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Mutual point-winning probabilities (MPW): a new performance measure for table tennis

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Abstract

We propose a new performance measure for table tennis players: the mutual pointwinning probabilities (MPW) as server and receiver. The MPWs quantify a player's chances to win a point against a given opponent, and hence nicely complement the classical match statistics history between two players. We shall describe the MPWs, explain the statistics underpinning their calculation, and show via a Monte Carlo simulation study that our estimation procedure works well. As an illustration of the MPWs' versatile use, we use it as an alternative ranking method in two round-robin tournaments of ten respectively eleven table tennis players that we have ourselves organized.

Key words: Bradley-Terry model, Maximum likelihood estimation, Round-robin tournament, Sport performance analysis, Strength model

1 Introduction

The use of advanced statistical techniques in sports has become increasingly popular over the past years. This fact is noticeable both at the highest sport competition level, where several data analysts get hired by the major professional teams, as well as at the level of scientific research. Numerous papers have been written about soccer (Groll et al. 2015, Koopman and Lit 2015, Tutz and Schauberger 2015), tennis (Baker and McHale 2014, Harris 2016), basketball (Martín-Gonzáleza et al. 2016, Vračar et al. 2016), baseball (Heumann 2016, Peach et al. 2016) or badminton (Percy 2009, Paindaveine and Swan 2011), to cite but these, and international scientific research projects such as the "Big Data Analytics in Sports"¹ have been created. The mutual fertilization between the world of sports and academic statistics is undoubtable, and one of the reasons why *sport analytics* play such an important role nowadays. We refer the reader to the webarticle Steinberg (2015) to get insight into this quick development.

Quite surprisingly, the available scientific literature about table tennis is relatively scarce. To the best of the authors' knowledge, only two papers have been devoted to the topic, and both papers have proposed a stochastic analysis of the game. Schulman and Hamdan (1977) investigated the probabilities of winning a set in the former scoring system (5 services in a row by each player, first to reach 21 points wins the set, at 20-20 the players take turns in services),

¹http://bodai.unibs.it/BDSports/

while Dominicy et al. (2013) have modelled the match-winning chances in the old as well as the new scoring system (2 services in a row by each player, first to reach 11 points wins the set, at 10-10 the players take turns in services). Dominicy et al. (2013) have furthermore compared the two scoring systems and concluded that the change implied (i) a better control of the length of a match and (ii) an augmentation of the number of crucial points without influencing the winning probabilities too heavily. Both papers, Schulman and Hamdan (1977) and Dominicy et al. (2013), are purely theoretical and are not making use of any real data.

The recent surge in computing power has enabled the creation of devices that can collect, store and transfer increasingly complex and large data sets. We are living in the "Big Data" era and the strong need to properly analyze these data has even led to the creation of a new research field called "Data Science" (Diggle 2015, Ley and Bordas 2017). All fields are facing these new challenges, be it science, business, industry, medicine, politics or sports. Sport analytics have become essential to competitive teams, but also odd-setters need them for their forecasts, the fans are greedy of sport statistics, and the media rely on them for their coverage. In the present paper, we wish to propose a novel strength model for table tennis players that shall be of interest to all parties.

Suppose two players face each other and the result of a match is 3-0 in favor of Player A. Looking in detail at the match statistics sheet is more revealing: a victory by 11-2, 11-1 and 11-5 shows a clear dominance of Player A, while a victory by 11-9, 11-8 and 13-11 indicates a relatively close match where Player A was able to win the crucial points. Our new model is designed to provide two numbers that measure the relative strength of Player A with respect to Player B. After each game between the same players, these numbers shall be updated. Besides the classical match results (say A has a record of 5-1 against B), these numbers will reveal how dominant Player A was when playing against Player B.

