I'll never forget my first cigarette:

A revealed preference analysis of the 'habits as durables' model*

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Abstract

We provide a revealed preference analysis of the 'habits as durables' model. This approach avoids the need to impose a functional form on the underlying utility function. We show that our characterization is testable by means of linear programming methods and we demonstrate its practical usefulness by means of an application to cigarette consumption using a Spanish household consumption data set. We find that the 'habits as durables' model has better empirical fit in terms of predictive success compared to the 'short memory habits' and life cycle models.

JEL Classification: C6, D9

Keywords: rational habit formation, rational addiction, revealed preference, intertemporal consumption, habits as durables, short memory habits

1 Motivation

In the literature, there are two main ways to model rational habit formation. The 'short memory habits' model departs from the assumption that the instantaneous utility or felicity function depends on a *finite* number of lagged consumption quantities of the addictive good. On the other hand, the 'habits as durables' model assumes that *all* past consumption decisions enter the instantaneous utility function through a single 'stock of addiction' variable. By establishing the revealed preference characterization for the 'habits as durables' model, we complement and complete the research by Crawford (2010) who developed the revealed preference conditions for the 'short memory habits' model. As revealed preference characterizations are entirely nonparametric, our analysis is robust with respect to specification errors. Further, we demonstrate that our results generalize the revealed preference characterization for the life cycle model, as given by Browning (1989), and the short memory habits model with one lag. At a practical level, we show that our restrictions can be verified using elementary linear programming techniques. An application to a data set drawn from the Encuesta Continua de Presupuestos Familiares (a Spanish household budget survey) demonstrates the empirical relevance of our results. We find that the 'habits as durables' model has better empirical fit in terms of predictive success compared to the 'short memory habits' and life cycle models.

Models of rational habit formation. Models of rational habit formation — or rational addiction — offer an behavioral rationale for the seemingly inconsistent behavior of addictive consumption (see, for example, Pollak (1970) and Spinnewyn (1981) for early contributions). Not only do these models demonstrate how a genuinely rational individual can still become addicted, but it also captures and explains addictive related behavior, such as cold turkey and binges. Models of rational habit formation explain addictive behavior as the result of a rational decision process. The consumer maximizes his lifetime utility, taking into account all future consequences of current and past addictive consumption, e.g. negative health effects.

In the literature, there are two main ways to model rational habit formation: the 'habits as durables' model (or HAD) and the 'short memory habits model (or SMH). Although the HAD and SMH models both explain addictive behavior in terms of a rational decision process, they differ in some crucial respects. The main difference between the two models lies in the durability of past addictive consumption. The SMH model assumes that addictive consumption only influences the consumer's welfare in a limited number of future periods. In other words, the instantaneous utility or felicity function depends on current consumption and a *finite* number of lagged consumption levels of the addictive good. The HAD model, initially developed by Becker and Murphy (1988), incorporates the intertemporal aspect of addiction by defining a 'stock of addiction' variable that enters the instantaneous utility function. The consumer is, as it were, investing in a stock of addictive substances. The way this stock is formed is very similar to an investment problem. Current and past consumption increases the future stock while in every time period, the stock is depreciated at a fixed rate. In this way, current addictive consumption potentially influences the consumer's welfare in *all* future periods. Below, we will show that SMH model with a single lag (the 1–lag SMH model) is in fact a special case of the HAD model. However, if the instantaneous utility function in the SMH model depends on the consumption of *more* than one lagged period, then the SMH and the HAD models are generally unrelated, i.e. they are independent or non-nested.

The empirical validation of the rational habit formation models —in casu the 1–lag SMH model— has been initiated by Becker, Grossman, and Murphy (1994) who use the theoretical prediction of this model to estimate the demand for cigarettes. More precisely, they estimate the demand for cigarettes as a linear function of past, current and future prices and past and future consumption. It is this last feature that distinguishes addictive consumption from regular, nonaddictive consumption. Unlike regular commodities, the purchase of cigarettes today depends on past consumption as well as on expected future consumption. Most empirical research since then has followed the same empirical framework. The rational habits model has been verified for various commodities (alcohol, cocaine, caffeine,...) and activities (gambling, cinema, eating, . . .).[2] As stressed by Ferguson (2000), most of the key theoretical predictions of the rational habit formation model appear to have been confirmed empirically virtually every time they have been tested.

[2]See, for example, Chaloupka (1991); Becker, Grossman, and Murphy (1994); Conniffe (1995); Labeaga (1999); Baltagi and Griffin (2001); Escario and Molina (2001); Fenn, Antonovitz, and Schroeter (2001); Bask and Melkersson (2003); Wan (2006) and Laporte, Karimova, and Ferguson (2010) for the case of cigarettes, Grossman, Chaloupka, and Sirtalan (1998); Bentzen, Eriksson, and Smith (1999); Baltagi and Griffin (2002) and Williams (2005) for alcohol, Grossman and Chaloupka (1998) for the case of cocaine, Olekalns and Bardsley (1996) for caffeine and Cameron (1999); Yamamura (2009); Sisto and Zanola (2010) for addictive behavior relating to activities, such as cinema.

However, the framework of Becker, Grossman, and Murphy (1994) also poses some difficulties. Baltagi (2007) discusses the possible econometric problems when estimating the intertemporal demand equations, and Auld and Grootendorst (2004) demonstrate that most tests of rational habit formation tend to yield spurious evidence in favor of the model when serially correlated aggregate data is used.[3] This spurious evidence would also explain why in many researches, estimates of the subjective discount rate are either implausibly high or implausibly low, or even negative.[4] Moreover, it should also be stated that the underlying structural assumptions that are imposed in order to derive the particular linear functional form for the demand equation are quite strong. In particular, Becker, Grossman, and Murphy (1994) assume that all nonaddictive goods can be aggregated into a single numeraire good (such that one can restrict the analysis to a partial demand system), that the utility function is quadratic, that there are no credit constraints or capital market imperfections and that the subjective discount rate equals the interest rate. Finally, most empirical analyses in the literature only focus on the 1–lag SMH model, which constitutes only one specific model of rational habit formation.[5] Taken together, these assumptions are quite strong and, when imposed simultaneously, create a very restrictive framework for testing the theory of rational habit formation.

The revealed preference approach. In this paper, we circumvent the need to impose these restrictive (and often unverifiable) assumptions by employing the revealed preference methodology. This approach, which was introduced by Samuelson (1948), Houthakker (1950), Afriat (1967); Afriat (1973) and Varian (1982), allows to test for the existence of a well–behaved utility function that is compatible with observed (addictive)

[3]This serial correlation would also explain their finding that seemingly nonaddictive goods, such as milk, are found to be more addictive than cigarettes.

[4]See Auld and Grootendorst (2004, table 1) for an overview of this fact.

[5]Chaloupka (1991) is a noteworthy exception.

consumer behavior, without the need to impose any functional structure on this utility function. In this way, we are able to obtain an exact test of the HAD model. To our knowledge, there are only two previous researches that apply the revealed preference methodology to an intertemporal decision context. First, Browning (1989) considers the basic life cycle model under the assumptions of perfect foresight and perfect capital markets. Hence, consumers can lend and borrow at the same interest rate as they please, implying perfect consumption smoothing over time. Furthermore, he also assumes consumption independence, implying that the instantaneous utility in any given period depends only on the consumed bundle in this period. As such, this life cycle model is too restrictive to capture habit formation. An important extension to this basic setting is given by Crawford (2010), who develops the revealed preference conditions for the SMH model. In this paper, we complement and complete this literature by presenting a revealed preference characterization of the HAD model.

Overview. In Section 2, we present the HAD model as a general intertemporal budget allocation problem. We also discuss the similarities and differences between the HAD model, the SMH model and the life cycle model. Furthermore, we show how it is possible to relax the assumption of perfect capital markets by considering the case where households are constrained in the amount of money they can transfer (borrow) between periods. In the final part of this section, we derive the revealed preference characterization of the HAD and we show how it relates to the revealed preference characterizations of the life cycle and SMH models.

In Section 3, we apply our revealed preference conditions to a real life data set. All tests were conducted on micro panel data drawn from the Encuesta Continua de Presupuestos Familiares. Since our revealed preference conditions are evaluated separately for each individual household, our tests fully account for interhousehold heterogeneity. Our results suggest that, for the data set at hand, the HAD model provides a better empirical fit than the 1–lag and 2–lag SMH models and the life cycle model in terms of improved predictive success.

Section 4 concludes the paper and hints at future research. The proofs are in the appendix.

