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# A Theory of Social Finance

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Keywords: Social Finance, Philanthropy, Foundations, Social Banks.

JEL Classifications: G21, D63, G24, H25.

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# A Theory of Social Finance*

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**Abstract**

Myriad different types of institutions are involved in social finance. This paper attempts to make sense of the diverse ways of operationalizing the delivery of funds by social financial institutions (SFIs). It explores the continuum of feasible SFIs, which range from foundations offering pure grants to social banks supplying soft loans. The in-between category includes "quasi-foundations" granting loans that require partial repayment only. In our model, the SFIs face information asymmetries and trade off costly social screening against social contributions, under the budget constraint that depends on the generosity of their funders. We characterize the SFIs' optimal strategy and suggest that quasi-foundations can be efficient vehicles for social finance, especially when social screening costs are relatively low.

## 1. Introduction

Philanthropy and charitable giving have existed since time immemorial. In the 12th century, Maimonides established an eight-level classification of charitable giving (*tsedakah* in Hebrew), which places pure gifts and zero-interest loans on an equal footing, together with business partnerships.[1] Today's economic theory promotes a more segmented view: charitable giving by individual donors and philanthropic institutions is typically separate from lending at favorable conditions by social banks and other socially-oriented financial intermediaries (Bekkers & Wiepking, 2011), such as microfinance institutions (Armendariz & Morduch, 2010; Hudon & Sandberg, 2013).[2] This line of reasoning regrettably leaves unaddressed the intermediate situations where a social financial institution (SFI) asks back from the beneficiary less than 100% of the capital it provides. Yet this missing middle of the segment that extends between a full gift (nothing to be reimbursed) and loans from commercial banks (full reimbursement plus the market interest rate) offers attractive opportunities for philanthropic and socially-oriented actions. Therefore, we argue that acknowledging the full spectrum of social finance not only enriches economic theory; it also helps rationalize poorly understood phenomena, such as so-called blending, which combines grants and loans. So far, the optimal design of money transfer to beneficiaries—the "loan versus grant" debate—has been confined at the macro level to foreign-aid programs and supranational funding schemes, whether from a theoretical perspective (e.g. Schmidt, 1964; Cohen et al., 2007)

[1] The classification is provided in Chapter 10 of *Laws on Gifts for the Poor.* See Raphael (2003) for a Hebrew-English annotated translation of Moses Maimonides' treatise on philanthropy, Gifts for the Poor by Rabbi Joseph Meszler.

[2] Yet, identifying the difference between favorable and regular lending conditions is not always obvious, for intertwined reasons, such as regulatory factors (e.g. usury rate ceilings, creditor protections) and reputational considerations (banking relationship, progressive lending), which affect lenders' interest-rate setting (Fehr & Zehnder, 2009).

or in practical terms. This paper aims to go one step further and develop a unified theory of social finance that applies to any type of SFI.

Charitable giving is massively present in English-speaking countries: in the U.S., 90% of citizens give money to charities (Della Vigna et al., 2012). The financial literature offers increasing evidence of social funders who willingly require below-market returns for capital that serves socially-responsible purposes. For example, Hesse and Čihák (2007) and Becchetti et al. (2011) report anecdotal evidence on the funders of stakeholder banks (i.e. cooperative and savings banks) and social banks. Cornée et al. (2017) extend these findings to the European banking industry. Cull et al. (2009, 2016) show that the cost of capital is typically lower for microfinance institutions than for conventional banks. Similar patterns are observable in alternative financial activities, such as crowdfunding. Thanks to digitalization, crowdfunding allows social projects to collect funds from a multitude of individuals. The diversity of compensation mechanisms embedded in crowdfunded projects blurs the boundaries between giving and lending. These mechanisms often include non-cash compensation, such as illiquid equity or future products (Agrawal et al., 2014; Mollick, 2014). SFIs may be depicted as "delegated philanthropic intermediaries" (Bénabou & Tirole, 2010) or "value-based intermediaries" (Scholtens, 2006), be they real, like social banks, or virtual, like crowdfunding platforms (Heminway, 2015).[3] In line with the contemporary theory of financial intermediation (Diamond, 1984; Bhattacharya & Thakor, 1993), the SFIs' mission consists in channeling capital from prosocial investors to their beneficiaries using efficient selection and monitoring mechanisms. All these initiatives come within the scope of social finance.

[3] Arguably, crowdfunding platforms act as partial intermediaries since they pre-screen projects, but do not allocate funds.

In our framework, social finance emerges as soon as some individuals, which we shall refer to as social funders, are ready to relinquish part of their financial return or donate some of their capital to deserving ventures or enterprises, and they delegate the tasks of selecting the projects and taking care of the funding practicalities to a financial intermediary (Riedl & Smeets, 2017). Depending on the circumstances, social funders can be pure donors, lenders, depositors, or equity holders. They are typically motivated by certain values, insofar as they require the service of double bottom-line intermediaries, namely SFIs whose mission is to obtain both social and financial returns. The spread between the market return and the funders' required return is a signal for the SFI of the relative expected weight of each bottom line. Since the lending/giving activity is plagued by severe information asymmetries, SFIs rely on both social and financial screening to meet their funders' hybrid objective as much as possible. At the same time, the probabilities of success of SFI-supported projects depend on the interest rate charged. Ultimately, the problem for the SFI is basically to choose between engaging in costly screening and supplying cheap financing to the selected projects. This argument is the gist of our model.

One original feature of our approach is to consider pure gifts and subsidized credit both together. We argue that separating philanthropy (or grants) from subsidized lending is artificial, since access to financial instruments blending grants and loans helps charitable institutions maximize their social impact. The only restriction on SFIs is the budget constraint dictated by their funders. An SFI sits somewhere between foundations that are funded solely by donors and make pure grants, on the one hand, and commercial banks funded at market prices and supplying loans at market rates, on the other hand. We split the full SFI segment into two categories: first, quasi-foundations delivering grant-plus-loan contracts, which are equivalent to loans with negative interest rates; and second, social banks supplying soft loans, which represent debt with a positive, but below-market, interest rate. Depending on the funders' financial sacrifice, the cost of social screening and the characteristics of the environment, our model endogenously determines the optimal design of the supplied financial products and hence the category to which the SFI belongs.

Ultimately, we aim to build a conceptual framework that makes sense of the various ways of delivering charitable funding in a market economy. In so doing, we explore the continuum of feasible funding solutions, ranging from pure grants to market-rate loans. Our results show that the optimal design of social finance depends on the extent of both information asymmetries and social screening costs. The quasi-foundation strategy of granting loans with negative interest rates is optimal either when information asymmetries are low or when social screening is cheap. Under these circumstances, disregarding "giving plus lending" opportunities in social finance would negatively affect mission fulfillment. In sum, this paper claims that altruistic organizations would be more effective if they adopted a flexible, inclusive approach to funding social enterprises. Combining grants and loans is a fruitful avenue for effective altruism.

The rest of the paper is organized as follows. Section 2 reviews the literature on SFIs. Section 3 presents a novel and comprehensive taxonomy of these institutions. Section 4 introduces the main features of our model. Section 5 solves the model. Section 6 concludes.

## 2. Social Financial Institutions as Delegated Philanthropic Intermediaries

The starting point of our SFI taxonomy lies on the funders' side. The very difference between what we call non-profit and hybrid SFIs lies in the magnitude of their funders' financial sacrifice. The funders of non-profit SFIs are willing to give part of their money away.[4] By contrast, the funders of hybrid SFIs donate no capital, since they require a positive, yet below-market, return. The type of SFI determines its flexibility to choose between costly social screening and preferential credit conditions. Thanks to their softer budget constraint, non-profit SFIs are more flexible. They can target social beneficiaries more accurately than their hybrid counterparts can, and offer them more attractive borrowing conditions. Only non-profit SFIs can supply credit with a negative interest rate.

Philanthropic intermediaries transfer the financial sacrifices of their motivated funders to final beneficiaries. The funders are willing—to varying extents—to forgo capital and/or revenues in exchange for social returns. The motivation of pro-social investors stems from a mixture of drivers, such as ideological obedience and value-based solidarity. Yet, our model works for any underlying social motivation. To document the significance of motivations in social finance, this section lists various drivers identified by the economic literature.

Pro-social behaviors are governed by other-regarding preferences. A large proportion of individuals—between 40% to 60% according to Fehr and Schmidt (2003)—not only care for their own self-interest, but also exhibit concern for the well-being of others (Gintis et al., 2004). So-called strongly reciprocal—or, alternatively, purely altruistic—individuals are prone to sacrifice their own resources in order to encourage positive action or punish negative action (Fehr & Gächter, 2000; Gintis, 2003). Typically, they expect that the norms of fairness will be respected as a result of their actions (Fehr & Schmidt, 1999; Bolton & Ockenfels, 2000; Charness & Rabin,

[4] Non-profit SFIs have simply been ignored in the literature. See Ghosh and Van Tassel (2013) for a recent example.

2002). In finance, strongly reciprocal individuals require a lower financial return when investing in social projects that deserve to be pushed forward (Starr, 2008).

Less noble forces can also be at play in pro-social behavior. According to Bénabou and Tirole (2010), people may exhibit fairness and generosity not because of intrinsically other-regarding motivations, but because of material incentives, such as tax-deductibility, and image-based motivations, such as self-esteem and warm glow behavior. People do not want to appear unfair, either to others or to themselves. The quest for social prestige is part of the reason for engaging in conspicuous, estimable deeds (Glazer & Konrad, 1996).[5] Likewise, people enjoy the warm glow of giving, which avoids self-deception and reinforces self-esteem (Andreoni, 1990). But moral wiggle room makes image-based motivations shallow and circumstantial: People who are generous when the action-outcome relationship is clear tend to change their behavior when the link is less transparent (Dana et al., 2007; Bénabou & Tirole, 2006 & 2011).

A third type of pro-social motivations stem from social identity, i.e. a person's sense of self, derived from perceived membership in a social group (Chen & Li, 2009). With variable salience, social identities are associated with factors such as gender, ethnicity, nationality, social class, and corporate culture. They produce norms and encourage close connections and networking between individuals sharing similar characteristics (McPherson et al., 2001; Akerlof & Kranton, 2000 & 2005). In the case of social lending, identity-sharing between funder and borrower can explain why

[5] This view is confirmed by experiments with volunteer firemen (Carpenter & Myers, 2007), blood donors (Lacetera et al., 2012), and door-to-door fundraising (Della Vigna et al., 2012). Glazer and Konrad (1996) report that anonymous donations are scarce.

preferential credit conditions afforded to the latter bring non-pecuniary utility to the former. In addition, social identification boosts reciprocal and altruistic motivations (Chen & Li, 2009).[6]

The literature has identified real-life value-based financial intermediaries. Social (or ethical) banks target socially-oriented borrowers on the basis of their foundational principles. But the preferential credit conditions they can offer to borrowers are constrained by their funders' financial expected returns. Therefore, both sides of the intermediation process contribute to the success of social banking. Cornée et al. (2016) identify four conditions that preserve social values throughout the intermediation chain. The first two concern lending. First, selectivity implies that the screening of loan applicants must be both financial and social (Cornée & Szafarz, 2014). Second, relationship lending helps to effectively address information asymmetries, which are deemed stronger in social projects than in conventional ones (Cornée, 2017). The last two conditions relate to management principles. First, accountability to motivated funders is facilitated by carrying out simple and transparent financial operations (Cornée et al., 2016). Second, a stakeholder governance structure can help prevent breaches in the moral contract (San Jose et al., 2011).