In Section 2, we shall describe in detail our measures of relative strength, the MPWs, and the statistical methods to obtain them. A Monte Carlo simulation study further shows that our estimation methods are efficient. In Section 3 we illustrate the potential use of MPWs through two round-robin tournaments that we have ourselves organised with table tennis players from Belgium. A final conclusion is provided in Section 4.

2 The new performance measure

2.1 Mutual point-winning probabilities (MPW)

We consider a match between Player A and Player B. Each player is assigned two strength parameters, (p_{As}, p_{Ar}) for Player A and (p_{Bs}, p_{Br}) for Player B, where the index s stands for server and r for receiver. These parameters represent winning probabilities and hence vary between 0 and 1: p_{As} is the probability of Player A winning a rally he/she initiates, while p_{Ar} is his winning probability as receiver. Obviously, the parameters of both players are mutually dependent in the sense that

$$p_{As} + p_{Br} = 1$$
 and $p_{Ar} + p_{Bs} = 1$.

This explains our terminology of *mutual point-winning probabilities*, which we abbreviate MPW. At the end of a match, the server and receiver MPWs provide additional information to the classical final results and can be averaged over several matches between the two players. Since the MPWs are probabilities, they allow predicting the outcome of the next match between two players. Let us illustrate MPWs via a toy example. Suppose that Player A won against Player B in four sets, 11-3, 11-7, 8-11 and 11-5. Our MPWs would further reveal that $p_{As} = 0.71$ and

 $p_{Ar} = 0.50$, hence that Player A won 71% of the rallies he/she initiated and that both players were equally strong when Player B served. MPWs thus represent a brief and efficient summary that nicely complements the game statistics.

This probabilistic strength model is a slightly modified version of the popular Bradley-Terry model for paired comparisons (Bradley and Terry 1952). In our model, we assume that every rally played is independent of the previous rallies. This assumption is based on findings by Klaassen and Magnus (2003) who show that, within the framework of tennis, independence of exchanges yields a good approximation to real games.

2.2 Calculation of the MPWs

We shall estimate the MPWs by making use of a detailed point-by-point result sheet after a match. Let P_{iA} , respectively P_{iB} , stand for the probability that Player A, respectively Player B, wins rally *i* among the total of N rallies needed to complete a match (depending on who serves, P_{iA} thus equals either p_{As} or p_{Ar}). A particularity of our model is that we assign different weights w_i to each rally, reflecting their respective importances. Indeed, winning a rally at 9-2 is less crucial than at 8-8. The weighting scheme will be detailed below. The resulting overall likelihood function of our model then takes the form

$$L = \prod_{i=1}^{N} \prod_{j=A,B} P_{ij}^{y_{ij} \cdot w_i}$$

$$\tag{2.1}$$

where y_{ij} equals 1 if Player *j* has won rally *i*, and 0 else. We attract the reader's attention to the fact that the parsimonious way of writing the likelihood (2.1) is simplified thanks to the fact that a point is scored on every exchange and that there is a fixed rule of service changes (contrary to former rules in badminton, for example). The strength parameters (p_{As}, p_{Ar}) and (p_{Bs}, p_{Br}) are then estimated as maximizers of the likelihood function (2.1).

Consider two rallies with respective weights 2 and 6. The latter will, compared to the former, have three times more impact on the estimation of the strength parameters. We shall now expand on how we choose the weights w_i . They are the sum of the following three importance factors:

• Total points played in a set: we assign distinct importance factors to a point depending on the number of rallies x played to reach the related score. We propose the following scheme:

Total Points - Importance Factor =
$$\begin{cases} 1 & \text{for } x < 14 \\ 2 & \text{for } 14 \le x < 18 \\ 3 & \text{for } 18 \le x. \end{cases}$$

This scoring reflects well the fact that points played at the beginning of a set have less importance, as well as rallies at intermediate scores like 9-2.