2 Testable implications of the habits as durables model

In this section, we present the HAD model as introduced in the seminal contribution of Becker and Murphy (1988). In order to keep the notation and the analysis simple, we focus on the case of a single addictive good. Extensions to multiple addictive good are presented at the end of Section 3. Subsection 2.1 presents the HAD model and defines when a data set is consistent with this model. Subsection 2.2 discusses the life cycle model and the SMH models. Finally, Subsection 2.3 presents the revealed preference characterization of the considered models.

2.1 The habits as durables model

Consider an individual (or household) who consumes in each period $t \in \mathbb{N}$ a vector $\mathbf{q}_t \in \mathbb{R}_+^K$ of nonaddictive goods at prices $\mathbf{p}_t \in \mathbb{R}_{++}^K$ and an amount $Q_t \in \mathbb{R}_+$ of an addictive good at price $P_t \in \mathbb{R}_{++}$.[6] We assume that our consumer receives in each period t an exogenous income $Y_t \in \mathbb{R}_+$, which can be more or less than the consumption expenditure for this period. If consumption in period t amounts to less than Y_t, the difference is added to savings, yielding a net return of r_t between period t and $t+1$. If current income Y_t is insufficient to purchase the demanded bundle, the deficit is borrowed

[6]We remark that the infinite horizon formulation is not crucial. In fact, assuming a finite time horizon would lead to the same revealed preference conditions.

at the same interest rate, with the additional constraint that the borrowed amount in period t cannot exceed some upper limit b_t (i.e. we allow for imperfect capital markets). If we denote total savings in period t by S_t, this implies that $S_t \geq -b_t$. Note that there is no need for the value of b_t to be identical for different consumers. The special case without credit constraints is obtained by setting b_t sufficiently high for all periods t. The intertemporal budget constraint for period $t \geq 1$ is then determined by the following equation.

$$\mathbf{p}_t \, \mathbf{q}_t + P_t \, Q_t + S_t = Y_t + (1 + r_{t-1}) \, S_{t-1}$$

We take initial savings as given; $S_0 \equiv \overline{S_0}$. Then, given the consumption quantities, the prices, the income stream and the interest rates, we have that S_t is fully determined for all t.

We denote by A_t, the addictive stock that has been built up by the past consumption of the addictive good (i.e. $Q_1, \ldots, Q_{t-1}$). Further, the HAD model posits the existence of a depreciation rate, $\delta \in]0, 1]$ which measures how fast the physical and psychological effects of past consumption of the addictive good wear off over time. In each time period $t \geq 2$, the addictive stock is then determined by the following equation.

$$A_t = (1 - \delta) \, A_{t-1} + Q_{t-1}.$$

If $\delta \in]0, 1[$, the effects of past consumption decrease progressively as time proceeds. This becomes more obvious if we solve the linear equation recursively.

$$A_t = (1 - \delta)^{t-1} A_1 + \sum_{j=1}^{t-1} (1 - \delta)^{t-1-j} Q_j.$$

Observe that we exclude the case where $\delta = 0$, since this would imply that the effects of past consumption would never wear off (i.e. A_t could never decrease after once consuming the addictive good). Evidently, such a case would rule out so-called 'cold turkey' quitting behavior, which is often observed with addicts who try to get rid of their harmful addiction by abstaining from the addictive good until the physical and psychological effects have worn off over time.

Let $u(\mathbf{q}_t, Q_t, A_t) : \mathbb{R}_+^{K+2} \to \mathbb{R}_+$ represent the instantaneous utility function of the consumer. We assume that u is strictly increasing in $\mathbf{q}_t$ and Q_t, continuous and concave in all its arguments. The addictive commodity can either be detrimental for the individual, such as tobacco or alcohol, but can also be beneficial, such as healthy leisure expenditures for practicing sports. If the addictive good is detrimental, which we will assume from now on, then u should be decreasing in A_t.[7] Finally, the HAD model assumes that the consumer is endowed with a subjective discount factor, which we represent by $\beta \in]0, 1]$.

Following Browning (1989) and Crawford (2010), we maintain the assumption that there is perfect foresight concerning future prices, incomes and interest rates. By putting everything together, the agent chooses her optimal consumption path by solving the following maximization problem, given an initial stock of addiction $\overline{A}_1$.

OP-HAD:

$$\max_{\mathbf{q}_t, Q_t} \sum_{t=1}^{\infty} \beta^{t-1} u(\mathbf{q}_t, Q_t, A_t)$$

$$\text{s.t. for all } t \geq 1$$

$$\mathbf{p}_t \mathbf{q}_t + P_t Q_t + S_t = Y_t + (1 + r_{t-1}) S_{t-1},$$

$$A_{t+1} = (1 - \delta) A_t + Q_t,$$

$$S_t \geq -b_t, \qquad \text{and,}$$

$$S_0 = \overline{S}_0, \quad A_1 = \overline{A}_1.$$

The optimization program OP-HAD requires that the consumer takes all future con-

[7]For a beneficial addictive good, we would have that u is increasing in A_t.

sequences of her addiction (e.g. utility losses due to illness or depression, craving or withdrawal caused by past consumption of harmful substances) into account when deciding on her optimal consumption path. This is the main assumption that makes the models of rational habit formation different from other models of (irrational) addictive behavior.

The first order conditions for the problem OP-HAD are stated as follows.[8]

$$\beta^{t-1}\frac{\partial u(\mathbf{q}_t, Q_t, A_t)}{\partial q_k} = \lambda_t p_{t,k} \qquad \forall k \leq K, t \geq 1, \tag{A.1}$$

$$\beta^{t-1}\frac{\partial u(\mathbf{q}_t, Q_t, A_t)}{\partial Q} = \mu_t + \lambda_t P_t \qquad \forall t \geq 1, \tag{A.2}$$

$$-\beta^{t}\frac{\partial u(\mathbf{q}_{t+1}, Q_{t+1}, A_{t+1})}{\partial A} = -(1-\delta)\mu_{t+1} + \mu_t \qquad \forall t \geq 1, \tag{A.3}$$

$$\lambda_{t+1}(1+r_t) \leq \lambda_t \qquad \forall t \geq 1, \tag{A.4}$$

$$A_{t+1} = (1-\delta)A_t + Q_t \qquad \forall t \geq 1. \tag{A.5}$$

Condition (A.1) presents the first order conditions for the private goods $\mathbf{q}_t$, where λ_t is the Lagrange multiplier for the intertemporal budget constraint. Condition (A.2) gives the first order condition for the addictive good. The additional term μ_t on the right hand side, which is positive for a harmful addictive good and negative for a beneficial addictive good, gives the marginal effect of Q_t on lifetime utility due to the increase in the stock of addiction. It is the Lagrange multiplier for the stock of addiction equation. In particular, μ_t measures the marginal decrease in (future discounted) lifetime utility due to a marginal increase in the stock of addiction A_{t+1}. Given that $\mu_t > 0$, we see that the consumer will have a lower consumption of the harmful addictive good compared to the case where she does not take this negative effect into account. Condition (A.3) decomposes this marginal (future) welfare loss μ_t into two components. The first com-

[8] We omit the necessary transversality conditions as they do not really matter for the remaining part of this paper.

ponent, $-\beta^t\, \partial u(\mathbf{q}_{t+1}, Q_{t+1}, A_{t+1})/\partial A$, is the negative welfare effect on the instantaneous utility $u(\mathbf{q}_{t+1}, Q_{t+1}, A_{t+1})$ due to an increase in A_{t+1}. The second component, $(1-\delta)\mu_{t+1}$, gives the marginal future welfare loss due to the increase in the future stock of addiction A_{t+2} (caused by the increase in A_{t+1}). Next, condition (A.4) gives the intertemporal optimality condition corresponding to the amount of savings S_t. Notice that condition (A.4) holds with equality only if there is no liquidity constraint at period t, i.e. when $S_t > -b_t$. Finally, condition (A.5) gives the recursive equation that determines the stock of addiction.

