Iterative RToF-based Localization and Time Synchronization in WLAN-like Systems

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Iterative localization is currently arising as a solution to localize a Mobile Station (MS) in a cellular network. We recently showed that iterating between the conventional delay estimation and multi-lateration steps allows one to approach the performance of direct localization algorithms. Until now, the method has only been applied to the case of networks where the access points are perfectly synchronized with each other. In this letter, we present a localization and time synchronization iterative algorithm suitable for networks where access points are not synchronized. We show numerically that iterating between the two conventional steps brings a significant performance gain.

Introduction: Time-of-Arrival (ToA) based localization relies on the estimation of the distance between the Mobile Station (MS) and a set of Access Points (APs). In the conventional two-step localization approach, the distance estimation is followed by a multi-lateration step providing the user location. Another localization methodology outperforming this two-step approach is the Direct Position Estimation (DPE). It selects the position in a grid that corresponds the best to the observed signals, at the cost of an increased computational complexity [**1**]. It has been demonstrated in [**2**] that in a strictly synchronized cellular system, iterating between both distance estimation and multi-lateration steps allows to approach the performance of DPE at a reduced computational cost.

This letter extends the results of [**2**] to the case of a network in which the Access Points (APs) are not synchronized with each other. We develop a Round Trip Time of Flight (RToF) based localization and time synchronization system and prove by simulation that our system outperforms the two-step conventional method in this scenario.

System model: We consider a WLAN-like network composed of non-synchronized APs communicating with Orthogonal Frequency Division Multiplexing (OFDM) signals. The MS is simultaneously connected to K neighbouring APs and operates on a communication bandwidth B using Q sub-carriers centred around the carrier frequency f_c. A single path channel introducing a delay $\tau_k(x,y)$ between the MS and AP k is assumed. Coordinates x and y denote the MS position. This delay corresponds to the absolute signal time of flight and is therefore equal to $\tau_k(x,y) = d_k(x,y)/c$ where $d_k(x,y)$ is the distance between the MS and AP k and c is the speed of light. The MS reference clock is assumed to have a time offset T_{0k} with respect to AP k.

We use the method developped in [**3**] to measure the RToF. This method does not require any synchronization between the MS and the APs but assumes that fixed Fast Fourier Transform (FFT) window time slots are used for transmission and reception. AP k initiates the measurement by sending one OFDM symbol defined in the frequency domain as $\mathbf{S} = \left[S_1, ..., S_Q\right]^T$. The corresponding time domain symbol $\mathbf{s}$ is obtained by taking a Q-point Inverse Fast Fourier Transform (IFFT) of $\mathbf{S}$. The first and second half of $\mathbf{s}$ are respectively denoted by $\mathbf{s}_1$ and $\mathbf{s}_2$. As illustrated in Figure 1(a), the time domain signal transmitted by AP k is composed of the signal $\mathbf{s}$ which is extended using $\mathbf{s}_1$ as cyclic postfix and $\mathbf{s}_2$ as cyclic prefix. The length of those cyclic extensions is assumed larger than the delays.

At the MS side, the FFT reception window observes a shifted version of $\mathbf{s}$. This time shift can be expressed as

$$t_{1k} = \tau_k(x,y) + T_{0k}. \tag{1}$$

After a processing time T_p of an integer multiple of the reception window size, the MS sends back the signal received on its reference window after adding the cyclic prefix and postfix extensions. AP k receives the reflected signal which turns out to be shifted by the RToF $t_{2k} = 2\tau_k$ in the corresponding reception FFT window.

Iterative localization: The user location is retrieved using an iterative positioning method inspired by the one developed in [**2**]. Figure 1(b) illustrates the functional blocks involved in this algorithm, including information exchanged between each step. As done in [**3**], we use the