• Difference in points: In case of tied or very close intermediate scores, the next point is obviously more important than if the point difference were four or more. Writing d the absolute point difference, we adopt the following rule:

Difference in Points - Importance Factor =
$$\begin{cases} 1 & \text{for } d \ge 4 \\ 2 & \text{for } d = 2 \text{ or } 3 \\ 3 & \text{for } d = 0 \text{ or } 1. \end{cases}$$

Table 2.1: Ten imaginary intermediate scores, ranked according to their importance for the final match outcome by 29 table tennis players (column Rank) and evaluated according to our weighting scheme (column Weight).

Intermediate score (sets played, intermediate score)	Rank	Weight
2-2, 5-5	5	7
0-1,12-11	2	8
2-0, 1-6	7	4
1-0, 2-0	8	4
0-0, 10-7	6	5
1-1, 8-8	4	7
2-2, 11-10	1	9
0-0, 9-1	10	2
2-1, 0-0	9	4
2-1, 9-7	3	7

• <u>Current set</u>: rallies exchanged in the first set are considered less important than rallies in later sets, especially sets 4 and 5. Writing s for the set currently played, this lead to

Current Set - Importance Factor =
$$\begin{cases} 1 & \text{for } s = 1 \\ 2 & \text{for } s = 2 \text{ or } 3 \\ 3 & \text{for } s = 4 \text{ or } 5. \end{cases}$$

On top of this weighting scheme we add the following two binding rules:

- If the point difference is 8 or more, the final weight w_i shall be assigned the minimum of 2.
- As long as the total number of points scored is 4 or less, the weight w_i cannot exceed 4 during the first three sets and not exceed 5 during the last two sets.

For the sake of illustration, a score of 5-4 in the fourth set is assigned the weight $w_i = 1+3+3 = 7$. We stress here that, at the evaluation of the score 5-4, our system does not take into account how we got to that point. This is totally in line with our assumption of independence between the different rallies played.

In order to confirm the validity of our weighting scheme, we have asked 29 competitive table tennis players from Belgium to rank ten imaginary intermediate scores according to their importance for the final match outcome. Their answers are summarized in Table 2.1. As we can see, our weighting scheme matches these players' feelings quite well. This confirms that our rules are simple, but effective.

2.3 A Monte Carlo simulation study

In this section we show by means of Monte Carlo simulations that our maximum likelihood estimation procedure effectively works, i.e., that the estimated values are consistent estimates of the true (unknown) probabilities. To this end, we have simulated 10,000 times entire table tennis matches for six distinct settings of MPWs: (0.5, 0.5, 0.5, 0.5), (0.5, 0.5, 0.2, 0.8), (0.5, 0.5, 0.8, 0.2), (0.85, 0.15, 0.25, 0.75), (0.85, 0.15, 0.9, 0.1) and (0.15, 0.85, 0.2, 0.8). These need to be read as

Table 2.2: Consistency study of our likelihood-based performance model. For each setting of MPWs (p_{As}, p_{Ar}) , we have simulated 10,000 times 1 match respectively 2 matches and recorded the estimated parameter values. Below are indicated the averaged estimates as well as the Root Mean Squared Errors (RMSE) for both settings.

True MPWs	Estimates 1 Match	RMSE	Estimates 2 Matches	RMSE
(0.5000, 0.5000)	(0.4845, 0.4951)	(0.0683, 0.0662)	(0.4975, 0.5000)	(0.0467, 0.0467)
(0.5000, 0.8000)	(0.4845, 0.7935)	(0.0668, 0.0547)	(0.5048, 0.7863)	(0.0504, 0.0443)
(0.5000, 0.2000)	(0.5082, 0.2056)	(0.0673, 0.0575)	(0.5131, 0.2034)	(0.0559, 0.0424)
(0.8500, 0.7500)	(0.8438, 0.7375)	(0.0676, 0.0767)	(0.8512, 0.7277)	(0.0492, 0.0680)
(0.8500, 0.1000)	(0.8618, 0.1111)	(0.0357, 0.0342)	(0.8485, 0.0959)	(0.0295, 0.0245)
(0.1500, 0.8000)	(0.1580, 0.8060)	(0.0394, 0.0453)	(0.1481, 0.7979)	(0.0310, 0.0342)