Interestingly, the success of philanthropy relies on similar conditions. The typical recipients of the philanthropic projects are non-profit organizations, with religious institutions capturing the lion's share of donations (Havens et al., 2007). Charities have to comply with accountability standards about raising and allocating funds (Steinberg, 1994; Jegers, 2015). Transparency requirements are attested by the emphasis on watchdog agency evaluations (Silvergleid, 2003). According to Bowman (2006), charities allocate around 80% of the donations they collect to programs and grants. The remainder goes to operational and screening costs.

[6] In cooperative banks, social identification may enhance membership commitment, whereby equity holders and depositors are ready to receive a return lower than that from other banks (Fulton, 1999).

In sum, the literature reveals that the mission of social finance is adequately theorized as maximizing a social contribution under the return constraint imposed by motivated funders. Our model determines the optimal strategy of an SFI facing costly social screening and maximizing its social contribution. This strategy relies on two instruments: the level of social screening, and the interest charged to borrowers. Before introducing our model, we develop a taxonomy of SFIs intended to guide intuition in later theoretical derivations.

## 3. A Taxonomy of Social Finance Institutions

The loan market is overwhelmingly dominated by profit-maximizing, or 'normal', banks. SFIs are small players with hardly any influence on the market price of credit. They value social contributions such as poverty alleviation, inclusion of disadvantaged people through work (via work integration social enterprises, for example), promoting ethical trade and providing community services such as human and health services, renewable energies, recycling, subsidized housing, education and employment skills training (Dart, 2004; Di Domenico et al., 2010). SFIs also put emphasis on more value-laden issues. We henceforth assume, for simplicity, that all institutions are risk-neutral, and that the demand for funding exceeds the capacity of SFIs, but not that of normal banks. The level of social contribution enabled by SFIs chiefly depends on the extent of the financial sacrifices their funders are willing to make.

We use the following assumptions and notations. In equilibrium, normal banks receive an average repayment of $\mathcal{R}^m$ for each unitary loan granted to their borrowers and pay $(R^m - 1)$ on a dollar to their shareholders. The way $\mathcal{R}^m$ and $R^m$ are determined is irrelevant to our purpose, but we have $\mathcal{R}^m > R^m$ because screening applicants is costly, and some borrowers will inevitably default. At the other end of the social finance spectrum are grant-making foundations, whose social funders are pure donors.

Between the two polar cases, the social funders of SFIs are willing to make financial sacrifices to let the institutions produce a social contribution. They require only a unit return of $0 < R < R^m$. The funders' sacrifice is inversely captured by parameter $\rho$ :

$$R = \rho R^m, \ 0 < \rho < 1 \quad (1a)$$

We define two categories of SFIs depending on the sacrifice made by social funders (see Table 1). When R exceeds zero but falls short of 1 ($0 < \rho < \frac{1}{R^m}$), the social funders require a return smaller than the capital they provide. We refer to the SFIs falling in this category as 'non-profit' because even though the funders recover some of their capital, they cannot derive any profit from the institutions' activity. By contrast, if $R$ exceeds 1, and therefore $\rho \geq \frac{1}{R^m}$ , the institution is a hybrid SFI supplying loans to pay the below-market return expected by its funders.

**Table 1: Characterization of Social Finance Institutions Depending on Funders' Sacrifice**

| Variable \ Institution | Grant-making foundation | **Non-profit SFI** | **Hybrid SFI** | For-profit loan-making bank |
|---|---|---|---|---|
| Funders' expected return: $R$ | $R = 0$ | $\mathbf{0 < R < 1}$ | $\mathbf{1 \leq R < R^m}$ | $R = R^m$ |
| (Inverse of) sacrifice: $\rho$ | $\rho = 0$ | $\mathbf{0 < \rho < \frac{1}{R^m}}$ | $\mathbf{\frac{1}{R^m} \leq \rho} < 1$ | $\rho = 1$ |

To understand how the two categories of SFIs can co-exist, assume that their social funders have utility functions $U(\rho, SC)$, where $\rho$ is defined by Equation (1) $\left(\frac{\partial U}{\partial \rho} < 0\right)$, and $SC$ is the expected social contribution generated by the SFI $\left(\frac{\partial U}{\partial SC} > 0\right)$. Funders can have different relative weights of $\rho$ and $SC$. Moreover, $SC$ depends on $\rho$ since a higher sacrifice from its funders allows the SFI to generate higher social performance, so that: $SC = SC(\rho)$. Potential funders can be grouped in three categories: (i) the funders for which $U(\rho, SC)$ is maximized for $\rho = 0$ $\left(U^* = U\left(0, SC^*(0)\right)\right)$, and among the others $\left(U^* = U\left(\rho^*, SC^*(\rho^*)\right), \rho^* > 0\right)$, (ii) those for which $0 < \rho^* < \frac{1}{R^m}$, and (iii) those for which $\frac{1}{R^m} \leq \rho^* < 1$. These funders will opt for: (i) foundations, (ii) non-profit SFIs, and (iii) hybrid SFIs, respectively.

Beyond the mathematical argument of completeness, we contend that the mere presence of non-profit SFIs is beneficial for generating social contributions. There are two arguments for taking non-profit SFIs seriously. First, in the absence of non-profit SFIs some potential social funders of these institutions (in group ii) might prefer to decline any contribution rather than funding foundations or hybrid SFIs. If it is impossible to form a non-profit SFI, the funders for which $U^* = U\left(\rho^*, SC^*(\rho^*)\right), 0 < \rho^* < \frac{1}{R^m}$ could still opt grant-making foundations or hybrid SFIs. Resorting to such foundations leads to higher social contributions, but that would be feasible only if the diminished utility still exceeds the reservation utility of the potential funders. It is realistic to think this will not always be the case: Potential funders requiring a positive return, however small or partial, might be fiercely opposed to simply giving away their money, thus implying that, for them, $U^* = U\left(0, SC^*(0)\right)$ would fall short of their reservation utility. A similar argument applies to hybrid SFIs. It is therefore plausible that, for some potential funders, only non-profit SFIs are attractive enough to invest in. The alternative is no social investment at all.

Second, from a social standpoint, non-profit SFIs are more efficient than hybrid ones. All else equal, the financial sustainability of SFIs is related to their funders' sacrifice. Grant-making foundations are always viable even though high screening costs would significantly reduce their social contribution. In some market configurations, by contrast, hybrid and non-profit SFIs cannot develop a sustainable business because they are unable to meet the cost of social screening while charging below-market interest rates. Yet, the conditions necessary for the existence of hybrid SFIs are stronger that those required for their non-profit counterparts, which have more generous funders. Foundations aside, there are situations where non-profit SFIs are the only viable institutions. Thus, excluding them from the picture implies excluding any form of social lending to sectors with high information asymmetries on borrowers' social characteristics.

To formalize the argument, we introduce the novel notion of quasi-foundations, which fills the gap between foundations giving grants and social banks supplying soft loans. We use the terms quasi-foundations for SFIs delivering negative-interest loans, and social banks for the other SFIs, which grant loans with positive but preferential rates. While the non-profit/hybrid SFI split originates from the funders' requirement, the quasi-foundation/social bank split relates to the financial products supplied.

Let us consider an SFI providing a single financial product comprising loans and/or grants, and maximizing its social contributions, given the expected return imposed by its funders in Equation (1a). A key policy instrument is the—possibly negative—interest rate charged to their borrowers. Let us denote by $\mathcal{R}$ the repayment required by the SFI on a one-dollar loan:

$$\mathcal{R} = \delta \mathcal{R}^{m}, \quad 0 < \delta < 1 \tag{1b}$$

where $\mathcal{R}^{m}$ is the requested repayment of a one-dollar loan granted by a normal bank. A grant-making foundation would set $\delta = 0$ by definition, whereas otherwise $0 < \delta < 1$. When $\delta = 1$, the borrowers are charged the normal bank's rate. The situation where $\delta \geq 1$ makes the SFI financially unattractive, even for socially-oriented borrowers. Note that $\delta$ need not be equal to $\rho$. Table 2 describes the possible $\rho$ - $\delta$ combinations.[7] In our model, the product supplied by the SFI is determined endogenously, so that the formal proofs of compatibility sketched in Table 2 will be provided in Section 5.2. A key, yet preliminary, message from Table 2 is that only non-profit SFIs can structure themselves as quasi-foundations since the sacrifice made by a hybrid SFI's funders is insufficient to give partial grants to beneficiaries. At best, hybrid SFIs manage to be social banks.

**Table 2: Feasible $\boldsymbol{\rho}$ - $\boldsymbol{\delta}$ Combinations**

| **Interest charged** / **Funder's sacrifice** | $\delta = 0$ | $0 < \delta < \frac{1}{\mathcal{R}^m}$ | $\frac{1}{\mathcal{R}^m} \leq \delta < 1$ | $\delta = 1$ |
|---|---|---|---|---|
| $\rho = 0$ | Grant-making foundation | | | |
| **Non-profit SFI:** $\boldsymbol{0 < \rho < \frac{1}{R^m}}$ | | **Quasi-foundation** | **Social bank** | |
| **Hybrid SFI:** $\boldsymbol{\frac{1}{R^m} \leq \rho < 1}$ | | | **Social bank** | |
| $\rho = 1$ | | | | For-profit loan-making bank |

Quasi-foundations have funders who want to recover their investment but are willing to donate part of it. Depending on all the parameters, primarily the cost of screening, non-profit SFIs can charge either a negative interest rate and be a quasi-foundation $\left(0 < \delta < \frac{1}{\mathcal{R}^m}\right)$ or a positive below-market rate and be a social bank ($\frac{1}{\mathcal{R}^m} \leq \delta < 1$). Social banks can also result from funders requiring a

[7] A formal proof of the impossibilities is provided in Section 4 (Proposition 2).

positive return on their investment, meaning that borrowers charge a positive interest rate. Our model determines the set of viable SFIs and their optimal screening strategy (see Section 5).

## 4. Model Set-Up

In our model, the SFI's mission is to fund projects yielding social value, and it has the expertise to do so. To carry out that mission, it chooses between two tools: social screening and interest rate setting. To focus on its decision, we assume that the loan conditions provided by normal banks are all identical, and that the positive market interest rate they charge takes all costs into account, except the cost of social screening, which concerns SFIs only.

