Introduction to Focus Issue: Time-delay dynamics

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Introduction to Focus Issue: Time-delay dynamics

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The field of dynamical systems with time delay is an active research area that connects practically all scientific disciplines including mathematics, physics, engineering, biology, neuroscience, physiology, economics, and many others. This Focus Issue brings together contributions from both experimental and theoretical groups and emphasizes a large variety of applications. In particular, lasers and optoelectronic oscillators subject to time-delayed feedbacks have been explored by several authors for their specific dynamical output, but also because they are ideal test-beds for experimental studies of delay induced phenomena. Topics include the control of cavity solitons, as light spots in spatially extended systems, new devices for chaos communication or random number generation, higher order locking phenomena between delay and laser oscillation period, and systematic bifurcation studies of mode-locked laser systems. Moreover, two original theoretical approaches are explored for the so-called Low Frequency Fluctuations, a particular chaotical regime in laser output which has attracted a lot of interest for more than 30 years. Current hot problems such as the synchronization properties of networks of delay-coupled units, novel stabilization techniques, and the large delay limit of a delay differential equation are also addressed in this special issue. In addition, analytical and numerical tools for bifurcation problems with or without noise and two reviews on concrete questions are proposed. The first review deals with the rich dynamics of simple delay climate models for El Nino Southern Oscillations, and the second review concentrates on neuromorphic photonic circuits where optical elements are used to emulate spiking neurons. Finally, two interesting biological problems are considered in this Focus Issue, namely, multi-strain epidemic models and the interaction of glucose and insulin for more effective treatment. *Published by AIP Publishing.*
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Delay differential equations (DDEs) are equations where the evolution of a dependent variable depends not only on its actual value but also on its value at time τ in the past. They appear in all scientific disciplines including mathematics, engineering, biology, physiology, economics, and finance. In the life sciences, time delays can arise in the nervous system because of axonal conduction velocities, and distances between neurons are finite in cell biology because of cell maturation times, or in molecular biology because of the time required for transcription and translation. These delays may contribute to the generation of robust, clock-like oscillations or, on the contrary, affect normal physiological functions. In the manufacturing industry, delays in the metal cutting process are responsible for chatter instabilities characterized by violent vibrations, loud noise, and poor quality of surface finish. For lasers, optical feedback refers to undesirable delayed feedback into the optical chain. Feedback is introduced into a laser when some portion of the optical output is reinjected back into the device. It comes from optical elements like micro-lenses in fiber-coupled modules, fiber ends, fiber combiners, and also radiation from other sources. Even very small portions of the light reflected can destabilize the laser and produce different kinds of regular or irregular pulsating outputs.

I. INTRODUCTION

To the best of our knowledge, the first published formulation of a DDE was by Airy[1] in 1830 who was trying to understand the operation of the human voice. The industrial revolution in Europe led to the invention of automatic control devices which were later described in terms of DDEs during the second half of the 20th century. Since the beginning of this century, DDE problems have gained increasing interest, thanks to the performance of our computers. In 2009–2010, the first pluridisciplinary meetings on the applications of DDEs took place and special issues of journals were edited.[2–4] Today, the ways we explore DDE problems strongly depend on our educational experience as illustrated by the variety of books which appeared since 2009.[5–11]

In engineering, numerous control systems involve a physical delay which proves to be problematic in the design and tuning of feedback control laws. These delays result from the fact that sensors and actuators are rarely co-located, for example, in processes that involve the transport of

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materials, such as mixing processes for liquid or gaseous fluids and chemical reactors.[12,13]

The delay does not need to be large to generate dynamical instabilities. In optics, semiconductor lasers are popular for our everyday applications (codebar reading at the supermarket, CD and DVD players, laser printer, and telecom applications). However, they are highly sensitive to optical feedback.[6,14,15] They can suffer damage that may be immediately apparent through loss in power and they may lose their spectral characteristics such as center wavelength and linewidth. Although the round-trip time of the light leaving and returning to the laser is small (10^{-9} s), it is much larger than the laser photon lifetime (10^{-12} s).

