From Poincaré's Divergences to Quantum Mechanics with Broken Time Symmetry *

Ilya Prigogine

International Solvay Institutes for Physics and Chemistry, U.L.B. CP-231, 1050-Brussels, Belgium and Center for Studies in Statistical Mechanics and Complex Systems, The University of Texas at Austin, Austin, TX 78712, USA

Z. Naturforsch. **52a**, 37–45 (1997)

We discuss the spectral property of unstable dynamical systems in both classical and quantum mechanics. An important class of unstable dynamical systems corresponds to the Large Poincaré Systems (LPS). Conventional perturbation technique leads then to divergences. We introduce methods for the elimination of Poincaré divergences to obtain a solution of the spectral problem analytic in the coupling constant. To do so, we have to enlarge the class of permissible transformations, to include non-unitary transformations as well as to extend the Hilbert space. A simple example refers to the Friedrichs model, which was studied independently by George Sudarshan and his co-workers. However, our main interest is the irreducible representations in the Liouville space. In these representations the central quantity is the density matrix, and the eigenfunctions of the Liouville operator cannot be expressed in terms of the wave functions. We suggest that this situation corresponds to quantum chaos. Indeed, classical chaos does not mean that Newton's equation becomes "wrong" but that trajectories loose their operational meaning. Similarly, whenever we have an irreducible representation in the Liouville space this means that the wave function description looses its operational meaning. Additional statistical features appear. A simple example corresponds to persistent interactions in the scattering problem which cannot be treated in the frame of usual S-matrix theory.

1. Introduction

It is a privilege to participate in this symposium honoring George Sudarshan. We met for the first time at a Solvay meeting in 1961. That is more than thirty years ago. Since then our personal and scientific relations have become increasingly close. To prepare this contribution, I glanced on George's list of publications. I knew of course that he was deeply interested in Indian culture, metaphysics and philosophy. Still I was surprised to find that the number of publications dealing with these problems was nearly of the same order as for his publications in science proper. This is indeed quite unusual.

George notes [1]

> "Most scientists are allergic to metaphysics and most metaphysical systems deal with archaic and irrelevant science. Perhaps this too is transient. But I for one have been a practicing theoretical physicist for the past four decades and find no contradictions between my science and metaphysics"

However, George is not alone in this view which tries to incorporate philosophy, metaphysics and science. He in fact belongs to the tradition of Boltzmann and Schrödinger for whom science and philosophy are complementary tools to reach a better understanding of the surprising world in which we happen to live.

Schrödinger wrote [2]

> "... there is a tendency to forget that all science is bound up with human culture in general, and that scientific findings even those which at the moment appear the most advanced and esoteric and difficult to grasp, are meaningless outside their cultural context."

This is also the point of view of George. Again and again he emphasizes that science is part of culture. It is a spiritual experience "akin" to art [3]. Sudarshan shares the opinion of Paul Valery for whom on the highest level of the creativity there is no difference between scientific activity and poetry. In both cases the mind goes from "disorder" to "order".

It is a fact that we are still living in a world of two cultures. Isaish Berlin has clearly decribed the schism between science and humanities [4]:

* Presented at a Workshop in honor of E. C. G. Sudarshan's contributions to Theoretical Physics, held at the University of Texas in Austin, September 15–17, 1991.

Reprint requests to Prof. Ilya Prigogine at Brussels.

0932-0784 / 97 / 0100-0037 $ 06.00 © – Verlag der Zeitschrift für Naturforschung, D-72072 Tübingen

"The specific and unique versus the repetitive and the universal, the concrete versus the abstract, perpetual movement versus rest, the inner versus the outer, quality versus quantity, culture-bound versus timeless principles, ..."

I think Isaish Berlin is right. The main contrast between the two cultures lies in the concept of time. It is impossible to conceive humanities without evolutionary time. In contrast, the basic laws of physics, be they classical, quantum or relativistic, are deterministic and time reversible. It is therefore not astonishing that the problem of time plays an important role in scientific work of George Sudarshan. I have specially in mind his important contribution on the so-called "Zeno paradox", on analytical continuation as well as his work on density matrices. It was unavoidable that in spite the difference of starting points there would develop points of contact between his work and the work of our group. George came from quantum field theory, I came from physical chemistry and thermodynamics. But basically our conclusion was the same. In the traditional view, irreversibility is the result of approximations, of some "coarse graining" which we add to the exact laws of motion, be they classical or quantum. We are now forced to give up this approach. Indeed, over the last decades we have becomes aware of the creative role of time, specially in the study of non-equilibrium structures as well as in the study of classical unstable dynamical systems. It is therefore clear that we need now to incorporate time in the basic laws of nature and no more to consider the arrow of time as a result of some subjective features.

