ToA-based Iterative Localization in Rich Multipath Channels

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***Abstract*—Iterative localization is arising as a promising solution to determine the position of a mobile station in a cellular network. We recently showed that in a perfect line-of-sight environment, iterating between the conventional delay estimation and multi-lateration steps allows to approach the performance of the direct localization based on the observation of the received signals. In this paper we extend our iterative localization method to operate in rich multipath environments. Simulation results prove that given some prior knowledge on the power delay profile of the channel, the proposed iterative algorithm is robust to harsh propagation environments and performs very close to the direct localization approach.**

I. Introduction

Cellular communication networks are continuously evolving towards geo-located services [1]. In 4G, localization has even become an essential part of the network and may replace the need for global navigation satellite system (GNSS) positioning. In particular, 4G includes a specific Positioning Reference Signal (PRS) in its protocol to finely estimate the signal time-of-flight (ToF) between a base station (BS) and a mobile station (MS). This PRS is defined as an Orthogonal Frequency Division Multiplexing (OFDM) signal spread in time and frequency [2]. The signal ToF estimation constitutes the first step of the conventional two-step localization approach. A multi-lateration step then combines the ToF measurements to determine the position of the Mobile Station (MS) [3].

One of the main causes of inaccuracies in ToA-based cellular localization systems is multipath propagation. In urban environments, Line-of-Sight (LOS) condition can often not be guaranteed between the MS and all the BSs involved in the localization process and many dense replicas of the transmitted signal are received. In this case the delay estimation step generally consists in estimating the arrival time of the first path and the bias induced by a potential none-line of sight (NLOS) propagation. Many delay estimation schemes have been developed in the literature, such as the Generalized Maximum Likelihood (GML) approach of [4] which jointly estimates all multipath coefficients and their arrival times in an iterative manner. Another approach is the frequency domain super-resolution ToA estimation of [5]. This subspace method uses an estimation of the signal autocorrelation which requires a large number of independent signal observations with the same time-of-arrival. Papers [6], [7] rather rely on the central limit theorem for random vectors to formulate an approximate Maximum Likelihood delay estimator in dense multipath. Given some prior knowledge on the shape of the channel power delay profile, this approach is shown to outperform super-resolution and GML methods in practical environments [6].

Two-step localization is clearly suboptimal since information is lost by transferring only the time-of-arrival estimates to the multi-lateration step. Another methodology to estimate the user position is the Direct Position Estimation (DPE) that directly estimates the position coordinates from the digitized received signal. Paper [8] analytically demonstrates that DPE always outperforms two-step positioning. At the time of writing, there are only a few DPE algorithms designed to operate in a frequency-selective multipath environment. In this paper, we will use the DPE method developed in [9]. This method relies on the same approximate Maximum Likelihood formulation as papers [6] and [7] and requires some prior knowledge on the shape of the channel power delay profile (PDP).

While DPE turns out to be the optimal localization solution, it suffers from a significant complexity increase and requires the full knowledge of the received signal at the central fusion center. We recently demonstrated that the performance of DPE can be approached by iterating between the two conventional steps [10]. This iterative approach allows to reduce the computational burden and the communication overhead compared to DPE but is up to now only designed for ideal line-of-sight propagation conditions. In this paper, we therefore propose to extend the iterative localization method of [10] to be able to operate in a rich multipath environment, typical of dense urban and indoor environments. We resort to the approach developed in [6], [7] to estimate the propagation delay of the signal using some prior knowledge of the channel PDP.

The rest of this paper is organized as follows. Section II introduces the OFDM signal model. The iterative positioning algorithm for rich multipath environments is described in Section III. Section IV numerically assesses the performance of the proposed algorithm and compares it to DPE and two-step approaches. Throughout the text vectors and matrices are identified by lowercase and uppercase bold letters respectively. A vector containing all elements of $\boldsymbol{x}$ excepting the k^{th} one is written as $\boldsymbol{x}^k$.