follows: in the last setting Player A has a chance of 15% to win a rally that he/she initiates, and 80% winning chance as he/she is the receiver. We have thus considered very diverse situations, ranging from a totally equalized game over a service-dominated game to a game where one player dominates his opponent both on service and reception. For each simulated match, we have estimated the corresponding parameters in our model (2.1). Then, in a second step, we have considered exactly the same settings but this time we have simulated two matches each time, in order to illustrate the capacity of our MPWs to be updated after every match. Since the number of exchanges in two matches is roughly double the amount of a single match, we expect the resulting estimates to lie closer to their true values. All simulations were carried out in Matlab; to ensure their replicability, we used the same seed for all simulations. As initial values for the optimisation procedure we used (0.5, 0.5, 0.5, 0.5) (we tried out other initial values and the results were very similar).

The simulation results are presented in Table 2.2, where we show the MPWs of Player A. Whatever the settings considered, the estimation procedure yields good results. As expected, playing 2 matches improves the precision of our estimators in 9 out of 12 settings.

3 Alternative MPW-based ranking in a round-robin tournament

In order to appreciate the additional information provided by the MPWs, we have organised ourselves two round-robin table tennis tournaments, one with strong competitive players and the other with less strong competitive players. This organisation was simplified thanks to the contacts of the third author, who is himself a competitive player in Belgium. In what follows we shall describe the progress of the tournament and data collection (Section 3.1), and analyse the results for the high-level players (Section 3.2) as well as the low-level players (Section 3.3). The analysis shall be based on both the classical ranking and an alternative ranking based on our MPWs.

3.1 Data collection and tournament conduct

We have invited some dozens of randomly chosen competitive table tennis players to participate in two round-robin tournaments. Twenty-one players responded to this invitation, all are mem-

Name	Ranking	Years of experience
AH	C4	8
BH	C4	10
CH	C2	14
DH	C0	30
\mathbf{EH}	C4	21
\mathbf{FH}	C2	10
GH	C4	13
HH	C2	9
IH	C0	6
$_{\rm JH}$	C0	6
KH	C2	15

Table 3.3: Overview of high-level players taking part in the first organized round-robin tournament.

Table 3.4: Overview of low-level players taking part in the second organized round-robin tournament.

Name	Ranking	Years of experience
AL	E2	9
BL	E4	6
CL	E0	10
DL	E2	6
EL	E2	9
FL	E4	9
GL	E0	9
HL	E4	4
IL	E4	5
JL	E2	9

bers of different official table tennis clubs in West Flanders, Belgium. Ten players are ranked as E0, E2 or E4, which corresponds to a low local level. Eleven players are ranked as C0, C2 or C4, corresponding to a high local level. An overview of the characteristics of these players can be found in Tables 3.3 and 3.4. We have anonymised the players' names and hence refer to them as Players AH, BH, ... respectively AL, BL, ..., where H and L respectively stand for high- and low-level.

Participants of both groups knew each other, and hence had an idea of their respective strengths and weaknesses, especially those playing for the same team. Players were aware that they are participating in a scientific study, because there are no such round-robin tournaments organised by the Flemish Table Tennis Association (VTTL). They did however ignore the key scientific questions of this study and were told to play every match like a normal competitive match. Furthermore, no coaches, fans or participants were allowed to talk with or support the players during the games.