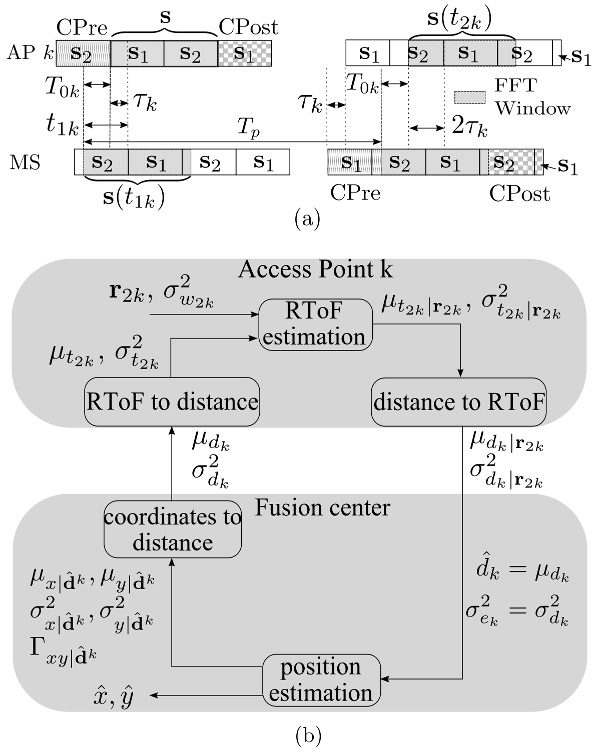

Fig. 1 *Algorithm principle: (a) RToF measurement scheme, (b) RToF iterative localization. The gray zone in (a) is the FFT observation window.*

RToF t_{2k} to localize the MS since it allows to deduce the position without the need of synchronizing the APs.

RToF estimation

Each AP implements one instance of the *RToF estimation* functional block. This block refines prior information received on the RToF (mean and variance) based on the observation of $\mathbf{R}_{2k}$ making use of the Bayesian framework.

The k^{th} AP estimates the RToF t_{2k} by observing its received signal on the set $\mathcal{P} = \{q_1, ..., q_P\}$ of loaded sub-carriers. The signal received at the access point can be expressed in the frequency domain as:

$$\mathbf{R}_{2k} = \mathbf{S}(t_{2k}) + \mathbf{W}_{2k} \tag{2}$$

where

$$\mathbf{R}_{2k} = \left[R_{2kq_1}, \cdots, R_{2kq_P}\right]^T \tag{3}$$

$$\mathbf{W}_{2k} = \left[W_{2kq_1}, \cdots, W_{2kq_P}\right]^T \tag{4}$$

and

$$\mathbf{S}(t_{2k}) = \left[S_{q_1} e^{-j2\pi \frac{q_1}{Q}\frac{t_{2k}}{T}}, \cdots, S_{q_P} e^{-j2\pi \frac{q_P}{Q}\frac{t_{2k}}{T}}\right]^T \tag{5}$$

for $k = 1, ..., K$. Parameter $T = 1/B$ denotes the sample period and W_{2kq} is Additive White Gaussian Noise (AWGN) of variance $\sigma^2_{W_{2k}}$ affecting sub-carrier q at AP k.

We know from (2) that $\mathbf{R}_{2k}$ is Gaussian distributed of mean $\mathbf{S}(t_{2k})$ and variance equal to the variance of the noise $\mathbf{W}_{2k}$. Assuming that t_{2k} is also a Gaussian distributed random variable of mean and variance known as prior information, we have all the elements to compute the posterior distribution of the RToF:

$$p(t_{2k}|\mathbf{R}_{2k}) = \frac{p(\mathbf{R}_{2k}|t_{2k})p(t_{2k})}{\int_{-\infty}^{\infty} p(\mathbf{R}_{2k}|t_{2k})p(t_{2k})dt_{2k}}. \tag{6}$$

The posterior mean and variance of t_{2k} are assessed by numerically computing statistical expectations $\mu_{t_{2k}|\mathbf{R}_{2k}} = \mathcal{E}\{t_{2k}|\mathbf{R}_{2k}\}$ and $\sigma^2_{t_{2k}|\mathbf{R}_{2k}} = \mathcal{E}\{(t_{2k} - \mu_{t_{2k}|\mathbf{R}_{2k}})^2|\mathbf{R}_{2k}\}$ based on the knowledge of posterior PDF (6). They respectively provide the Minimum Mean Square Error (MMSE) RToF estimate [**4**] together with an indication of its reliability. RToF mean and variance are easily converted into a distance information: $\mu_{d_k|\mathbf{R}_{2k}} = c\mu_{t_{2k}|\mathbf{R}_{2k}}/2$ and $\sigma^2_{d_k|\mathbf{R}_{2k}} = c^2\sigma^2_{t_{2k}|\mathbf{R}_{2k}}/4$.