II. Signal Model

We consider a cellular system operating in OFDM. We assume that the mobile station is simultaneously connected and strictly time synchronized to K neighboring base stations. It operates on a communication bandwidth B centered around the carrier frequency f_c. The OFDM modulation splits the communication bandwidth in Q orthogonal sub-carriers allocated to data or pilot symbols. A cyclic prefix (CP) is inserted in each multi-carrier block. This CP allows to maintain orthogonality among the sub-carriers when the signal undergoes a time dispersive channel. We assume a rich multipath channel between the MS and BS k. As long as the duration of the channel impulse response is shorter than the cyclic prefix, the signal received on the sub-carrier q at the base station k can be expressed as:

$$r_{kq} = s_q h_{kq} + w_{kq} \tag{1}$$

for $k = 1 \cdots K$ and $q = 0 \ \cdots \ Q-1$. In (1), s_q is the data/pilot symbol transmitted on the sub-carrier q and w_{kq} is the noise corrupting sub-carrier q at base station k. Additive white Gaussian noise (AWGN) of variance $\sigma^2_{w_k}$ is assumed. The frequency domain channel coefficient h_{kq} affecting sub-carrier q at base station k given by:

$$h_{kq} = \sum_{l=0}^{L-1} \alpha_{kl} e^{-j2\pi q(\tau_k + \nu_{kl})/QT}. \tag{2}$$

where $T = 1/B$ is the sample duration and $\tau_k + \nu_{kl}$ is the delay of the l^{th} path at BS k. We choose to set $\nu_{0k} = 0$ such that the delay of the first path is the propagation delay $\tau_k = d_k(x, y)/c$ where c is the speed of light and $d_k(x,y) = \sqrt{(x - x_k)^2 + (y - y_k)^2}$. Coordinates $\{x, y\}$ and $\{x_k, y_k\}$ respectively denote the position of the MS and of base station k. Coefficient α_{kl} denotes the complex gain associated to the l^{th} path of the channel impulse response for base station k. Complex channel gains α_{kl} are circular random variables and are conditionally independent. Those coefficients are of zero mean and their conditional second order moments are given by:

$$\mathcal{E}\left\{\alpha_{kl}\alpha^*_{l'k} \middle| \nu_{kl}\nu_{l'k}\right\} = \begin{cases} \sigma^2_{\alpha_k}(\nu_{kl}) & l = l' \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where operator $\mathcal{E}\{\}$ denotes the statistical expectation.

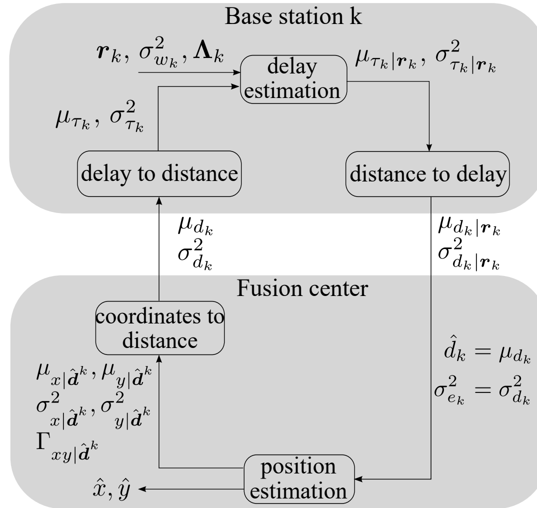

Fig. 1. Principle of the iterative localization algorithm.

Gathering the received signal on the set of pilot sub-carriers $\mathcal{P} = \{q_1, \cdots q_P\}$, we have the following vector model for the received signal:

$$\boldsymbol{r}_k = \sum_{l=0}^{L-1} \alpha_{kl} \boldsymbol{\Gamma}_{\tau_k + \nu_{kl}} \boldsymbol{s} + \boldsymbol{w}_k \tag{4}$$

where

$$\boldsymbol{r}_k = \left[r_{kq_1}, \cdots, r_{kq_P}\right]^T \tag{5}$$

$$\boldsymbol{s} = \left[s_{q_1}, \cdots, s_{q_P}\right]^T \tag{6}$$

$$\boldsymbol{w}_k = \left[w_{kq_1}, \cdots, w_{kq_P}\right]^T \tag{7}$$

and

$$\boldsymbol{\Gamma}_\nu = \text{diag}\left\{e^{-j2\pi\nu q_1/QT}, \cdots, e^{-j2\pi q_P \nu/QT}\right\}. \tag{8}$$