Let us start with some remarks on classical theory of unstable dynamic systems.

2. Classical Dynamics

We would like to show that "chaos" in classical dynamics leads to the breakdown of the concept of the trajectory. As well known, classical chaos is related to "sensitivity to initial conditions", that is to the existence of positive Lyapounoff exponents. As the simplest example let us consider the Bernoulli map:

$$x_{n+1} = 2x_n \pmod 1 . \tag{2.1}$$

It can be shown that two neighbouring points deviate exponentailly in time. After n iterations [5]

$$(\delta x)_n = (\delta x)_0 \quad e^{n \log 2} . \tag{2.2}$$

The Lyapounoff time is hence $t_L = 1/\log 2$. After a sufficient time, trajectories become "incomputable" whatever the (finite) specification of the initial conditions. The Bernoulli map is not invertible, as $x_{n+1} = \frac{1}{2}x_n$ leads to the attractor $x = 0$. But our remarks can be easily extended to dynamical systems proper (such as K-flows, K for Kolmogoroff, a simple example is the baker transformation [6]).

As the result of the Lyapounoff divergence it is natural to turn to a statistical description $\varrho(x)$ in terms of the so-called Perron-Frobenius operator U. The Bernoulli map leads to the relation [5]

$$\begin{aligned}\varrho_{n+1} &= U\varrho_n \\ &= \frac{1}{2}\left[\varrho\left(\frac{x}{2}\right) + \varrho\left(\frac{x+1}{2}\right)\right].\end{aligned} \tag{2.3}$$

Many properties of ϱ are known. For example

$$\varrho_n(x) \xrightarrow[n \to \infty]{} 1 \quad \text{over} \quad [0, 1] . \tag{2.4}$$

Whatever the initial condition, we reach uniformity in the future. There is a remarkable analogy between chaos and Brownian motion (described by a diffusion-type equation). However, when we turn to spectral theory there are also essential differences.

It is easy to find the eigenfunctions and eigenvalues of U. For example we have

$$U\left(x - \frac{1}{2}\right) = \frac{1}{2}\left(x - \frac{1}{2}\right). \tag{2.5}$$

Therefore $(x - \frac{1}{2})$ is an eigenfunction corresponding to the eigenvalue $\frac{1}{2}$ (related to Lyapounoff time). More generally it has been shown recently that the right eigenfunctions of U are the Bernoulli polynomials $B_n(x)$, corresponding to the eigenvalues $(\frac{1}{2})^n$ [7, 8]. Consider then the spectral decomposition of U

$$U = \sum_n |\varphi_n\rangle \frac{1}{2^n} \langle\tilde{\varphi}_n| . \tag{2.6}$$

This spectral decomposition cannot hold in the usual Hilbert space of square integrable functions [0, 1], as U^+ is an isometry and therefore the eigenvalues should be of module one. This shows that $\langle\tilde{\varphi}_n|$ should be distributions which have a meaning with a suitable choice of test functions [7, 9]. It is remarkable that even in this simple classical problem we have to introduce a "rigged Hilbert space" formalism to include the Lyapounoff time in the spectrum. Tasaki and Antoniou have shown that the test functions are formed by the space of all polynomials [9]. As the result, a δ-function is not in the domain of test functions.

In short, to describe the approach to equilibrium of statistical distributions $\varrho(x)$ we have to consider smooth functions and we have to go beyond the framework of classical mechanics based on the concept of trajectories.

Note also that there exist other spectral decompositions of U (corresponding to other spaces) [9]. However (2.6) is unique in the sense that it includes in the spectrum the Lyapounoff time responsible for instability and chaos. We shall show that similar considerations apply to quantum mechanics and lead to the necessity to extend the framework of conventional quantum theory.

