formulas on isotropic Riemannian symmetric spaces of non-compact type	80
2.4 Spherical transform and Fourier transform	83
2.4.1 Spherical functions and spherical transform	84
2.4.2 Fourier transform on Riemannian symmetric spaces of non-compact type	86

3 Integral geometry on Lorentzian symmetric spaces	89
3.1 Pseudo-spherical integrals on the pseudo-Euclidean vector space	90
3.2 Orbital integrals on semisimple pseudo-Riemannian symmetric spaces	91
3.3 Orbital integrals on Cahen-Wallach spaces	93
3.4 Limit formulas on even-dimensional Lorentzian symmetric spaces with constant sectional curvature	95
3.5 Limit formulas on rank-one semisimple pseudo-Riemannian symmetric spaces	101
3.6 Limit formulas on indecomposable Lorentzian symmetric spaces	110
3.6.1 Limit formulas on Cahen-Wallach spaces	111
3.6.2 Conclusion in the Lorentzian case	116
4 Integral geometry on symplectic symmetric spaces	117
4.1 Orbital integrals on Ricci-type symplectic symmetric spaces	118
4.1.1 Limit formulas on Ricci-type symplectic symmetric spaces	121
4.2 Double fibrations in a symplectic framework	124
4.2.1 Orbits of connected symplectic totally geodesic submanifolds	125
4.2.2 Orbits of connected Lagrangian totally geodesic submanifolds	130
4.3 Symplectic totally geodesic Radon transforms on $\mathbb{P}^n(\mathbb{C})$ and $\mathbb{H}^n(\mathbb{C})$	135
4.3.1 Inversion formulas on complex projective spaces	136
4.3.2 Inversion formulas on complex hyperbolic spaces	139
Conclusion and prospects	143
A Invariant differential operators	149
A.1 Invariant differential operators on reductive homogeneous spaces	149
A.2 Invariant differential operators on pseudo-Riemannian symmetric spaces . .	150
A.2.1 On quasi-isotropic pseudo-Riemannian symmetric spaces	152
A.2.2 On Cahen-Wallach spaces	159
B Riemann-Liouville integrals and Riesz potentials	165
B.1 Riemann-Liouville integrals	165
B.2 Riesz potentials	166
Bibliography	169