Introducing the vector $\boldsymbol{x}_k = \sum_{l=0}^{L-1} \alpha_{kl} \boldsymbol{\psi}(\boldsymbol{s}, \nu_{kl})$ with $\boldsymbol{\psi}(\boldsymbol{s}, \nu_{kl}) = \boldsymbol{\Gamma}_{\nu_{kl}} \boldsymbol{s}$, expression (4) can be more compactly written as:

$$\boldsymbol{r}_k = \boldsymbol{\Gamma}_{\tau_k} \boldsymbol{x}_k + \boldsymbol{w}_k. \tag{9}$$

III. Iterative Positioning

The principle of the iterative localization system is very similar to [10]. The algorithm relies on the Bayes framework [11] to take into account prior knowledge from the previous iteration. As illustrated in Fig. 1, delay and position estimates are transmitted together with an estimate of their variance between the two steps. The position computed during a step of the algorithm is translated to a delay used as prior information by the next iteration.

A. Time-of-arrival Estimation

We resort to a Bayesian delay estimator to take prior knowledge on the time-of-arrival into account. The idea is to deduce the time-of-arrival together with its reliability from the posterior distribution

$$p(\tau_k|\boldsymbol{r}_k) = \frac{p(\boldsymbol{r}_k|\tau_k)p(\tau_k)}{\int_{-\infty}^{+\infty} p(\boldsymbol{r}_k|\tau_k)p(\tau_k)d\tau_k}. \quad (10)$$

In (10), we assume the prior distribution of the delay to be normally distributed:

$$p(\tau_k) = C_{\tau_k} \exp\left(-\frac{1}{2\sigma_{\tau_k}^2}(\tau_k - \mu_{\tau_k})^2\right). \quad (11)$$

where C_{τ_k} is a constant.

To obtain the likelihood $p(\boldsymbol{r}_k|\tau_k)$, we adopt the approach developed in [6], [7], [9].

Assuming that the impulse response of the channel has many dense arrivals, we realize from (4) that the received signal vector $\boldsymbol{r}_k$ is a sum of a large number of statistically independent vectors. Under such assumptions, we can apply the Central Limit Theorem for random vectors [12] and *approximate* $\boldsymbol{r}_k$ as Gaussian:

$$p(\boldsymbol{r}_k|\tau_k) \approx C_{\boldsymbol{r}_k} \exp\left(-\boldsymbol{r}_k^H \boldsymbol{\Sigma}_{\boldsymbol{r}_k}^{-1}(\tau_k)\boldsymbol{r}_k\right) \quad (12)$$

where $C_{\boldsymbol{r}_k}$ is a constant.

The co-variance matrix of the received signal computes:

$$\begin{aligned}\Sigma_{\boldsymbol{r}_k}(\tau_k) &= \mathcal{E}\left\{\boldsymbol{r}_k\boldsymbol{r}_k^H|\tau_k\right\} \quad (13)\\ &= \boldsymbol{\Gamma}_{\tau_k}\boldsymbol{\Sigma}_{\boldsymbol{x}_k}\boldsymbol{\Gamma}_{\tau_k}^H + \sigma_{w_k}^2\mathbb{I}. \quad (14)\end{aligned}$$