3. Poincaré's Divergences

As we shall be mainly concerned with quantum systems it is important to start with a Hamiltonian formulation. Here Poincaré's classification [10] into "integrable" and "non-integrable" systems plays an essential role. As Petrosky and I have shown recently, this classification applies as well to quantum systems with continuous spectrum [11]. Consider a Hamiltonian of the form, for a quantum system

$$H = H_0 + \lambda V, \tag{3.1}$$

where λ is a coupling constant. We suppose that the eigenfunction $|\alpha\rangle$ and the eigenvalues of H_0 are known:

$$H_0 |\alpha\rangle = \omega_\alpha |\alpha\rangle . \tag{3.2}$$

How can we use this knowledge to construct the eigenfunctions and the eigenvalues of H?

$$H |\varphi_\alpha\rangle = E_\alpha |\varphi_\alpha\rangle . \tag{3.3}$$

We would like to find solutions we could expand in powers of λ to apply perturbative techniques. However, Poincaré's theorem shows that this is impossible as the result of divergences due to resonances between the unperturbed frequencies ω_α. Poincaré's non-integrable systems with continuous spectrum and continuous sets of resonances are quite common in physics. These are the systems we called Large Poincaré Systems (LPS).

For these systems there exists in general no constructive spectral theory [12]. It may even be shown that the solution of the spectral problem then becomes undecidable in the sense of Gödel's theory [13].

A simple example of LPS is the well-known. Friedrichs model in which a discrete state is coupled to a field with a continuous spectrum. The Hamiltonian is of the form (virtual transitions are neglected)

$$\begin{aligned} H &= H_0 + \lambda V \\ &= \omega_1 |1\rangle\langle 1| + \sum_k \omega_k |\boldsymbol{k}\rangle\langle \boldsymbol{k}| + \lambda \sum_k V_k (|\boldsymbol{k}\rangle \\ &\quad \cdot \langle 1| + |1\rangle\langle \boldsymbol{k}|) . \end{aligned} \tag{3.4}$$

There exists an exact solution of the spectral problem due to Friedrichs [14]

$$H = \sum_k \omega_k |\phi_k^F\rangle\langle\phi_k^F| . \tag{3.5}$$

The states $|\phi_k^F\rangle$ from an orthonormal and complete set.

The basic property of the Friedrichs solution is that the discrete state $|1\rangle$ is eliminated. Only the continuum modes remain. There are, however, a number of conceptual difficulties which are due to the fact that the Friedrichs solution is not analytic in the coupling constant λ. For $\lambda \to 0$ we obtain

$$\left[\sum_k \omega_k |\phi_k^F\rangle\langle\phi_k^F|\right]_{\lambda=0} = \sum_k \omega_k |\boldsymbol{k}\rangle\langle\boldsymbol{k}| \neq H_0 . \tag{3.6}$$

We see that $\lambda = 0$ is a singular point. Whatever small the value of λ, the discrete state disappears. For stable states the distinction between "bare states" (the eigenfunctions of H_0) and "dressed states" (the eigenfunctions of H) is essential. However, for unstable states we could only speak of "bare states". This leads to strange consequences. As well known, the decay of the "bare" state $|1\rangle$ can be subdivided into three periods [15]: First the Zeno time of the order $1/\omega$, then an exponential decay, and finally a long tail. If we identify the "bare" particle with the "physical" unstable particle, we come to a problem: we could indeed distinguish young and old particles according to their mode of decay. How to reconcile this with quantum indiscernability [16]?

We come therefore to our central problem. The elimination of Poincaré's divergences is necessary to obtain a solution of the spectral problem which would be analytic in the coupling constant λ. To do so, we have to enlarge the class of permissible transformations, to include non-unitary transformations as well as to extend the Hilbert space.

Let us first summarize the results of our approach for the Friedrichs model.

