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Part 2 : The main contributions

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List of Abbreviations

ARWU = Academic Ranking of World Universities

AHP = Analytical Hierarchy Process

AR = Assurance Regions

BCC = Banker, Charnes, Cooper

BFS = Basic Feasible Solution

CCA = Canonical Correlation Analysis

CCR = Charnes, Cooper, Rhodes

CI = Consistency Index

CRS = Constant Return to Scale

CSW = Common Set of Weights

CV = Coefficient of Variation

D = Dual

DEA = Data Envelopment Analysis

DM = Decision Maker

DMU = Decision Making Unit

DRS = Decreasing Returns to Scale

ELECTRE = ELimination Et Choix Traduisant la REalité/ ELimination and Choice Translating Reality

FDH = Free Disposal Hull

FMS = Flexible Manufacturing System

GAIA = Geometrical Analysis for Interactive Aid

GDEA = General Data Envelopment Analysis

GDP = Gross Domestic Product

GM = Geometric Mean

H & M = Hennes & Mauritz

HDI = Human Development Index

IMDb = Internet Movie Database

IMP = IMProved

I-O = Input-Oriented

IRS = Increasing Returns to Scale

IWR = Input Weight Restriction

LP = Linear Programming

MACBETH = Measuring Attractiveness by a Categorical Based Evaluation Technique

MAUT = Multi-Attribute Utility Theory

MCDA = Multiple Criteria Decision Aid/ Multicriteria Decision Aid

MCDEA = Multi-Criteria Data Envelopment Analysis

MPG=Miles Per Gallon

MS = Management Science

NLP = Non-Linear Programming

O-O = Output-Oriented

OR = Operations Research

OWR = Output Weight Restriction

P = Primal

PCA = Principal Component Analysis

PIIWCCR= PROMETHEE II Weight restricted CCR model

PPS = Production Possibility Set

PROMETHEE = Preference Ranking Organization METHod for Enrichment Evaluation

PTE = Pure Technical Efficient

RI = Random Index

RTS = Returns to Scale

SBM = Slack Base Measurement

SE = Scale Efficient

SEI = Sustainable Energy Index

TE = Technical Efficient

TSP = Travelling Salesman Problem

VEA = Vertex Enumeration Algorithm

VRS = Variable Return to Scale

WCCR = Weight restricted CCR

List of Notations

Chapter 1

- The optimal solution is: $x^* = argmax\ \{f(x) | x \epsilon \mathcal{A}\}$;
- An input vector $X_j = (x_{1j}; x_{2j}; \dots; x_{ij}; \dots; x_{mj})^T$; x_{ij} is the quantity of input i used by DMU_j;
- An output vector $Y_j = (y_{1j}; y_{2j}; \dots; y_{rj}; \dots; y_{sj})^T$; y_{rj} is the quantity of output r obtained by DMU_j; where $j = 1,2, \dots, n$, $i = 1,2, \dots, m$ and $r = 1,2, \dots, s$;
- DMU_j is the unit j;
- Virtual input= $\sum_{i=1}^{m} v_i^* X_j, j = 1, \dots, n$;
- Virtual output= $\sum_{r=1}^{s} u_r^* Y_j, j = 1, \dots, n$;

where $v^* = (v_1^*, \dots, v_m^*)$: non-negative input optimal weight vector and $u^* = (u_1^*, \dots, v_s^*)$: non-negative output optimal weight vector;

- $\geq$ in $X_j \geq 0$ shows a non-negative input vector; $\exists j: x_{ij} \neq 0$ means at least one of the input vector's elements is not zero;
- $\geq$ in $Y_j \geq 0$ shows a non-negative input vector; $\exists j: y_{rj} \neq 0$ means at least one of the output vector's elements is not zero;
- (X, Y) Activity ;
- $X' \geq X$ and $Y' \leq Y$: any activity with input no less than X in any component and with output no greater than Y in any component;

In this thesis, vectors are compared in their magnitude.

- Convex set: $(X, Y) \epsilon PPS$, $(X', Y') \epsilon PPS$ and $\lambda \epsilon (0,1) \Rightarrow \lambda(X, Y) + (1 - \lambda)(X', Y') \epsilon PPS$;
- $PPS_{CRS} = \{(X, Y) | X \geq \sum_{j=1}^{n} X_j \lambda_j, Y \leq \sum_{j=1}^{n} Y_j \lambda_j, \lambda_j \geq 0\}$;
- $PPS_{VRS} = \{(X, Y) | X \geq \sum_{j=1}^{n} X_j \lambda_j, Y \leq \sum_{j=1}^{n} Y_j \lambda_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0\}$;
- θ is the efficiency score of DMU_j: the optimal solution of primal LP (CCR, BCC and Additive I-O multiplier models);
- Z is the efficiency score of DMU_j: the optimal solution of dual LP (CCR, BCC and Additive I-O envelopment models);
- u_r is the weight of output r;
- v_i is the weight of input i;

- (Z^*, λ^*) is an optimal solution for the CCR I-O envelopment model;
- $(\phi^*, \widehat{\lambda^*})$ is optimal for the corresponding CCR O-O model;
- λ_j the dual variable;
- u_o the dual variable of BCC model;
- s_i^- is the shortage quantity of input i in DMU_j;
- s_r^+ are and extra quantity of output r in DMU_j;
- $\varepsilon > 0$, where ε is a non-Archimedean element smaller than any positive real number;
- Q is the efficiency score of the O-O multiplier BCC model;
- $(\widetilde{x_o}, \widetilde{y_o})$ The projected point on the efficient frontier in CCR and BCC models.