Those values are transmitted to the fusion centre to deduce the user position.

Position estimation

Similarly to [**2**], one instance of the *position estimation* functional block is implemented per AP at the fusion center. It produces an estimate of the user position together with its reliability based on the distance information provided by all access points but the current one. The deduced prior information can therefore be assumed independent from the received signal at the current access point. Gathering the ranging estimates used for the *position estimation* block corresponding to the k^{th} AP we get the following model:

$$\hat{\mathbf{d}}^k = \mathbf{d}^k(x, y) + \mathbf{e}^k \tag{7}$$

with

$$\hat{\mathbf{d}}^k = \left[\hat{d}_1, ..., \hat{d}_{k-1}, \hat{d}_{k+1}, ..., \hat{d}_K\right]^T \tag{8}$$

$$\mathbf{d}^k(x, y) = [d_1(x, y), ..., d_{k-1}(x, y), d_{k+1}(x, y), ...d_K(x, y)]^T \tag{9}$$

$$\mathbf{e}^k = [e_1, ..., e_{k-1}, e_{k+1}, ..., e_K]^T \tag{10}$$

where superscript k indicates that information from AP k is excluded from the vector. Elements of the error vector $\mathbf{e}^k$ are assumed of zero mean and variance $\sigma^2_{e_k} = \sigma^2_{d_k|\mathbf{R}_{2k}}$. Ranging estimates in (8) are given by $\hat{d}_k = \mu_{d_k|\mathbf{R}_{2k}}$. Position mean terms $\mu_{x|\hat{\mathbf{d}}^k}, \mu_{y|\hat{\mathbf{d}}^k}$ and (co)variance terms $\sigma^2_{x|\hat{\mathbf{d}}^k}, \sigma^2_{y|\hat{\mathbf{d}}^k}$ and $\Gamma_{xy|\hat{\mathbf{d}}^k}$ are deduced with a similar Bayesian estimator to the RToF using the posterior PDF of the coordinates:

$$p(x, y|\hat{\mathbf{d}}^k) = \frac{p(\hat{\mathbf{d}}^k|x, y)p(x, y)}{\int_{-\infty}^{+\infty} p(\hat{\mathbf{d}}^k|x, y)p(x, y)dxdy} \tag{11}$$

where $p(\hat{\mathbf{d}}^k|x, y)$ is a Gaussian PDF with mean $\mathbf{d}^k(x, y)$ and covariance matrix $C_{\mathbf{e}^k} = \text{diag}\{\sigma^2_{e_1}, ..., \sigma^2_{e_K}\}$. Coordinates can be assumed uniformly distributed on $[x_{\min}, x_{\max}]$ and $[y_{\min}, y_{\max}]$. The final coordinate estimates $(\hat{x}, \hat{y})$ are obtained at each iteration by averaging the position estimates available from the K instances of the *position estimation* block:

$$\hat{x} = \frac{1}{K}\sum_{k=1}^{K} \mu_{x|\hat{\mathbf{d}}^k}, \qquad \hat{y} = \frac{1}{K}\sum_{k=1}^{K} \mu_{y|\hat{\mathbf{d}}^k}. \tag{12}$$

Position to RToF conversion

As shown in Figure 1(b), information about distances is extracted from the position estimator. This conversion relies on a first order approximation of the relation between true distance and user coordinates:

$$\mu_{d_k} \approx d_k(\hat{x}, \hat{y}) \tag{13}$$

$$\sigma^2_{d_k} \approx \frac{1}{d_k^2(\hat{x}, \hat{y})} \begin{bmatrix} x_k - \hat{x} \\ y_k - \hat{y} \end{bmatrix}^T \cdot \begin{bmatrix} \sigma^2_{x|\hat{\mathbf{d}}^k} & \Gamma_{xy|\hat{\mathbf{d}}^k} \\ \Gamma_{xy|\hat{\mathbf{d}}^k} & \sigma^2_{y|\hat{\mathbf{d}}^k} \end{bmatrix} \cdot \begin{bmatrix} x_k - \hat{x} \\ y_k - \hat{y} \end{bmatrix}. \tag{14}$$

It is then converted into RToF mean and variance:

$$\mu_{t_{2k}} = 2\mu_{d_k}/c, \qquad \sigma^2_{t_{2k}} = 4\sigma^2_{d_k}/c^2. \tag{15}$$

Those two first order moments are fed back into the Bayesian RToF estimator and used as prior information in order to refine the RToF estimation.

