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Behavioral Banking: A Theory of the Banking Firm with Time-Inconsistent Depositors

Carolina Laureti and Ariane Szafarz

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JEL Classifications: G21, E21, D53, D91, G28.

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Abstract

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1. Introduction

Reserve requirements for liquidity management spur banks to supply illiquid savings accounts. If their clients prefer liquid rather than illiquid accounts, the equilibrium remuneration of illiquid savings is necessarily higher (Miller, 1975). In this respect, the presence of time-inconsistent agents can make a difference because they value commitment devices to discipline their future selves (Strotz, 1955; Laibson, 1997), and so push up the prices of rigid contracts, such as illiquid savings accounts (DellaVigna & Malmendier, 2004). In the banking literature, surprisingly little is known about how time-inconsistent agents influence deposit interest schemes. This paper fills the gap by investigating how time-inconsistent agents affect the pricing of savings contracts. We build an equilibrium model that determines the liquidity premium, in other words, the interest spread between illiquid and liquid deposits.

Why is banking different from other industries when it comes to gauging the impact of time-inconsistent clients? To answer this question, we consider a bank supplying both liquid and illiquid deposit accounts. Holders of liquid accounts may withdraw any amount at any time, provided their outstanding balance remains non-negative, whereas illiquid holdings, such as time deposits, cannot be taken out before a given maturity. This setting allows us to pinpoint two key differences between banking and other industries. First, all savers—including time-inconsistent ones—appreciate liquid savings to hedge against future shocks (Amador *et al*., 2006). In non-banking industries, by contrast, time-inconsistent agents have little to gain in making flexible contracts (Oster & Scott Morton, 2005; DellaVigna & Malmendier, 2006). Second, regulatory reserve requirements drive a supply effect specific to banks. Banks are more motivated to attract illiquid deposits than liquid ones. That is why the impact of savers' time-inconsistency on the pricing of banking products deserves a special attention.

Much has already been written on the influence of time-inconsistent agents in the economy. The left side of Table 1 presents the economic literature, which concentrates on the design of optimal commitment contracts for banks and non-banking firms, and on the pricing of non-banking products and services. Exceptions include Heidhues and Kőszegi (2010; 2017) and Ru and Schoar (2016), who examine the pricing of credit contracts. This paper adds the pricing of savings products to the picture.

Table 1: Literature on Commitment Devices with Time-inconsistent Agents

	Non-banking products		**Banking products**[c]	
Topic:	Theory	Empirics	Theory	Empirics
Optimal commitment devices[a]	Duflo *et al.* (2011) Hwang and Möllerström (2017)	Gine *et al.* (2010) Duflo *et al.* (2011) Acland and Levy (2015) Royer *et al.* (2015)	Amador *et al.* (2006)[b] Carroll *et al.* (2009) Ambrus and Egorov (2013)[b] Beshears *et al.* (2014a)[b] Beshears *et al.* (2015a)[b] Bond and Sigurdsson (2015)[b] Galperti (2015)[b] Laibson (2015)[b]	Angeletos *et al*. (2001) Madrian and Shea (2001) Thaler and Bernatzi (2004) Ashraf *et al*. (2006) Carroll *et al.* (2009) Meyer (2010) Dupas and Robinson (2013) Beshears *et al.* (2015a)[b] Labie *et al.* (2017)[b]
Pricing of commitment devices	DellaVigna and Malmendier (2004) Eliaz and Spiegler (2006) Esteban *et al.* (2007) Gottlieb (2008)	Oster and Scott Morton (2005) DellaVigna and Malmendier (2006)	Heidhues and Kőszegi (2010; 2017)	Ru and Schoar (2016)

[a] for a profit-maximizing firm, a welfare-maximizing government, or a utility-maximizing consumer.
[b] discusses the commitment/flexibility tradeoff.
[c] includes personal finance products such as retirement savings and credit cards.

Evidence in the field makes the case that time-inconsistent agents abound in various markets. Gine *et al.* (2010) show that agents engage in pure commitment devices to stop smoking. Acland and Levy (2015) and Royer *et al.* (2015) find similar evidence for gym attendance. Hwang and Möllerström (2017) argue that voters' time-inconsistency explains why political reforms are actuated with delays. Duflo *et al.* (2011) demonstrate that time-inconsistent preferences lead poor Kenyans to under-invest in fertilizers. The financial markets are no exception: Behavioral anomalies consistent with the lack of self-control in saving and borrowing attitudes include procrastinating and postponing the taking-up of optimal savings plans (Madrian & Shea, 2001), over-borrowing in the short term (Meyer, 2010), and saving excessively for the long term (Angeletos *et al*., 2001). Likewise, some savers are willing to pay for no-interest illiquid savings accounts (Beshears *et al*., 2015a). Time-inconsistent preferences can also explain why automatic enrollment in pension savings schemes increases participation (Madrian & Shea, 2001), and why fixed savings plans increase individuals' propensity to save (Thaler & Benartzi, 2004; Ashraf *et al.*, 2006; Dupas & Robinson, 2013; Labie *et al.*, 2017).

Theoretical articles determine the pricing of non-banking contracts with time-inconsistent agents (DellaVigna & Malmendier, 2004; Eliaz & Spiegler, 2006; Esteban *et al.*, 2007; Gottlieb, 2008). Yet, the empirical evidence on the cost of commitments is still controversial. Oster and Scott Morton (2005) and DellaVigna and Malmendier (2006) provide examples of firms offering costly commitments to time-inconsistent consumers. Laibson (2015) argues, however, that pure commitments are rare in real life because they generate low perceived welfare gains, and time-inconsistent agents are reluctant to pay for loss of flexibility. Our model addresses Laibson's criticism by considering the commitment embedded in illiquid savings accounts, which are typically rewarded by banks.

Close to our topic are the theoretical contributions on optimal commitment devices for banking contracts. Most articles focus on savings accounts with penalties for early withdrawal (Amador *et al.*, 2006; Ambrus & Egorov, 2013; Beshears *et al.*, 2014a; Galperti, 2015). Beshears *et al.* (2015a) scrutinize the demand elasticity of time-inconsistent agents to the early withdrawal penalty. Carroll *et al.* (2009) explore the socially optimal enrolment regime in voluntary retirement savings plans. The literature offers two ways to model the trade-off between commitment and flexibility. Amador *et al.* (2006), Ambrus and Egorov (2013) and Bond and Sigurdson (2015) assume that the agents' degree of time inconsistency is observable, and the trade-off results from contract design. Galperti (2015) opts for an information-driven trade-off, where agents' demand for flexibility depends on their privately known level of self-control. The trade-off between commitment and flexibility in our model is designed into the contract, but we depart from Amador *et al.* (2006) by introducing asymmetric information, as in Galperti (2015). We consider time-inconsistent agents who are sophisticated, meaning that they know their own degree of time inconsistency.[1] We successively study the cases of homogenous agents—either time-consistent or time-inconsistent—and symmetric information, and heterogeneous agents and asymmetric information.

Our benchmark model features a monopolistic bank, a pool of time-consistent savers, and symmetric information. The model has three periods. At time zero, the bank offers both liquid and illiquid accounts to all the agents. Withdrawal in time one is possible only to holders of flexible accounts. An adverse shock can occur in time one, and make time-one consumption more valuable. Alternatively, there is no shock, and consumption in time two is more valuable. In this benchmark setting, the equilibrium liquidity premium must be (non-negative and) sufficiently high to attract savers to illiquid accounts; otherwise, the time-consistent savers will prefer to stick to flexible accounts.