In (14), $\mathbb{I}$ denotes the identity matrix and $\boldsymbol{\Sigma}_{\boldsymbol{x}_k} = \mathcal{E}\left\{\boldsymbol{x}_k\boldsymbol{x}_k^H\right\}$ is the $P \times P$ co-variance matrix of the time-of-arrival independent part of the channel. Using the relation $\boldsymbol{\Gamma}_{\tau_k}\boldsymbol{\Gamma}_{\tau_k}^H = \boldsymbol{\Gamma}_{\tau_k}^H\boldsymbol{\Gamma}_{\tau_k} = \mathbb{I}$, we get:

$$\Sigma_{\boldsymbol{r}_k}^{-1}(\tau_k) = \boldsymbol{\Gamma}_{\tau_k}\left(\boldsymbol{\Sigma}_{\boldsymbol{x}_k} + \sigma_{w_k}^2\mathbb{I}\right)^{-1}\boldsymbol{\Gamma}_{\tau_k}^H. \quad (15)$$

Defining the matrix $\boldsymbol{D} = \left(\boldsymbol{\Sigma}_{\boldsymbol{x}_k} + \sigma_{w_k}^2\mathbb{I}\right)^{-1}$, the argument of the exponential of the likelihood function (12) reads:

$$\begin{aligned}-\boldsymbol{r}_k^H\boldsymbol{\Sigma}_{\boldsymbol{r}_k}^{-1}(\tau_k)\boldsymbol{r}_k &= -\boldsymbol{r}_k^H\boldsymbol{\Gamma}_{\tau_k}\boldsymbol{D}\boldsymbol{\Gamma}_{\tau_k}^H\boldsymbol{r}_k \quad (16)\\ &= -\boldsymbol{\gamma}_{\tau_k}^H\boldsymbol{C}\boldsymbol{\gamma}_{\tau_k} \quad (17)\end{aligned}$$

where $\boldsymbol{\gamma}_{\tau_k}$ is the vector composed by the diagonal elements of $\boldsymbol{\Gamma}_{\tau_k}$ and $\boldsymbol{C} = \boldsymbol{R}_k^H\boldsymbol{D}\boldsymbol{R}_k$ with $\boldsymbol{R}_k = \text{diag}\{\boldsymbol{r}_k\}$. It is shown in [6] that (17) can be efficiently computed for all the delays on the search grid as:

$$2\mathcal{R}\left\{\text{IFFT}\left([0.5u_1, u_2, \cdots, u_P]\right)\right\} \quad (18)$$

where the u_i is the the sum of the i^{th} diagonal of $\boldsymbol{C}$ and $\mathcal{R}\{\}$ denotes the real part operator. The search grid for τ_k is refined by zero-padding the argument of the Inverse Fast Fourier Transform (IFFT).

To determine the co-variance matrix $\boldsymbol{\Sigma}_{\boldsymbol{x}_k}$, we assume similarly to [6] the multipath arrivals to be confined to a finite grid with a sufficiently fine resolution Δ. Under this assumption, $\boldsymbol{x}_k$ can be expressed by:

$$\boldsymbol{x}_k = \boldsymbol{\Psi}\boldsymbol{\alpha}_k \quad (19)$$

where

$$\boldsymbol{\Psi} = [\boldsymbol{\psi}(\boldsymbol{s}, 0), \boldsymbol{\psi}(\boldsymbol{s}, \Delta), \cdots, \boldsymbol{\psi}(\boldsymbol{s}, T_D)] \quad (20)$$

and

$$\boldsymbol{\alpha}_k = [\alpha_{0k}, \alpha_{\Delta k}, \cdots, \alpha_{T_D k}]^T. \quad (21)$$

In the previous expressions, T_D denotes the maximum delay spread of the channel. Substituting (19) into the definition of $\boldsymbol{\Sigma}_{\boldsymbol{x}_k}$, we get:

$$\begin{aligned}\boldsymbol{\Sigma}_{\boldsymbol{x}_k} &= \mathcal{E}\left\{\boldsymbol{x}_k\boldsymbol{x}_k^H\right\} \quad (22)\\ &= \boldsymbol{\Psi}\boldsymbol{\Lambda}_k\boldsymbol{\Psi}^H \quad (23)\end{aligned}$$

where $\boldsymbol{\Lambda}_k = \text{diag}\left\{\sigma_{\alpha_k}^2(0), \sigma_{\alpha_k}^2(\Delta), \cdots, \sigma_{\alpha_k}^2(T_D)\right\}$. The diagonal of $\boldsymbol{\Lambda}_k$ is the channel power delay profile.