DDEs also found their way in the modeling of neuron collective behaviors.[16,17] Our brain is the place of intense electrochemical activity. About 100×10^9 neurons are each firing off 5–50 messages (action potentials) per second. A great variability is found in the velocity of the propagation of action potentials. The propagation velocity of the action potentials in nerves can vary from 100 m/s to less than a tenth of a meter per second. These propagation velocities depend on the length, thickness, or the extend of myelination of axons.[18]

Other classical areas where time delays are taken into account are, for example, population dynamics[19] (gestation periods and maturation times) and car following models for traffic flow simulation[20] (driver reaction time). Although time delays were initially considered as a nuisance, they are now viewed as a resource that can be beneficially exploited. For example, they have been used for secure communication, random number generators, and chaos control. Some of these applications are described in this focus issue.

In the simplest case of a scalar DDE of the form

$$\frac{dx}{dt} = f(x, x(t-\tau)),$$

where x describes the state of the system and $\tau > 0$ is the time delay, the solution may become quite complicated. The equation

$$\frac{dx}{dt} = \frac{1}{1 + x(t-\tau)^n} - bx, \quad (b > 0 \text{ and } n > 0) \tag{1}$$

describes the effect of a strong negative feedback for large n and admits regular periodic oscillations. It was originally proposed by Mackey and Glass in 1977 (Ref. 21) for an autoimmune disease that causes periodic crashes in circulating red blood cells. Because a negative feedback is a key regulatory mechanism in biology, Eq. (1) frequently appears in the literature.[22] A variation of this equation called the Mackey-Glass equation[21] displays both periodic and chaotic oscillations. The latter has led to the concept of dynamical diseases when normal physiological controls exhibit anomalous clinical disorders.

Another scalar DDE that had a strong impact is Ikeda equation[23]

$$\frac{dx}{dt} = a[1 + 2b\cos(x(t-\tau) - x_0)] - cx, \quad (a, b, c, x_0 > 0). \tag{2}$$

By the end of the seventies and early eighties, all laboratories studying the dynamics of lasers and other optical devices were looking for chaotic outputs. In 1979, Ikeda considered a nonlinear absorbing medium subject to a long delayed feedback and modeled his problem by Eq. (2). He showed numerically that chaotic regimes were possible, although the experimental demonstration came later in 1983. Ikeda equation and Lang and Kobayashi equation[14] in 1980 for a semiconductor laser subject to optical feedback from a distant mirror stimulated research activities not only in optics but also in applied mathematics, for example, when exploring the limit of large delays of a DDE.[15] Probably, the first DDE model for a laser with optical feedback from a distant mirror was considered by Rozanov in 1975.[24]

Starting from the middle of the 20th century, the basics of the mathematical theory of delay differential equations have been developed.[25–31] One of the most influential publications dealing with nonlinear dynamics of delay systems was the paper of Farmer,[32] which studied chaotic attractors in delay systems, and especially their dependence on time-delay. It was observed that the complexity of the attractors grows with the increasing of the delay. It is meanwhile known that:[15]

1. Time delay systems exhibit dynamical phenomena that can be arbitrary high-dimensional.
2. The dimensionality of the observed phenomena is proportional to the time delay τ.
3. A proper description of the high-dimensional dynamics in delay systems requires methods, that, in some cases, are different from those used for low-dimensional systems. In particular, the normal forms that describe the destabilization processes are sometimes similar to those for spatially extended systems (partial differential equations).

Despite the important achievements in the study of delay systems, there are still many challenging problems related to both theory and applications. Many theoretical and experimental groups employ time-delayed systems in their research. Within one focus issue, it is unrealistic to collect the contributions from all groups. However, the editors have made an attempt to make a representative collection where the approaches from mathematics, theoretical physics, and experiments are present, interconnected, and all together lead to a better understanding of the subject. The contributions include new developments in the theory of delay systems,[33–37] time-delayed control,[37–42] interplay of noise and time-delays,[39,43,44] applications to laser dynamics,[38,42,44–54] biological and neural systems,[19,48,53] random-number generation and chaos communication,[45,54] and epidemic and climate models.[55,56] The following Secs. II–VII provide some more detailed overview.

