Towards a Unified Description of Magnetar Crusts

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Abstract. The properties of the crust of a neutron star can be significantly altered by the presence of a high magnetic field. The effects of Rabi quantization of electron motion on the equation of state and on the composition of the crust are studied. Both the outer and inner regions are described in a unified and consistent way by extending the neutron-star crust models developed by the Brussels-Montreal collaboration. The first numerical results obtained for different magnetic field strengths are presented.

INTRODUCTION

Formed from the catastrophic gravitational-core collapse of massive stars during supernova explosions, neutron stars can be endowed with extremely high magnetic fields [1]. In particular, surface magnetic fields up to a few times 10^{15} G have been measured in soft-gamma ray repeaters (SGRs) and anomalous x-ray pulsars (AXPs) [2, 3, 4]. Astrophysical observations provide evidence for the existence of even higher magnetic fields in the interior of these stars [5, 6, 7, 8]. Numerical simulations have confirmed that the internal magnetic field could reach $\sim 10^{18}$ G [9, 10].

We have previously shown that the composition and the equation of state of the outer crust of a neutron star can be drastically modified by such high magnetic fields due to Rabi quantization of electron motion [11, 12, 13, 14]. In this paper, we present new results using updated experimental atomic mass data. We also extend our investigations of highly-magnetised matter to the inner crust, where neutron-proton clusters coexist with free neutrons.

EFFECTS OF THE MAGNETIC FIELD ON MAGNETAR CRUST PROPERTIES

Outer crust

We determine the properties of the outer crust of a magnetar using a model that we have previously presented in detail in Ref. [11]. This model assumes that atoms, arranged in a perfect body-centered cubic lattice, are fully ionized by the pressure, and are embedded in an homogeneous electron gas at zero temperature. It is well-known that the electron motion perpendicular to the magnetic field is quantized into Landau orbitals. If the magnetic field strength exceeds the characteristic value

$$B_{\rm rel} = \frac{m_e^2 c^3}{e\hbar} \simeq 4.41 \times 10^{13}\,\mathrm{G}\,, \tag{1}$$

as encountered in SGRs and AXPs, the electron motion becomes relativistic. The energy levels of a relativistic electron gas in a magnetic field were first calculated by Rabi [15]. Expressions for the electron pressure and energy density can be found in Ref. [11]. The equilibrium properties of any crustal layer at pressure P and magnetic field strength $B_\star \equiv B/B_{\rm rel}$ are determined by minimizing the Gibbs free energy per nucleon g (see, e.g., Appendix of Ref. [16]). We consider only pure layers, i.e. layers containing only one single species of nuclei (A, Z) with mass number A and charge number Z.

TABLE 1. Sequence of equilibrium nuclides with increasing depth in the outer crust of a magnetar with a magnetic field strength $B_\star = 2000$ using two differents version of the the Atomic Mass Evaluation (AME) supplemented with the HFB-24 theoretical mass table. Nuclides with experimentally measured masses are indicated in boldface.

2012 AME	2016 AME
$^{56}\mathbf{Fe}$	$^{56}\mathbf{Fe}$
$^{62}\mathbf{Ni}$	$^{62}\mathbf{Ni}$
$^{88}\mathbf{Sr}$	$^{88}\mathbf{Sr}$
$^{86}\mathbf{Kr}$	$^{86}\mathbf{Kr}$
$^{84}\mathbf{Se}$	$^{84}\mathbf{Se}$
$^{82}\mathbf{Ge}$	$^{82}\mathbf{Ge}$
$^{132}\mathbf{Sn}$	$^{132}\mathbf{Sn}$
$^{80}\mathbf{Zn}$	$^{80}\mathbf{Zn}$
$^{130}\mathbf{Cd}$	–
^{128}Pd	^{128}Pd
^{126}Ru	^{126}Ru
^{124}Mo	^{124}Mo
^{122}Zr	^{122}Zr
^{121}Y	–
–	^{124}Zr
^{120}Sr	^{120}Sr
^{122}Sr	^{122}Sr
^{124}Sr	^{124}Sr

We have determined the properties of the outer crust of strongly magnetized neutron stars with $B_\star = 2000$ making use of the experimental atomic mass data from the 2016 Atomic Mass Evaluation (AME) [17, 18] supplemented by recent mass measurements of copper isotopes [19]. For the masses that have not been measured, we have implemented the theoretical nuclear mass table HFB-24 from the BRUSLIB database[1]. These masses were obtained from self-consistent deformed Hartree-Fock-Bogoliubov calculations using the generalized Skyrme functional BSk24 [20]. This microscopic model was fitted to the 2353 measured masses of nuclei with N and $Z \geq 8$ from the 2012 AME [21], with a root-mean-square deviation of 0.549 MeV. This model provides an equally good fit to the 2408 measured masses of nuclei with N and $Z \geq 8$ from the 2016 AME. In Table 1, we compare these new results against those we obtained with the 2012 AME data [21] and which were published in Ref. [12]. We now find that ^{130}Cd is no longer present in the crust, and ^{121}Y is replaced by ^{124}Zr.