In our single-period model, the SFI maximizes its social contribution under the budget constraint imposed by its funders. Its two decision variables are the interest rate it charges and the effort it exerts in running social screening. The SFI does not differentiate loans and grants *ex ante*. Yet, Table 2 shows that viable hybrid SFIs necessarily follow the social-bank business model. By contrast, non-profit SFIs can be either quasi-foundations granting loans with negative interest rates, or social banks charging positive rates. The sign of the interest rate will emerge endogenously as a consequence of the parameter configuration describing the environment.

### *4.1 Project Characteristics*

The SFI can finance two types of projects: normal projects, which have a low probability of delivering a social contribution, and social projects having a high probability of generating a social contribution. Social projects may be viewed as endeavors set up in pursuit of the common good (Peredo & Chrisman, 2006). Typically, they are supplied by social enterprises and non-profits, which vary substantially in size, scale, and purpose due to the broad spectrum of products and services they cover (Borzaga & Defourny, 2001). The social projects undertaken by mission-driven organizations are less likely than normal projects to be financially successful since their sponsors prioritize social change over private wealth creation (Kerlin, 2006). Profit maximization is not their prime objective (Defourny & Nyssens, 2008 & 2010)—although generating profits can be legitimate (Wilson & Post, 2013). Social enterprises tend to internalize social costs and create positive externalities (Doherty et al. 2014). These characteristics justify the assumption that the *ex ante* probability of default is higher for social projects than for normal ones.

Normal projects are identified by index $n$. Their prevalence in society is $(1-\sigma)$, with $0 \leq \sigma \leq 1$ . Each such project has either a financial return that can repay a loan at market rate $\mathcal{R}^m$ (with probability $p_{nf} < 1$), or no financial return at all (with probability $\left(1-p_{nf}\right) < 1$). The ability to generate the repayment is not affected by the value of $\mathcal{R}^m$ as such, but only by the initiator's ability (reflected by $p_{nf}$) to make the project sustainable under market conditions. In addition, a normal project can also generate either an unintended social contribution (with probability $p_{ns} < 1$), or no social contribution at all (with probability $(1-p_{ns}) < 1$).

The prevalence in society of social projects, identified by index $s$, is $\sigma$. The financial return of a social project is above $\mathcal{R}^m$ with probability $p_{sf} < p_{nf}$ , and zero with an *a priori* probability $\left(1-p_{sf}\right)$. A social return occurs with probability $p_{ss} > p_{ns}$ while $(1-p_{ss})$ is the probability of no social return at all. Together, the two inequalities $p_{ss} > p_{ns}$ and $p_{sf} < p_{nf}$ make sense of the different natures of both project categories. Since the prime intention of social projects is to generate social contributions, their probability of social success $p_{ss}$ exceeds that of normal projects while their probability of financial success $p_{sf}$ is smaller than for normal projects.

For each project, we assume that financial success and social success are drawn from independent distributions. In the whole set of projects, however, financial success and social contributions are negatively correlated since social projects have a relatively higher probability of social success and a higher probability of default. For both types of project, we exclude strategic default and assume limited liability.

A key assumption in our model is that offering loans at below-market interest rates $\mathcal{R} = \delta\mathcal{R}^m$ ($0 \leq \delta < 1$) increases the probability of financial success and makes it easier for the borrowers to meet their financial obligations. The main reason for the first effect is that a lower interest rate reduces the borrower's financial burden. A further argument for SFIs getting better *ex post* repayment scores than regular banks stems from the reciprocity effect observed in social banking. Specifically, borrowers with social projects appreciate dealing with a socially-motivated lender (Barigozzi & Tedeschi, 2015) and spontaneously increase their efforts to meet financial obligations, regardless of their *ex ante* creditworthiness. Cornée and Szafarz (2014) provide evidence that reciprocity in social banking is effective against moral hazard, confirming previous experimental results obtained by Brown and Zehnder (2007), Brown et al. (2009), and Cornée et al. (2012). Reciprocity works only between an SFI and a social borrower.

In our model, the credit market is competitive, and normal banks charge borrowers the market rate. Only SFIs can charge below-market interest rates to their borrowers, because their funders have below-market profitability requirements. We let the parameter $\delta$ stand for the ratio between the interest rate charged by the SFI and the corresponding market rate ($\delta = 0$ for a foundation; $\delta = 1$ for a normal bank).

Softer financial requirements from SFI funders leave borrowers with higher average profits, *ceteris paribus*. The surplus can, in turn, be used for additional social endeavors. We therefore represent the social contribution generated by funded, socially successful projects as $(f - \delta)$, where $f(> 1)$ is the highest possible social contribution generated per unit loan. Like $\sigma$, $f$ is exogenous.

To express that the interest charged on borrowers has a positive impact on the *ex ante* probability of success, we write the probability of financial success as $p_{nf}(\delta)$, where:

$p_{nf}(1) = p_{nf}$ , $\frac{\partial p_{nf}(\delta)}{\partial \delta} < 0$, and $\lim_{\delta \to 0} p_{nf}(\delta) = 1$.

We chose the following simple explicit form for $p_{nf}(\delta)$:

$$p_{nf}(\delta) = p_{nf} + (1-\delta)(1-p_{nf}) = \delta p_{nf} + (1-\delta) = 1 - \delta(1-p_{nf}) \qquad (2)$$

As $\delta \leq 1$, negative values are excluded for $p_{nf}(\delta)$. We impose on $p_{sf}(\delta)$ a similar structure, considering that $p_{sf}(\delta) < p_{nf}(\delta)$:

$$p_{sf}(\delta) = 1 - (1-\varepsilon)\alpha\delta(1-p_{nf}) = 1 - (1-\varepsilon)\alpha\delta(1-p_{nf}) - \delta(1-p_{nf}) + \delta(1-p_{nf})$$

$$= p_{nf}(\delta) + (1 - (1-\varepsilon)\alpha)\delta(1-p_{nf}) \qquad (3)$$

Parameter $\alpha > 1$ reflects that social projects achieve financial success less often than normal ones, whereas parameter $\varepsilon$ describes the positive reciprocal impact on the probability of financial success for social projects borrowing from SFIs. As $p_{sf}(\delta) < p_{nf}(\delta)$ except when $\delta = 0$, $(1-\varepsilon)\alpha$ should exceed 1, implying that the reciprocity effect is low enough to keep the probability of financial success of a social project below that of a normal project charged the same rate. As $p_{sf}(1) = p_{sf} > 0$, we also have: $(1-\varepsilon)\alpha < \frac{1}{1-p_{nf}}$. Finally, we assume that the probabilities of social success, $p_{ns}$ and $p_{ss}$, are insensitive to $\delta$. Consequently, the overall probabilities of financial success and social success of an unidentified project are respectively:

$$p_f(\delta) = (1-\sigma)p_{nf}(\delta) + \sigma p_{sf}(\delta) = p_{nf}(\delta) + \sigma(1-(1-\epsilon)\alpha)(1-p_{nf}) \qquad (4)$$

$$p_s = (1-\sigma)p_{ns} + \sigma p_{ss} \qquad (5)$$

### *4.2 Social Screening*

SFIs do not observe whether projects are social. Neither do they know which projects will achieve financial success. What they do know are the proportions of social and normal projects in the pool of applicants, as well as the probabilities that social and normal projects deliver a social contribution. From a financial standpoint, SFIs face the same information asymmetries as any normal bank, so that the cost of financial screening is already priced into the market rate that serves here as the benchmark rate and is represented as the case where $\delta = 1$ in our model (see Table 2). SFIs depart from normal banks in that they face additional asymmetric information on the projects' social contribution.

To address information asymmetries on the social contribution of the projects, the SFI engages in social screening, intended to assess the social status of the applicant. Screening effort $e \in [0,1]$ leads to probability $\pi \in [0,1]$ of correctly identifying the applicant's social status, where:

$$\pi = \frac{1+e}{2} \qquad 0 \leq e \leq 1 \qquad (6)$$

If no screening effort is exerted ($e = 0$), the selection of loan applicants is random, with probability $\pi = \frac{1}{2}$ for any applicant to be granted a loan. Social screening efforts improve social selectivity, and so provide indirect information on a project's *ex ante* social default probability. The SFI exerting effort $e$ will fund a project delivering a social contribution (be it a normal project or a social project) with probability $\pi$, and a socially unproductive project it fails to recognize as such with probability: $1 - \pi = \frac{1-e}{2}$. Therefore, the expected number of unitary loans granted per application is:

$$P_s = \pi \{(1-\sigma)Prob[\text{social contribution by normal project}] + \sigma Prob[\text{social contribution by social project}]\} + (1-\pi)\{(1-\sigma)Prob[\text{no social contribution by normal project}] + \sigma Prob[\text{no social contribution by social project}]\}$$

$$= (1+e)\frac{(1-\sigma)p_{ns}+\sigma p_{ss}}{2} + (1-e)\frac{(1-\sigma)(1-p_{ns})+\sigma(1-p_{ss})}{2}$$

$$= (1+e)\frac{p_s}{2} + (1-e)\frac{1-p_s}{2} \qquad (7)$$

where $p_s$ is defined by Equation (5). We determine the cost of screening with effort $e$ as:

$$C(e) = \frac{c}{1-e}, \text{ with } 0 < c < 1. \qquad (8)$$

Parameter *c* is normalized to reflect the cost of social screening per currency unit available for both lending and screening. We impose that $c < 1$ because otherwise, even with no effort, the cost would exceed the available amount of funds. $C(e)$ has the following properties:

$$\lim_{e \to 0} C(e) = c \text{ and } \lim_{e \to 1} C(e) = +\infty \qquad (9)$$

Our operationalization of the social screening cost conveys the realistic idea that perfect screening $(e = 1)$ comes at an infinite cost, a feature not captured by the customary expression $C(e) = \frac{ce^2}{2}$. Further, *c* can be interpreted as the semi-fixed cost of screening (for $e = 0$, but also for each available currency unit), making the screening cost a realistic combination of fixed and variable components, another characteristic not implied by specification $C(e) = \frac{ce^2}{2}$. Examining a French social bank, Cornée and Szafarz (2014) establish that labor—a fixed cost—is a substantial part of social screening costs. Evidently, $e$ should not exceed $1 - c$ as otherwise social screening would cost more than the funds available.

### *4.3 Expected Social Contribution*

Conditional on having invested in a project, it follows from Equation (7) that the expected social contribution per unit invested, $SCU,$ equals:

$$SCU = (f - \delta)\frac{\frac{1+e}{2}p_s}{\mathrm{P_s}} = (f - \delta)\frac{(1+e)p_s}{(1+e)p_s+(1-e)(1-p_s)} \tag{10}$$

Appendix 1 demonstrates that, as intuitively expected, $\frac{\partial SCU}{\partial e} > 0$, implying that, *ceteris paribus*, $SCU$ increases in line with social screening efforts. Once the decision to invest has been taken, more social screening logically improves the detection of socially contributing projects. Further, it follows from Equation (10) that, again *ceteris paribus*, a lower interest rate also improves the amount of social contributions: A lower interest rate raises the level of contributions made by a borrowing project.