Chapter 2

- $F = \{f_1, \dots, f_k, \dots, f_q\}$ Family of criteria;
- $\mathcal{A} = \{a_1, \dots, a_j, \dots, a_n\}$ Set of alternatives;
- $f_k(a)$ The evaluation of action a (alternative a) according to criterion j;
- aPb a is prefered to b;
- aIa a is indifferent to b;
- aRb a is incomparable to b;
- $a_j \neg P a_i$: P is "asymmetric";
- $a_i \neg R\ a_i$: R is irreflexive;
- $S = (P \cup I)$ an outranking relation;
- $aDb \leftrightarrow f_k(a) \geq f_k(b)$: $a, b \epsilon \mathcal{A}, k = \{1,2, \dots, q\}$: a dominates b;
- $\exists k \epsilon \{1,2, \dots, q\}$: $f_k(a) > f_k(b)$: $a, b \epsilon \mathcal{A}$; a is efficient in comparison with b;
- $U(x)$ is the utility function;
- v_{nq} : The cells of the matrix contain estimates of the performance of each alternative on each of the criteria provided by an expert or various experts in MAUT;
- $U(a) > U(b) \Leftrightarrow a \succ b$ (a is preferred to b);
- $U(a) = U(b) \Leftrightarrow a = b$ (a is indifference to b);
- U_k is the utility function of criterion k: $k = \{1,2, \dots, q\}$;

- $C_{ik}: i, k = 1,2, \dots, q$ expresses the relative importance of the criterion i over the criterion k in AHP;
- C the weight matrix in AHP;
- CI consistency index;
- $c(a_i S a_j)$ Concordance index in ELECTRE;
- $d(a_i S a_j)$ Discordance index;
- v discordance threshold;
- $\forall a_j \epsilon \mathcal{A} \backslash A'$ the solutions, which are not belongs to $\mathcal{A}$;
- s_1 and $s_2$concordance thresholds;
- $v_k(f_k(a_i)$ veto threshold ;
- $S(a_i, a_j)$ the credibility degree;
- q_k and p_k are indifference and preference thresholds;
- $d_k(a_i, a_j) = f_k(a_i) - f_k(a_j)$, the differences between each pair of alternatives on each criterion in PROMETHEE;
- $P_k(a_i, a_j)$ predefined preference function;
- $P_k(d_k(a, b))$ Function of preference of one action over another;
- $\pi_k(a_i, a_j)$ unicriterion preference degrees;
- $\pi(a_i, a_j)$ outranking degree;
- $\emptyset(a) = \emptyset^+(a) - \emptyset^-(a)$, The *net outranking flow* is defined as the difference between the positive flow and the negative one;
- $\emptyset_k(a_j)$ the unicriterion net flow score of alternative a_j ;
- (S^+, I^+) and (S^-, I^-) be the complete pre-orders obtained from the positive and negative flows;
- (P^I, I^I, R^I) correspond to the preference, indifference and incomparability of each pair of alternatives in PROMETHEE I;
- (P^{II}, I^{II}) are the preference and indifference relations between each pair of alternatives in PROMETHEE II;
- $\emptyset$ matrix of unicriterion net flow scores;
- α_j is the coordinate vector of each alternative;

- e_k is an axis for each criterion;
- w is the weight vector;
- π the decision stick;
- δ is the amount of information preserved by GAIA plane;
- $\Delta(a_i, a_j) = \emptyset(a_i) - \emptyset(a_j)$;
- $\Delta'(a_i, a_j) = \emptyset'(a_i) - \emptyset'(a_j)$;
- $\Delta_k(a_i, a_j) = \emptyset_k(a_i) - \emptyset_k(a_j)$;
- α and β the factor of rearranging weights ;
- α_k^- the lower bound of α ;
- $\Omega^- = \{(a_i, a_j) \in \mathcal{A} \times \mathcal{A}, s.t. \Delta(a_i, a_j)\, \Delta_k(a_i, a_j) < 0\}$;
- α_k^+ the upper bound of α ;
- $\Omega^+ = \{(a_i, a_j) \in \mathcal{A} \times \mathcal{A}, s.t. \Delta(a_i, a_j)\, \Delta_k(a_i, a_j) > \Delta^2(a_i, a_j)\}$;
- $\Omega^0 = \{(a_i, a_j) \in \mathcal{A} \times \mathcal{A}, s.t. \Delta(a_i, a_j) = 0 \; and \; \Delta_k \neq 0\}$.

