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List of Abbreviations

ARWU = Academic Ranking of World Universities
AHP = Analytical Hierarchy Process
AR = Assurance Regions
BCC = Banker, Charnes, Cooper
BFS = Basic Feasible Solution
CCA = Canonical Correlation Analysis
CCR = Charnes, Cooper, Rhodes
CI = Consistency Index
CRS = Constant Return to Scale
CSW = Common Set of Weights
CV = Coefficient of Variation
D = Dual
DEA = Data Envelopment Analysis
DM = Decision Maker
DMU = Decision Making Unit
DRS = Decreasing Returns to Scale
ELECTRE = ELimination Et Choix Traduisant la REalité/ ELimination and Choice Translating Reality
FDH = Free Disposal Hull
FMS = Flexible Manufacturing System
GAIA = Geometrical Analysis for Interactive Aid
GDEA = General Data Envelopment Analysis
GDP = Gross Domestic Product
GM = Geometric Mean
H & M = Hennes & Mauritz
HDI = Human Development Index
IMDb = Internet Movie Database
IMP = IMProved
I-O = Input-Oriented
IRS = Increasing Returns to Scale
IWR = Input Weight Restriction
LP = Linear Programming
MACBETH = Measuring Attractiveness by a Categorical Based Evaluation Technique
MAUT = Multi-Attribute Utility Theory
MCDA = Multiple Criteria Decision Aid/ Multicriteria Decision Aid
MCDEA = Multi-Criteria Data Envelopment Analysis
MPG = Miles Per Gallon
MS = Management Science
NLP = Non-Linear Programming
O-O = Output-Oriented
OR = Operations Research
OWR = Output Weight Restriction
P = Primal
PCA = Principal Component Analysis
PIIWCCR = PROMETHEE II Weight restricted CCR model
PPS = Production Possibility Set
PROMETHEE = Preference Ranking Organization METHod for Enrichment Evaluation
PTE = Pure Technical Efficient
RI = Random Index
RTS = Returns to Scale
SBM = Slack Base Measurement
SE = Scale Efficient
SEI = Sustainable Energy Index
TE = Technical Efficient
TSP = Travelling Salesman Problem
VEA = Vertex Enumeration Algorithm
VRS = Variable Return to Scale
WCCR = Weight restricted CCR
List of Notations

Chapter 1

- The optimal solution is: \( x^* = \text{argmax} \ {f(x)}_{x \epsilon A} \);
- An input vector \( X_j = (x_{1j}; x_{2j}; \ldots; x_{ij}; \ldots; x_{mj})^T \); \( x_{ij} \) is the quantity of input \( i \) used by \( DMU_j \);
- An output vector \( Y_j = (y_{1j}; y_{2j}; \ldots; y_{rj}; \ldots; y_{sj})^T \); \( y_{rj} \) is the quantity of output \( r \) obtained by \( DMU_j \); where \( j = 1, 2, \ldots, n; i = 1, 2, \ldots, m \) and \( r = 1, 2, \ldots, s \);
- \( DMU_j \) is the unit \( j \);
- Virtual input= \( \sum_{i=1}^{m} v_i^* X_j, j = 1, \ldots, n \);
- Virtual output= \( \sum_{r=1}^{s} u_r^* Y_j, j = 1, \ldots, n \);

where \( v^* = (v_1^*, \ldots, v_n^*) \): non-negative input optimal weight vector and \( u^* = (u_1^*, \ldots, u_s^*) \): non-negative output optimal weight vector;
- \( \geq \text{ in } X_j \geq 0 \) shows a non-negative input vector; \( \exists j: x_{ij} \neq 0 \) means at least one of the input vector’s elements is not zero;
- \( \geq \text{ in } Y_j \geq 0 \) shows a non-negative input vector; \( \exists j: y_{rj} \neq 0 \) means at least one of the output vector’s elements is not zero;
- \((X, Y) \) Activity ;
- \( X \geq X \) and \( Y \leq Y \): any activity with input no less than \( X \) in any component and with output no greater than \( Y \) in any component;