When agents are time-inconsistent, a trade-off arises because agents with flexible accounts are tempted to withdraw and consume in time one, regardless of whether the shock occurs. The illiquid account acts as a protection against over-consumption. As a result, the liquidity premium required to make agents select the illiquid account can have either sign, depending chiefly on the probability of an adverse shock. When the probability of the shock is high, the equilibrium liquidity premium is positive. The bank subject to reserve requirements leading to a preference for illiquidity chooses to compensate time-inconsistent agents for holding illiquid accounts, so that time-inconsistent savers do not have to pay for commitment. Conversely, when the probability of the shock is low enough, commitment has more value than flexibility, and the equilibrium liquidity premium can be negative,

[1] This paper focuses on time-inconsistent agents who are aware of their condition, referred to as "sophisticated" (O'Donoghue & Rabin, 1999; 2001). Appendix E discusses the cases of naïve and partially naïve time-inconsistent agents. Except in this appendix, we use the term "time-inconsistent" to designate sophisticated time-inconsistent agents.

meaning that the bank pays higher interest on liquid deposit accounts than on illiquid ones. In this case, the bank supplies illiquid accounts only. Yet, no such situation has ever been reported for real-life banking institutions.

To understand why the liquidity premium is always positive on formal financial markets, we generalize the model to heterogeneous agents and asymmetric information. The bank knows only the share of time-inconsistent agents in the market. Solving the model shows that when the share of time-consistent agents in the market exceeds a given threshold, the equilibrium liquidity premium is always positive, even though time-inconsistent agents would agree to pay for the commitment embedded in illiquid accounts. This case underscores the adverse-selection problem arising in an equilibrium where time-inconsistent agents have access to costless commitment devices. From the bank's standpoint, time-inconsistent agents who are willing to accept low liquidity premia represent an opportunity for cheap liquidity management.

The rest of the paper is organized as follows. Section 2 outlines our model. Section 3 derives the equilibrium liquidity premium for homogenous agents. Section 4 solves the model with heterogeneous agents and asymmetric information. Section 5 concludes.

2. The Model

The model features three periods ($t = 0, 1, 2$) and a bank that offers two types of savings accounts: liquid, or flexible, accounts ($s = F$) permitting costless withdrawal in $t = 1$; and illiquid, or commitment, accounts ($s = C$) forbidding withdrawals before maturity ($t = 2$). The N agents can be one of two types: time-consistent ($i = TC$) or time-inconsistent ($i = TI$). As in Galperti (2015), the time-inconsistent agents are heterogeneous in their degree of time inconsistency. Their present bias, $\beta < 1$, measures the strength of temptation toward current consumption. In period 0, the bank sets its interest-rate policy for the two types of account and makes it public. We assume for simplicity that both accounts deliver zero interest in 0 and 1, and that interest matures in period 2. Rates r_F and r_C represent the period-2 interest on the liquid account and the illiquid account, respectively. We also assume that the liquid account is a perfect substitute for cash holding, so that the liquid interest rate is zero: $r_F = 0$. Hence, the illiquid interest rate, r_C, boils down to the liquidity premium. To keep the model simple, we first assume that the bank is monopolistic, and later discuss how our results can be extended to perfect competition.

Uncertainty. The agents choose their savings accounts in period 0 under uncertainty. Only the holders of a liquid account will have the opportunity to smooth future consumption through early withdrawal. We study an environment where time-inconsistent agents face a trade-off between commitment and flexibility (Amador *et al.*, 2006). The need for commitment, i.e. no early withdrawal, arises from the fact that time-inconsistent agents are tempted to over-consume in period 1. The preference for flexibility (Kreps, 1979; Beshears *et al*., 2014b), where early withdrawal is permissible, stems from the possible occurrence with probability π of an adverse shock in period 1. This shock is common to all agents; for example, it could be a natural disaster that increases the marginal utility of consumption in period 1 in the same way for all agents. Consumption is known to be more valuable in difficult times than when conditions are good. The no-shock and shock situations are referred to as the good state (*G*) and the bad state (*B*) of nature, respectively.

Depositors. Each agent has a one-dollar initial endowment. We assume that the participation constraint is met, i.e. the reservation utility is sufficiently low to ensure that all agents allocate the endowment to liquid and illiquid accounts. No consumption takes place in period 0, thus ruling out any self-control problems in period 0. The state of nature is revealed in period 1. Consumption in periods 1 and 2 may thus be contingent on this state. Let $c_t^i(s, \omega)$ denote the consumption of agent *i* $(i = TC, TI)$ in period t $(t = 1, 2)$ of account s $(s = F, C)$ when the revealed state of nature is ω $(\omega = G, B)$. In line with Beshears *et al.* (2015a), we use a linear specification for the utility function to keep the model as simple as possible. We assume that all agents are risk-neutral and share the same linear instantaneous utility in periods 1 and 2. The instantaneous utility in period 1 depends on the state of nature in the following way:

$u_1^i(s,\omega) = (1+\theta_\omega)\, c_1^i(s,\omega),$

where $1+\theta_\omega$, the marginal utility of consumption in period 1, is given by:

$$1+\theta_\omega = \begin{cases} 1+\theta_G, & \text{if } \omega = G \\ 1+\theta_B, & \text{if } \omega = B \end{cases} \qquad 1+\theta_\omega > 0,\ \theta_G < \theta_B.$$

Consumption in period 2 delivers the following instantaneous utility:

$u_2^i(s,\omega) = c_2^i(s,\omega).$

Intertemporal preferences. The two types of agents differ in discounting instantaneous utilities: time-consistent agents use exponential discounting while time-inconsistent ones use quasi-hyperbolic discounting. Quasi-hyperbolic discounting (Phelps & Pollack, 1968; Laibson, 1997; Diamond & Köszegi, 2003) is a standard tool for modeling time-inconsistency in intertemporal decision-making.[2] It entails a present bias resulting in over-valuation of immediate consumption with respect to future consumption. We assume for simplicity that the long-run discount factor is $\delta = 1$ for all agents.[3] The four corresponding intertemporal utility functions in periods 0 and 1 are given by:

$$U_0^{TC}(u_1^{TC}, u_2^{TC}) = U_1^{TC}(u_1^{TC}, u_2^{TC}) = u_1^{TC} + u_2^{TC}; \tag{1a}$$

$$U_0^{TI}(u_1^{TI}, u_2^{TI}) = \beta(u_1^{TI} + u_2^{TI});\ U_1^{TI}(u_1^{TI}, u_2^{TI}) = u_1^{TI} + \beta u_2^{TI}, \qquad \beta \in (0,1). \tag{1b}$$

Eq. (1b) describes how time-inconsistent agents weigh their period-1 and period-2 instantaneous utilities: the intensity of the trade-off is 1 in period 0, and $\beta < 1$ in period 1. This encourages agents to consume more in period 1 than they had planned to do in period 0. Sophisticated time-inconsistent agents know from period 0 that their future selves will be tempted to over-consume in period 1, and

[2] Alternative behavioral approaches to model time-inconsistency, lack of self-control, or temptation are proposed by Gul and Pesendorfer (2001); Fudenberg and Levine (2006).

[3] Time-inconsistent agents have two sources of discounting, β and δ. They are more impatient than time-consistent ones when trading-off current and future utilities. This assumption is common in hyperbolic consumption models (Angeletos *et al.*, 2001).

that this may prove detrimental to their period-2 consumption. They are inclined to use commitment devices to discipline their future selves.