From each dollar made available by its funders, the SFI invests $\left(1 - \frac{c}{1-e}\right)$ in the projects it selects, which leads to the following expected social contribution:

$$SC = \left(1 - \frac{c}{1-e}\right) SCU = \left(1 - \frac{c}{1-e}\right) (f - \delta)\frac{(1+e)p_s}{(1+e)p_s+(1-e)(1-p_s)} \tag{11}$$

Parameter $\sigma$ represents the prevalence of social projects. A higher value of $\sigma$ means that social projects are more prevalent *ex ante* in the pool of loan applicants. Parameter $\sigma$ can also be interpreted as the SFI's capacity of stimulate self-selection among social projects. According to this view, an SFI able to attract an above-average share of social projects boosts the prevalence $\sigma$, and so increases $p_s$, *f,* and *SC*. One might expect this to occur if some social enterprises were prone to trade with SFIs whose values they share.

Importantly, Equation (11) gives the expected social contribution conditional on having invested in a project, considering that the number of such projects decreases with screening efforts. In the SFI's decision making, the costs associated with selected *and* rejected projects should be taken into consideration, since both divert funds away from investments in the projects considered as social. Moreover, the optimization is subject to the budget constraint imposed by the social funders. The next section addresses this issue and solves the model.

## 5. Optimal Social Screening Effort and Interest Discount

### *5.1 Benchmark Case: Grant-Making Foundations*

As a first step in the discussion of SFIs' optimal business model and social contribution, we consider the programs pursued by foundations, which are sensitive to social performance only. By assumption, foundations receive full donations (i.e. funders request zero financial return) and make pure grants. In doing as much good as possible, their only decision variable is the screening effort that helps select projects with high social contributions. Since grant-making foundations charge no interest, their mathematical program is simpler than those of non-profit and hybrid SFIs.

As displayed in Table 2, grant-making foundations are characterized by $\rho = \delta = 0$. They face a simple trade-off between the social contribution per project funded, which increases with screening effort, and the number of projects they invest in, which decreases with the screening effort. They maximize the expected social contribution, *SCF*, derived from Equation (11) without any constraint:[8]

$$SCF^* = \max_e SCF = \max_e \left(1 - \frac{c}{1-e}\right) \frac{f(1+e)p_s}{(1+e)p_s + (1-e)(1-p_s)} \quad (12)$$

The first factor decreases in *e*, whereas the second increases in both *e* and $p_s$ (see Appendix 1). For any *e* < 1, higher values of $p_s$ imply higher values of *SCF*. We have the following result:

*Proposition 1:*

*The optimal strategy of grant-making foundations involves screening if and only if:*

$$c < \hat{c} = \frac{2(1-p_s)}{3-2p_s} \quad (13)$$

*In this case, the optimal screening effort* $e^*$ *is:*

$$e^* = \frac{K - 2\sqrt{c\left(Kp_s + c(1-2p_s)\right)}}{K + 2c\,(1-2p_s)} \quad (14)$$

*where*: $K = 1 + (1-c)\,(1-2p_s)$ (15)

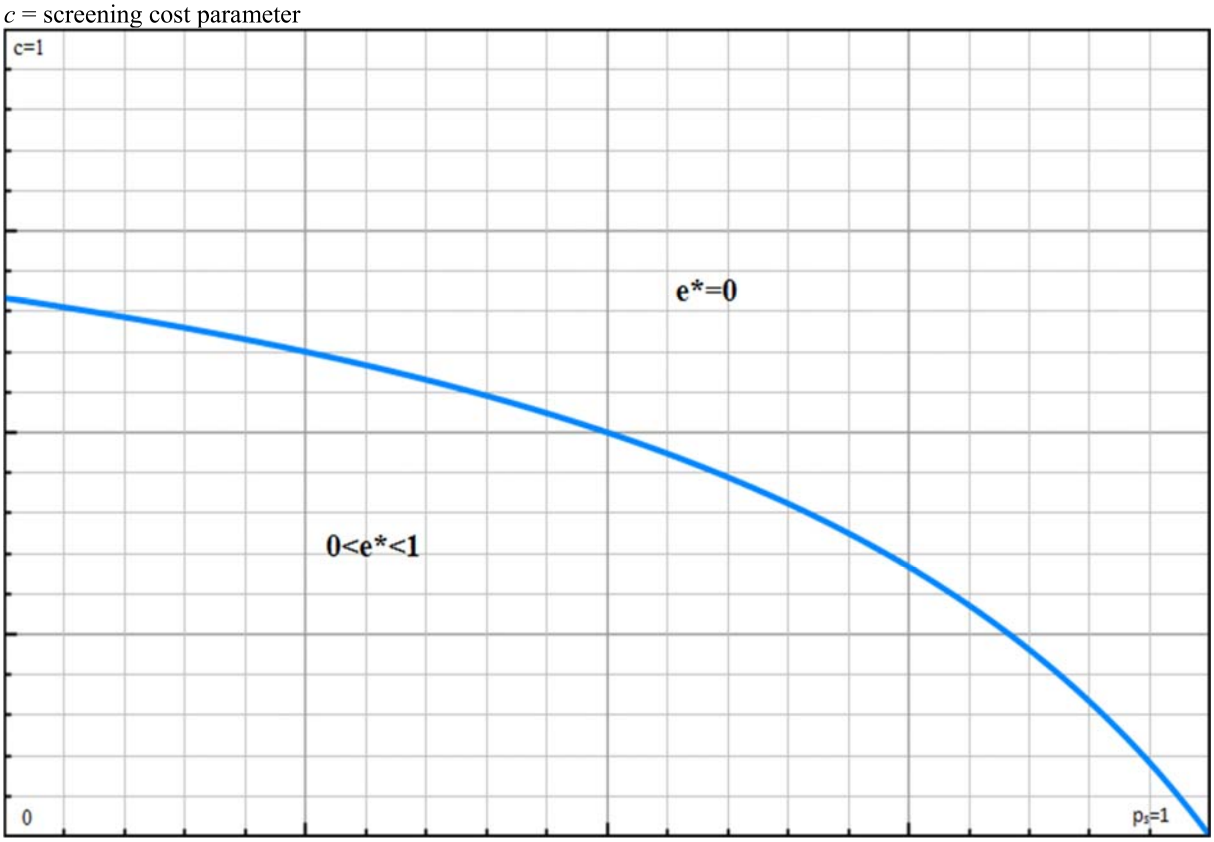
*Proof:* see Appendix 2

Proposition 1 gives the condition on *c* and $p_s$ under which the maximization of *SCF* imposes zero social screening. This condition, represented in Figure 1, takes the form of a lower bound $\hat{c}$ on screening cost *c*. Threshold $\hat{c} \in [0,1)$ in Equation (13) crucially depends on $p_s$, the probability that a random applicant holds a project generating a social contribution. When $p_s$ is relatively high (right side of Figure 1), the optimal strategy of the foundation excludes any screening ($e^* = 0$). This is especially relevant when the cost *c* of social screening is high. By contrast, foundations

[8] To differentiate social contributions generated by foundations from social contributions from SFIs, we label the former SCF and the latter SCSFI.

confronted with low values for $p_s$ and $c$ (bottom-left quadrant of Figure 1) are better-off screening their grant applicants. Appendix 2 also proves that $\hat{c}$ depends negatively on $p_s$.

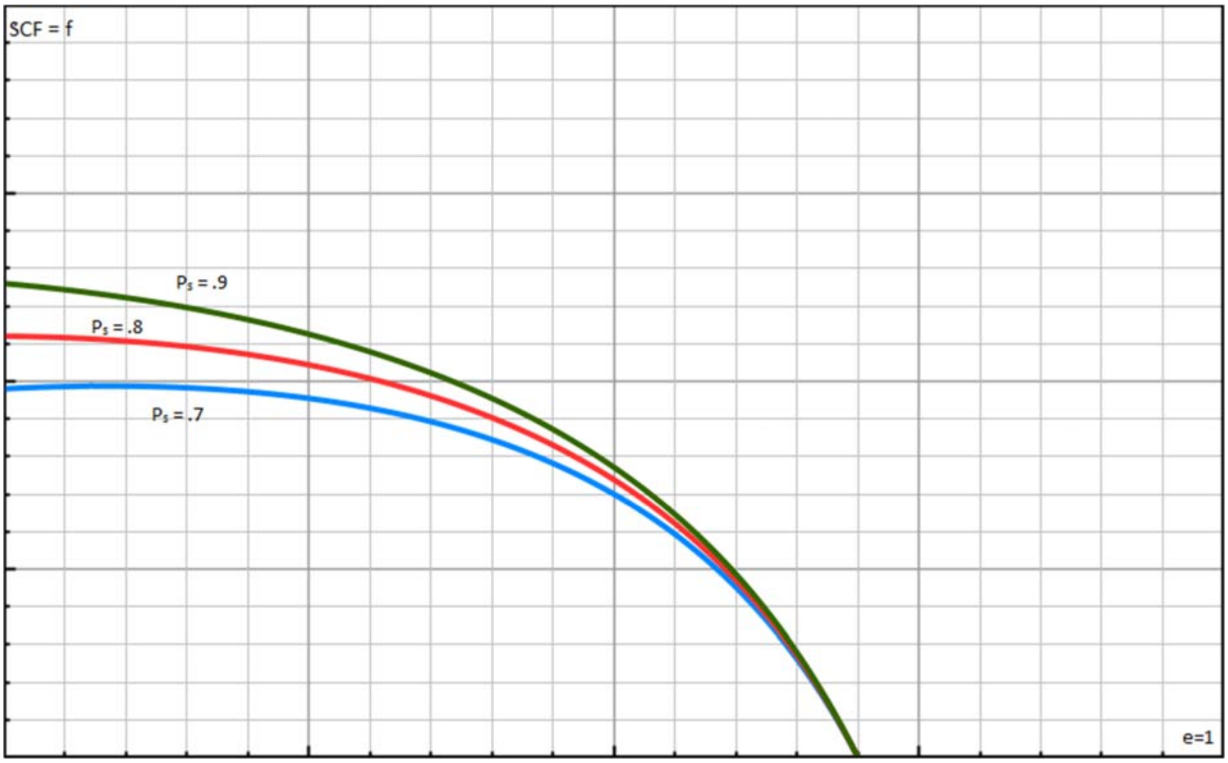
**Figure 1: Foundation's Optimal Strategy: Parameter Configurations With $(\boldsymbol{e}^* > 0)$ or Without $(\boldsymbol{e}^* = \mathbf{0})$ Social Screening**




Practically speaking, since $c$ is hardly modifiable, a foundation willing to reduce the burden of costly social screening might wish to increase $p_s$, that is the *ex-ante* proportion of applicants introducing social projects. The idea is to make redistribution more efficient by generating a higher level of self-selection among applicants (see also Section 4.5). Hypothetically, the success of such a policy depends on the foundation's social mission. Transparent missions aligned with more prevalent social projects have a better chance of making screening unnecessary. In our model, however, all the foundations and SFIs face a fixed sunk cost $c$, since $\lim_{e \to 0} C(e) = c$, regardless of whether their optimal strategy involves screening. One interpretation is that foundations have to maintain a minimal screening capacity to address any changes in environmental parameters. This capacity also serves as a credible threat to fend off potential cheaters.