Both tournaments were organised in a local sports hall of the table tennis club in Zonnebeke,

Rank	Name	Matches won	Matches lost	Sets won	Sets lost	Sets even ranked
1	JH	10	0	30	10	
2	IH	8	2	28	11	
3	DH	7	3	25	14	
4	BH	6	4	23	18	
5	CH	5	5	21	18	4/3
6	GH	5	5	19	23	4/4
7	\mathbf{EH}	5	5	19	22	3/4
8	KH	4	6	21	25	
9	HH	3	7	13	26	
10	\mathbf{FH}	2	8	18	27	
11	AH	0	10	7	30	

Table 3.5: Official ranking of the round-robin tournament played by the high-level players. It displays the matches won and lost, the sets won and lost and the mutual results between players that have won an equal amount of matches (sets even ranked).

Belgium. The games were played at Stiga Optimum 30 tables, using plastic 40+ mm Xushaofa balls. Both groups played a round-robin tournament, on two different days. The games were played in a completely random sequence. The (2,11,3)-scoring system was used, which is the scoring system of the national and local competitions in Belgium. A toss decided which player served first at the beginning of each match. Three matches were played simultaneously. Games were led by a referee, as this is the case in competitions and official tournaments. Those referees were either volunteers or players of the study. Participants were told they could rest long enough between the games, so that fatigue could be neglected as much as possible. The players were aware that each game was recorded by a camera, which allowed us to precisely aggregate the point-by-point data after the tournament.

3.2 Analysis of the tournament of high-level players

Eleven players formed the high-level group, leading to a total of 55 matches. The final ranking of the tournament is displayed in Table 3.5. JH played some exciting games and managed to win them all. IH finished second but lost only one set more than JH. Only AH was not playing at a high level and lost all matches.

We now analyse the results via our MPWs. For every game played, each player has been given service and receiver MPWs. In Table 3.6 we indicate the resulting rankings on service as well as on return. There is a strong correlation between the MPW rankings and the official ranking, with a Spearman ρ coefficient of 0.855 for service and 0.909 for return. Most remarkably, player FH is ranked much higher on the service ranking than in the official ranking, which reflects the fact that he has been playing several close matches and is a good server. This is one of the aspects where our new performance measure sheds interesting new light on match outcomes.

Since we are in a round-robin tournament, it is sensible to add up these MPWs for every player and to rank them according to the resulting sums. Moreover, since the service alternates every two points, both service and return are equally important to distinguish an excellent player from a good player. Table 3.7 provides the ranking based on the sum of both strengths. The Spearman correlation with the tournament ranking is 0.909, revealing that the combined

Table 3.6: Ranking of high-level players based on the MPW on service (left) and on return (right). The value in brackets shows the ranking of the high-level player in the tournament (see Table 3.5).

Rank	Name	MPW on service	Name	MPW on return
1	IH(2)	0.592	DH(3)	0.553
2	JH(1)	0.581	JH(1)	0.542
3	BH(4)	0.577	IH(2)	0.541
4	CH(5)	0.561	CH(5)	0.492
5	$\mathrm{EH}(7)$	0.555	BH(4)	0.473
6	DH(3)	0.539	KH(8)	0.464
7	FH(10)	0.538	EH(7)	0.457
8	GH(6)	0.504	GH(6)	0.450
9	KH(8)	0.480	FH(10)	0.428
10	HH(9)	0.466	HH(9)	0.410
11	AH(11)	0.437	AH(11)	0.365

Table 3.7: Ranking of high-level players based on the sum of both server and receiver MPWs.

Rank	Name	MPW Sum
1	IH(2)	1.133
2	$\mathrm{JH}(1)$	1.123
3	DH(3)	1.091
4	CH(5)	1.054
5	BH(4)	1.050
6	$\mathrm{EH}(7)$	1.012
7	FH(10)	0.966
8	GH(6)	0.955
9	KH(8)	0.944
10	HH(9)	0.877
11	AH(11)	0.802

Rank	Name	Matches won	Matches lost	Sets won	Sets lost	Sets even ranked
1	HL	9	0	27	7	
2	DL	8	1	26	14	
3	AL	6	3	22	14	
4	GL	5	4	19	16	3/2
5	CL	5	4	20	15	2/3
6	JL	4	5	18	20	
7	BL	3	6	15	23	
8	IL	3	6	16	20	
9	EL	2	7	9	21	
10	FL	0	9	6	27	