To produce a meaningful self-control problem, we impose the following constraints on our model parameters:

$$\beta < 1 + \theta_G \leq 1 \leq 1 + \theta_B. \qquad (2)$$

First, assuming $1 + \theta_G \leq 1 \leq 1 + \theta_B$ implies that withdrawal from the liquid account is *ex ante* (in t = 0) utility-maximizing in the bad state of nature, but not in the good state. Second, since $\beta < 1 + \theta_\omega (\omega = G, B)$, the period-1 time-inconsistent agents withdraw all their savings from liquid accounts, regardless of the state of nature. In the good state, these agents' present bias is strong enough to generate temptation to consume in period 1. This is a key assumption that makes period-1 agents willing to withdraw more than period-0 agents would have wished for, and so gives value to illiquidity.

The utilities in periods 1 and 2 are deterministic because they are computed once the state of nature is revealed. In period 0, all agents maximize their expected utility:

$$E\left[U_0^{TC}(s)\right] = \pi\left[(1 + \theta_B)c_1^{TC}(s, B) + c_2^{TC}(s, B)\right] + (1 - \pi)\left[(1 + \theta_G)c_1^{TC}(s, G) + c_2^{TC}(s, G)\right],$$

$$E[U_0^{TI}(s)] = \beta\{\pi[(1 + \theta_B)c_1^{TI}(s, B) + c_2^{TI}(s, B)] + (1 - \pi)[(1 + \theta_G)c_1^{TI}(s, G) + c_2^{TI}(s, G)]\}.$$

Bank. The supply side of the model is inspired by the monopolistic banking model known as the Klein-Monti model (Klein, 1971; Monti, 1972), summarized among others by Freixas and Rochet (2008). The bank collects savings through two vehicles, liquid and illiquid accounts, and allocates these funds to earning assets A, consisting of loans and cash reserves, R. For simplicity, we assume that equity is zero and we leave aside insolvency risk (Dermine, 1986). The loans pay an exogenous net rate of return r_A in period 2. The net rate of return on cash reserves is zero. As stated by Klein (1971), the return on these reserves is implicit because any increase would reduce the likelihood of a liquidity shortage, which can be costly to the bank. The reserve requirement depends only on the proportions of liquid and illiquid savings accounts, not on the state-contingent withdrawal that can take place in period 1. Total reserves are split into two components derived from liquid and illiquid savings accounts, respectively:

$$R = \rho_F D_F + \rho_C D_C \qquad \rho_F, \rho_C \in [0,1], \qquad \rho_F \geq \rho_C,$$

where D_F and D_C represent the total amounts collected through liquid and illiquid accounts, respectively. The corresponding cash reserve ratios, ρ_F and ρ_C, are the proportions of savings balances assigned to cash reserves. The cash reserve ratios—or reserve requirements—are fixed by banking regulations.

The withdrawal option makes liquid accounts more volatile than illiquid accounts. For the bank, liquid accounts create higher liquidity risks than illiquid ones, which is why regulatory reserve requirements are typically stricter for the former than for the latter. Consequently, we impose the following condition: $\rho_F \geq \rho_C$ (Miller, 1975; Calomiris *et al*., 2015; DeYoung & Jang, 2016). The level of cash reserves depends not only on the total volume of savings collected but also on their allocations (Baltensperger, 1980). Stemming from liquidity management, this argument rationalizes the bank's preference for illiquid deposits, all else equal. As a consequence, the value of $(\rho_F - \rho_C)$ is expected to play a key role in equilibrium.[4]

The bank is a risk-neutral price-setter. Its decision variable is r_C, the liquidity premium. Its revenues come from loans while its costs consist of the interest paid on illiquid accounts. We neglect all other costs, such as management outlays. In period 0, the bank knows with certainty the amounts of interests to be paid on the illiquid accounts in period 2. For the liquid accounts, interest rates are zero regardless of the total amount withdrawn in period 1. We assume that, similarly to the depositors, the bank uses a unit time-discounting factor. Its profit to be maximized in period 0 is:

[4] In the simplified Klein-Monti model, liquidity problems come only from the liability side of the balance sheet. In reality, however, the asset side matters as well (for example, maturity and default risk), and banks have to comply with capital adequacy regulations.

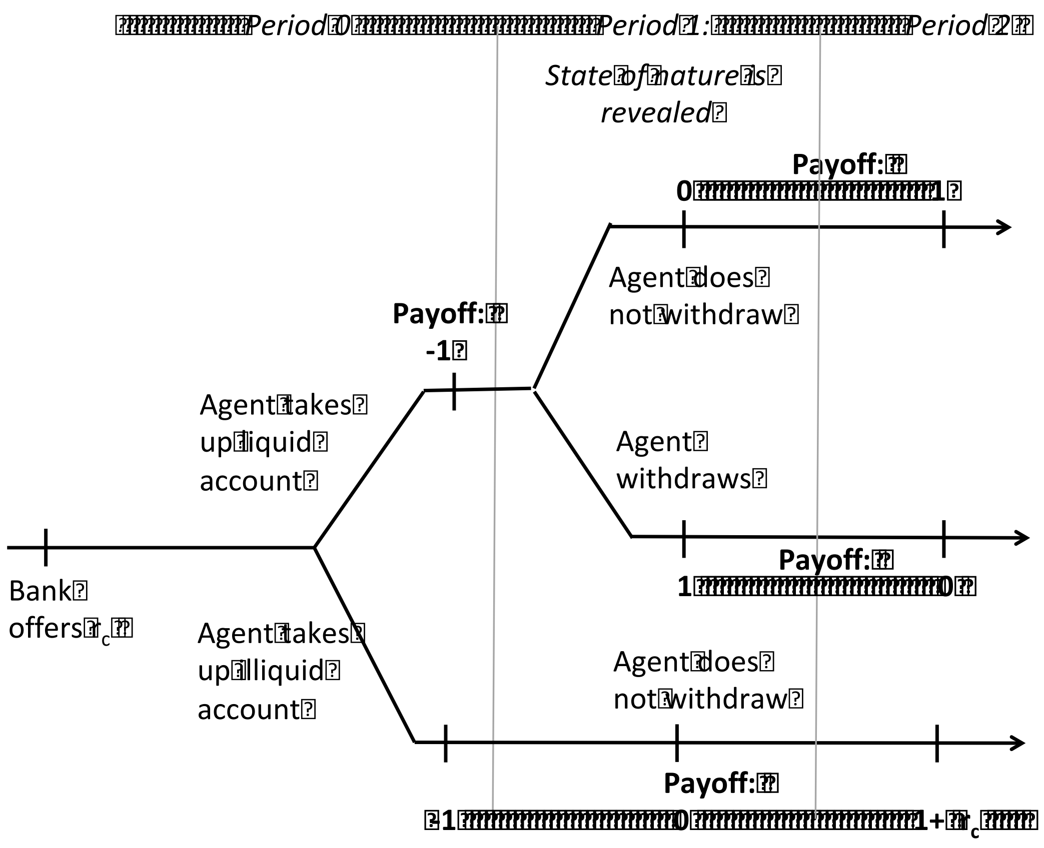
$$P_0(r_C) = r_A\, A - r_C\, D_C, \tag{3}$$

under the balance-sheet constraint:

$$A = (1 - \rho_F)D_F + (1 - \rho_C)D_C. \tag{4}$$

Timing of the game. The bank announces its liquidity premium, r_C, in period 0. Subsequently, each agent chooses a single account. In period 1, the state of nature is revealed. The agents holding liquid accounts determine the amount they wish to withdraw, and allocate it to current consumption. The others are left with no choice but to keep their savings on illiquid accounts. In period 2, all agents recoup their remaining capital plus interest and consume it all.