Location aware synchronization: After the last iteration of the positioning algorithm, the final position estimate $(\hat{x}, \hat{y})$ is transmitted to the MS. The clock offset T_{0k} is deduced at the MS from the observation of $\mathbf{R}_{1k}$. Taking advantage of the available location estimate and making use of relation (1), $\mathbf{R}_{1k}$ can be expressed similarly to (2) as

$$\mathbf{R}_{1k} = \mathbf{S}(\hat{\tau}_k + T_{0k}) + \mathbf{W}_{1k} \tag{16}$$

where the variance of the noise vector $\mathbf{W}_{1k}$ is $\sigma^2_{W_{1k}} = \sigma^2_{W_{2k}}/2$ and $\hat{\tau}_k = d_k(\hat{x}, \hat{y})/c$. The posterior distribution of T_{0k} is therefore

$$p(T_{0k}|\mathbf{R}_{1k}) = \frac{p(\mathbf{R}_{1k}|T_{0k})p(T_{0k})}{\int_{-\infty}^{\infty} p(\mathbf{R}_{1k}|T_{0k})p(T_{0k})dT_{0k}}. \tag{17}$$

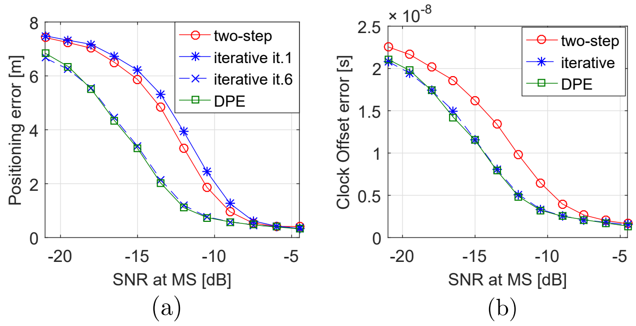

Fig. 2 *Iterative algorithm performance. Average positioning error (a) and average clock offset error (b) as a function of the SNR.*

The prior distribution $p(T_{0k})$ is assumed to be uniform: $T_{0k} \backsim U\left[T_{0k_{\min}}, T_{0k_{\max}}\right]$. The clock offset estimate is finally obtained as $\hat{T}_{0k} = \mathcal{E}\{T_{0k}|\mathbf{R}_{1k}\}$.

Numerical results: The performance of the localization and time synchronization iterative algorithm is investigated in a scenario consisting of four APs located on the corners of a 20 m sided square. The location of the user terminal is randomly chosen inside the square and clock offsets T_{0k} are arbitrarily chosen between 0 and $0.1\mu s$. The transmitted signals consist of a single OFDM block $\mathbf{S}$ with cyclic extensions spanning a 40 MHz bandwidth using 128 sub-carriers. Similarly to the IEEE 802.11ac standard [**5**], only sub-carriers -58 to -2 and 2 to 58 are loaded in the transmitted symbol $\mathbf{S}$.

Figures 2(a) and 2(b) respectively depict the positioning and average synchronization error as a function of the SNR. Results are obtained from 1000 independent runs per SNR. For each run, the synchronization error is averaged over the four APs. The iterative position estimation is compared to the conventional two-step estimation and to the DPE method.

It appears that the iterative method significantly improves the conventional two-step method for both localization and time synchronization and performs very close to DPE.

Conclusion: In this letter, we propose to extend the iterative localization method presented in [**2**] to achieve both time synchronization and localization in the case of a WLAN-like non-synchronized network. Localization is achieved using an iterative approach that refines the position estimate using a Bayesian Round Trip Time of Flight estimation at the access point side. Time synchronization is then performed at the mobile station side making use of the location estimate. Numerical results show that the proposed algorithm significantly improves the conventional two-step method.

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