Inner crust

At high enough pressure, neutrons drip out of the nuclei ^{124}Sr marking the transition to the inner crust. Because free neutrons are in equilibrium with those bound in nuclei, the inner crust matter should be treated consistently. We have implemented the effects of Rabi quantization in the computer code developed by the Brussels-Montreal collaboration [22, 23]. This code is based on the fourth-order extended Thomas-Fermi method with proton shell corrections

[1] http://www.astro.ulb.ac.be/bruslib/

added perturbatively using the Strutinsky integral theorem. This ETFSI method is a computationally very fast approximation to the fully self-consistent Hartree-Fock plus Bardeen-Cooper-Schrieffer equations. Nuclear clusters are supposed to be unaffected by the presence of the magnetic field, and are further assumed to be spherical. The Coulomb lattice is described following the approach of Wigner and Seitz. Nucleon density distributions in the Wigner-Seitz cell are parameterised as

$$n_q(r) = n_{B,q} + n_{\Lambda,q}\left\{1 + \exp\left[\left(\frac{C_q - R_c}{r - R_c}\right)^2 - 1\right]\exp\left(\frac{r - C_q}{a_q}\right)\right\}^{-1} \tag{2}$$

where $q = n, p$ for neutrons or protons respectively, while $n_{B,q}$, $n_{\Lambda,q}$, C_q, a_q, and R_c are geometrical parameters of the Wigner-Seitz cell. The equation of state of nuclear clusters and free neutrons is calculated from the same nuclear energy density functional BSk24 [20], as that underlying the nuclear mass model HFB-24 used in the outer crust. This functional was not only fitted to nuclear masses but was also constrained to reproduce the microscopic neutron-matter equation of state labelled 'V18' in [24], as obtained from the Brueckner Hartree-Fock approach using realistic two- and three-body forces. We thus believe that this functional is well suited for describing the neutron-rich matter of the inner crust of a neutron star. The analytical approximations implemented in the routines developed by Potekhin and Chabrier [25] were adopted to calculate the equation of state of the cold magnetized electron gas.

TABLE 2. Comparison between the outer- and inner-crust codes at the neutron-drip point; results for the latter code are indicated in parentheses. $B_\star$ is the magnetic field strength, $\bar{n}_{\rm drip}$ and $P_{\rm drip}$ are mean baryon number density and the pressure at the neutron drip-point respectively, Z and N are the proton and neutron numbers of the equilibrium nucleus.

$B_\star$	$\bar{n}_{drip}$ [fm^{-3}]	Z	N	$P_{\rm drip}$ [MeV fm^{-3}]
1	2.56×10^{-4}	38 (40)	86 (94)	5.14×10^{-4} (4.96×10^{-4})
10	2.56×10^{-4}	38 (40)	86 (94)	5.13×10^{-4} (4.96×10^{-4})
100	2.57×10^{-4}	38 (40)	86 (94)	5.14×10^{-4} (4.96×10^{-4})
1000	2.91×10^{-4}	38 (41)	86 (96)	6.87×10^{-4} (6.64×10^{-4})

TABLE 3. Inner crust properties at different mean baryon number densities $\bar{n}$, as obtained with the two versions of our inner crust code: the one for ordinary neutron stars and the other for magnetars (in parenthesis). For comparison, the magnetar code was run with the small magnetic field value $B_\star = 1$. Z is the charge number of clusters, P is the pressure, and e is the energy per nucleon.