In this thesis, vectors are compared in their magnitude.
- Convex set: \( (X, Y) \epsilon PPS, (X', Y') \epsilon PPS \) and \( \lambda \epsilon (0,1) \Rightarrow \lambda(X, Y) + (1 - \lambda)(X', Y') \epsilon PPS \);
- \( PPS_{CRS} = \{(X, Y) | X \geq \sum_{j=1}^{n} X_j \lambda_j, Y \leq \sum_{j=1}^{n} Y_j \lambda_j, \lambda_j \geq 0 \} \);
- \( PPS_{VRS} = \{(X, Y) | X \geq \sum_{j=1}^{n} X_j \lambda_j, Y \leq \sum_{j=1}^{n} Y_j \lambda_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \} \);
- \( \theta \) is the efficiency score of \( DMU_j \): the optimal solution of primal LP (CCR, BCC and Additive I-O multiplier models);
- \( Z \) is the efficiency score of \( DMU_j \): the optimal solution of dual LP (CCR, BCC and Additive I-O envelopment models);
- \( u_r \) is the weight of output \( r \);
- \( v_i \) is the weight of input \( i \);
• \((Z^*, \lambda^*)\) is an optimal solution for the CCR I-O envelopment model;
• \((\phi^*, \tilde{\lambda}^*)\) is optimal for the corresponding CCR O-O model;
• \(\lambda_j\) the dual variable;
• \(u_o\) the dual variable of BCC model;
• \(s_i^-\) is the shortage quantity of input \(i\) in \(DMU_j\);
• \(s_r^+\) are and extra quantity of output \(r\) in \(DMU_j\);
• \(\varepsilon > 0\), where \(\varepsilon\) is a non-Archimedean element smaller than any positive real number;
• \(Q\) is the efficiency score of the O-O multiplier BCC model;
• \((\tilde{x}_{o}\tilde{y}_{o})\) The projected point on the efficient frontier in CCR and BCC models.

Chapter 2
• \(F = \{f_1, ..., f_k, ..., f_q\}\) Family of criteria;
• \(\mathcal{A} = \{a_1, ..., a_j, ..., a_n\}\) Set of alternatives;
• \(f_k(a)\) The evaluation of action \(a\) (alternative \(a\)) according to criterion \(j\);
• \(aPb\) \(a\) is preferred to \(b\);
• \(aIa\) \(a\) is indifferent to \(b\);
• \(aRb\) \(a\) is incomparable to \(b\);
• \(a_j \neg P a_i\): \(P\) is “asymmetric”;
• \(a_i \neg R a_i\): \(R\) is irreflexive;
• \(S = (P \cup I)\) an outranking relation;
• \(aDb \leftrightarrow f_k(a) \geq f_k(b): a, b \in \mathcal{A}, k = \{1,2, ..., q\} : a\) dominates \(b\);
• \(\exists k \epsilon \{1,2, ..., q\}: f_k(a) > f_k(b): a, b \in \mathcal{A}; a\) is efficient in comparison with \(b\);
• \(U(x)\) is the utility function;
• \(v_{n,q}\): The cells of the matrix contain estimates of the performance of each alternative on each of the criteria provided by an expert or various experts in MAUT;
• \(U(a) > U(b) \iff a > b\) (\(a\) is preferred to \(b\));
• \(U(a) = U(b) \iff a = b\) (\(a\) is indifference to \(b\));
• \(U_k\) is the utility function of criterion \(k\): \(k = \{1,2, ..., q\}\).
\[ C_{ik} ; i, k = 1, 2, ..., q \] expresses the relative importance of the criterion \( i \) over the criterion \( k \) in AHP;