In order to enhance the intuition behind the two key conditions (A.2) and (A.3), we define the following discounted shadow prices for the addictive good and the stock of addiction.

$$\widetilde{P}_t^Q \equiv \beta^{t-1} \frac{\partial u(\mathbf{q}_t, Q_t, A_t)}{\partial Q_t} \qquad \forall t \geq 1,$$
$$\widetilde{P}_t^A \equiv \beta^{t-1} \frac{\partial u(\mathbf{q}_t, Q_t, A_t)}{\partial A_t} \qquad \forall t \geq 1.$$

The positive variable $\widetilde{P}_t^Q$ gives the discounted marginal utility gain caused by an increase in the consumption of the addictive good, Q_t. The negative variable $\widetilde{P}_t^A$ gives the discounted marginal utility cost of an increase in the stock of (detrimental) addiction, A_t. Then, if we substitute condition (A.2) in (A.3), we obtain the following expression.

$$\widetilde{P}_{t+1}^A = (1-\delta)(\widetilde{P}_{t+1}^Q - \lambda_{t+1} P_{t+1}) - (\widetilde{P}_t^Q - \lambda_t P_t) \tag{1}$$

This condition can be solved recursively to obtain the following equilibrium condition.

$$\widetilde{P}_t^Q = \lambda_t P_t - \sum_{j=1}^{\infty} (1-\delta)^{j-1} \widetilde{P}_{t+j}^A. \tag{2}$$

Equation (2) provides the equilibrium condition for an addicted consumer in her choice of the optimal quantity Q_t. If good Q were not addictive, then the right hand side of this equation would be equal to $\lambda_t P_t$. In other words, for a regular good, rational behavior requires that the marginal benefit of an additional unit of Q_t must be equal to its marginal cost. This marginal cost is equal to the marginal utility of income, given by λ_t, times the prevailing price P_t. However, in the case of detrimental addiction, the rational consumer should also take into account all future costs incurred by this marginal increase in Q_t. This additional cost is equal to the total utility loss caused by the increase in all future stocks of addiction $(A_{t+1}, A_{t+2}, \dots)$, stemming from this increase in Q_t.

From an empirical perspective, it is nearly impossible to observe the entire stream of consumption bundles. In reality, we only have a subsample of this stream. Our framework assumes that we observe a finite set of nominal interest rates, nominal prices and consumed quantities $D = \{(r_t, \mathbf{p}_t, P_t; \mathbf{q}_t, Q_t)\}_{t \leq T}$. We follow Browning (1989) and Crawford (2010) and define rationalizability of a data set in terms of its consistency with respect to the first order conditions.

Definition 1 (Rationalizability)**.** *The data set $D = \{r_t, \mathbf{p}_t, P_t; \mathbf{q}_t, Q_t\}_{t \leq T}$ is rationalizable by (or consistent with) the* HAD *model if and only if there exist a well-behaved (sub)differentiable utility function u, numbers $\delta, \beta \in]0, 1]$, and for all $t \leq T$, there exist numbers $\mu_t \geq 0, \lambda_t > 0$ and $A_t \geq 0$ such that such that conditions* (A.1)-(A.5) *are satisfied.*

The above definition states that a data set is consistent with the HAD model if there exists a well–behaved instantaneous utility function that satisfies the set of first order conditions. In other words, the data set is rationalizable if we can find some utility function which provides a perfect within–sample fit of the observed consumption data. Consistency is tested separately for each observed household in the sample, relieving the need to impose additional conditions on interhousehold preference heterogeneity.

2.2 The short memory habits and life cycle models

For the SMH model, the instantaneous utility function depends on the current consumption bundle and on a finite number of lagged consumption quantities of the addictive good. In other words, the instantaneous utility function can be presented by $u(\mathbf{q}_t, Q_t, Q_{t-1}, \dots, Q_{t-R})$. The SMH model can be subdivided by the number of lags that are included in the utility function. If there are R lags, we call this the R–lag SMH model. The set of first order conditions for this model are given by:

$$\beta^{t-1}\frac{\partial u(\mathbf{q}_t, Q_t, \dots, Q_{t-R})}{\partial q_k} = \lambda_t p_{t,k} \quad \forall k \leq K, t \geq 1, \tag{B.1}$$

$$\beta^{t-1}\frac{\partial u(\mathbf{q}_t, Q_t, \dots, Q_{t-R})}{\partial Q_t} + \sum_{j=1}^{R} \beta^{t-1+j}\frac{\partial u(\mathbf{q}_{t+j}, Q_{t+j}, \dots, Q_{t-R+j})}{\partial Q_t} = \lambda_t P_t \quad \forall t \geq 1, \tag{B.2}$$

$$\lambda_{t+1}(1 + r_t) = \lambda_t \qquad \forall t \geq 1. \tag{B.3}$$

Conditions (B.1) and (B.3) are similar to the first order conditions (A.1) and (A.4) for the SMH model. In order to capture the intuition behind condition (B.2), let us define the following discounted shadow prices:

$$\begin{aligned}
\widetilde{P}_t^{Q} &\equiv \beta^{t-1}\frac{\partial u(\mathbf{q}_t, Q_t, \dots, Q_{t-\ell})}{\partial Q_t} && \forall t \geq 1,\\
\widetilde{P}_t^{Q-j} &\equiv \beta^{t-1+j}\frac{\partial u(\mathbf{q}_{t+j}, Q_{t+j}, \dots, Q_{t-R+j})}{\partial Q_t} && \forall t \geq 1; 1 \leq j \leq R.
\end{aligned}$$

Then, condition (B.2) can be rewritten as:

$$\widetilde{P}_t^Q = \lambda_t P_t - \sum_{j=1}^{R} \widetilde{P}_t^{Q_{-j}}. \tag{3}$$

Equation (3) states that at equilibrium, the marginal benefit of an additional increase in Q_t should equal its marginal cost. The left hand side of (3) gives the instantaneous benefit of a marginal increase in Q_t. The first term on the right hand side, $\lambda_t P_t$, gives the additional monetary cost of this increased consumption (in utility terms). The second part of the right hand side gives the marginal cost on future welfare caused by the increase in Q_t. It is interesting to compare this equilibrium condition with the equilibrium condition (2) for the HAD model. Although the two conditions have a similar structure, they differ in two crucial respects. Both differences relate to the term that reflects the marginal future welfare cost, i.e. the second part of the right hand side. First of all, for the SMH model, the negative welfare effects involve a finite sum compared to an infinite sum for the HAD model. This shows that for the HAD model, addictive consumption might influence the welfare in *all* future periods, while for the SMH model, current addictive consumption only influences future welfare in a limited number of periods. Second, for the HAD model, the negative welfare effects propagate through a single 'stock of addiction' variable, A_t. On the other hand, the SMH model imposes no restrictions on the way that current addictive consumption influences future utility.

Concerning the first difference, we see that the HAD model is more general than the SMH model. On the other hand, for the second distinction, the SMH model is more general. This shows that, in general, the R–lag SMH model is independent from the HAD model, i.e. they will impose different restrictions on observed consumption behavior. However, there is one special case for which the two are nested. This occurs when the instantaneous utility function in the SMH model depends on only one lag of the addictive good, i.e. the 1–lag SMH model. Indeed, if we set δ equal to one in the HAD model, we obtain that A_t coincides with Q_{t-1}, and we immediately see that the HAD model reduces to the 1–lag SMH model.

The life cycle model can be obtained from the HAD model by assuming that that the instantaneous utility function is independent of A_t. In other words, the life cycle model assumes that the instantaneous utility function takes on the form $u(\mathbf{q}_t, Q_t)$. The first order conditions for this model are given by:

$$\beta^{t-1}\frac{\partial u(\mathbf{q}_t, Q_t)}{\partial q_k} = \lambda_t p_{t,k} \qquad \forall k \leq K, t \geq 1, \tag{C.1}$$

$$\beta^{t-1}\frac{\partial u(\mathbf{q}_t, Q_t)}{\partial Q} = \lambda_t P_t \qquad \forall t \geq 1, \tag{C.2}$$

$$\lambda_{t+1}(1+r_t) = \lambda_t \qquad \forall t \geq 1. \tag{C.3}$$

Conditions (C.1) and (C.3) are similar to conditions (A.1) and (A.4) for the HAD model or conditions (B.1) and (B.3) for the SMH model. To compare condition (C.2), let us again introduce the notation:

$$\widetilde{P}_t^Q \equiv \beta^{t-1}\frac{\partial u(\mathbf{q}_t, Q_t)}{\partial Q_t} \qquad \forall t \geq 1,$$

Then, we can rewrite condition (C.2) as:

$$\widetilde{P}_t^Q = \lambda_t P_t. \tag{4}$$

This condition can readily be compared to the equilibrium conditions for the HAD model (equation (2)) and the SMH model (equation (3)). The difference with condition (4) lies in the fact that for the life cycle model, there are not future welfare costs associated with an increase in the consumption of Q_t.

2.3 Revealed preference characterizations

We are now ready to provide the revealed preference conditions that characterize the collection of data sets that are consistent with the HAD model. The proof can be found in appendix A.