Table 3.8: Official ranking of the round-robin tournament played by the low-level players. It displays the matches won and lost, the sets won and lost and the mutual results between players that have won an equal amount of matches (sets even ranked).

information about both service and return MPWs is highly in line with the classical point system. Again the most remarkable change is Player FH who is ranked 7th according to MPWs, while he was 10th in the classical ranking. Other position changes like those involving Players IH and JH or Players CH and BH are based on only minor differences in MPWs. The latter result from similar numbers of sets won and lost, see Table 3.5. An unsurprising fact is the ranking change of Player HH, who loses one place in our ranking compared to the official ranking, given that he only won a low amount of sets. In this respect, the MPW ranking seems even fairer than the traditional ranking.

3.3 Low-level players

The analysis here follows along the same lines as in the previous section. HL, the winner, was the best player in the competition conceding only 7 sets in 9 games. FL lost all matches.

The Spearman correlations between the official ranking and the MPW rankings on service, return, and the sum of both components are respectively 0.806, 0.818 and 0.980. Quite interestingly, this means that the service and return rankings for low-level players differ more from the official ranking than for high-level players, while the sum of service and return MPWs yields a nearly identical picture to the official ranking.

The MPW rankings reveal the following insightful facts. Player IL has been the weakest on the service, but ranked 4th on the receiver ranking. The overall winner, Player HL, dominates the service ranking by quite some margin, and ranks second as receiver. Players GL and CL, who could only be distinguished on the official ranking through the number of even ranked sets, exchange their positions on the combined MPW ranking.

4 Conclusion

We have proposed an alternative performance measure for table tennis: the mutual pointwinning probabilities, abbreviated MPWs. They are based on a statistical strength model whose parameters are estimated by means of maximum likelihood estimation. Through a thorough Monte Carlo simulation study we have shown that our estimation procedure works very well.

Table 3.9:	Ranking	of low-level	players	based of	on the	MPW	on service	(left)	and	on	return
(right). Th	e value in	brackets she	ows the	ranking	of the	low-lev	el player in	the to	ourna	mer	nt (see
Table 3.8).											

Rank	Name	MPW on service	Name	MPW on return
1	HL(1)	0.597	AL(3)	0.555
2	GL(4)	0.555	HL(1)	0.543
3	$\mathrm{CL}(5)$	0.552	DL(2)	0.541
4	DL(2)	0.543	IL(8)	0.530
5	JL(6)	0.541	CL(5)	0.490
6	AL(3)	0.521	GL(4)	0.468
7	$\mathrm{EL}(9)$	0.504	BL(7)	0.466
8	BL(7)	0.477	JL(6)	0.450
9	FL(10)	0.445	EL(9)	0.416
10	IL(8)	0.440	FL(10)	0.367

Table 3.10: Ranking of low-level players based on the sum of both server and receiver MPWs.

Rank	Name	MPW Sum
1	HL(1)	1.140
2	DL(2)	1.084
3	AL(3)	1.076
4	CL(5)	1.042
5	GL(4)	1.024
6	JL(6)	0.991
7	IL(8)	0.970
8	BL(7)	0.943
9	EL(9)	0.919
10	FL(10)	0.811

We have applied our MPWs to two round-robin tournaments that we organized with both high-level and lower-level competitive table tennis players from Belgium. We draw two main conclusions from this experiment:

- MPW-based rankings, be it on service, return or the combination of both, are highly in line with the classical tournament ranking, hence are reasonable performance measures.
- MPW-based rankings offer insightful additional information about the players' strength, and they remain easy to read and interpret.

Wrapping up, we believe that MPWs are interesting, easily interpretable and efficient performance measures for table tennis players that can nicely complement the traditional match statistics and rankings.

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