4. Quantum Theory of Non-Integrable Systems: Friedrichs Model

Let us go back to the Friedrichs model (3.4) and look for solutions which are analytic in the coupling constant [24]

$$(H_0 + \lambda V)\,|\varphi_\alpha\rangle = z_\alpha\,|\varphi_\alpha\rangle \qquad (4.1)$$

with

$$|\varphi_\alpha\rangle = |\alpha\rangle + \sum_{n=1}^{\infty} \lambda^n\,|\varphi_\alpha^{(n)}\rangle, \quad z_\alpha = \omega_\alpha + \sum_{n=1}^{\infty} \lambda^n z_\alpha^{(n)}. \qquad (4.2)$$

This leads immediately to the Poincaré's divergences

$$\langle\beta|\varphi_\alpha^{(n)}\rangle = \frac{1}{\omega_\beta - \omega_\alpha + i\,\varepsilon_{\beta\alpha}} \cdot \left[\langle\beta|V|\varphi_\alpha^{(n-1)}\rangle - \sum_{l=1}^{n} z_\alpha^{(\ell)}\langle\beta|\varphi_\alpha^{(n-\ell)}\rangle\right]. \qquad (4.3)$$

To eliminate these divergences we time order the quantum transitions

$$\varepsilon_{\alpha\beta} = \begin{cases} +\varepsilon & \text{for } 1 \leftarrow \boldsymbol{k}, \\ -\varepsilon & \text{for } \boldsymbol{k} \leftarrow 1, \\ -\varepsilon & \text{for } \boldsymbol{k}' \leftarrow \boldsymbol{k}. \end{cases} \qquad (4.4)$$

We use retarded solutions for the transition from the excited states to the ground state and advanced solutions for the transitions from the ground state to the excited states. This is a special case of the "$i\varepsilon$-rule" introduced by George [25] (see also [17]). Analytically this corresponds to the rule (4.4). The use of both retarded and advanced solutions eliminates Poincaré's divergence and leads to the eigenstates

$$|\varphi_1\rangle = N_1^{1/2}\left[|1\rangle - \sum_k \frac{\lambda V_k}{(\omega_k - \tilde{\omega}_1 - z)^{+}_{-i\gamma}}\,|\boldsymbol{k}\rangle\right]. \qquad (4.5)$$

The symbol $+$ means that we have to consider $|\varphi_1\rangle$ as defined in the upper complex plane and then continued analytically till the point $(-i\gamma)$ into the lower half plane. We see that $|\varphi_1\rangle$ is a distribution with broken time symmetry. In this formula γ^{-1} is the life time of the unstable state. In addition to $|\varphi_1\rangle$ we have a second eigenfunctions $\langle\tilde{\varphi}_1|$ corresponding to the same eigenvalue. Moreover the solution of the eigenvalue problem is given by

$$H|\varphi_1\rangle = (\tilde{\omega}_1 - i\gamma)|\varphi_1\rangle, \quad \langle\tilde{\varphi}_1|H = (\tilde{\omega}_1 - i\gamma)\langle\tilde{\varphi}_1|, \qquad (4.6)$$

$$H|\varphi_k\rangle = \omega_k|\varphi_k\rangle, \quad \langle\tilde{\varphi}_k|H = \omega_k\langle\tilde{\omega}_k|. \qquad (4.7)$$

The eigenfunctions are complete and orthogonal as they satisfy the relations

$$|\varphi_1\rangle\langle\tilde{\varphi}_1| + \sum_k |\varphi_k\rangle\langle\tilde{\varphi}_k| = 1, \quad \langle\tilde{\varphi}_\beta|\varphi_\alpha\rangle = \delta_{\alpha\beta}. \qquad (4.8)$$

We obtain therefore the complex spectral representation

$$H = (\tilde{\omega}_1 - i\gamma)|\varphi_1\rangle\langle\tilde{\varphi}_1| + \sum_k \omega_k|\varphi_k\rangle\langle\tilde{\varphi}_k| \qquad (4.9)$$

which should be compared with the Friedrichs representation (3.5). The main difference is that the unstable state appears in the complex spectral representation and that the eigenvalue contains both the energy $\tilde{\omega}$ and the life time γ^{-1}. The bare eigenstate is in this way replaced by one of the two "dressed" eigenstates $|\varphi_1\rangle$ or $|\tilde{\varphi}_1\rangle$.

Each of those two states has a broken time symmetry as

$$e^{-iHt}|\varphi_1\rangle = e^{-i\tilde{\omega}_1 t - \gamma t}|\varphi_1\rangle \qquad (4.10)$$

and

$$e^{-iHt}|\tilde{\varphi}_1\rangle = e^{-i\tilde{\omega}_1 t - \gamma t}|\tilde{\varphi}_1\rangle. \qquad (4.11)$$

The requirement that the unstable state vanishes in our future (for $t \to \infty$) singles out $|\varphi_1\rangle$ as the "physical" dressed state.