Figure 1: Timing of the Game



3. Homogenous Agents

The equilibrium model proposed in this section determines the impact of homogenous agents— either time-consistent or time-inconsistent—on the pricing of fixed-interest savings contracts. The first case where all agents are time-consistent is the standard textbook situation that serves as a benchmark. In the second, all agents are time-inconsistent, and their present bias is identical and public knowledge. Examining the price differences in equilibrium of the two polar cases will give a first hint about the situation where a bank is facing a mix of savers and information is asymmetric.

When the *N* agents are homogenous, they all demand the same kind of savings account. Their decision depends on whether the liquidity premium is sufficient to make them choose an illiquid account with time-2 maturity. Since time-consistent and time-inconsistent agents have different utility functions, the minimum required premium depends on type of agent. Let $r_C^{i,min}$ be the cut-off premium for type *i* agents. In both homogenous cases, the aggregate demand function is:

$$\begin{cases} if \quad r_C < r_C^{i,min} & \Rightarrow \quad D_F^i = N,\ D_C^i = 0 \\ if \quad r_C \geq r_C^{i,min} & \Rightarrow \quad D_F^i = 0,\ D_C^i = N \end{cases} \quad , \quad i = TC, TI \tag{5}$$

where D_F^i and D_C^i are the demand schedules of type *i* agents for liquid accounts and illiquid accounts, respectively. Threshold $r_C^{i,min}$ is equivalent to the consumers' reservation price (Armstrong & Porter, 2007), and the opposite of the willingness to pay for commitment.

3.1 Benchmark Case: Time-Consistent Agents

Time-consistent agents use exponential discounting. We solve their maximization problem backwards in time. In period 2, the agent consumes the remaining capital plus interest. In period 1, she observes state of nature ω, and fixes consumption plan $\{c_1^{TC}(s,\omega), c_2^{TC}(s,\omega)\}$. For time-consistent agents holding a liquid account, the plan is contingent on the state of nature:

$$c_1^{TC,*}(F,\omega) = \begin{cases} 1 & if\ \omega = B \\ 0 & if\ \omega = G \end{cases};$$

$$c_2^{TC,*}(F,\omega) = 1 - c_1^{TC,*}(F,\omega), \qquad \omega = G, B.$$

By contrast, holders of an illiquid account are bound to keep their savings until maturity, and they consume $1 + r_C$ in period 2:

$$c_1^{TC,*}(C,\omega) = 0;\; c_2^{TC,*}(C,\omega) = 1 + r_C, \qquad \omega = G, B.$$

In period 0, agents contemplate the two possible accounts and make their decision under uncertainty, knowing probability π of an adverse shock. The presence of uncertainty makes flexibility valuable. In the bad state of nature, i.e., when a shock is observed in period 1, agents are better-off withdrawing the cash from their savings accounts. The optimization problem is written:

$$Max\,\{\pi(1+\theta_B) + (1-\pi); (1+r_C)\}.$$

As expected, time-consistent agents need to be compensated for holding an illiquid account, which would prevent them from hedging against the adverse shock. The non-negative minimal liquidity premium they require for holding the illiquid account is:

$$r_C^{TC,min} = \pi\theta_B\;\; (\geq 0).$$

Meanwhile, the bank maximizes its profit, $P_0(r_C)$ in Eq. (3) under the balance-sheet constraint in Eq. (4), yielding the following result.[5]

Proposition 1:

If the bank is monopolistic and the pool of savers is made up of time-consistent agents, the equilibrium quantities of savings accounts, D_F^* and D_C^*, and the equilibrium liquidity premium r_C^*, are given by:

(i) $r_A(\rho_F - \rho_C) < \pi\theta_B \quad \Rightarrow \quad D_F^* = N,\; D_C^* = 0$, and $r_C^* = -\infty$;

(ii) $r_A(\rho_F - \rho_C) \geq \pi\theta_B \quad \Rightarrow \quad D_F^* = 0,\; D_C^* = N$, and $r_C^* = \pi\theta_B$.

Proof: see Appendix A.

[5]For expositional facility, we conventionally fix the liquidity premium of inexistent accounts as $r_{C,I}^* = -\infty$.

According to Proposition 1, when the pool of savers is composed of time-consistent agents, the bank has two possibilities: it supplies them with liquid accounts, or with illiquid accounts with premium $\pi\theta_B$ (≥ 0). The bank's decision depends on parameters r_A and $(\rho_F - \rho_C)$, which influence the earning potential associated with a clientele shift from liquid to illiquid accounts. In case (i), the bank's preference for illiquid accounts is low and the agents pick liquid accounts. Alternatively, in case (ii), the bank has a strong regulatory incentive for supplying illiquid accounts, so it offers liquidity premium $\pi\theta_B \geq 0$ that is just enough to make agents select these accounts.

3.2 Time-inconsistent Agents

Time-inconsistent agents use quasi-hyperbolic discounting with $\beta < 1$. Regardless of the state of nature, those holding a liquid account consume their unit endowment in period 1, while holders of an illiquid account behave like their time-consistent counterparts. Their respective consumption plans in period 1 are:

$c_1^{TI,*}(F,\omega) = 1; c_2^{TI,*}(F,\omega) = 0, \quad \omega = G, B;$

$c_1^{TI,*}(C,\omega) = 0; c_2^{TI,*}(C,\omega) = 1 + r_C, \quad \omega = G, B.$

The optimization problem in period 0 is now:

$Max\ \{\beta(1 + E\theta); \beta(1 + r_C)\},$

where $E\theta = \pi\theta_B + (1 - \pi)\theta_G$, the expected value of θ, has no predetermined sign since $\theta_B \geq 0$ and $\theta_G \leq 0$. It is negative when the expected marginal utility of period-1 consumption, $(1 + E\theta)$, is lower than 1.[6]

Solving the optimization problem yields $r_C^{TI,min}$, the minimal liquidity premium demanded by time-inconsistent agents to hold an illiquid account:

$$r_C^{TI,min} = E\theta = \pi\theta_B + (1 - \pi)\theta_G. \quad (6)$$

[6] In their demand-sided model, Beshears *et al.* (2015a) assume that $E\theta < r_F$, where the interest rate on liquid accounts, r_F, is exogenous. Under this assumption, time-inconsistent agents are always willing to pay for commitment.

Time-inconsistent agents face a trade-off between commitment and flexibility. On the one hand, commitment protects them against over-consumption in period 1 in the good state of nature. On the other, flexibility offers a hedge against the bad state of nature.