Theorem 1. *Consider a finite data set $D = \{r_t, \mathbf{p}_t, P_t; \mathbf{q}_t, Q_t\}_{t=1,\dots,T}$. The following conditions are equivalent:*

- *The data set D is rationalizable by the HAD model.*
- *For all $t \leq T$, there exist positive numbers u_t and A_t, strictly positive numbers λ_t and $\widetilde{P}_t^Q$, a non–positive number $\widetilde{P}_t^A$ and numbers $\delta, \beta \in]0,1]$, such that:*

$$u_t - u_v \leq \frac{1}{\beta^{v-1}} \begin{bmatrix} \lambda_v \mathbf{p}_v (\mathbf{q}_t - \mathbf{q}_v) + \widetilde{P}_v^Q (Q_t - Q_v) \\ + \widetilde{P}_v^A (A_t - A_v) \end{bmatrix}, \quad \forall t, v \leq T \tag{G.1}$$

$$\widetilde{P}_{t+1}^A = (1-\delta)(\widetilde{P}_{t+1}^Q - \lambda_{t+1} P_{t+1}) - (\widetilde{P}_t^Q - \lambda_t P_t), \quad \forall t \leq T-1 \tag{G.2}$$

$$\lambda_{t+1}(1 + r_t) \leq \lambda_t, \quad \forall t \leq T-1 \tag{G.3}$$

$$A_{t+1} = (1-\delta) A_t + Q_t \quad \forall t \leq T-1 \tag{G.4}$$

Condition (G.1) is a generalization of the Afriat inequalities for this intertemporal setting. Condition (G.2) is an immediate translation of expression (1). Finally, conditions (G.3) and (G.4) are obtained from conditions (A.4) and (A.5). It is interesting to note that by replacing the inequality in condition (G.3) with an equality, we can test whether the data is consistent with a model without credit constraints. In fact, for our empirical application, we will make a distinction between the model where the borrowing constraints are possibly binding, i.e. where (G.3) holds with weak inequality and the case where these constraints are not binding, i.e. where (G.3) holds with equality. Also, if we would like to obtain revealed preference conditions for the case where the addictive good is beneficial, it suffices to impose that $\widetilde{P}_t^A$ is positive for all t. Conditions (G.2)–(G.4) relate observations from period t to observations from period $t+1$. This means that we only have $T-1$ distinct inequalities. In other words, we always lose the first observation in the revealed preference test of the HAD model.

Let us now have a look at the two special cases considered in the previous section; the SMH and life cycle models. The definition of rationalizability for these models parallels Definition 1 for the HAD model (see also Definition 1 of Crawford (2010) and the first Definition of Browning (1989)). For the SMH model, we obtain the following set of revealed preference conditions (see also Crawford (2010)).

Theorem 2. *Consider a data set $D = \{r_t, \mathbf{p}_t, P_t; \mathbf{q}_t, Q_t\}_{t=1,\dots,T}$. The following conditions are equivalent:*

- *The data set D is rationalizable by the SMH model with R lags,*
- *For all $t \leq T$, there exist a positive number u_t, strictly positive numbers λ_t and $\widetilde{P}_t^Q$ and non–positive numbers $\widetilde{P}_t^{Q-j}$ $(j = 1, \dots, R)$ such that:*

$$u_t - u_v \leq \frac{1}{\beta^{v-1}} \left[\begin{array}{c} \lambda_v \mathbf{p}_v (\mathbf{q}_t - \mathbf{q}_v) + \widetilde{P}_v^Q (Q_t - Q_v) \\ + \sum_{j=1}^R \widetilde{P}_v^{Q-j} (Q_{t-j} - Q_{v-j}) \end{array} \right] \quad \forall t, v \leq T - R \tag{G.5}$$

$$\lambda_t P_t = \widetilde{P}_t^Q + \sum_{j=1}^R \widetilde{P}_{t+j}^{Q-j} \quad \forall t \leq T - R \tag{G.6}$$

$$\lambda_{t+1}(1 + r_t) = \lambda_t \quad \forall t \leq T - 1 \tag{G.7}$$

Condition (G.5) is an Afriat inequality, similar in spirit to (G.1). Condition (G.6) is a translation of condition (3). If we relax (G.7) to condition (G.3) we can account for possible binding borrowing constraints in the SMH model.

For the 1–lag SMH model (i.e. $R = 1$), we see that the revealed preference conditions coincide with the revealed preference conditions for the HAD model if $\delta = 1$ (with $\widetilde{P}_t^{Q_{-1}} = \widetilde{P}_t^A$). As such, we see that the 1–lag SMH model is indeed a special case of the HAD model. However, for $R \geq 2$, the two models are independent in the sense that some data set might pass the revealed preference conditions for one model but not for the other.

Finally, observe that conditions (G.5) and (G.6) use observations from R different time periods. As such, we only retain $T - R$ distinct inequalities, which actually implies that we lose the first R observations in the revealed preference test of the R–lag SMH model. This provides another difference with the HAD model where we only lose a single observation (as with the 1–lag SMH model).

In our empirical application, we will focus on the 1–lag and 2–lag SMH models, i.e. the cases where $R = 1$ and $R = 2$.

Finally, let us then consider the revealed preference conditions for the life cycle model (see also Browning 1989).

Theorem 3. *Consider a data set $D = \{r_t, \mathbf{p}_t, P_t, \mathbf{q}_t, Q_t\}_{t=1,\ldots,T}$. The following conditions are equivalent:*

- *The data set D is rationalizable by the life cycle model,*
- *For all $t \leq T$ there exist a positive number u_t, a strictly positive number λ_t and a number $\beta \in]0, 1]$ such that:*

$$u_t - u_v \leq \frac{\lambda_v}{\beta^{v-1}} \left[\mathbf{p}_v \left(\mathbf{q}_t - \mathbf{q}_v\right) + P_v \left(Q_t - Q_v\right)\right] \qquad \forall t, v \leq T \qquad \text{(G.8)}$$

$$\lambda_{t+1}(1 + r_t) = \lambda_t \qquad \forall t \leq T - 1 \qquad \text{(G.9)}$$

We see that these revealed preference conditions coincide with the revealed preference conditions of the HAD model if $\widetilde{P}_t^A = 0$ for all t. This effectively imposes that the instantaneous utility function does not depend on the stock of addiction. Again, if we replace condition (G.9) with (G.3) we can allow for binding credit constraints.

3 Empirical application

In this section, we illustrate our revealed preference results by applying it to a data set drawn from the Encuesta Continua de Presupuestos Familiares. Subsection 3.1 discusses the methodology employed to verify the revealed preference conditions. In subsection 3.2, we present the data and our results.

3.1 Verification

The set of revealed preference conditions (G.1)–(G.4) is highly nonlinear, and therefore, difficult to verify. This problem can be solved by conducting a grid search on the values of δ and β. Since these values are restricted to lie in the interval $]0, 1]$, a grid search on these parameters can be done quite efficiently. In practice, we consider 6 evenly spaced values for β between 0.95 and 1 ($\beta = 0.95, 0.96, \dots, 1$), and 4 evenly spaced values of δ between 0.7 and 1 ($\delta = 0.7, 0.8, 0.9, 1$).[9]

Now, keeping the values of δ and β fixed, we see that only condition (G.1) remains nonlinear, where the variables A_t and A_v interact with the variable $\widetilde{P}_v^A$. This nonlinearity can be resolved by fixing a value for the initial stock of addiction. Indeed, given the value of A_1, we can use condition (G.4) and the known value of δ to compute A_t for all

[9] The considered values of β are quite high because our application deals with quarterly data. Similarly, the values of δ are reasonably high, which is motivated by the fact that the physical and mental effects of past tobacco consumption usually wear off fairly quickly (see, for example, Hughes 2007). We performed several robustness results by considering other ranges for the grid search. Because these did not significantly change our results, we refrain from presenting these results. However, outcomes for these alternative scenarios are available upon request.

values of $t \leq T$.