**Figure 2: Foundation's Social Contribution with High Screening Costs ($c = 0.3$, $p_s \in \{0.7, 0.8, 0.9\}$)**

*SCF*: Foundation's social contribution



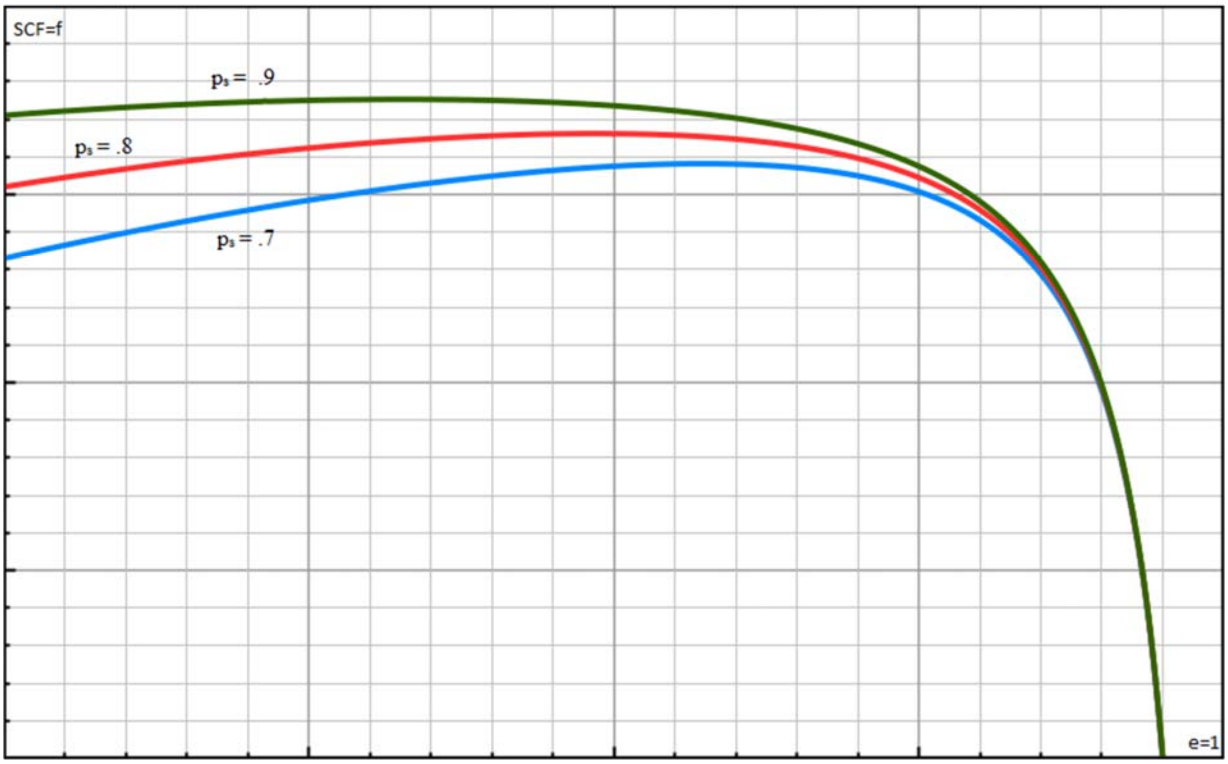

*e*: Social screening effort

Finally, Appendix 3 proves that the optimal screening effort decreases when either the probability of social success increases $\left(\frac{\partial e^*}{\partial p_s} < 0\right)$, or the screening cost increases $\left(\frac{\partial e^*}{\partial c} < 0\right)$. Both results are intuitively appealing: When social benefits are more probable, screening has a lower marginal effect with respect to its cost, whereas when screening costs are large, their marginal cost relative to the marginal social benefit increases. These observations supplement the findings illustrated in

Figure 1 showing that, for large values of $c$ and $p_s$, no social screening is optimal ($e^* = 0$). The simulation results reported in Figures 2 and 3 illustrate the effects. Figure 2 corresponds to the high screening cost of $c = 0.3$. Three values of $p_s$ are used ($p_s \in \{0.7, 0.8, 0.9\}$). For each of them, we determine the corresponding threshold $\hat{c}$ defined by Equation (13). When $p_s$ equals 0.8 or 0.9, $\hat{c}$ stays below $c = 0.3$ (the respective values are 0.286 and 0.167), implying that the optimal level of screening is 0 ($e^* = 0$), so that social contribution *SCF* decreases with $e$, as is visible on Figure 2. Moreover, as the optimal social contribution in Equation (11) is $SCF^* = 0.7fp_s$, it depends positively on $p_s$. Last, for $p_s = 0.7$, one has: $\hat{c} = 0.375 > c = 0.3$ and $e^* = 0.09 > 0$.

**Figure 3: Foundation's Social Contribution with Low Screening Costs ($c = 0.05$, $p_s \in \{0.7, 0.8, 0.9\}$)**

*SCF*: Foundation's social contribution




*e*: Social screening effort

Figure 3 depicts the situation of a low screening cost ($c = 0.05$), which is below threshold $\hat{c}$ for any value of $p_s \in \{0.7, 0.8, 0.9\}$. The optimal screening efforts $e^*$ are 0.33, 0.48, and 0.57, respectively. These values mean that the foundation makes a substantial screening effort before delivering grants.

### 5.2 *The Profitability Constraint of Social Finance Institutions*

In our model, SFIs decide upon both the values for the screening effort *e* and the charged interest rate $\mathcal{R} = \delta\mathcal{R}^{\mathrm{m}}$ to maximize their social contribution while delivering the required expected return $R = \rho R^m (\rho > 0)$ to their funders. Hence, the budget constraint writes:

$$\rho R^m = \frac{\left(1-\frac{c}{1-e}\right)}{P_s}\left[\delta\mathcal{R}^{\mathrm{m}}\left((1-\sigma)[(1+e)p_{ns} + (1-e)(1-p_{ns})]\frac{p_{nf}(\delta)}{2} + \sigma[(1+e)p_{ss} + (1-e)(1-p_{ss})]\frac{p_{sf}(\delta)}{2}\right)\right]$$

or equivalently:

$$P_s\rho R^m = \left(1-\frac{c}{1-e}\right)\left[\delta\mathcal{R}^{\mathrm{m}}\left(P_s p_{nf}(\delta) + \sigma P_{ss}[1-(1-\varepsilon)\alpha]\delta\left(1-p_{nf}\right)\right)\right] \qquad (16)$$

where:

$$P_{ss} = \frac{1+(2p_{ss}-1)e}{2},\ P_{ns} = \frac{1+(2p_{ns}-1)e}{2}, \text{ and } P_s = (1-\sigma)P_{ns} + \sigma P_{ss} \qquad (17)$$

Equation (16) is binding: the RHS being smaller than the LHS would leave the funders financially unsatisfied, whereas the RHS exceeding the LHS would leave funds unused by the SFI. Since $(1-\varepsilon)\alpha > 1$ and $1 - \frac{c}{1-e} \leq 1$, Equation (16) implies that $\delta\mathcal{R}^{\mathrm{m}} \geq \rho R^m$. This in turn explains the impossibility announced in Table 2 for the case in which $0 < \delta < \frac{1}{\mathcal{R}^{\mathrm{m}}} (\Rightarrow \delta\mathcal{R}^{\mathrm{m}} < 1)$ and $\frac{1}{R^m} < \rho < 1 (\Rightarrow \rho R^m > 1)$ because it would violate the necessary condition $\delta\mathcal{R}^{\mathrm{m}} \geq \rho R^m$. In addition, the situation where funders of social banks require zero interest on their capital $\left(\rho = \frac{1}{R^m}\right)$ is incompatible with asking the borrowers to repay exactly the amount borrowed $\left(\delta = \frac{1}{\mathcal{R}^m}\right)$. We have thus proved the next proposition that provides the first formal link between the *ex-ante* types of SFIs, defined by their funders' request (non-profit and hybrid), and the *ex-post* types resulting from the interest charged (quasi-foundation and social bank; see Table 2):

*Proposition 2: The SFI budget constraint implies the following conditions:*

*(i)* $\delta\mathcal{R}^{\mathrm{m}} \geq \rho R^m$;

*(ii)* *Only non-profit SFIs are compatible with quasi-foundations providing loans with negative interest rates;*

*(iii)* *Hybrid SFIs are bound to be social banks and charge their borrowers positive interest rates.*

The distinction between social banks and quasi-foundations relies on parameter $\delta$. Quasi-foundations require negative interest corresponding to $0 < \delta < \frac{1}{\mathcal{R}^m}$. By contrast, social banks charge a positive but below-market rate with: $\frac{1}{\mathcal{R}^m} \leq \delta < 1$. The value of $\delta$ is always smaller for quasi-foundations than for social banks. To make the case for quasi-foundations, we must examine how a lower $\delta$ can make it easier to achieve higher social performance. Put differently, how much better can quasi-foundations perform socially than social banks do?

So far, however, it remains unclear whether a higher $\delta$ (a positive interest rate with higher default probability) or a lower $\delta$ (a partial donation, implying a negative interest rate, combined with a lower default probability) are feasible, let alone optimal. Evidently, when the funders require some return on investment ($\rho > 0$), the SFI borrowers must repay ($\delta > 0$), even if the social screening cost is zero ($c = 0$). Only foundations can distribute pure grants. Determining how the extra budget made available to quasi-foundations, relative to social banks, is used boils down to examining the link between the two decision variables: interest charged $\delta$ and screening effort $e$, at the optimum. The intuition is that the sign and intensity of this link depend on the population parameters. Proposition 3 confirms that intuition and gives a sufficient condition for a positive correlation between the two variables when the budget constraint is met:

*Proposition 3: Under the budget constraint (16), the following condition:*

$$p_{nf}\left(1-\sigma\frac{p_{ss}}{p_s}\right)>\frac{1}{2} \tag{18}$$

*implies that* e *and* δ *are positively correlated.*

*Proof:* see Appendix 4

The circumstances of condition (18) are realistic as they correspond to profitable normal projects (high values of $p_{nf}$), relatively infrequent social project (low values of $\sigma$), and low social return of normal projects (low values of $p_{ns}$). All these conditions tend to make screening more needed. Under condition (18), more generous funders allow quasi-foundations to screen applications more effectively than social banks do. From now on, we simplify the computations by assuming this condition to hold. As Proposition 3 states, working under this condition makes $e$ and $\delta$ positively related.