Eq. (6) shows that $r_C^{TI,min}$ does not depend on the present-bias, β. This outcome results from the combination of quasi-hyperbolic discounting and absence of consumption in time 0. In our model, the only role of parameter β is to encourage time-inconsistent agents to withdraw their full endowment in period 1, regardless of the state of nature. By contrast, $r_C^{TI,min}$ varies with probability π of an adverse shock. When π is low enough to yield $E\theta < 0$, threshold $r_C^{TI,min}$ is negative and savers are willing to pay for commitment because flexibility has a lower value than commitment. When the probability of the shock is high ($E\theta \geq 0$), flexibility is more valuable than commitment, and agents require a reward to hold illiquid accounts. Parameter $E\theta$ measures the intensity of the trade-off between flexibility and commitment faced by time-inconsistent agents. The lower $E\theta$, the lower the equilibrium liquidity premium. Maximizing the bank's profits determines the equilibrium outcomes as shown in Proposition 2:

Proposition 2:

If the bank is monopolistic and the pool of savers is made up of sophisticated time-inconsistent agents, the equilibrium quantities of savings accounts, D_F^* and D_C^*, and the equilibrium liquidity premium r_C^*, are given by:

(i) $r_A(\rho_F - \rho_C) < E\theta \quad \Rightarrow \quad D_F^* = N,\ D_C^* = 0$, and $r_C^* = -\infty$;

(ii) $r_A(\rho_F - \rho_C) \geq E\theta \quad \Rightarrow \quad D_F^* = 0,\ D_C^* = N$, and $r_C^* = E\theta$.

Proof: see Appendix B.

The bank serving a homogenous pool of time-inconsistent savers chooses between two types of saleable accounts: liquid accounts and illiquid accounts with premium $E\theta$. Proposition 2 describes how the decision is made depending on the position of $r_A(\rho_F - \rho_C)$ with respect to $E\theta$. When $E\theta < 0$, case (ii) always prevails and time-inconsistent agents end up paying for commitment. This case fits with the evidence both for the gym industry (DellaVigna & Malmendier, 2006) and for the newspaper business (Oster & Morton, 2005). When $E\theta \geq 0$ and $r_A(\rho_F - \rho_C) \geq E\theta$, the time-inconsistent agents are rewarded for making the commitment.

Let us now compare the model outcome for time-inconsistent savers in Proposition 2 with the benchmark situation in Proposition 1. Three features stand out. First, the minimal liquidity premium required by time-inconsistent agents for holding illiquid savings accounts is not larger than that of their time-consistent counterparts: $r_C^{TI,min} \leq r_C^{TC,min}$ This is because time-consistent agents face no trade-off. Commitment is useless to them: in the good state of nature, time-consistent agents with a liquid account stick to their initial plan of consuming their total wealth in period 2. The second difference concerns the sensitivity to parameter θ_G (≤ 0) , which is related to the marginal utility they get from period-1 consumption in the good state of nature. Time-consistent agents are insensitive to θ_G since they never consume during period 1 in the good state of nature. By contrast, θ_G matters to time-inconsistent agents with a liquid account, who consume in period 1. The closer θ_G to zero, the smaller the gap between the minimal liquidity premiums that the two types of agents require for holding illiquid accounts. In addition, Eq. (2) implies that $|\theta_G|$ is the lower bound for $(1-\beta)$. Hence, $|\theta_G|$ represents the minimal level of present bias needed to push time-inconsistent agents to consume in period 1 when the state of nature is good. Last, in both propositions the assumption that $(\rho_F - \rho_C) > 0$ is key to our argument that all agents can be rewarded for making the commitment. If the bank is indifferent between liquid and illiquid accounts (i.e., $\rho_F = \rho_C$) and if agents demand a positive liquidity premium ($E\theta > 0$, which implies also $\pi\theta_B > 0$), then commitment contracts do not exist in equilibrium.

3.3 Perfect Competition

Under perfect competition, banks reward deposits at their marginal benefit, i.e., $r_C^* = r_A(\rho_F - \rho_C)$, and the equilibrium liquidity premium is unaffected by the aggregate demand functions in Eq. (5). The following Proposition summarizes the results.

Proposition 3:

Under perfect competition, if the pool of savers is made up of time-consistent (resp., sophisticated time-inconsistent) agents, the equilibrium liquidity premium r_C^* and the equilibrium quantities of savings accounts, D_F^* and D_C^*, are given by:

(i) $r_A(\rho_F - \rho_C) < \pi\theta_B\,(resp., E\theta) \quad \Rightarrow \quad D_F^* = N,\ D_C^* = 0,\ r_C^* = r_A(\rho_F - \rho_C);$

(ii) $r_A(\rho_F - \rho_C) \geq \pi\theta_B\,(resp., E\theta) \quad \Rightarrow \quad D_F^* = 0,\ D_C^* = N, r_C^* = r_A(\rho_F - \rho_C).$

Two situations can occur. In case (i), the equilibrium is like the one arising in the monopolistic situation described in Propositions 1 and 2: The reserve requirement differential is small and the bank is better-off offering liquid accounts only. In case (ii), compared with the monopolistic situation, the bank pays a relatively high (positive) liquidity premium to its savers. The agents receive a higher-than-requested premium for holding illiquid accounts. This is because, under perfect competition, the bank cannot capture any profit surplus.

In the remainder of the paper, we will focus on the situation of the monopolistic bank. This is a more challenging task from our standpoint since the bank has the possibility to exploit the savers' reservation prices.

4. Heterogeneous Agents and Asymmetric Information

This section studies a monopolistic setting where time-inconsistent and time-consistent agents coexist. Real banks typically reward holders of illiquid accounts with a positive liquidity premium. In our monopolistic model with time-inconsistent agents, we find a positive liquidity premium in equilibrium only if two conditions are met: first the probability of the shock is high, so time-inconsistent agents value flexibility over commitment; and second, high reserve requirements or high returns on assets encourage the bank to pay the minimal liquidity premium that the agents require. Another possible explanation for a positive liquidity premium is that time-inconsistent agents are not alone in savings markets. We extend our model to acknowledge the possibility that demand for savings accounts emanates from a heterogeneous set of agents, including both time-consistent and time-inconsistent savers. We assume that the bank does not observe each agent's degree of time inconsistency but knows the proportion of agents of each type in the pool of N depositors, namely share q of time-inconsistent agents and share $(1-q)$ of time-consistent agents.[7] This generalization should help elucidate why time-inconsistent savers do not pay for commitment in the banking market.

Again, the bank announces its unique liquidity premium to be offered uniformly to all agents. Let us determine the demand function emanating from the mix of savers. Table C2 in Appendix C summarizes the consumption plans of both time-consistent and time-inconsistent agents holding each kind of account. The position of r_C with respect to $r_C^{TC,min}$ and $r_C^{TI,min}$ determines the demand of each type of agents for the two kinds of savings accounts, and hence the aggregate demand function:

$$\begin{cases} if \quad r_C < E\theta & \Rightarrow \quad D_F = N, D_C = 0 \\ if \quad E\theta \leq r_C < \pi\theta_B & \Rightarrow \quad D_F = (1-q)N, D_C = qN, \\ if \quad r_C \geq \pi\theta_B & \Rightarrow \quad D_F = 0, D_C = N \end{cases} \tag{7}$$

where D_F and D_C are the demand schedules for liquid and illiquid accounts, respectively. Maximizing the bank's profits delivers both separating and pooling equilibria, described respectively in Propositions 4 and 5.

[7] Appendix E explains why restricting the set of agents to two categories of depositors (time-consistent and sophisticated time-inconsistent) is not an actual limitation of our model.

Proposition 4: Separating Equilibrium

If the pool of savers is made up of proportion $q \in (0,1)$ *of time-inconsistent agents and proportion* $(1-q)$ *of time-consistent agents, and*

$$E\theta \leq r_A(\rho_F - \rho_C) < \frac{\pi\theta_B - qE\theta}{1-q}, \qquad (8)$$

then the unique equilibrium is separating: $D_F^* = (1-q)N$, $D_C^* = qN$, and $r_C^* = E\theta$.