II. ADVANCES IN THE THEORY OF DELAY SYSTEMS

Currently, the normal form computation is implemented for constant delays in the bifurcation software DDE-Biftool.[57] The work of Sieber[33] contributes toward the extension of the software to state-dependent delays. Wang and Campbell[34] study the delay-induced bifurcations leading to the emergence

of cluster solutions in lattices (with Z_N-symmetry) of delay coupled systems.

The papers[35–37] are more applied and propose numerical solutions of certain problems. In particular, Zou *et al.*[35] explore how time delays modify the amplitude death and oscillation death regimes in a Stuart-Landau system with time-delayed feedback. Other forms of synchronization are examined by Mirasso *et al.*[36] who study anticipated and zero-lag synchronization in network motifs of delay-coupled systems with small delays.

III. LASERS WITH DELAYED FEEDBACK

Laser dynamics is one of the fields where models with time-delays are frequently proposed. Quantitative comparisons between theory and experiments are possible and have contributed to our understanding of dynamical phenomena induced by a delayed feedback. Today, the study of lasers is becoming more and more motivated by their test-bed character for delay systems in general. Hence, a larger part of articles in this focus issue contributes to this application field. The applications include random-number generation, chaos communication, dynamics of localized solutions, mode-locked lasers, lasers with phase-conjugate and long-delayed feedback, and Low-Frequency fluctuations.

A. Random-number generation and chaos communication

Verschaffel *et al.*[45] consider an integrated laser with a long on-chip optical feedback section. By controlling the amount of feedback, the authors achieve chaotic dynamics which is sufficiently complex in order to generate random bits based on the chaotic intensity fluctuation at a rate of 500 Mbit/s.

Oden *et al.*[54] propose a new device for optical chaos communications. It consists of a time-delayed optoelectronic oscillator with a customized three-wave fiber interferometer. The system is shown to produce broadband optical phase chaos that can be synchronized with negligible residual noise. As a result, a successful multi-Gbit/s optical chaos communication is possible.

B. Cavity solitons

Cavity solitons are localized spots of light in the transverse section of passive and active optical devices, broad area lasers, or wide-aperture semiconductor cavities. Using a combination of analytical and numerical methods, Schemmelmann *et al.*[38] analyze the bifurcation structure of stationary and moving cavity solitons. They identify different types of traveling localized solutions corresponding to either slow or fast motion.

Tlidi *et al.*[47] investigate the Lugiato-Lefever equations with time-delayed feedback. They study the effects of time delays on the formation of temporal cavity solitons, self-pulsating cavity solitons, and rogue waves in a fiber cavity. Interactions, collisions, and binding of solitons in a semiconductor laser with optical injection and feedback are examined in detail by Garbin *et al.*[52]

C. Mode-locked lasers

Jaurigue *et al.*[42] investigate the dynamics and bifurcations arising in a mode-locked semiconductor laser model under the influence of time-delayed optical feedback. The considered system of delay-differential equations contains two delays corresponding to the laser cavity length and the feedback loop.

A simpler delay model for a mode-locked laser with one delay is studied by Kovalev and Viktorov[51] with particular attention to the bifurcations of steady states and periodic orbits.

D. Laser with phase-conjugate optical feedback

Weicker *et al.*[46] investigate the bifurcation diagram of a semiconductor laser subject to phase-conjugate optical feedback. In particular, they explore the parameter dependence of so-called external cavity modes, defined as delay-induced periodic solutions for the laser intensity.