Inserting expressions (17) and (23) into (12) gives the approximate likelihood function $p(\boldsymbol{r}_k|\tau_k)$. The only required knowledge is the channel power delay profile $\sigma_{\alpha_k}^2(\nu)$ which can be available from an appropriate channel model[1]. This likelihood function can be used to deduce the posterior distribution (10) from which it is possible to numerically compute the mean and the variance of the time-of-arrival τ_k:

$$\mu_{\tau_k|\boldsymbol{r}_k} = \int_{-\infty}^{+\infty} \tau_k p(\tau_k|\boldsymbol{r}_k)d\tau_k \quad (24)$$

and

$$\sigma_{\tau_k|\boldsymbol{r}_k}^2 = \int_{-\infty}^{+\infty} (\tau_k - \mu_{\tau_k|\boldsymbol{r}_k})^2 p(\tau_k|\boldsymbol{r}_k)d\tau_k. \quad (25)$$

The mean and variance of the posterior distribution (10) respectively provide the Minimum Mean Square Error (MMSE) delay estimate [11] together with an indication of its reliability. Delay mean and variance are then converted into a distance information by scaling them by the speed of light:

$$\begin{aligned}\mu_{d_k|\boldsymbol{r}_k} &= c\mu_{\tau_k|\boldsymbol{r}_k} \quad (26)\\ \sigma_{d_k|\boldsymbol{r}_k}^2 &= c^2\sigma_{\tau_k|\boldsymbol{r}_k}^2. \quad (27)\end{aligned}$$

Those values are transmitted to the fusion center to deduce the user position.

B. Position Estimation

The position estimation step is very similar to the one developed in [10]. One instance of the position estimation block is implemented per base station at the fusion center. It generates an estimate of the user position together with its reliability based on the distance information provided by all base stations but the current

[1]If such a channel model is not available, a good approximation of the channel PDP can also be obtained from the observation of pilot sub-carriers spread on successive OFDM symbols (e.g [13]).

one. This allows to make the deduced prior information independent from the received signal at the current base station. If we gather the distance estimates used for the position estimation block of the k^{th} BS we get the following model:

$$\hat{\boldsymbol{d}}^k = \boldsymbol{d}^k(x,y) + \boldsymbol{e}^k \tag{28}$$

with

$$\hat{\boldsymbol{d}}^k = \left[\hat{d}_1, ..., \hat{d}_{k-1}, \hat{d}_{k+1}, ..., \hat{d}_K\right]^T \tag{29}$$

$$\boldsymbol{d}^k(x,y) = [d_1(x,y), ..., d_{k-1}(x,y), \\ d_{k+1}(x,y), ... d_K(x,y)]^T \tag{30}$$

$$\boldsymbol{e}^k = [e_1, ..., e_{k-1}, e_{k+1}, ..., e_K]^T \tag{31}$$

where superscript k indicates that BS k is excluded from the vector. Elements of the error vector $\boldsymbol{e}^k$ are assumed of zero mean and variance $\sigma^2_{e_k} = \sigma^2_{d_k|\boldsymbol{r}_k}$. Distance estimates in (29) are given by $\hat{d}_k = \mu_{d_k|\boldsymbol{r}_k}$. Position mean terms $\mu_{x|\hat{\boldsymbol{d}}^k}$, $\mu_{y|\hat{\boldsymbol{d}}^k}$ and (co)variance terms $\sigma^2_{x|\hat{\boldsymbol{d}}^k}$, $\sigma^2_{y|\hat{\boldsymbol{d}}^k}$ and $\Gamma_{xy|\hat{\boldsymbol{d}}^k}$ are deduced from the posterior PDF of the coordinates:

$$p(x,y|\hat{\boldsymbol{d}}^k) = \frac{p(\hat{\boldsymbol{d}}^k|x,y)p(x,y)}{\int_{-\infty}^{+\infty} p(\hat{\boldsymbol{d}}^k|x,y)p(x,y)dxdy} \tag{32}$$

where $p(\hat{\boldsymbol{d}}^k|x,y)$ is a Gaussian PDF with mean $\boldsymbol{d}^k(x,y)$ and covariance matrix $C_{\boldsymbol{e}^k} = \text{diag}\{\sigma^2_{e_1}, ..., \sigma^2_{e_K}\}$. Coordinates are assumed uniformly distributed on $[x_{\text{min}}, x_{\text{max}}]$ and $[y_{\text{min}}, y_{\text{max}}]$. We obtain the final coordinate estimates $(\hat{x}, \hat{y})$ at each iteration by averaging the position estimates available from the K instances of the position estimation block:

$$\hat{x} = \frac{1}{K}\sum_{k=1}^{K} \mu_{x|\hat{\boldsymbol{d}}^k}, \qquad \hat{y} = \frac{1}{K}\sum_{k=1}^{K} \mu_{y|\hat{\boldsymbol{d}}^k}. \tag{33}$$

C. Position to Time-of-arrival

To extract information about distances from the position estimator, we rely on a first order approximation of the relation between true distance and user coordinates:

$$\mu_{d_k} \approx d_k(\hat{x}, \hat{y}) \tag{34}$$

$$\sigma^2_{d_k} \approx \tfrac{1}{d_k^2(\hat{x},\hat{y})} \begin{bmatrix} x_k - \hat{x} \\ y_k - \hat{y} \end{bmatrix}^T \cdot \begin{bmatrix} \sigma^2_{x|\hat{\boldsymbol{d}}^k} & \Gamma_{xy|\hat{\boldsymbol{d}}^k} \\ \Gamma_{xy|\hat{\boldsymbol{d}}^k} & \sigma^2_{y|\hat{\boldsymbol{d}}^k} \end{bmatrix} \cdot \begin{bmatrix} x_k - \hat{x} \\ y_k - \hat{y} \end{bmatrix}. \tag{35}$$

It is then converted into delay mean and variance:

$$\mu_{\tau_k} = \mu_{d_k}/c, \qquad \sigma^2_{\tau_k} = \sigma^2_{d_k}/c^2. \tag{36}$$

Those two first order moments are fed back into the Bayesian delay estimator and used as prior information in order to refine the time-of-arrival estimation.

IV. Numerical Results

To estimate the performance of our iterative localization scheme, we assume a system composed of four base stations at the corners of a 100 m sided square. The mobile station lies at arbitrary positions inside the square and communicates with the base stations over a bandwidth of 20 MHz. At each base station, the time-of-arrival is estimated using a single OFDM symbol containing 64 equispaced pilot sub-carriers among 1024. Algorithm performances are averaged over 1000 MS position, channel and noise realizations.

We assume that the base stations have a perfect knowledge of the noise variance $\sigma^2_{w_k}$. The channel between the MS and the BSs is generated from the 5G TDL-C channel model of [14] with a root mean square delay spread of 350 ns. This delay spread is indicated in [14] to characterize a normal delay spread for the Urban Macro-cell scenario at a carrier frequency of $f_c = 2$ GHz. The PDP of this channel model is illustrated in Fig. 2. The base stations have a prior knowledge of the power delay profile of the channel defined at the rate $(1/T)$ of the system.

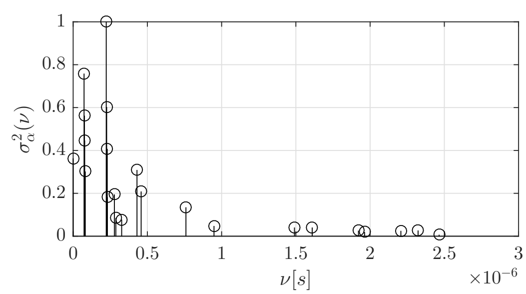

Fig. 2. Power delay profile of the ETSI TDL-C channel model [14].