Notice that if we would observe the entire past consumption pattern of the addictive good, it would actually be possible to reconstruct the value of A_1 (given the value of δ). Unfortunately, our data set does not contain this kind of information. As such, the initial stock must be estimated by some other means. The single other research in the literature that deals with the same problem of estimating the stock of addiction is from Chaloupka (1991). He estimates the stock of addiction by using information on the number of years that an individual has been smoking and on the average consumption level over this past period. For this, he uses the identity (with initial value $A_0 = 0$):

$$A_r = \sum_{j=0}^{r-1} (1-\delta)^{r-j-1} Q_j = \sum_{j=0}^{r-1} Q_j + \sum_{j=0}^{r-1} D(j) + r \operatorname{cov}\left[Q_j, D(j)\right],$$

where $D(j) = (1-\delta)^{r-j-1}$. Then, given that the covariance term is relatively small, A_r can be approximated using information on the mean of past addictive consumption and the number of years (quarters) that the individual has been smoking. Unfortunately, our data set does not contain this kind of information.

Therefore, we propose to estimate A_1 in a different way. Consider condition (A.5) evaluated at $t = 1$:

$$A_2 = (1-\delta) A_1 + Q_1.$$

Conditional on the grid search of δ, this condition gives us a single equation in two unknowns (A_1 and A_2). Next, define g as the growth rate of the stock of addiction in the initial period:

$$\frac{A_2}{A_1} = 1 + g$$

Using this, we find that:

$$(1+g) = \frac{A_2}{A_1} = (1-\delta) + \frac{Q_1}{A_1}$$

Solving this in terms of A_1 gives:

$$A_1 = \frac{Q_1}{\delta + g}$$

As such, we obtain that the initial stock of addiction can be written as a function of initial consumption Q_1, the depreciation rate δ and the initial growth rate of the stock of addiction, g. As we do not observe g, we choose to approximate it by assuming that the growth rate of the stock of addiction is the same over the first two observed periods, i.e. $A_3/A_2 = A_2/A_1$. Then, we obtain that:

$$g = \frac{Q_2 - Q_1}{Q_1}.$$

In other words, our assumption is equivalent to the condition that the growth rate of the addiction stock in the initial observed period equals the growth rate of addictive consumption in the initial observed period. As a robustness exercise, we conducted a (small) grid search on g around this benchmark.[10]

The HAD, 1–lag SMH, 2–lag SMH and life cycle models both with and without possible binding borrowing constraints gives us eight different models to test. For completeness, we consider one additional ninth model which is frequently used in revealed preference analysis, namely the static optimization model (see Afriat (1967), Diewert (1973) and Varian (1982) for and extensive discussion). The static model model assumes that the consumer maximizes (in each period) her instantaneous utility function subject to the current period budget constraint. In other words, the individual solves the following

[10] To be precise, we considered the values $[g - 0.1, g, g + 0.1]$. Of course, for A_1 to be positive, we only considered values of g such that $\delta + g > 0$. Together with the grid search on β and δ, this gives us a maximum of 72 possible combinations of parameters to consider for each test of the HAD model.

problem in each period t:

$$\max_{\mathbf{q},Q} u(\mathbf{q}, Q) \qquad \text{s.t. } \mathbf{p}_t\mathbf{q} + P_tQ \leq Y_t$$

This model coincides with the life cycle model if there are no monetary transfers between periods, i.e. there is no borrowing or saving. The revealed preference characterization for this model is given in the following theorem (see Afriat 1967, for a proof).

Theorem 4. *Consider a data set $D = \{\mathbf{p}_t, P_t, \mathbf{q}_t, Q_t\}_{t\leq T}$. Then the following are equivalent.*

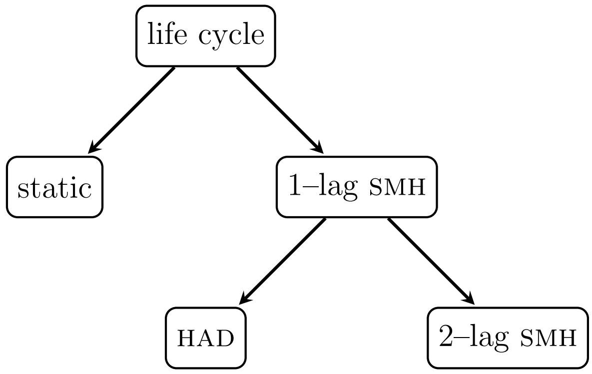
- *The data set is rationalizable by the static utility maximization model.*
- *There exist numbers u_t and $\gamma_t > 0$ such that for all t and $v \leq T$:*

$$u_t - u_v \leq \gamma_v \left[\mathbf{p}_v(\mathbf{q}_t - \mathbf{q}_v) + P_v(Q_t - Q_v)\right].$$

Observe that these Afriat conditions coincide with the testable implications for the life cycle model when condition (G.9) is discarded (and λ_t/β^{t-1} is replaced by γ_t).

Although we consider nine distinct models, several of these models are empirically nested. This nestedness is illustrated by Figure 1. At the top of the figure, we find the life cycle model which is the strongest (most restrictive) model. In other words, if a data set is consistent with this model, then it is consistent with all other models. On the second level, we find the static model, which is weaker than the life cycle model but unrelated to the SMH and HAD models. Next, the 1–lag SMH model is weaker than the life cycle model but stronger than both the HAD and 2–lag SMH models. Finally, we see that the HAD and 2–lag SMH models are unrelated. Furthermore, each model (except for the static one) has a variant with and without credit constraints, the former case being less restrictive than the latter.

Figure 1: Logical implication for data consistency with the different models



3.2 Application

Our empirical illustration uses the Encuesta Continua de Presupuestos Familiares. This data set contains detailed information on consumed quantities and prices for a large sample of Spanish households. We refer to Browning and Collado (2001) and Crawford (2010) for a more detailed explanation of this data set. The observations range from 1985 until 1997 and are gathered on a quarterly basis. Every new quarter, new households are participating in the moving panel and others are dropped, with a maximum of eight consecutive observations per household. We consider the following 14 nondurable commodity categories: (1) Food and non-alcoholic drinks at home, (2) Alcohol, (3) Tobacco, (4) Energy at home, (5) Services at home, (6) Nondurables at home, (7) Nondurable medicines, (8) Medical services, (9) Transportation, (10) Petrol, (11) Leisure, (12) Personal services, (13) Personal non–durables, (14) Restaurants and bars. We take tobacco to be the addictive good and, for matters of comparability of empirical results and sample homogeneity, we will only focus on the subset of households for which the wife is outside of the labor market and for which we have observations for all eight quarters. We further restrict the sample to households which have strict positive consumption for the addictive good in all periods. This procedure still leaves a sizeable sample of 671 households. Since tobacco is a detrimental good, the variables $\widetilde{P}_t^A$ (and

$\widetilde{P}_t^{Q-j}$) are restricted to be non-positive.

Table 1 contains the goodness–of–fit results (i.e. the pass rates), given by the percentage of households that pass the revealed preference tests for the nine models under consideration. The static model fits the data relatively well, and evidently yields the same results whether we allow for binding borrowing constraints or not. On the contrary, the life cycle model is heavily rejected, which is not surprising given the strong underlying assumptions for this model. By allowing that tobacco consumption has lasting utility effects that persist for only one period (i.e. the 1–lag SMH model), we are able to attain a fairly high goodness–of–fit of 74%, but only if we simultaneously relax the assumption that there are no binding borrowing constraints. The pass rate drops to 26% if we drop this condition. If we further relax the model towards the HAD model, we see that the model can rationalize a vast majority of the observed behavior, given that we allow for the presence of borrowing constraints (87%). Compared to the 1–lag SMH model, the assumption of perfect capital markets appears to be less strong for the HAD model, as the pass rate still remains reasonably high if we assume perfect capital markets (nearly 53%). Finally, the best fit is obtained by the 2–lag SMH model which rationalizes nearly all households in the data set (more than 99% for the case with imperfect capital markets and 94% for the model without borrowing constraints). Of course, due to the nestedness of the different models (see Figure 1), it should come as no surprise that the HAD and 2–lag SMH models outperform the more restrictive life cycle and 1–lag SMH models in terms of goodness–of–fit.

Table 1: Pass rates and power

	static	life cycle	1–SMH	2–SMH	HAD
pass rates					
no credit constraints	0.918	0.001	0.259	0.940	0.526
credit constrained		0.051	0.742	0.997	0.869
power					
no credit constraints	0.094	0.999	0.774	0.043	0.624
credit constrained		0.960	0.295	0.005	0.211

In order to account for this nestedness, it is crucial to perform a power analysis. The power of a model is defined as the probability of rejecting the model when this model is not the true data generating process. The power is computed as the probability that the revealed preference conditions reject seemingly irrational (or random) behavior. Towards this end, we consider a power measurement procedure introduced by Bronars (1987), based on a model of irrational behavior from Becker (1962). For each household, we simulate 1000 random time series of consumption choices over the eight time periods, by drawing random consumption shares from the (intertemporal) budget hyperplane. The power of the model for each household is then obtained as one minus the proportion of these 1000 randomly generated consumption streams that are consistent with the rationalizability condition under evaluation.[11] Table 1 gives the average power over all 671 households for each considered model. The highest power is obtained for the life cycle model, which rejects almost all random data sets. Also, the 1–lag SMH model has a reasonably high power for the case with no credit constraints. The HAD model has a power of 62% for the setting with no credit constraints and drops to 21% for the model with credit constraints. The lowest power is obtained for the 2–lag SMH model which fails to reject almost all randomly generated data sets.