<u>Chapter 3</u>

- $\succcurlyeq$ and $\succ$ mean $\geq$ and $>$;
- Δ Objective function of GDEA model;
- $\tilde{d}_j$ is the maximum of deviation between weighted investigated DMU and other DMUs.
- Y_r^* is the output of the ideal point ;
- s_j is the relative distance to the ideal point.

<u>Chapter 4</u>

- $(\alpha_i, \beta_i, \gamma_r, \delta_i, \eta_i, \theta_r, k_i, \rho_i, \sigma_r, \tau_r, \varphi_i)$ are constants which imposed to weights in different weight restricted DEA model (5-2);
- $V = \{v | Cv \geq 0\}$ convex cone for the inputs weights in Cone-Ratio DEA model: intersection form;
- $U = \{u | Du \geq 0\}$ convex cone for the outputs weights in Cone-Ratio DEA model: intersection form;
- R_m^+ the non-negative real numbers: domain of inputs weights;
- R_s^+ the non-negative real numbers: domain of outputs weights;

- $\hat{V}$ convex cone for the inputs weights in Cone-Ratio DEA model: sum form;
- $\hat{U}$ convex cone for the outputs weights in Cone-Ratio DEA model: sum form;
- w_i^{c*} the central weights in the vector W ;
- $\emptyset_l(a_j) = 1$: the dummy input added to unicriterion net flow score matrix ;
- v_l associated weight to dummy input;
- α_k, β_k and λ_j are the dual variables of the model PIIWCCR;
- $\emptyset_k$ the output vector of dual form of PIIWCCR;
- $\emptyset_l$ is the dummy input vector of dual form of PIIWCCR;
- λ, A and B are the vectors of dual variables of PIIWCCR;
- W^+ and W^- are also lower and upper bounds vectors;
- $(Z_o^*, C_o{}^*, \lambda^*, s^{+*}, s^{-*}, \mathrm{A}^*, \mathrm{B}^*)$ an optimal solution of dual PIIWCCR ;
- E_o reference set ;
- $(\widetilde{\emptyset_{\boldsymbol{lo}}}, \widetilde{\emptyset_{\boldsymbol{ko}}})$ the improved activity in dual of PIIWCCR ;
- P_{imp} : the primal form according to the improved activity;
- D_{imp} : the dual form according to the improved activity;
- $(Z_{oimp}^* = 1, \lambda_{imp}^* = \lambda^*, s_{imp}^{-*}, s_{imp}^{+*}, \boldsymbol{A}_{imp}^*, \boldsymbol{B}_{imp}^* = 0)$ the optimal solution of dual problem according to improved activity;
- (v_{imp}^*, w_{imp}^*) the optimal solution of primal problem according to improved activity.

<u>Chapter 5</u>

- v_{ik}^* and u_{rk}^* : the optimal weights of inputs and outputs resulted by running CCR;
- E_{kj} : the related score of DMU_j, using weights of DMU_k ;
- $\overline{E_k}$: the average cross efficiency scores;
- M_k: maverick index;
- ρ_o^r: the benchmark score of efficient DMU_o;
- Z_j and W_j : input and output composites, respectively ;
- r_{ZW}: the coefficient correlation between composite input and output;
- S_{xx}, S_{yy}: the matrices of the sums of squares of the input and output variables, respectively;
- S_{xy}: the matrix of the sums of products of the input and output variables;

- T_j: DEA scores of DMUs of canonical correlation analysis technique;
- a_{jk}: the evaluation of unit j over unit k in the pairwise comparison matrix in AHP;
- v_q: the weights of the criteria in the Jablonsky's model;
- w_{ij}: the preference indices of efficient DMUs;
- E^*_{AB} and E^*_{BA}: the cross efficiency scores of each pair of A and B ;
- E^*_{ij}: the cross efficiency score of unit i in comparison with unit j;
- E^*_i and E^*_{i+k}: the efficiency score of unit i and $i+k$, respectively, in comparison between n units with a single input and a single output;
- $\mathrm{A}_i = \frac{x_j}{y_j}$;
- $E^*_{i,i+k}, E^*_{i+k,i}$: the efficiency scores of DMU_i and DMU_{i+k}, respectively;
- $E^{*\prime}{}_i, E^{*\prime}{}_{i+k}$: the efficiency scores of DMU_i and DMU_{i+k} after adding α to their outputs, respectively;
- $\emptyset'(a_i), \emptyset'(a_{i+k})$: the net flow scores of DMU_i and DMU_{i+k} after adding α to the outputs in DEA model.

<u>Chapter 6</u>

- w_k^- and w_k^-: weight intervals within which the values are likely to vary, determined by DM;
- $\Delta_{ij} = \emptyset_k(a_i) - \emptyset_k(a_j)$;
- P is a convex polyhedron ;
- R^q q-dimentional space of polyhedron ;
- v is a vertex of polyhedron P;
- B is the set of basic points ;
- N is the set of co-basic points;
- $M = c(n, 2)$ is the number of constraints resulted by a super-efficient ranking;
- $M' = c(n, 2) - c(N(efficiency = 1), 2)$ is the number of constraints resulted by a CCR ranking;
- w_{kj} is the weight of criterion k in the constraint j;
- GM the geometric mean in weight matrix.