The important point is that starting with a Hamiltonian H which is invariant in respect to time inversion we obtain two complete sets of solutions which have a broken time symmetry [26]. Irreversibility appears therefore as a selection principle. The universe is less symmetric than it seems to follow from the Hamiltonian description. This situation is reminiscent of spontaneous symmetry breaking as it occurs in ferromagnetism or in the particle-antiparticle problem.

Our method shows that, while standard perturbation theory diverges in the "real", it converges in the "complex". In short, Poincaré's divergences are lifted when dissipation is taken into account.

We shall not go here into the interesting mathematical aspects [12]. Obviously the transformation from the initial "bare states" $|1\rangle, |\boldsymbol{k}\rangle$ to the new states $|\varphi_1\rangle, |\varphi_k\rangle$ is not a unitary one. Also the space in which these states live cannot be the usual Hilbert space as in this space H, which is a Hermitian operator, cannot have complex eigenvalues. Let us mention that we deal here with the so called "rigged Hilbert space" [27, 28] (which admits states with zero norm).

We see that LPS may have more than one spectral representation. As we have seen in Sect. 2, this is also true for classical mechanics.

There is a deep analogy between the Friedrichs model and classical chaotic systems such as the Bernoulli map studied in Section 2. In both cases to include temporal characteristics such as Lyapounoff time or the quantum life time into the spectrum we have to go beyond the usual Hilbert space and use a complex spectral representation.

The Friedrichs model is soluble. Therefore there is no difficulty to describe the evolution of observables, and from this point of view nothing new can be expected. But we can now define states with broken time symmetry. We may even introduce a dynamic analogue of Boltzmann's $\mathscr{H}$-function. This is given by the operator [24]

$$\mathscr{H} = |\tilde{\varphi}_1\rangle\langle\tilde{\varphi}_1|, \tag{4.12}$$

which in conjunction with "test functions" defined in the Hilbert space satisfies the inequalities

$$\langle\Psi|\mathscr{H}|\Psi\rangle \geq 0, \quad \frac{\mathrm{d}}{\mathrm{d}t}\langle\Psi|\mathscr{H}|\Psi\rangle \leq 0. \tag{4.13}$$

Similar results have been obtained by Sudarshan, Chiu and Gorini [29] in their important paper "Decaying states as complex energy eigenvalues in generalized quantum mechanics". According to Sudarshan and his coworkers, their paper was inspired by our earlier work in Friedrichs type of models [30]. Sudarshan et al. use a more traditional method of analytical continuation. The interesting point is that we can extend our method of time ordering to more general situations and show that then the elimination of Poincaré's divergences forces us to go to an irreducible description in terms of density matrices. This is the situation we shall consider now.

5. Irreducible Representations in the Liouville Space

In the case of Friedrichs-like models the elimination of Poincaré's divergence is relatively easy as the time ordering of the propagators can be performed on the level of wave functions. This is no more so in general. We may, for example consider scattering. The Hamiltonian for two-body scattering is

$$H = \sum_k \omega_k |k\rangle\langle k| + \lambda \sum_{kk'} V_{kk'} |k\rangle\langle k'| \tag{5.1}$$

As the result of its symmetry it has no physical meaning to introduce a time ordering between the states $|k\rangle$ and $|k'\rangle$ (in addition this leads to difficulties which are discussed in [31]). Also observables such as scattering cross sections cannot be obtained as eigenvalues in a spectral theory in the Hilbert space as they do not satisfy the Ritz-Rydberg principle: they cannot be expressed as differences as it is the case for energy levels.

We turn therefore to a statistical description in terms of density matrices ϱ. As is well known, ϱ satisfies the Liouville-von Neumann equation

$$i\frac{\partial\varrho}{\partial t} = L_H\varrho \quad \text{with} \quad L_H = H \times 1 - 1 \times H, \tag{5.2}$$

where L_H is a "superoperator" acting on ϱ (it is the commutator with L_H). We look then for the spectral representation of L_H. For integrable systems this is of course trivial. The eigenfunctions of L_H are products of eigenfunctions of H and the eigenvalues are differences of the corresponding energy levels.