Arguably, these numbers only give a concise presentation of the empirical perfor-

[11] We refer to Andreoni and Harbaugh (2008) for a general discussion on alternative procedures to evaluate power in the context of revealed preference tests such as ours.

mance of each model. We present two approaches to improve our ability to assess and rate models in a more coherent manner. The first approach is to look at the entire distribution of the household–specific power estimates, which will show us much more than the averages from Table 1. The second approach will reconcile the empirical performance in terms of pass rates and power into a single measure, which is convenient for directly comparing different models in terms of overall empirical performance.

Power distribution Table 2 presents the quartiles of the power distribution for our sample of 671 households, for each considered model. For comparison, we repeat the pass rates as stated in Table 1. Figure 2 presents the kernel densities of the power distribution for the 1–lag SMH, the 2–lag SMH and the HAD models where there are no credit constraints and Figure 3 does the same for the models with possible credit constraints. These give a more detailed overview of how much the discriminatory power of each model varies between 0 and 1 on the household level. Since the models are nested, the more general models (i.e. the HAD and 2–lag SMH models) always have lower power and, thus, a density with higher weight to the left in comparison to the 1–lag SMH model. Similarly, allowing for credit constraints reduces the power of all considered intertemporal models. These notions are confirmed by looking at the quartile values in Table 2, and become even more apparent when looking at Figures 2 and 3.

When looking at Figure 2, we notice the power of the 1–lag SMH model is centered around 0.78. The more general HAD model has a density peak at a slightly lower power value of 0.62. The power of the 2–lag SMH is considerably lower and peaks close to zero (around 0.04), indicating that this model is very difficult to reject for the data set at hand.[12] These results broadly confirm the basic finding from Table 1, which is that

[12]For matters of comparison between the habit formation models, we did not include the kernel density for the power of the static and life cycle models. As already apparent from Table 2, the power density of the static model has most of its mass around 0.10, while the life cycle model has most mass

models with higher goodness–of–fit (i.e. pass rate) typically have lower power and vice versa.

Table 2: Power distribution

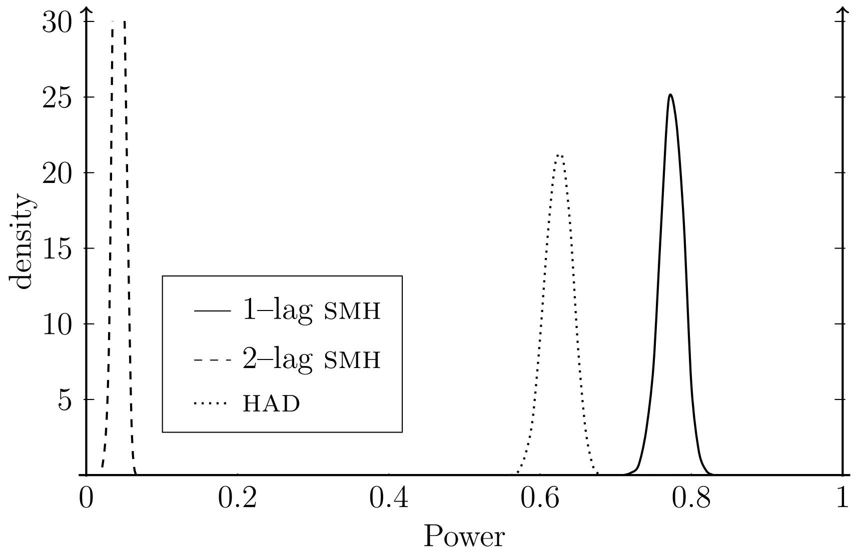
	pass rate	power				
		Min	1st quart.	median	3rd quart.	max
static	0.918	0.041	0.083	0.098	0.108	0.141
life cycle						
no credit constraints	0.000	0.997	1.000	1.000	1.000	1.000
credit constrained	0.051	0.924	0.954	0.961	0.967	0.980
1–SMH						
no credit constraints	0.259	0.724	0.764	0.774	0.785	0.816
credit constrained	0.742	0.186	0.259	0.304	0.327	0.376
2–SMH						
no credit constraints	0.940	0.022	0.038	0.043	0.047	0.063
credit constrained	0.997	0.000	0.003	0.005	0.007	0.018
HAD						
no credit constraints	0.526	0.577	0.613	0.625	0.636	0.669
credit constrained	0.869	0.132	0.190	0.216	0.234	0.280

When we look at the power distribution for the models with borrowing constraints in Figure 3, we see a qualitatively similar picture. The 1–lag SMH model has the highest power with a distribution centered around 0.3. The peak of the HAD model is somewhat lower around 0.22. Finally, the power of the 2–lag SMH model again has most of its mass close to zero. So far, the HAD and 2–lag SMH models outperform the 1–lag SMH model in terms of goodness-of-fit (pass rate), but the opposite is true when looking at discriminatory power.

Predictive success Up to now, we have focused our empirical assessment on the pass rates and discriminatory power of the various models. How can we reconcile both (often inversely related) performance measures into a single index, such that they can be used as a reliable criterion for comparing different but possibly nested models? Beatty and

at 1.

Figure 2: Kernel density of power for models with no credit constraints



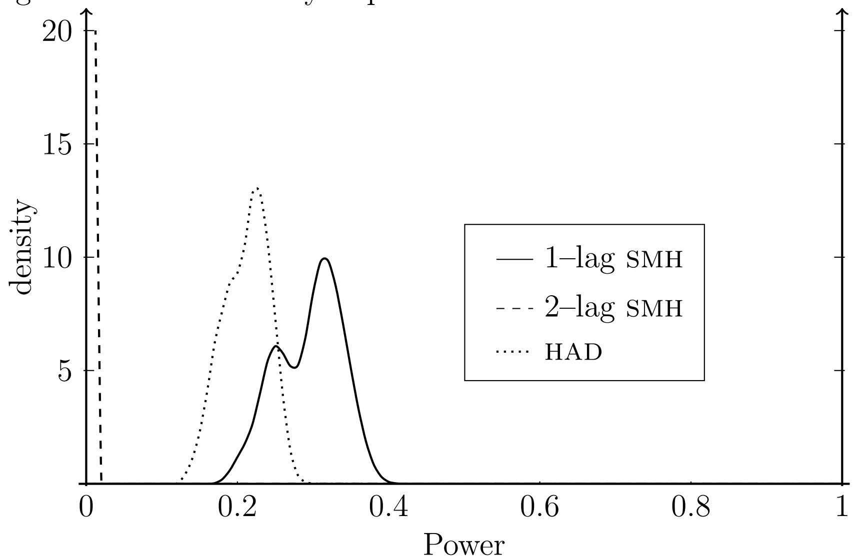
Crawford (2011) suggest a measure which is based on an original idea of Selten (1991). The measure is called *p*redictive success and is defined in terms of pass rate and power in the following way:

$$\textit{predictive success} = \textit{pass rate} - (1- \textit{power})$$

Beatty and Crawford (2011) show that this measure has an interesting axiomatic characterization which provides a convincing theoretical foundation.[13] The measure takes on values between -1 and 1. Negative values of predictive success (i.e. low pass rate together with low power) suggest that the model is rather inadequate for describing observed consumer behavior, since it is at least as good at explaining random behavior. On the other hand, positive values (i.e. high pass rate together with high power) point to a potentially useful model that is able to reject irrational behavior while explaining the actual observed behavior. Table 3 presents the quartiles for the predictive success of all models, together with the mean across all households. Notice that it is possible to measure predictive success for every household separately (with the pass rate being

[13]We do not give a formal definition of these axioms here, but refer to the study of Beatty and Crawford for a detailed discussion.

Figure 3: Kernel density of power for models with credit constraints



either 1 or 0).

It appears that, based on Selten's criterion, the HAD model without credit constraints provides, on average, the best fit with a mean predictive success of 0.15, which is significantly higher than any other model considered. Furthermore, if we look at the models with credit constraints, we see that the quartile values reveal a similarly good performance of the 1–lag SMH and the HAD models.