Proof: see Appendix D.

In the separating equilibrium, the differential in reserve requirements is high enough to motivate the bank to offer a liquidity premium equal to $E\theta$. This premium encourages only time-inconsistent agents to opt for illiquid accounts. In contrast, time-consistent agents prefer liquid accounts because the liquidity premium does not compensate for forgoing the protection that flexibility offers against the occurrence of the adverse shock. In this case, the bank offers a menu of prices and products that allow clients to self-select, with only time-inconsistent agents opting for the illiquid account. This selection is favorable to the bank, because low-value, time-consistent clients are selected out of the market for illiquid savings. Thanks to time-inconsistent depositors, the bank earns a discount on the liquidity premium.

Proposition 5: Pooling Equilibria

If the pool of savers is made up of proportion $q \in (0,1)$ *of time-inconsistent agents and proportion* $(1-q)$ *of time-consistent agents, and if condition* (33) *is not met, then there are two possible pooling equilibria:*

(i) $r_A(\rho_F - \rho_C) < E\theta \quad \Rightarrow \quad D_F^* = N$, $D_C^* = 0$, *and* $r_C^* = -\infty$;

(ii) $r_A(\rho_F - \rho_C) \geq \frac{\pi\theta_B - qE\theta}{1-q} \quad \Rightarrow \quad D_F^* = 0$, $D_C^* = N$, and $r_C^* = \pi\theta_B$ (≥ 0).

Proof: see Appendix D.

In case *(i)*, all savers opt for liquid accounts. In case *(ii)*, the bank offers a non-negative liquidity premium equal to $\pi\theta_B$, the threshold needed for both types of agent to opt for illiquid accounts. The presence of time-consistent savers permits time-inconsistent savers to obtain a higher liquidity premium than the one they would otherwise be offered. The presence of time-consistent agents prevents the bank from exploiting the time-inconsistency of their fellows.

Propositions 4 and 5 allow us to characterize the equilibria and develop comparative statics. First, probability π of an adverse shock influences the equilibrium through its impact on $E\theta$. When π is high enough to drive $E\theta \geq 0$, time-inconsistent agents value the opportunity of early withdrawal more than they do commitment. Hence, they demand a non-negative premium for binding themselves through illiquid accounts. Still, the minimal premium they require is equal to or lower than the one required by time-consistent agents, $\pi\theta_B$. When $E\theta \geq 0$, all three situations depicted in Propositions 4 and 5 are possible. The final decision belongs to the bank and depends on its reserve requirements, the profitability of its lending activity, and share q of time-inconsistent agents in the pool of savers. In contrast, when π is low and $E\theta < 0$, flexibility is less valuable and time-inconsistent agents consent to pay for commitment. The bank is keen to seize this opportunity. As a result, in equilibrium all time-inconsistent agents end up with illiquid accounts, and case (i) of Proposition 5 disappears. Still, two cases are possible. In the first, the bank reaches its optimum by supplying illiquid accounts to time-inconsistent agents only, and the equilibrium is separating. Accordingly, the monopolistic bank captures the profit surplus that time-inconsistent agents pay for commitment. In the second case, this surplus is low and the bank prefers to supply costlier illiquid accounts to all agents, and the equilibrium is pooling. The additional stable funds the bank receives from time-consistent agents are worth giving up in return for the surplus associated with time-inconsistent savers. The winners are time-inconsistent agents, who end up being rewarded for holding illiquid savings accounts they would have agreed to pay for.

Second, Propositions 4 and 5 highlight that the equilibrium liquidity premium depends on the composition of the pool of depositors. The next corollary gives the cut-off value $\tilde{q}$ between the separating and the pooling equilibria under the condition that the bank supplies illiquid accounts.

Corollary 1:

If $E\theta \leq r_A(\rho_F - \rho_C)$, *then:*[8]

(i) *For* $q > \tilde{q}$, *the equilibrium is separating and* $r_C^* = E\theta = \pi\theta_B + (1-\pi)\theta_G$;

(ii) *For* $q \leq \tilde{q}$, *the equilibrium is pooling and:* $r_C^* = \pi\theta_B$,

where $\tilde{q} = \frac{r_A(\rho_F-\rho_C)-\pi\theta_B}{r_A(\rho_F-\rho_C)-E\theta}$.

Corollary 1 shows that the nature of prevailing equilibria boils down to an inequality between q, the proportion of time-inconsistent agents in the market, and $\tilde{q}$, a parameter summarizing the influence of all the other structural parameters. The bank is more reluctant to attract time-consistent agents to illiquid accounts when the share of time-inconsistent agents in the market is high ($q > \tilde{q}$). While share $q\epsilon[0,1]$, the cut-off value $\tilde{q}$ can take any sign. When $\tilde{q}$ is negative, the inequality $q > \tilde{q}$ is not binding and the equilibrium is separating, irrespective of the share of time-inconsistent agents in the market. Corollary 2 features the impacts on $\tilde{q}$ of the structural parameter of the model.

Corollary 2:[9]

If $E\theta \leq r_A(\rho_F - \rho_C)$:

a) Impact of bank productivity: $\frac{\partial\tilde{q}}{\partial r_A} = (\rho_F - \rho_C)\frac{\pi\theta_B - E\theta}{[r_A(\rho_F-\rho_C)-E\theta]^2} \geq 0.$

b) Impact of banking regulation: $\frac{\partial\tilde{q}}{\partial(\rho_F-\rho_C)} = r_A\frac{\pi\theta_B - E\theta}{[r_A(\rho_F-\rho_C)-E\theta]^2} \geq 0.$

c) Impact of (minimum level of) present bias of time-inconsistent agents:

[8] The assumption rules out the situation where the bank fails to supply illiquid accounts.
[9] Corollary 2 is a direct consequence of derivation rules.

$$\frac{\partial\tilde{q}}{\partial|\theta_G|} = -\frac{(1-\pi)[r_A(\rho_F-\rho_C)-\pi\theta_B]}{[r_A(\rho_F-\rho_C)-E\theta]^2}\begin{cases}< 0 & if\ r_A(\rho_F-\rho_C) > \pi\theta_B \\ \geq 0 & if\ r_A(\rho_F-\rho_C) \leq \pi\theta_B\end{cases}.$$

d) Impact of the probability of an adverse shock:

$$\frac{\partial\tilde{q}}{\partial\pi} = \frac{-\theta_G[r_A(\rho_F-\rho_C)-\theta_B]}{[r_A(\rho_F-\rho_C)-E\theta]^2}\begin{cases}< 0 & if\ r_A(\rho_F-\rho_C) > \theta_B \\ \geq 0 & if\ r_A(\rho_F-\rho_C) \leq \theta_B\end{cases}.$$