### *5.3 To Screen or Not To Screen?*

All SFIs maximize their social contribution by funding projects. But, contrary to foundations, they must meet the budget constraint in Equation (16), resulting from the return requested by their funders. Hence, in addition to $e$, the screening effort, SFIs have a second decision variable, $\delta$, driving the interest charged to borrowers ($\delta\mathcal{R}^{\mathrm{m}}$). The lower the value of $\delta$, the higher the expected financial surplus of the financially successful projects and the expected social contributions from the social project. Hence, the program for SFIs writes:

$$SCSFI^{*} = \max_{\substack{e\in[0,1)\\ \delta\in(0,1)}} \left(1-\frac{c}{1-e}\right)\frac{(f-\delta)(1+e)p_s}{(1+e)p_s+(1-e)(1-p_s)} \tag{19}$$

subject to Equation (16).

Proposition 4 states that $c > \hat{c}$ is a sufficient condition for the "no social screening" decision to be optimal, assuming that the financial constraint can be met:

*Proposition 4: Under the budget constraint (16):*

$$c > \hat{c} = \frac{2(1-p_s)}{3-2p_s} \Longrightarrow e^{*} = 0$$

*Proof:* see Appendix 5

Comparing Propositions 1 and 4 shows that the threshold on screening cost, $\hat{c}$, under which zero social screening is optimal is the same for SFIs and for grant-making foundations. Yet, this time the condition holds under the budget condition. Thus the question is whether (or when) there is an admissible value of δ that makes Equation (16) hold. Proposition 5 gives a necessary condition on $\rho$ for the existence of that value.

*Proposition 5: If* c > ĉ, *a necessary condition for a screening-free optimal SFI is:*

$$\frac{\rho R^{m}}{(1-c)\mathcal{R}^{m}} \leq \frac{1}{4L} \tag{20}$$

*where*: $L = \left(1-p_{nf}\right)[1-\sigma(1-(1-\varepsilon)\alpha)] \in (0,1)$ (21)

*and in this case, the optimal interest rate to be charged to borrowers is:*[9]

$$\delta^* = \frac{1-\sqrt{\frac{1-4\rho L R^m}{(1-c)\mathcal{R}^m}}}{2L} \tag{22}$$

*Proof:* see Appendix 6

In sum, when $c > \hat{c}$, the only solution—if any—involves zero screening. But, if equation (20) does not hold, there is no solution. Proposition 5 shows that the absence of a solution can result from a high value of $\rho$, the return required by the SFI's funders, which is nevertheless lower than the market return. All else equal, quasi-foundations are affected less than social banks by this impossibility result.

Condition (20) can be rewritten as:

$$\rho \leq \hat{\rho} = \frac{(1-c)\mathcal{R}^m}{4R^m(1-p_{nf})[1-\sigma(1-(1-\varepsilon)\alpha)]} .$$

The limiting case of $\rho = \hat{\rho}$ corresponds to the minimal financial sacrifice needed from the funders to make an SFI viable. The SFI's technology is then screening-free and the interest charged its borrowers is $\delta^* = \frac{1}{2L}$. To further interpret Proposition 5, we rewrite the limiting value $\hat{\rho}$ as:

$$\hat{\rho} = \frac{1}{R^m} \cdot \left[\frac{(1-c)\mathcal{R}^m}{4(1-p_{nf})[1-\sigma(1-(1-\varepsilon)\alpha)]}\right]$$

---

[9] When $1 - L < \frac{\rho R^m}{(1-c)\mathcal{R}^m} < \frac{1}{4L}$ and $L > \frac{1}{2}$, there is also a second critical value: $\delta^* = \frac{1+\sqrt{\frac{1-4\rho L R^m}{(1-c)\mathcal{R}^m}}}{2L}$ but substitution in the objective function (23) shows that equation (22) gives the optimum.

From Table 1 we learn that if the second factor exceeds one, the case $\rho = \hat{\rho}$ corresponds to a hybrid SFI, and otherwise to a non-profit SFI. The first situation occurs when $p_{nf}$ is sufficiently high, combined with a $\sigma$ sufficiently small, which matches assumption (18) derived for Proposition 3. In that case, we have $\frac{(1-c)\mathcal{R}^m}{4L} > 1$, implying that $\frac{1}{\mathcal{R}^m} < \frac{1}{(1-c)\mathcal{R}^m} < \frac{1}{4L} < \frac{1}{2L}$, and showing that, in accordance with Table 2, the hybrid SFI acts as a social bank. If $\rho = \hat{\rho}$ and the SFI is a non-profit, then $\frac{(1-c)\mathcal{R}^m}{4L} < 1$, and it results in $\frac{(1-c)}{2}\frac{1}{2L} < \frac{1}{\mathcal{R}^m}$. There are two possible cases depending on whether $\frac{1}{2L}$ is larger[10] or smaller than $\frac{1}{\mathcal{R}^m}$. In the first case the non-profit SFI acts as a social bank, in the second as a quasi-foundation.

Let us now consider the situation where the optimal level of social screening is strictly positive $(e^* > 0)$. In this case, solving the optimization problem (19) leads to further analytical complexity. We therefore look at special cases, featured as a conjecture, to show that greater financial sacrifices by the SFI's funders do indeed result in better social performance, without having to explicitly solve the program. A comprehensive analysis will be made in a subsequent paper. The main technical issue relates to the generality of the reciprocity effect formalized by $(1-\varepsilon)\alpha$, which influences the probability of success $p_f(\delta)$ in Equation (4). By construction, we have $1 < (1-\varepsilon)\alpha < \frac{1}{1-p_{nf}}$. To prove that any additional financial sacrifice by the SFI's funders results in a social improvement, we rely on a continuity argument. We develop in separate lemmas the argument for the two limiting cases: $(1-\varepsilon)\alpha \to 1$ and $(1-\varepsilon)\alpha \to \frac{1}{1-p_{nf}}$ and then invoke the continuity of the objective function with respect to $(1-\varepsilon)\alpha$ to derive the final conjecture.

[10] As $c > \hat{c}$, the value of $(1-c)$ is rather small here.

*Lemma 1: If* $(1-\varepsilon)\alpha \rightarrow 1$, *then* $\frac{\partial SCSFI^*}{\partial \rho} < 0$

*Proof of Lemma 1:*

If $((1-\varepsilon)\alpha \rightarrow 1)$, the budget constraint (16) becomes:

$$\rho R^m = \left(1 - \frac{c}{1-e}\right)\left[\delta \mathcal{R}^{\mathrm{m}}\left(1 - \delta\left(1 - p_{nf}\right)\right)\right] \tag{23}$$

Assume, for a given $\rho$, that we have values of *e* and $\delta$ that meet Equation (23) and maximize SCSFI (Equation (19)). Lowering $\rho$ can be compensated in Equations (23) by decreasing $\delta$ (leaving *e* unchanged), as Proposition 3 leads us to assume that $p_{nf} > \frac{1}{2}$, from which:

$$\frac{\partial}{\partial \delta} \delta \mathcal{R}^{\mathrm{m}}\left(1 - \delta\left(1 - p_{nf}\right)\right) = \mathcal{R}^{\mathrm{m}}\left(1 - 2\delta\left(1 - p_{nf}\right)\right) < 0$$

A decreasing $\delta$ results in an increasing social contribution, as becomes clear from Equation (19): The additional financial sacrifice by the SFI's funders results in a social improvement.

QED

*Lemma 2: If* $(1-\varepsilon)\alpha \rightarrow \frac{1}{1-p_{nf}}$, *then* $\frac{\partial SCSFI^*}{\partial \rho} < 0$

*Proof of Lemma 2:* see Appendix 7.

*Conjecture: If a solution exists, then* $\frac{\partial SCSFI^*}{\partial \rho} < 0$.

Our model supports the relevance of the classification proposed in Table 2, suggesting that the lack of quasi-foundations generates a market gap in social finance. Since quasi-foundations are SFIs that require only partial reimbursement of loans, the gap is potentially prejudicial. A social finance market without quasi-foundations can leave part of the pro-social supply of funds unused. If our conjecture is validated, a segment of generous non-profit funders—those with a low but non-zero $\rho$—can end up funding social banks even though they could have optimally funded quasi-foundations that are more socially efficient.

## 7. Conclusion

The main innovation of this paper is to show how the social contribution made by financial institutions depends on their funders' return requirements. We also emphasize that funders' financial sacrifice is a prerequisite for achieving a much higher social outcome than conventional banks, which can accidently produce social returns. Only below-market funding opportunities enable SFIs to deliver significant social contributions by choosing optimally between social screening and preferential lending rates. In addition, we show that social screening is necessary to attract social funders who would otherwise be reluctant to trade financial sacrifices for social outcomes. In this regard, our model confirms that social screening is key to the credibility and accountability of social institutions (Laufer, 2003).

Yet, social screening sometimes proves counterproductive. Our model establishes the conditions under which this happens. Intuitively, these conditions combine high screening costs and high probabilities of random social outcomes. Since the cost of screening depends on the social dimension to be investigated and the information asymmetry in the targeted credit market, SFIs operating with expensive screening technology may assign the bulk of their financial surplus to interest-rate rebates. For example, microfinance institutions, which serve large pools of similar, needy borrowers, can find it optimal to cut social screening to the core. More generally, our theoretical model helps rationalize the institutional trajectories and strategies of alternative financial organizations, such as social banks, credit cooperatives, microfinance institutions, and crowdfunding platforms.

But do we really need social financial institutions? Is it not enough to have two polar types of institutions, namely philanthropic foundations and commercial banks? By transposing Modigliani and Miller's (1958) argument to social finance, one may argue that socially-minded investors can separate their profit-maximizing investment strategies from their impact-maximizing charitable donations. This argument is close in spirit to the tenets of effective altruism that recommends doing the "most good you can do" (Singer, 2015). Yet, the institutional relevance of SFIs should not be underestimated. In addition to being able to address social and financial information asymmetries, SFIs are useful in coordinating social funders. The task of coordination that SFIs have to undertake to address the indivisibility of investment projects is even more decisive and arduous than that of mainstream financial intermediaries. In addition to synchronizing funders’ financial requirements, as mainstream financial intermediaries do (Diamond, 1984), SFIs coordinate an unmet heterogeneous social demand for value-based services. By filling this gap, SFIs are welfare-increasing, both for socially-minded individuals and for society as a whole.

Our theory of social finance helps explain why institutions supply debt at below-market interest rates. Aside from pure donations made by foundations, our model formalizes the decision-making process of the burgeoning yet understudied realm of social banking. It also reveals the conditions for the existence of quasi-foundations, which require only partial reimbursement of the capital they supply. Their presence in the market for philanthropic funding can significantly increase global welfare, especially where pure foundations would be too restrictive to attract socially-minded donors, whose alternative option would be to make no donation at all. Those donors would be better-off—as would society as a whole—with quasi-foundations in the market.