For clarity and comparison, Figure 4 shows the densities for the models with perfect capital markets, and Figure 5 shows the models in the case where we allow for possible credit constraints.[14]

If we look at Figure 4, we see that the 2–lag SMH model has a high and narrow peak close to zero. This seems to indicate that the restrictions of the 2–lag SMH model are too weak when imposed on our sample of tobacco addicted consumers. The 1–lag SMH

[14]Graphical results for the static and life cycle models are again omitted. Predictive success for the life cycle model peaks highly around 0 whether or not we allow for credit constraints. For the static model, we observe a large peak around 0.10, with a smaller peak close to -1 for the households that were not consistent with the model.

Table 3: Distribution of predictive success

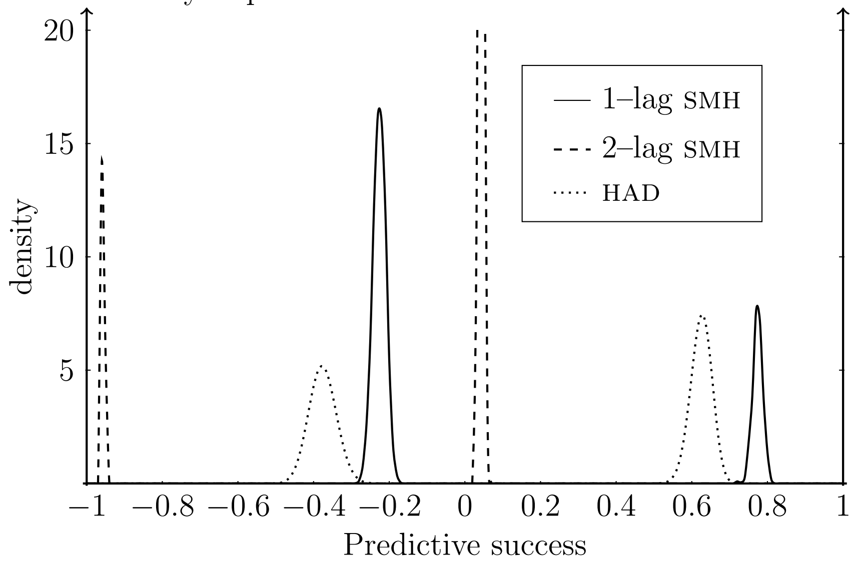
	mean	quartiles				
		min	1st quartile	median	3rd quartile	max
static	0.012	-0.955	0.075	0.095	0.108	0.141
life cycle						
no credit constraints	0.000	-0.003	0.000	0.000	0.000	0.000
credit constrained	0.011	-0.076	-0.045	-0.038	-0.032	0.971
1–lag SMH						
no credit constraints	0.033	-0.274	-0.233	-0.218	0.748	0.812
credit constrained	0.037	-0.795	-0.639	0.265	0.314	0.376
2–lag SMH						
no credit constraints	-0.017	-0.967	0.037	0.042	0.047	0.063
credit constrained	0.002	-0.995	0.003	0.005	0.007	0.018
HAD						
no credit constraints	0.150	-0.421	-0.374	0.598	0.627	0.669
credit constrained	0.080	-0.854	0.175	0.206	0.229	0.267

model without credit constraints has a small peak at 0.78 and a higher peak around -0.22, which was not apparent from Table 3. The HAD model has a larger (wider) peak at 0.63 and a smaller peak around -0.38. Hence, even under the strict assumption of perfect capital markets, the HAD model manages to adequately capture the rational behavior of a sizeable subset of households.

For the 1–lag SMH model which allows for borrowing constraints, we observe a broad peak around 0.3 and a somewhat smaller peak at -0.7 in Figure 5. The distribution of the HAD model with binding borrowing constraints has one peak around 0.22 and a small peak at -0.8. Again, the 2–lag SMH model has most mass close to zero.

Since we can identify the subset of households located in the rightmost peak, the HAD model can be used to adequately describe the behavior of this set of households, and predict outcomes in new market situations (e.g. changing commodity prices, incomes, interest rates, tobacco excises,...). In order to find out whether there are (observable) individual characteristics that can significantly explain whether or not the individual is consistent with any specification of the HAD model, we estimated a probit model of the

Figure 4: Kernel density of predictive success for models with no credit constraints



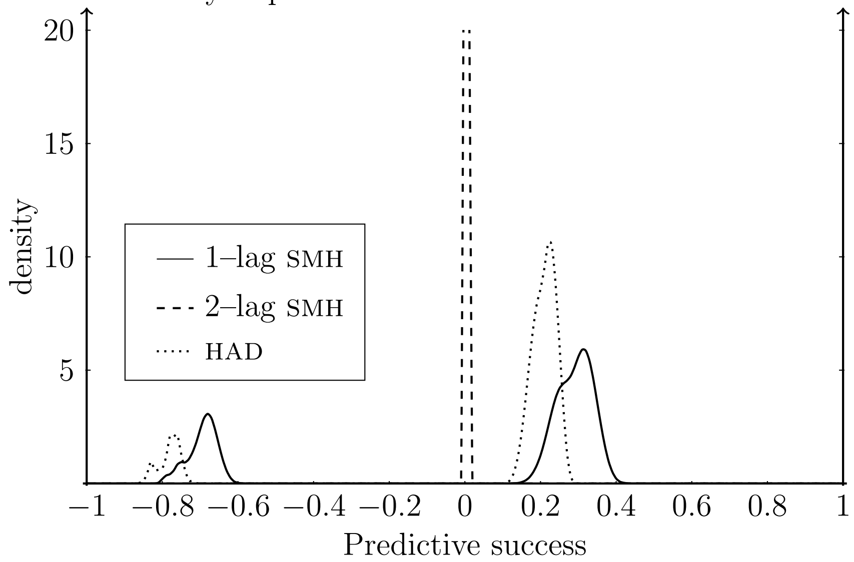
pass rate with respect to several observable characteristics.[15] Unfortunately, very few coefficients were (statistically) significant. As a sole exception, we found that the pass rate of the HAD model with credit constraints was significantly and positively related to the age of the family head. We do not have an intuitive explanation for this finding.[16]

Multiple addictive goods As a final exercise, we consider an extension of the different rational habit formation models for multiple addictive goods. Considering our empirical setting, we choose alcohol to be our second addictive good. For the HAD model, the extension to two addictive goods requires the introduction of a second stock of addiction for alcohol, A_t^a, and a second depreciation rate, δ^a. Denoting by Q_t^a the

[15]Our data set provides information on age of the family head and age difference with the other partner, education and occupation of the family head, number of children in various age categories and housing tenure. Detailed results from these estimates are available from the authors upon request.

[16]However, somewhat related, Chaloupka (1991) finds some evidence that younger and less educated individuals tend to have a higher rate of time preference (i.e. a lower beta), implying they do not fully internalize the future costs of current addictive consumption.

Figure 5: Kernel density of predictive success for models with credit constraints



consumption of alcohol in period t, we obtain the following 'investment' equation.

$$A^a_{t+1} = (1 - \delta^a)A^a_t + Q^a_t$$

Of course, the addition of a second addictive good requires an adjustment to the instantaneous utility function, which now takes into account the negative influence of A^a_t on the level of utility. This utility function can be represented by $u(\mathbf{q}_t, Q_t, A_t, Q^a_t, A^a_t)$. We refer to Appendix .1 for a statement of the necessary and sufficient revealed preference conditions for this more general model. For the R–lag SMH model, the introduction of a second addictive good changes the instantaneous utility function to take on the form $u(\mathbf{q}_t, Q_t, \ldots, Q_{t-R}, Q^a_t, \ldots, Q^a_{t-R})$. Introducing a second addictive good to our data set produces the results in Table 4.[17] Again, we see that the HAD outperforms the 1–lag SMH and 2–lag SMH model in terms of (mean) predictive success.

[17]We restricted our data set to households that have strict positive consumption for both alcohol and tobacco for all eight observations. This restricts the sample to 137 households.