E. Long-delayed feedback and effect of noise

Marino and Giacomelli[44] investigate the effect of noise both experimentally and theoretically in an excitable semiconductor laser with long-delayed feedback. Because of the large delay, a spatio-temporal representation is used that describes the dynamics on different timescales: the internal timescale of the solitary system (within one delay unit) and the timescale on delay units. It is shown that a spatial-like ordering of excitation pulse patterns is possible.

F. Low-frequency fluctuations

Ruschel and Yanchuk[48] propose a new view at the classical problem of Low Frequency Fluctuations (LFF) in semiconductor lasers with delayed feedback. LFF consist of chaotic laser amplitude fluctuations interrupted by irregular dropouts. The authors consider these regimes as bursting slow-fast oscillations where underlying slow and fast phases can be clearly identified using singular perturbation theory.

A different interpretation of LFF is proposed by Hicke *et al.*[49] The authors treat the feedback as an externally injected signal and show that the LFF regime can be associated with the dynamical injection-locking between the laser and its own delayed input.

Panozzo *et al.* investigate the possible regimes of a diode laser with time-delayed optical feedback.[50] The authors demonstrate that statistical and symbolic data analysis can be used to identify their onset. In terms of the laser current and feedback strength, parameter regions where LFFs or coherent collapse are computed.

IV. LASERS VS. NEURONS

Romeira *et al.*[53] review their recent achievements on neuromorphic optoelectronic nanoscale resonators. In particular, they concentrate on different excitable responses of single and delayed artificial neurons including all-or-none response, spike-based data encoding, storage, signal regeneration, and signal healing. In the presence of time delay, it is shown that the excitable pulses are regenerated after each

time delay and that these temporal excitations display all the signatures of localized structures.

V. TIME-DELAY CONTROL

The quantization of the controlling force occurs often in applications due to e.g., "round off" effects, A/D and D/A conversions, and it is known that such quantizations can introduce spurious oscillations or small fluctuations into the control dynamics. Stepan *et al*.[40] discuss another, less known side of quantization. They show that a "coarse-grained" signal quantization can stabilize unstable dynamical systems in the presence of feedback delays.

A network of nonlocally coupled excitable FitzHugh-Nagumo systems under the influence of time-delayed feedback is studied by Zakharova *et al*.[37] Previously, it was shown that such a system without delayed feedback exhibits a noise-induced chimera state. The authors study how time-delayed feedback influences the range of parameter values where the chimera states occur.

Delayed feedback control of steady states and periodic solutions of Mackey-Glass system is investigated by Kiss and Röst.[41] Various control mechanisms have been considered such as Pyragas control, proportional feedback, and state-dependent control.

VI. INTERPLAY BETWEEN NOISE AND DELAY

The work of René and Longtin[43] deals with general properties of systems with delays and randomness (stochastic delayed differential equations). In particular, the authors propose a computational approach to decompose the solution of linear stochastic delay differential equations into natural modes. They show that for single and uniformly distributed delays, only a few of these modes can be sufficient.

The combined effect of delay and noise is also the topic studied by Wang and Kuske.[39] The authors develop new semi-analytic approaches for a system subject to act-and-wait control. Specifically, it consists of periods when the control is on, followed by waiting periods when the control is off.

VII. FURTHER APPLICATIONS: BIOLOGICAL, EPIDEMIC, AND CLIMATE MODELS

Shi *et al*.[58] propose a delay differential model for an intravenous glucose tolerance test. It is shown that such a relatively simple model provides a good fit to patient data sets and reveals interesting dynamics.

A time-delayed epidemic model for multi-strain diseases with temporary immunity is studied by Bauer *et al*.[55] Using numerical simulations, they investigate systematically the effects of all-to-all and non-local coupling topologies. In particular, they show that cross-immunity can induce complex dynamical behaviors and synchronization patterns, including discrete traveling waves, solitary states, and amplitude chimeras.

Keane *et al*.[56] review DDEs that have been used as conceptual climate models. The authors describe some of their basic dynamical properties and emphasize numerical approaches for the bifurcation analysis.

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