- \( C \) the weight matrix in AHP;
- \( CI \) consistency index;
- \( c(a_i S a_j) \) Concordance index in ELECTRE;
- \( d(a_i S a_j) \) Discordance index;
- \( v \) discordance threshold;
- \( \forall a_j \in A \setminus A' \) the solutions, which are not belongs to \( A \);
- \( s_1 \) and \( s_2 \) concordance thresholds;
- \( v_k(f_k(a_i)) \) veto threshold;
- \( S(a_i, a_j) \) the credibility degree;
- \( q_k \) and \( p_k \) are indifference and preference thresholds;
- \( d_k(a_i, a_j) = f_k(a_i) - f_k(a_j) \), the differences between each pair of alternatives on each criterion in PROMETHEE;
- \( P_k(a_i, a_j) \) predefined preference function;
- \( P_k(d_k(a, b)) \) Function of preference of one action over another;
- \( \pi_k(a_i, a_j) \) unicriterion preference degrees;
- \( \pi(a_i, a_j) \) outranking degree;
- \( \phi(a) = \phi^+(a) - \phi^-(a) \), The net outranking flow is defined as the difference between the positive flow and the negative one;
- \( \phi_k(a_j) \) the unicriterion net flow score of alternative \( a_j \);
- \( (S^+, I^+) \) and \( (S^-, I^-) \) be the complete pre-orders obtained from the positive and negative flows;
- \( (P^I, I^I, R^I) \) correspond to the preference, indifference and incomparability of each pair of alternatives in PROMETHEE I;
- \( (P^{II}, I^{II}) \) are the preference and indifference relations between each pair of alternatives in PROMETHEE II;
- \( \phi \) matrix of unicriterion net flow scores;
- \( \alpha_j \) is the coordinate vector of each alternative;
• \(e_k\) is an axis for each criterion;
• \(w\) is the weight vector;
• \(\pi\) the decision stick;
• \(\delta\) is the amount of information preserved by GAIA plane;
• \(\Delta(a_i, a_j) = \emptyset(a_i) - \emptyset(a_j);\)
• \(\Delta'(a_i, a_j) = \emptyset'(a_i) - \emptyset'(a_j);\)
• \(\Delta_k(a_i, a_j) = \emptyset_k(a_i) - \emptyset_k(a_j);\)
• \(\alpha\) and \(\beta\) the factor of rearranging weights;
• \(\alpha^-\) the lower bound of \(\alpha;\)
• \(\Omega^- = \{(a_i, a_j) \in A \times A, s.t. \Delta(a_i, a_j) \Delta_k(a_i, a_j) < 0\};\)
• \(\alpha^+\) the upper bound of \(\alpha;\)
• \(\Omega^+ = \{(a_i, a_j) \in A \times A, s.t. \Delta(a_i, a_j) \Delta_k(a_i, a_j) > \Delta^2(a_i, a_j)\};\)
• \(\Omega^0 = \{(a_i, a_j) \in A \times A, s.t. \Delta(a_i, a_j) = 0 and \Delta_k \neq 0\}.\)

Chapter 3

• \(\geq\) and \(>\) mean \(\geq\) and \(>\);
• \(\Delta\) Objective function of GDEA model;
• \(\bar{d}_j\) is the maximum of deviation between weighted investigated DMU and other DMUs.
• \(Y^*_r\) is the output of the ideal point;
• \(s_j\) is the relative distance to the ideal point.