$\bar{n}$ [fm^{-3}]	Z	P [MeV fm^{-3}]	e [MeV]
3.038×10^{-4}	40 (40)	5.467×10^{-4} (5.472×10^{-4})	-1.5240×10^{0} (-1.5239×10^{0})
5.474×10^{-4}	40 (40)	7.477×10^{-4} (7.495×10^{-4})	-6.5492×10^{-1} (-6.5489×10^{-1})
1.777×10^{-3}	40 (40)	2.154×10^{-3} (2.152×10^{-3})	7.2042×10^{-1} (7.2038×10^{-1})
5.772×10^{-3}	40 (40)	9.605×10^{-3} (9.603×10^{-3})	2.3285×10^{0} (2.3285×10^{0})
1.874×10^{-2}	40 (40)	4.340×10^{-2} (4.340×10^{-2})	4.6393×10^{0} (4.6394×10^{0})
6.087×10^{-2}	40 (40)	1.778×10^{-1} (1.777×10^{-1})	7.6005×10^{0} (7.6005×10^{0})

Minimizing the Gibbs free energy per nucleon at fixed pressure is numerically more delicate in the inner crust than in the outer crust because the pressure also depends on the density of free neutrons. Instead, it is more convenient to minimize the energy per nucleon at fixed average baryon number density [22]. We have checked that the neutron-drip composition and the equation of state obtained for various magnetic-field strengths are compatible with those obtained using our outer crust code [13]. Some results are summarized in Table 2. As discussed in Ref. [22], we do not expect a perfect matching between the two codes since the outer and inner regions of the crust are described using different approximations. To further test our code, we have computed the properties of different crustal layers in the limit of a low magnetic field by setting $B_\star = 1$. As shown in Table 3, the results for weakly magnetized neutron stars are in very good agreement with those obtained with the code for strictly unmagnetized neutron stars (some results were previously published in Ref. [26]). The small deviations could be attributed to the interpolations implemented in the routines of Ref. [25]. Determining the equilibrium state in the weakly quantizing regime can be numerically tricky due to the occurrence of quantum oscillations (see, e.g., Ref. [14]).

The equation of state of the outer and inner regions of the crust is plotted in Figure 1 for different magnetic field strengths. As previously found in Ref. [11], the effects of the magnetic field are most prominent in the outermost region of the crust, and become less and less important with increasing density as more and more Rabi Levels are filled by electrons. In particular, the equation of state of the inner crust is barely changed for $B_\star \lesssim 1000$ and almost exactly matches that obtained in the absence of magnetic fields.

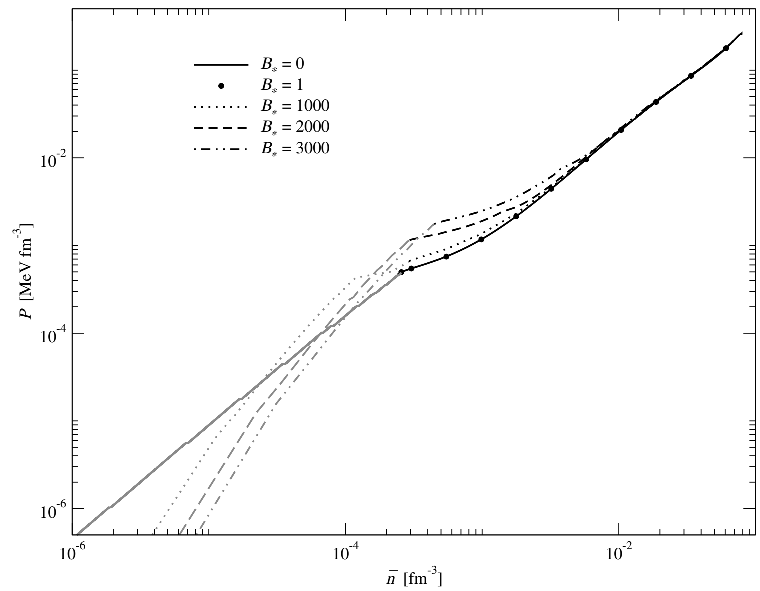

FIGURE 1. The pressure P as a function of the mean baryon number density $\bar{n}$ at different magnetic field strengths B_* for the outer (in gray) and inner (in black) crust.

CONCLUSIONS

We have studied the role of Rabi quantization of electron motion on the equilibrium properties of magnetar crusts, treating consistently both the outer and inner regions in the framework of the nuclear-energy density functional theory. For the outer crust, we have made use of experimental data from the 2016 AME and recent measurements of copper isotopes, supplemented with the HFB-24 atomic mass table. Compared to our previous calculations, the layers made of ^{130}Cd and ^{121}Y have disappeared whereas a new layer made of ^{124}Zr is now predicted to be present. For the inner crust, we have extended the ETFSI code developed by the Brussels-Montreal collaboration so as to account for the quantization of the relativistic electron gas. The presence of a high magnetic field leads to a very stiff equation of state at low densities, however the equation of state of the inner crust is only weakly altered if the magnetic field lies below a few times 10^{16} G.

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