In fact, surrogates for quasi-foundations already exist in public policies, albeit to a modest extent. The European Union uses innovative "blending financial instruments" as a way to enhance the efficiency of its poverty reduction agenda.[11] However, the development of such quasi-foundations—especially those that could take the form of digital crowdfunding platforms—is hampered by regulatory obstacles. The lending/giving products that quasi-foundations could offer do not fall into any category set by the prevailing tax and accounting rules, which are based on the deemed nature of the funder's financial interest (Heminway, 2017). Giving, lending, or investing transactions are governed by distinct operating provisions aimed at protecting funding interests from various threats, primarily financial risk. In this context, social enterprises are forced to secure access to capital by resorting to *ad hoc* strategies that are often unsatisfactory. Sometimes they create dual legal structures to accommodate resources stemming simultaneously from commercial equity and deductible donations (Di Domenico et al. 2010; Doherty et al. 2014). Yet, digital solutions are technologically sound tools for ensuring that capital will flow to the fast-growing sector of the social economy. Provided that the regulatory framework allows hybrid forms of funding, quasi-foundations offer promising avenues to foster and support the hybridization of innovative entrepreneurial forms.

Our results contrast with the claim that socially responsible investments (SRIs) can be as profitable as non-SRI ones. From a theoretical standpoint, our model of social finance departs from the SRI paradigm in several respects. First and foremost, our theory is anchored in an information asymmetry setting. SFIs are delegated financial intermediaries that grant loans on behalf of individual investors, and so specialize in gathering private information to assess borrowers' social deeds. Conversely, SRI funds are available on regular financial markets, and their valuation relies

[11] See: https://ec.europa.eu/europeaid/policies/innovative-financial-instruments-blending_en for more details.

on public information released in corporate social responsibility (CSR) reports and ratings. Second, the social finance approach diverges from SRI in the way it relates the cost of capital to social responsibility. The SRI literature, both theoretical and empirical, is still unclear about how CSR and financial performance are linked (Margolis & Walsh, 2012). In social finance, by contrast, the social contribution is unambiguously conditional on investors making financial sacrifices, thus enabling SFIs to access capital at lower cost. This result echoes growing strands of scholarship showing that firms generating more negative externalities incur a higher a cost of capital (El Ghoul et al., 2011; Chava, 2014).

Our work is only the first step towards a more comprehensive theory of social finance. One main limitation of our model stems from imposing one-period financial constraints, which may underestimate social contributions in the long run. The typical legal status of SFIs stipulates that they should retain a significant portion, if not all, of their profits in reserve to be reinvested in subsequent periods. Likewise, the capital that SFIs allocate may also spawn multi-periodic social effects (Borzaga & Defourny, 2004). The promising developments of our model include a dynamic setting acknowledging the long-term perspective in social finance, which may contrast with excessive short-termism in mainstream financial intuitions (Dallas, 2011). Another fruitful avenue for further investigation is determining the market characteristics under which sustainable social projects denied by normal banks can benefit from funding by SFIs. More generally, one could scrutinize the robustness of the credit-rationing approach *à la* Stiglitz and Weiss (1981) under the paradigm of social finance.

Overall, SFIs make a difference by materializing individual investors' social preferences in financial terms, on the one hand, and by serving social projects, on the other. Acting on such principles of value-based intermediation may indirectly incentivize normal projects to limit their negative externalities or foster their social contribution in order to tap less onerous capital (Cheng et al., 2014). In this regard, too, SFIs participate in a virtuous circle of aligning financial services with societal benefits.

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**Appendix 1: Derivatives of *SCU***

From Equation (10), we have:

$$\frac{\partial SCU}{\partial e} = \frac{(f-\delta)2\, p_s(1-p_s)}{[(1+e)p_s+(1-e)(1-p_s)]^2} > 0$$

Likewise:

$$\frac{\partial SCU}{\partial p_s} = \frac{(f-\delta)(1-e^2)}{[(1+e)p_s+(1-e)(1-p_s)]^2} > 0 \text{ for } e < 1$$

**Appendix 2: Proof of Proposition 1**

Following Equation (12), the problem is:

$$\max_e SCF = \max_e \left(1 - \frac{c}{1-e}\right) \frac{f(1+e)p_s}{(1+e)p_s+(1-e)(1-p_s)}$$

$$\frac{\partial SCF}{\partial e} = f\left(1 - \frac{c}{1-e}\right) \frac{2\, p_s(1-p_s)}{[(1+e)p_s+(1-e)(1-p_s)]^2} + \frac{fc(1+e)p_s}{(1-e)^2[(1+e)p_s+(1-e)(1-p_s)]}$$

$$= f\,\frac{[(1-e)^2-c(1-e)]\,[2p_s(1-p_s)]+(1-e)c(1+e)p_s}{(1-e)^2[(1+e)p_s+(1-e)(1-p_s)]^2} = f\,\frac{[(1-e)^2-c(1-e)]\,[2p_s(1-p_s)]+(1-e)^2cp_s}{(1-e)^2[(1+e)p_s+(1-e)(1-p_s)]^2}$$

Since the denominator, *f*, and $p_s$ are positive, the sign of $\frac{\partial SCF}{\partial e}$ is the sign of a degree-two polynomial, say $A + Be + Ce^2$, where:

$$A = -c + 2(1-c)(1-p_s)\,,\ B = 2[-1-(1-c)(1-p_s)]\,,\ C = 2(1-p_s) + c(1-2p_s)$$

(i) For $A > 0 \iff c < \hat{c} = \frac{2(1-p_s)}{3-2p_s}$, the social contribution *SCF* increases at $e = 0$.

Hence, since $\lim_{e\to 1} SCF = -\infty$, there must be a value $e^* \in (0,1)$ maximizing $SCF$. As $SCF$ is increasing and positive for $e = 0$, its maximal value must be positive as well: $e^* > 0$ and $SCF^* = SCF|_{e=e^*} > 0$.

(ii) For $A \leq 0 \iff c \geq \hat{c} = \frac{2(1-p_s)}{3-2p_s}$, we have:

$$SCF|_{e=0} - SCF|_{e\in(0,1)} = p_s(1-c) - p_s\left(1 - \frac{c}{1-e}\right)\frac{1+e}{1+e(2p_s-1)} > 0$$

because this expression has the same sign as:

$$2p_s - 2 + c(3 - 2p_s) > 2p_s - 2 + 2 - 2p_s = 0$$

Therefore, $SCF|_{e=0} > SCF|_{e\in(0,1)}$ implying that:

$e^* = 0$ and $SCF^* = SCF|_{e=0} > 0$.

In addition, we have: $\frac{\partial \hat{c}}{\partial p_s} = \frac{-2}{(3-2p_s)^2} < 0$

In case (i), we derive the expression for $e^* > 0$ by rewriting the first-order condition as:

$$[K + 2c(1 - 2p_s)]e^2 - 2Ke + (K - 2c) = 0$$

where $K = 1 + (1 - c)(1 - 2p_s) > 0$ since $|(1 - c)(1 - 2p_s)| < 1$.

In addition, $c < \hat{c} = \frac{2(1-p_s)}{3-2p_s} \Rightarrow K - 2c = -c - 2(1 - c)(1 - 2p_s) > 0$

The above first order condition is a quadratic equation. Its solutions are:

$$\frac{K \pm 2\sqrt{2c(1-p_s)}}{K+2c(1-2p_s)}$$

Since $2c(1 - p_s) > 0$ and $e^*$ must be smaller than one, we obtain:

$$e^* = \frac{K-2\sqrt{2c(1-p_s)}}{K+2c(1-2p_s)}$$

QED

**Appendix 3: Proof that $\boldsymbol{c < \hat{c}} \Rightarrow \frac{\partial e^*}{\partial p_s} < 0$ and $\frac{\partial e^*}{\partial c} < 0$**

We know that: $\boldsymbol{c < \hat{c} = \frac{2(1-p_s)}{3-2p_s}} \Longrightarrow e^* = \frac{K-2\sqrt{2c(1-p_s)}}{K+2c(1-2p_s)}$, where $K = 1 + (1 - c)(1 - 2p_s)$.

Since $e^*$ fulfills the first order condition, we have from Appendix 2:

$$-c + 2(1 - c)(1 - p_s) + 2[-1 - (1 - c)(1 - p_s)]\, e^* + [2(1 - p_s) + c(1 - 2p_s)]e^{*2} = 0$$

Deriving this condition w.r.t. *c* results after some manipulations into:

$$\frac{\partial e^*}{\partial c} = \frac{1+2(1-p_s) - 2(1-2p_s)\, e^* - (1-2p_s)e^{*2}}{2(-2+c+2p_s-2cp_s)+2(-2p_s+2+c-2cp_s)e^*}$$

(i) We first prove that the denominator of $\frac{\partial e^*}{\partial c}$, which is also the denominator of $\frac{\partial e^*}{\partial p_s}$, is negative.

We have: $-2 + c + 2p_s - 2cp_s = -1 + (1-c)(2p_{s-1}) < 0$ because $|1-c| < 1$ and $|2p_{s-1} - 1| < 1$, and since $0 < e^* < 1$

$$(-2 + c + 2p_s - 2cp_s) + (-2p_s + 2 + c - 2cp_s)e^* < (-2 + c + 2p_s - 2cp_s) + (-2p_s + 2 + c - 2cp_s) = 2\,c(1 - 2p_s)\begin{cases} < 0 & if p_s > 0.5 \\ \geq 0 & f p_s \leq 0.5 \end{cases}$$

Thus, if $p_s > 0.5 \Rightarrow \frac{\partial e^*}{\partial c} < 0$.

If $p_s \leq 0.5$, we use inequalities $e^* < (1-c)$ and $c < 1$ to derive:

$$\begin{aligned}(-2 + c + 2p_s - 2cp_s) &+ (-2p_s + 2 + c - 2cp_s)e^* \\ &< (-2 + c + 2p_s - 2cp_s) + (-2p_s + 2 + c - 2cp_s)(1-c) \\ &= 2cp_s - c^2 + 2c^2 p_s < 0\end{aligned}$$

(ii) Next, the numerator of $\frac{\partial e^*}{\partial c}$ positive since together $|1 - 2p_s| < 1$ and $|1 - 2e^* + e^{*2}| < 2$ imply that:

$$1 + 2(1 - p_s) - 2(1 - 2p_s)\, e^* - (1 - 2p_s)e^{*2} = 2 + (1 - 2p_s)(1 - 2e^* + e^{*2}) > 0$$

Let us now turn to the derivative of $e^*$with respect to $p_s$:

$$\frac{\partial e^*}{\partial p_s} = \frac{2(1-c) - 2(2-2c)\, e^* - (-2-2c_s)e^{*2}}{2(-2+c+2p_s-2cp_s) + 2(-2p_s+2+c-2cp_s)e^*}$$

We already know that the denominator is negative. As for the numerator, we have:

$$\begin{aligned}2(1-c) - 2(2-2c)\, e^* - (-2-2c_s)e^{*2} &= (2-2c)(1 - 2e^* + e^{*2}) \\ &= 2(1-c)(1-e^*)^2 > 0\end{aligned}$$

QED

**Appendix 4: Proof of Proposition 3**

Equation (16) can be written as follows:

$$\rho R^m = \left(1 - \frac{c}{1-e}\right)\left[\delta \mathcal{R}^{\mathrm{m}}\left(p_{nf}(\delta) + \sigma \frac{P_{ss}}{P_s}[1-(1-\varepsilon)\alpha]\delta\left(1-p_{nf}\right)\right)\right] \quad (26)$$

It is equivalent to:

$$\frac{P_{ss}}{P_s} = \frac{1+(2p_{ss}-1)e}{1+[(1-\sigma)(2p_{ns})+\sigma(2p_{ss})-1]e} = \frac{E+A}{E+B}$$

where: $E = \frac{1}{e},\ A = 2p_{ss} - 1,\ B = 2p_{ns}(1-\sigma) + 2p_{ss}\sigma - 1.$

As $p_{ns} < p_{ss}$ we have: $1 \geq A > B \geq -1$, making both numerator and denominator of $\frac{P_{ss}}{P_s}$ positive.