Corollary 2 (a and b) shows that the critical value $\tilde{q}$ is an increasing function of the supply-side parameters: the net rate of return on loans, r_A, and the spread of reserve ratios between liquid and illiquid accounts, $(\rho_F - \rho_C)$. All else equal, the higher r_A and/or $(\rho_F - \rho_C)$, the higher the probability that time-consistent agents hold illiquid accounts. Higher r_A and/or $(\rho_F - \rho_C)$ imply that illiquid accounts are more profitable to the bank, pushing it towards paying a higher liquidity premium and attracting time-consistent depositors. When $r_A\ (\rho_F - \rho_C)$ is high enough, the bank is not motivated to exploit the presence of time-inconsistent agents. 2c states that if $r_A(\rho_F - \rho_C) > \pi\theta_B$, then $\tilde{q}$ is a decreasing function of $|\theta_G|$, which relates to the marginal utility in period 1 in the no-shock situation and also represents the minimum level of present bias of time-inconsistent agents. The parameter $|\theta_G|$ pushes the minimal required premium of time-inconsistent agents downward. The higher $|\theta_G|$, the more the bank would prefer to sell the illiquid account to time-inconsistent agents only. Hence, a high $|\theta_G|$ translates into a low $\tilde{q}$ and consequently a small chance that time-consistent agents will end up with illiquid accounts. In contrast, when $r_A(\rho_F - \rho_C) \leq \pi\theta_B$, the equilibrium is separating irrespective of the proportion of time-inconsistent agents in the market. The equilibrium outcomes are insensitive to the (positive) impact of $|\theta_G|$ on $\tilde{q}$. According to Corollary 2d, the impact of probability π on $\tilde{q}$ depends on the net benefits that the bank gains by collecting illiquid deposits from time-consistent agents, $r_A(\rho_F - \rho_C) - \theta_B$. Uncertainty pushes the minimal required liquidity premium of time-consistent agents upwards proportionally to θ_B. When $r_A(\rho_F - \rho_C) > \theta_B$, providing illiquid accounts to time-consistent depositors is profitable: π impacts $\tilde{q}$ positively, increasing the likelihood of reaching a pooling equilibrium. In contrast, when $r_A(\rho_F - \rho_C) \leq \theta_B$, higher uncertainty yields a smaller $\tilde{q}$.

To sum up, the privacy of individual time inconsistency creates asymmetric externalities of each type of agent on the other. On the one hand, time-inconsistent agents are insensitive to (or benefit from) the presence of time-consistent agents because they obtain at least the minimal premium they demand for holding illiquid accounts. On the other hand, the presence of time-inconsistent agents makes it harder for their time-consistent counterparts to obtain illiquid accounts. In formal terms, for a pool of time-consistent savers the cut-off value for $r_A(\rho_F - \rho_C)$ is $\pi\theta_B$ (Proposition 1), which is lower than the cut-off value in the general case, $\frac{\pi\theta_B - qE\theta}{1-q}$. When q is large (i.e. close to 1), the spread can be huge. Thus, compared with banks serving a heterogeneous market, those dealing only with time-consistent savers are more likely to offer them illiquid accounts.

Banks are liquidity providers. Loans and deposits are meant to allow agents to smooth consumption when revenues are irregular or uncertain. As described in the model of Diamond and Dybvig (1983), liquid savings are optimal for agents locked in long-term investments that are costly to liquidate. The problem is that liquid savings are subject to bank panics, which are in turn amplified by asymmetric information and self-fulfilling expectations (Chari & Jagannathan, 1988; Calomiris & Gorton, 1991). Faced with the moral hazard issues associated with deposit insurance, financial regulators envisioned alternative measures such as reserve requirements aiming to reduce the supply of liquid accounts, which can trigger bank runs. Yet, restrictions on banks' activities can negatively affect their efficiency (Barth *et al.*, 2013) and should therefore be used with caution.

Our results in Propositions 4 and 5 emphasize that regulations on reserve requirements might be relaxed for banks fulfilling one of two conditions: either they manage to attract a significant share of sophisticated time-inconsistent savers—i.e., when q is high—or they serve a clientele experiencing rare aggregate shocks—when π is low. First, when q is high, cumbersome liquidity restrictions are likely counterproductive. Following the evidence reported by Bernheim *et al.* (2015) and Carvalho *et al.* (2016) that time-inconsistent savers are frequently found in poor populations, regulating institutions that serve the poor, such as microfinance institutions, can be effective with less stringent reserve requirements than those applicable to mainstream banks. Financial institutions targeting poor savers benefit from a sort of spontaneous hedge against liquidity risks. This point is especially relevant given that regulatory compliance is shown to curtail both the social and the financial performance of microfinance institutions (Cull *et al.*, 2014). Second, when π is low, the bank can earn a financial reward by offering the illiquid accounts it needs in order to hedge liquidity risk. We are, however, not aware of any real-life example of such situations in a formal financial market. The explanation may be that time-inconsistent agents are never alone in savings markets. If there are enough time-consistent—or naïve time-inconsistent—agents around to reach condition $q \leq \tilde{q}$, then Corollary 1 ensures that the equilibrium is pooling and $r_C^* = \pi\theta_B \geq 0$. Even though sophisticated time-inconsistent savers are willing to pay for commitment, the liquidity premium may be positive in equilibrium because there are time-consistent savers in the market. Time-consistent agents prevent the bank from extracting a monopoly rent from their time-inconsistent fellows. This argument may not hold for informal savings markets, where deposit collectors are able to observe their clients' degree of time inconsistency and offer individually tailored products with possibly negative liquidity premia (Rutherford, 2000).

5. Conclusion

This paper determines the impact of time-inconsistent savers on the equilibrium liquidity premium, and delivers predictions for the banking sector. First, the presence of time-inconsistent agents creates intrinsic demand for illiquid and stable deposits. Second, time-inconsistent agents demand commitment devices, such as illiquid assets, thus increasing their price and making illiquid accounts less attractive to their time-consistent counterparts. This is especially true when savers are dealing with a monopolistic bank that fails to develop a profitable lending activity and poorly hedges its liquidity risk. In addition we show that, compared with the situation where all savers are time-consistent, a mix of savers spontaneously reduces the bank's liquidity risk. The composition of the pool of savers is thus relevant to regulators when setting reserve requirements.

The banking sector provides a meaningful example showing that time-inconsistent agents do not necessarily have to pay for commitment contracts. Our model departs from the literature on pricing contract problems with time-inconsistent agents (DellaVigna & Malmendier, 2004; Gottlieb, 2008) by addressing a situation where the provider has an intrinsic motivation for favoring or rewarding commitment contracts. Further work could investigate whether similar situations exist outside the banking sector. For instance, as is the case for banks incurring liquidity risk, commitment contracts should be valuable to firms facing pervasive uncertainty. Still, the contracts in question should be nonexclusive, which therefore restricts their design (Gottlieb, 2008).

Our model suffers from several limitations. First, it considers only the most stringent commitment, which excludes early withdrawals. In fact, the relevance of finite penalties is still controversial. Amador *et al.* (2006) and Beshears *et al.* (2014a; 2015a) show that time-inconsistent agents prefer the 100% withdrawal penalty. Finite withdrawal penalties lead time-inconsistent agents to under-saving and money burning, which are costly to one of the parties but bring no direct benefits to the other (Ambrus & Egorov, 2015). In contrast, Ambrus and Egorov (2013) show that finite penalties make sense in theoretical settings where a rare but severe negative shock is possible. As Gilkeson *et al.* (1999), Amromin and Smith (2003), and Beshears *et al*. (2015b) put it, finite early withdrawal penalties are used for real-life bank deposits and retirement savings.

Second, we use linear objective functions and a three-period situation, where cash may be withdrawn from the liquid account in one period only. A multi-period model with non-linear utilities could deliver more nuanced results, by allowing clients to combine savings accounts. The same holds true had we allowed for idiosyncratic shocks. Still, we contend that adding a layer of complexity would have little effect on the qualitative outcomes of our model. The purpose of this stylized model is to pinpoint the impact of time-inconsistent savers on the market price for illiquid accounts. Our model is the first to make these points.