Table 4: Results for multiple addictive goods

	Pass–rate	power	predictive success
1–lag SMH			
no credit constraints	0.824	0.133	-0.043
credit constrained	0.993	0.014	0.007
2–lag SMH			
no credit constraints	1.000	0.000	0.000
credit constraints	1.000	0.000	0.000
HAD			
no credit constraints	0.963	0.204	0.167
credit constraints	0.985	0.178	0.163

What can we learn from all this? First of all, our application shows that from an empirical point of view, the models of rational habit formation are much more realistic than the standard life cycle model, whose testable implications are to strong for our data. Second, we find that (at least for our data set) the restrictions imposed by the 2–lag SMH model are very weak in the sense that almost all 'random' behavior can be rationalized by this model. On the other hand, the HAD model (with or without credit constraints) performs rather well compared to the other models of rational habit formation in terms of higher predictive success. The adequacy of the specification may be correlated with specific household characteristics. Unfortunately, our data did not allow us to identify which characteristics matter. Finally, from a more general perspective, we believe that our application convincingly shows the practical usefulness of the revealed preference approach for assessing the validity between several models of intertemporal decision making in a real life setting.

4 Conclusion

We developed a revealed preference methodology for assessing the validity of the HAD model as it was introduced by Becker and Murphy (1988). By generalizing the intertemporal consumption dependence underlying addictive behavior, our revealed preference characterization extends the life cycle model of Browning (1989) and the 1-lag SMH model of Crawford (2010). Moreover, we relax the assumption of perfect capital markets by allowing consumers to be credit constrained to some (unobserved) extent. We applied our tests on a sample of Spanish households to look whether consumers addicted to detrimental goods such as tobacco can still be considered as rational. The empirical analysis shows that the life cycle model is heavily rejected by the Spanish panel data. When rational habit formation is added to the model and, additionally, the possibility of credit constraints is introduced, we notice an improved empirical fit of these more general models. We complemented our analysis by calculating discriminatory power for the different models, and find that the higher pass rates of the HAD model compared to the 1–lag SMH model cannot be entirely attributed to the generality or permissiveness of this model, since the tests do not lack in power. Based on the measure of predictive success that was suggested by Selten (1991) and Beatty and Crawford (2011), we find that an additional and nontrivial subset of households can be rationalized by extending the life cycle and 1–lag SMH model towards the more general HAD framework. On the other hand, including more lags to the SMH model does not seem to provide a better fit when evaluated in terms of power and predictive success.

We see different avenues for follow-up research. First of all, it might be possible to use our framework to investigate which household characteristics drive the consistency with the HAD model. However, this would require a data set with richer information on household characteristics.

Second, in order to keep our application focused, we concentrated on characterizing the revealed preference conditions and testing consistency of household data with these conditions. This implies that we only considered 'sharp' rationality tests in the sense that a given household passes the test, which means that the behavior is consistent with the model, or the model is rejected and the household is considered irrational. On the other hand, Varian (1990) asserts this rather extreme notion could be attended to by investigating to what extent a so–called irrational household is not a perfect optimizer, by allowing for a small optimization error to enter the testable restrictions. The inclusion of such an optimization error into the HAD model could easily be done by modifying condition (G.1).

Third, given that observed behavior is consistent with the HAD a natural next question pertains to the recovery and identification of the underlying decision model that rationalizes the observed behavior, and to forecast behavior in new situations. We refer to Crawford (2010) who investigated such issues in the case of the SMH model.

Finally, future research could focus on relaxing several assumptions of our model. The perfect foresight assumption, which is maintained throughout this paper, is potentially too restrictive when imposed over longer periods of time. Also, our model assumes that household behavior can be represented by the maximization of a single utility function. However, most considered households from our empirical application consist of multiple individuals. We leave it up to future research to extend our approach towards a setting that explicitly allows for multiple (addicted) members within the same household.[18]

[18]See, for example, Mazzocco (2007) and Adams et al. (2011) for collective characterizations of intertemporal consumption models, who still maintain the assumptions of consumption independence and perfect capital markets.

Appendix

A. Proof of Theorem 1

(necessity) The function u is concave, hence for all t and $v \leq T$,

$$\begin{aligned} u(\mathbf{q}_t, Q_t, A_t) - u(\mathbf{q}_v, Q_v, A_v) \leq & \frac{\partial u(\mathbf{q}_v, Q_v, A_v)}{\partial \mathbf{q}}(\mathbf{q}_t - \mathbf{q}_v) \\ & + \frac{\partial u(\mathbf{q}_v, Q_v, A_v)}{\partial Q}(Q_t - Q_v) + \frac{\partial u(\mathbf{q}_v, Q_v, A_v)}{\partial A}(A_t - A_v) \end{aligned}$$

Here $\frac{\partial u(\mathbf{q}_v, Q_v, A_v)}{\partial \mathbf{q}}$, $\frac{\partial u(\mathbf{q}_v, Q_v, A_v)}{\partial Q}$ and $\frac{\partial u(\mathbf{q}_v, Q_v, A_v)}{\partial A}$ are suitable subdifferentials of the function $u(\mathbf{q}_v, Q_v, A_v)$. Setting $u(\mathbf{q}_t, Q_t, A_t) = u_t$ for all t, together with the first order conditions (A.1)–(A.5), establishes the necessity part.

(sufficiency) Consider a subset τ of observations and sum condition (G.1) across all observations within this subset. This gives

$$0 \leq \sum_{v,t \in \tau} \left(\frac{1}{\beta^{v-1}} \left[\lambda_v \mathbf{p}_v (\mathbf{q}_t - \mathbf{q}_v) + \tilde{P}_v^Q (Q_t - Q_v) + \tilde{P}_v^A (A_t - A_v) \right] \right).$$

This is a cyclical monotonicity condition (see Rockafellar 1970, theorem 24.8). This condition implies that there exists a concave utility function u, increasing in $\mathbf{q}$ and Q and there exist positive numbers λ_t such that

$$\begin{aligned} \frac{\partial u(\mathbf{q}_t, Q_t, A_t)}{\partial \mathbf{q}} &= \frac{1}{\beta^{t-1}} \lambda_t \mathbf{p}_t, \\ \frac{\partial u(\mathbf{q}_t, Q_t, A_t)}{\partial Q} &= \frac{1}{\beta^{t-1}} \tilde{P}_t^Q, \\ \frac{\partial u(\mathbf{q}_t, Q_t, A_t)}{\partial A} &= \frac{1}{\beta^{t-1}} \tilde{P}_t^A. \end{aligned}$$

Together with conditions (G.2)-(G.4) and taking into account condition (1), we obtain conditions (A.1)-(A.5).

B. Revealed preference conditions for multiple addictive goods

The following presents the revealed preference characterization for the HAD model with two addictive goods. We denote by P_t^a the price of the second detrimental addictive good and by Q_t^a its quantity.

Theorem 5. *The following statements are equivalent:*

- *The data set $D = \{r_t, \mathbf{p}_t, P_t, P_t^a, \mathbf{q}_t, Q_t, Q_t^a\}_{t \leq T}$ is rationalizable by the* HAD *model with two addictive goods.*
- *There exist numbers $\beta, \delta, \delta^a \in]0,1]$ and for all $t \leq T$ there exist positive numbers u_t, A_t, A_t^a, strict positive numbers $\widetilde{P}_t^Q, \widetilde{P}_t^{Q^a}, \lambda_t$ and negative numbers $\widetilde{P}_t^A, \widetilde{P}_t^{A^a}$ such that for all $t, v \leq T$:*

$$u_t - u_v \leq \frac{1}{\beta^{v-1}} \begin{bmatrix} \lambda_v \mathbf{p}_v \left(\mathbf{q}_t - \mathbf{q}_v\right) + \widetilde{P}_v^Q \left(Q_v - Q_t\right) \\ +\widetilde{P}_v^A \left(A_t - A_v\right) + \widetilde{P}_v^{Q^a}(Q_t^a - Q_v^a) \\ +\widetilde{P}_v^{A^a}(A_t^a - A_v^a) \end{bmatrix}, \tag{H.1}$$

$$\widetilde{P}_{t+1}^A = (1-\delta)(\widetilde{P}_{t+1}^Q - \lambda_{t+1} P_{t+1}) - (\widetilde{P}_t^Q - \lambda_t P_t), \tag{H.2}$$

$$\widetilde{P}_{t+1}^{A^a} = (1-\delta^a)(\widetilde{P}_{t+1}^{Q^a} - \lambda_{t+1} P_{t+1}^a) - (\widetilde{P}_t^{Q^a} - \lambda_t P_t^a) \tag{H.3}$$

$$\lambda_{t+1}(1 + r_t) \leq \lambda_t, \tag{H.4}$$

$$A_{t+1} = (1-\delta)A_t + Q_t, \tag{H.5}$$

$$A_{t+1}^a = (1-\delta^a)A_t^a + Q_t^a. \tag{H.6}$$

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