Chapter 4

• \((\alpha_i, \beta_i, \gamma_i, \delta_i, \eta_i, \theta_i, k_i, \rho_i, \sigma_i, \tau_i, \varphi_i)\) are constants which imposed to weights in different weight restricted DEA model (5-2);
• \(V = \{v| Cv \geq 0\}\) convex cone for the inputs weights in Cone-Ratio DEA model: intersection form;
• \(U = \{u| Du \geq 0\}\) convex cone for the outputs weights in Cone-Ratio DEA model: intersection form;
• \(\mathbb{R}^+_m\) the non-negative real numbers: domain of inputs weights;
• \(\mathbb{R}^+_s\) the non-negative real numbers: domain of outputs weights;
• \( \mathcal{V} \) convex cone for the inputs weights in Cone-Ratio DEA model: sum form;
• \( \mathcal{U} \) convex cone for the outputs weights in Cone-Ratio DEA model: sum form;
• \( w_i^* \) the central weights in the vector \( W \);
• \( \phi_i(a_j) = 1 \) : the dummy input added to unicriterion net flow score matrix ;
• \( v_i \) associated weight to dummy input;
• \( \alpha_k, \beta_k \) and \( \lambda_j \) are the dual variables of the model PIIWCCR;
• \( \phi_k \) the output vector of dual form of PIIWCCR;
• \( \lambda, A \) and \( B \) are the vectors of dual variables of PIIWCCR;
• \( W^+ \) and \( W^- \) are also lower and upper bounds vectors;
• \((Z^*_o, C_o^*, \lambda^*, s^{++}_i, s^{-+}_i, A^*, B^*)\) an optimal solution of dual PIIWCCR ;
• \( E_o \) reference set ;
• \((\tilde{O}_{to}, \tilde{O}_{ko})\) the improved activity in dual of PIIWCCR ;
• \( P_{imp} \) : the primal form according to the improved activity;
• \( D_{imp} \) : the dual form according to the improved activity;
• \((Z^{*\text{imp}}_{o_1} = 1, \lambda^{*\text{imp}} = \lambda^*, s^{*\text{imp}}_{i_1}, s^{*\text{imp}}_{i_2}, A^{*\text{imp}}, B^{*\text{imp}} = 0)\) the optimal solution of dual problem according to improved activity;
• \((v^{*\text{imp}}_i, w^{*\text{imp}}_i)\) the optimal solution of primal problem according to improved activity.

Chapter 5
• \( v^{*}_{ik} \) and \( u^{*}_{rk} \) : the optimal weights of inputs and outputs resulted by running CCR;
• \( E_{kj} \) : the related score of \( DMU_j \), using weights of \( DMU_k \);
• \( E_k \) : the average cross efficiency scores;
• \( M_k \) : maverick index;
• \( \rho_o^* \) : the benchmark score of efficient \( DMU_o \);
• \( Z_j \) and \( W_j \) : input and output composites, respectively ;
• \( r_{ZW} \) : the coefficient correlation between composite input and output;
• \( S_{XX}, S_{YY} \) : the matrices of the sums of squares of the input and output variables, respectively;
• \( S_{XY} \) : the matrix of the sums of products of the input and output variables;
• $T_i$: DEA scores of DMUs of canonical correlation analysis technique;
• $a_{jk}$: the evaluation of unit $j$ over unit $k$ in the pairwise comparison matrix in AHP;
• $v_q$: the weights of the criteria in the Jablonsky’s model;
• $w_{ij}$: the preference indices of efficient DMUs;
• $E_{AB}$ and $E_{BA}^*$: the cross efficiency scores of each pair of $A$ and $B$;
• $E_{ij}^*$: the cross efficiency score of unit $i$ in comparison with unit $j$;
• $E_i^*$ and $E_{i+k}^*$: the efficiency score of unit $i$ and $i + k$, respectively, in comparison between $n$ units with a single input and a single output;
• $A_i = \frac{x_j}{y_j}$;
• $E_{i,i+k}, E_{i+k,i}^*$: the efficiency scores of $DMU_i$ and $DMU_{i+k}$, respectively;
• $E''_{i}, E''_{i+k}^*$: the efficiency scores of $DMU_i$ and $DMU_{i+k}$ after adding $\alpha$ to their outputs, respectively;
• $\varnothing'(a_i), \varnothing'(a_{i+k})$: the net flow scores of $DMU_i$ and $DMU_{i+k}$ after adding $\alpha$ to the outputs in DEA model.

**Chapter 6**

• $w_k^-$ and $w_k^+$: weight intervals within which the values are likely to vary, determined by DM;
• $\Delta_{ij} = \varnothing_k(a_i) - \varnothing_k(a_j)$;
• $P$ is a convex polyhedron;
• $R^q$ $q$-dimentional space of polyhedron;
• $v$ is a vertex of polyhedron $P$;
• $B$ is the set of basic points;
• $N$ is the set of co-basic points;
• $M = c(n, 2)$ is the number of constraints resulted by a super-efficient ranking;
• $M' = c(n, 2) - c(N(\text{efficiency} = 1), 2)$ is the number of constraints resulted by a CCR ranking;
• $w_{kj}$ is the weight of criterion $k$ in the constraint $j$;
• $GM$ the geometric mean in weight matrix.