To see how $\frac{P_{ss}}{P_s}$ evolves with changing values of *e*, we derive it w.r.t. *E*. This results in a positive denominator and $B - A$ in the numerator, which is always negative. So, increasing values of *e*, implying decreasing *E*, implies increasing $\frac{P_{ss}}{P_s}$, or the expression between square brackets on the RHS of Equation (26) to decrease, as $[1-(1-\varepsilon)\alpha]$is negative. As $\left(1-\frac{c}{1-e}\right)$ also decreases in *e*, the RHS is decreasing in *e* while the LHS is independent from *e* (and $\delta$). Hence, to restore the equality (26), the impact of $\delta$ on the RHS should be positive. Partially deriving the RHS of Equation (26) gives, ignoring factors not affected by $\delta$:

$$1 - 2\delta(1-p_{nf}) + 2\sigma\frac{P_{ss}}{P_s}[1-(1-\varepsilon)\alpha]\delta\left(1-p_{nf}\right)$$

$$> 1 - 2\delta(1-p_{nf}) + 2\sigma\frac{P_{ss}}{P_s}\left[1-\frac{1}{1-p_{nf}}\right]\delta\left(1-p_{nf}\right)$$

$$= 1 - 2\delta\left(1-p_{nf}\right) - 2\sigma\frac{P_{ss}}{P_s}\left[p_{nf}\right]\delta$$

This expression is linear in $\delta$ and positive for $\delta$ = 0. For $\delta$ = 1, this expression becomes:

$$1 - 2(1 - p_{nf}) - 2\sigma \frac{P_{ss}}{P_s} [p_{nf}]$$

which is positive if, as a sufficient condition,

$$p_{nf}\left(1 - \sigma \frac{P_{ss}}{P_s}\right) > \frac{1}{2}$$

If this holds for $e = 1$, it will also hold for $0 < e < 1$, as $\frac{P_{ss}}{P_s}$ is decreasing in *e*.

**Appendix 5: Proof of Proposition 4**

Following the logic of Appendix 2 and acknowledging that $\delta$ is implicitly a function of *e* through the budget constraint, we have:

$$\frac{\partial SCFI}{\partial e} = (f - \delta) \frac{[(1-e)^2 - c(1-e)]\,[2p_s(1-p_s)] + (1-e)^2 c p_s}{(1-e)^2[(1+e)p_s + (1-e)(1-p_s)]^2} - \frac{d\delta}{de} \frac{(1+e)p_s}{(1+e)p_s + (1-e)(1-p_s)}.$$

From Proposition 3 we know that $\frac{d\delta}{de} > 0$, so that the first term of $\frac{\partial SCFI}{\partial e}$ being negative is sufficient for $e^* = 0$ to be optimal. As $(f - \delta) > 0$, the result follows from a proof identical to the one in Appendix 2.

**Appendix 6: Proof of Proposition 5**

We start the proof with a technical lemma:

Lemma:

If $L = (1 - p_{nf})[1 - \sigma(1 - (1 - \varepsilon)\alpha)]$ then, under assumptions in Sections 4.3:

(i) $0 < L < 1$

(ii) $1 - L < L < \frac{1}{4L}$

Proof of the Lemma:

(i) We know from Section 4.3 that $L > 0$, and $(1 - \varepsilon)\alpha < \frac{1}{1 - p_{nf}}$, implies that $L < 1$.

(ii) $1 - L - \frac{1}{4L} = \frac{4L - 4L^2 - 1}{4L} \leq 0$ (being equal to zero for $L = \frac{1}{2}$)

QED

Let us now prove Proposition 5. For $e^* = 0$, Equation (16) becomes:

$$P_s \rho R^m = (1-c)\left[\delta \mathcal{R}^{\mathrm{m}}\left(P_s p_{nf}(\delta) + \sigma P_{ss}[1-(1-\varepsilon)\alpha]\delta\big(1-p_{nf}\big)\right)\right]$$

Or equivalently using $L$ defined in the lemma:

$$\frac{\rho R^m}{(1-c)\mathcal{R}^{\mathrm{m}}} = \delta\big[1-\delta\big(1-p_{nf}\big) + \sigma[1-(1-\varepsilon)\alpha]\delta\big(1-p_{nf}\big)\big] = \delta(1-L\delta)$$

It is a quadratic equation in $\delta$:

$$\big(1-p_{nf}\big)[1-\sigma(1-(1-\varepsilon)\alpha)]\delta^2 - \delta + \frac{\rho R^m}{(1-c)\mathcal{R}^{\mathrm{m}}} = 0$$

Its discriminant is:

$$\Delta = 1 - 4\big(1-p_{nf}\big)[1-\sigma(1-(1-\varepsilon)\alpha)]\frac{\rho R^m}{(1-c)\mathcal{R}^{\mathrm{m}}}$$

which is positive if:

$$\frac{\rho R^m}{(1-c)\mathcal{R}^{\mathrm{m}}} < \frac{1}{4\big(1-p_{nf}\big)[1-\sigma(1-(1-\varepsilon)\alpha)]} = \frac{1}{4L}.$$

We have:

$$\delta(1-L\delta)|_{\delta=0} = 0\ ,\ \left.\frac{\partial[\delta(1-L\delta)]}{\partial\delta}\right|_{\delta=0} = 1\text{, and } \frac{\partial^2[\delta(1-L\delta)]}{\partial\delta^2} = -2L < 0.$$

Consequently, the first derivative of $[\delta(1-L\delta)]$ will decrease from $\delta = 0$ onwards and eventually become negative. As $\delta$ should be between zero and one, we distinguish two cases: either the derivative is still positive at $\delta = 1$, or it is not.

First, if the derivative is still positive at $\delta = 1$—that is when $\delta \in \left(0, \frac{1}{2}\right)$—and if $\delta(1-L\delta)|_{\delta=1} < \frac{\rho R^m}{(1-c)\mathcal{R}^{\mathrm{m}}}$, then there is no value of $\delta$ between zero and one meeting Equation (16). So, the condition needed for a solution is:

$\frac{\rho R^m}{(1-c)\mathcal{R}^{\mathrm{m}}} < \min\left\{1-L, \frac{1}{4L}\right\} = 1-L$. The solution corresponds to the lowest value of $\delta$ solving Equation (16).

Second, if the derivative is negative at $\delta = 1$—that is, when $\delta \in \left(\frac{1}{2}, 1\right)$—two cases stand out depending on the position of $\delta(1-L\delta)|_{\delta=1} = 1-L$ with respect to $\frac{\rho R^m}{(1-c)\mathcal{R}^{\mathrm{m}}}$ ,.

As in the previous case, for $\frac{\rho R^m}{(1-c)\mathcal{R}^{\mathrm{m}}} < 1-L$ the solution is the lowest value of $\delta$ solving Equation (16). Alternatively, if $1-L < \frac{\rho R^m}{(1-c)\mathcal{R}^{\mathrm{m}}} < \frac{1}{4L}$ ,there are up two admissible solutions to Equation (16). The lemma insures the existence of at least one solution, implying that the maximal value of $[\delta(1-L\delta)]$ minimally reaches $\frac{\rho R^m}{(1-c)\mathcal{R}^{\mathrm{m}}}$.

**Appendix 7: Proof of Lemma 2**

If $(1-\varepsilon)\alpha \to \frac{1}{1-p_{nf}}$, Equation (16) then turns into:

$$P_s\rho R^m = \left(1-\frac{c}{1-e}\right)\left[\delta\mathcal{R}^{\mathrm{m}}\left(P_s p_{nf}(\delta) + \sigma P_{ss}\left[1-\frac{1}{1-p_{nf}}\right]\delta\left(1-p_{nf}\right)\right)\right]$$

Let us first prove that it can be rewritten as:

$$\rho R^m = \left(1-\frac{c}{1-e}\right)\left[\delta\mathcal{R}^{\mathrm{m}}\left((1-\delta) + (1-\sigma)\,\frac{P_{ns}}{P_s}\delta p_{nf})\right)\right] \tag{27}$$

We have:

$$P_s\rho R^m = \left(1-\frac{c}{1-e}\right)\left[\delta\mathcal{R}^{\mathrm{m}}\left(P_s p_{nf}(\delta) + \sigma P_{ss}\left[1-\frac{1}{1-p_{nf}}\right]\delta\left(1-p_{nf}\right)\right)\right] = \left(1-\frac{c}{1-e}\right)\left[\delta\mathcal{R}^{\mathrm{m}}\left(P_s p_{nf}(\delta) - \sigma P_{ss}\delta p_{nf}\right)\right] = \left(1-\frac{c}{1-e}\right)\left[\delta\mathcal{R}^{\mathrm{m}}\left((P_s(1-\delta(1-p_{nf})) - \sigma P_{ss}\delta p_{nf}\right)\right] =$$

$\left(1-\frac{c}{1-e}\right)\left[\delta\mathcal{R}^{\mathrm{m}}\left(\left(P_s(1-\delta)+P_s\delta p_{nf}\right)-\sigma P_{ss}\delta p_{nf}\right)\right]= \quad \left(1-\frac{c}{1-e}\right)\left[\delta\mathcal{R}^{\mathrm{m}}\left(\left(P_s(1-\delta)+(1-\sigma)P_{ns}\delta p_{nf}\right)\right)\right]$

since $P_s=(1-\sigma)P_{ns}+\sigma P_{ss}$.

The sign of the partial derivative of the RHS of (27) w.r.t. $\delta$ is the sign of:

$$\frac{\partial\left[\delta(1-\delta)+(1-\sigma)\frac{P_{ns}}{P_s}\delta p_{nf}\right]}{\partial\delta}=1-2\delta\left[1-\left(1-\frac{\sigma P_{ss}}{P_s}\right)p_{nf}\right]$$

As we assume, after Proposition 3, that $\left(1-\frac{\sigma P_{ss}}{P_s}\right)p_{nf}>\frac{1}{2}$, the second term is smaller than 1 in absolute terms, making positive the partial derivative of the RHS of (27) w.r.t. $\delta$. Hence, lowering $\rho$ can be compensated by lowering $\delta$ keeping $e$ fixed, leading to an increase in social contribution.

QED