Fig. 3 depicts the average localization error as a function of the SNR. Our iterative algorithm is compared to the conventional two-step and direct approaches. After a few iterations, the performance of the iterative estimation comes very close to the performance of the optimal direct position estimation. At the first iteration, the iterative estimation performs slightly worse than the two-step approach. This is due to the fact that at the first iteration, the estimate is obtained with the four possible sets of three base stations, while the four base stations are directly used in case of the two-step estimation. The implemented delay estimation method allows all methods to perform very well at medium to high SNRs despite the harsh propagation conditions.

The convergence of the iterative solution is illustrated in Fig. 4. The performance gain becomes negligible after six iterations.

Fig. 5 finally compares the performance of the iterative approach developed in this paper with the one of [10] designed for strong LOS conditions only. Under the considered ETSI TDL-C channel [14], the iterative approach of [10] severely suffers from the multipath propagation and the average distance error remains above 35 m even at high SNR.

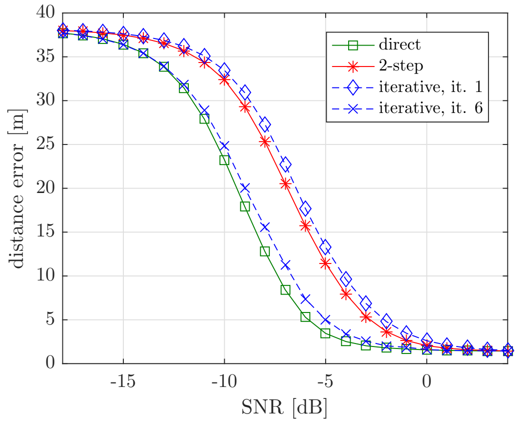

Fig. 3. Comparison of the positioning algorithm performances. Average distance error as a function of the SNR.

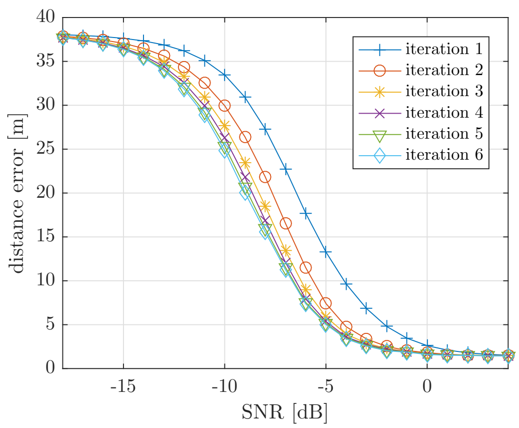

Fig. 4. Convergence of the iterative multi-lateration algorithm. Average distance error as a function of the SNR.

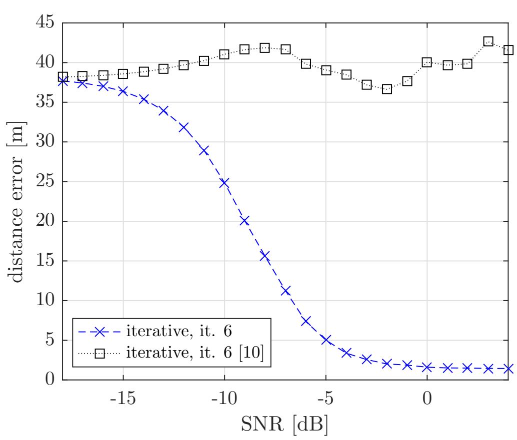

Fig. 5. Comparison of the iterative algorithm for rich multipath and the iterative approach of [10] under the ETSI TDL-C channel [14] after convergence.

V. Conclusion

In this paper, we propose a time-of-flight based iterative localization method robust to rich multipath propagation channels. Provided that the base stations have a good knowledge of the power delay profile of the channel, our iterative estimation scheme performs very well at medium to high SNRs. We showed by simulation that it outperforms the conventional two-step approach and performs very close to the optimal direct estimation.

Acknowledgment

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