Finally, the evidence that investors are subject to behavioral anomalies has long been recognized in the banking literature (Barberis & Thaler, 2003). Models of present bias were first developed to address savings and consumption, but they rapidly reached other topics such as housing and credit cards (Rabin, 2013). Strikingly, so-called irrational agents are often represented in financial models as noise traders or overly optimistic or pessimistic speculators. The lack of self-control is another, yet unaddressed, type of behavioral feature. We hope our approach will convince scholars to develop the theory of behavioral banking still further.

Appendix A: Proof of Proposition 1

We have the following optimal liquidity premium and profit:

$$r_C^* = \begin{cases} -\infty, & if \;\; r_C < \pi\theta_B \\ \pi\theta_B, & if \;\; r_C \geq \pi\theta_B \end{cases};$$

$$P_0{}^*(r_C) = \begin{cases} r_A(1-\rho_F)N, & if \;\; r_C = -\infty \\ [r_A(1-\rho_C) - \pi\theta_B\,]N, & if \;\; r_C = \pi\theta_B \end{cases}.$$

The bank optimization problem is:

$$\underset{r_C}{\text{Max}} \; \{r_A(1-\rho_F)N, [r_A(1-\rho_C) - \pi\theta_B\,]N\}.$$

We have:

$$[r_A(1-\rho_C) - \pi\theta_B\,] \geq r_A(1-\rho_F) \;\Leftrightarrow\; \pi\theta_B \leq r_A(\rho_F - \rho_C).$$

Thus, if $\pi\theta_B > r_A(\rho_F - \rho_C)$, the bank's profit from the illiquid account is smaller than that produced by liquid accounts, and in equilibrium the bank supplies liquid savings accounts only, so that $r_C^* = -\infty$. The agents have no other choice than to take up the liquid account: $D_F^* = N,\; D_C^* = 0$. Alternative, if $\pi\theta_B \leq r_A(\rho_F - \rho_C)$, the bank finds it profitable to reward illiquid accounts with the minimal liquidity premium required by savers to hold these accounts, which is $r_C^* = \pi\theta_B$, and the depositors take up illiquid savings accounts: $D_F^* = 0,\; D_C^* = N$.

QED

Appendix B: Proof of Proposition 2

We have the following optimal liquidity premium and profit:

$$r_C^* = \begin{cases} -\infty, & if \;\; r_C < E\theta \\ E\theta, & if \;\; r_C \geq E\theta \end{cases};$$

$$P_0{}^*(r_C) = \begin{cases} r_A(1-\rho_F)N, & if \;\; r_C = -\infty \\ [r_A(1-\rho_C) - E\theta]N, & if \;\; r_C = E\theta \end{cases}.$$

The rest of the proof is easily transposed from that of Proposition 1 by replacing $\pi\theta_B$ by $E\theta$.

QED

Appendix C: Heterogeneous Agents

Table C1: Optimal Consumption and Intertemporal Utility for Heterogeneous Agents

	Time-consistent agents ($i = TC$)			**Time-inconsistent agents** ($i = TI$)		
	Consumption in period 1: $c_1^{TC,*}(s,\omega)$	**Consumption in period 2:** $c_2^{TC,*}(s,\omega)$	**Intertemporal utility in period 1:** $U_1^{TC,*}(s,\omega)$	**Consumption in period 1:** $c_1^{TI,*}(s,\omega)$	**Consumption in period 2:** $c_2^{TI,*}(s,\omega)$	**Intertemporal utility in period 1:** $U_1^{TI,*}(s,\omega)$
Liquid account ($s = F$)						
No shock ($\omega = G$)	0	1	1	1	0	$1+\theta_G$
Shock ($\omega = B$)	1	0	$1+\theta_B$	1	0	$1+\theta_B$
Illiquid account ($s = C$)						
No shock ($\omega = G$)	0	$1+r_C$	$1+r_C$	0	$1+r_C$	$\beta(1+r_C)$
Shock ($\omega = B$)	0	$1+r_C$	$1+r_C$	0	$1+r_C$	$\beta(1+r_C)$

Appendix D: Proof of Proposition 4

Eq. (7) allow us to partition the possible values for $r_C \in \mathbb{R}$ into three zones:

Zone I: $r_C \in (-\infty, E\theta)$;

Zone II: $r_C \in [E\theta, \pi\theta_B)$;

Zone III: $r_C \in [\pi\theta_B, +\infty)$.

In zone *I*, all agents opt for liquid accounts. In zone *II*, time-inconsistent agents take illiquid accounts, and time-consistent agents take liquid accounts. In zone *III*, all agents opt for illiquid accounts. The bank's profits in zone j $(j = I, II, III)$ are:

$$r_{C,j}^* = \begin{cases} -\infty, & if \;\; j = I \\ E\theta, & if \;\; j = II \\ \pi\theta_B, & if \;\; j = III \end{cases}.$$

$$P_{0,j}^* = \begin{cases} r_A(1-\rho_F)N, & if\; j = I \\ [r_A(1-\rho_C) - E\theta]qN + r_A(1-\rho_F)(1-q)N, & if\; j = II. \\ [r_A(1-\rho_C) - \pi\theta_B]N\,, & if\; j = III \end{cases}$$

Next, the bank determines its overall optimum by comparing the zone-specific maximal profits. Its optimization problem becomes:

$Max\{r_A(1-\rho_F)N, [r_A(1-\rho_C)-E\theta]qN + r_A(1-\rho_F)(1-q)N, [r_A(1-\rho_C)-\pi\theta_B]N\}$.

We have:

$P_{II}^* \geq P_I^*$ if $E\theta \leq r_A(\rho_F - \rho_C)$;

$P_{III}^* \geq P_{II}^*$ if $\frac{\pi\theta_B - qE\theta}{1-q} \leq r_A(\rho_F - \rho_C)$;

$P_{III}^* \geq P_I^*$ if $\pi\theta_B \leq r_A(\rho_F - \rho_C)$.

These inequalities determine the bank's optimal liquidity premium for each parameter configuration.

QED

Appendix E: Naïve and Partially Naïve Time-Inconsistent Agents

The model presented in Section 4 includes two types of agents: time-consistent and sophisticated time-inconsistent. Here we show that adding partially or fully naïve time-inconsistent agents would not alter the model and subsequently the results on the equilibrium liquidity premium. Partially naïve time-inconsistent agents overestimate their time-consistency (O'Donoghue & Rabin, 1999; 2001). While their actual present bias is β, they perceive it as $\hat{\beta}$, with $\beta < \hat{\beta} < 1$. The difference between the perceived and actual present bias parameter, $\hat{\beta} - \beta$, reflects the agent's overconfidence about future self-control. Fully naïve time-inconsistent agents are unaware of their present bias ($\hat{\beta} = 1$);

Had we introduced fully naïve agents into our models, the consequences would not be noticeable because these agents actually believe they are time-consistent. In period 0, they behave in the same way as time-consistent agents do. By contrast, partially naïve agents can behave in two opposite ways depending on their perceived present bias $\hat{\beta}$. With a low perceived bias, i.e. $\hat{\beta} < 1 + \theta_G$ (see Eq. (6)), they behave as if they were sophisticated. Alternatively, if their perceived present bias is high, i.e. $\hat{\beta} \geq 1 + \theta_G$, partially naïve agents behave like their time-consistent counterparts. Ultimately, the depositors' beliefs in period 0 drive the aggregate demand functions, so that agent categorization boils down to two meaningful categories: sophisticated time-inconsistent and time-consistent depositors.

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