Now as the consequence of (3.1) we can also decompose the Liouville operator:

$$L_H = L_0 + \lambda L_V. \tag{5.3}$$

For L_0 we can construct a complete set of spectral projectors which satisfy the usual conditions

$$\overset{(\nu)}{P} L_0 = L_0 \overset{(\nu)}{P}, \quad \sum_\nu \overset{(\nu)}{P} = 1, \quad \overset{(\nu)}{P}\overset{(\nu')}{P} = \overset{(\nu)}{P}\delta_{\nu\nu'}, \quad \overset{(\nu)}{P} = \overset{(\nu)}{P}{}^\dagger. \tag{5.4}$$

However, as the result of Poincaré's divergences the corresponding projectors for L_H cannot be obtained through expansion in λ [31, 32]. But, we can obtain a complete set of projectors for L_H giving up the Hermiticity conditions and using an appropriate analytic continuation (or time-ordering). These satisfy the conditions [31, 32]

$$\overset{(\nu)}{\Pi} L_H = L_H \overset{(\nu)}{\Pi}, \quad \sum_\nu \overset{(\nu)}{\Pi} = 1, \quad \overset{(\nu)}{\Pi}\overset{(\nu')}{\Pi} = \overset{(\nu)}{\Pi}\delta_{\nu\nu'}, \quad \overset{(\nu)}{\Pi} \neq \overset{(\nu)}{\Pi}{}^\dagger. \tag{5.5}$$

Our rule of analytic continuation is the natural extension of the rule used for the Friedrichs model, but now in the Liouville space. It can be shown that the dynamics (5.2) associated to the Liouville operator can be expressed in terms of a "flow of correlations". Consider, for example, and N-body system such as studies in kinetic theory. Collisions between uncorrelated particles ("vacuum of correlations") lead to two-body correlations, subsequent collisions transfer them into 3-body, 4-body ... correlations. Our rule is then: transitions to the higher-order correlations are future-oriented, while transitions to lower-order correlations are past-oriented. As shown in [31, 32]. Poincaré's divergences are eliminated and we obtain well-defined expressions for the projection operators (we have called subdynamics this approach),

$$\overset{(\nu)}{\Pi} = (\overset{(\nu)}{P} + \overset{(\nu)}{C})\overset{(\nu)}{A}(\overset{(\nu)}{P} + \overset{(\nu)}{D}), \tag{5.6}$$

where

$$\overset{(\nu)}{A} = \overset{(\nu)}{P}\overset{(\nu)}{\Pi}\overset{(\nu)}{P}, \quad \overset{(\nu)}{C}\overset{(\nu)}{A} = \overset{(\nu)}{Q}\overset{(\nu)}{\Pi}\overset{(\nu)}{P}, \quad \overset{(\nu)}{A}\overset{(\nu)}{D} = \overset{(\nu)}{P}\overset{(\nu)}{\Pi}\overset{(\nu)}{Q}, \tag{5.7}$$

and $\overset{(v)}{Q} \equiv 1 - \overset{(v)}{P}$ is the projection operator orthogonal to $\overset{(v)}{P}$. Note that $\overset{(v)}{A}$ is diagonal in v, while $\overset{(v)}{C}$ corresponds to the creation of correlations v' out of v (its matrix elements are of the form $\overset{(v)}{C}_{v'v}$) while $\overset{(v)}{D}$ corresponds to the destruction of correlations v' (its matrix elements are $\overset{(v)}{D}_{vv'}$).

In this way the density matrix ϱ is decomposed into a sum of independent contributions:

$$\varrho(t) = \sum_v \overset{(v)}{\Pi} \varrho(t) = \sum_v (\overset{(v)}{P} + \overset{(v)}{C}) e^{-i\overset{(v)}{\theta} t} \overset{(v)}{A} (\overset{(v)}{P} + \overset{(v)}{D}) \varrho(0), \quad (5.8)$$

where

$$\overset{(v)}{\theta} \equiv \overset{(v)}{P} L_H (\overset{(v)}{P} + \overset{(v)}{C}) \overset{(v)}{P}. \quad (5.9)$$

Each component $\overset{(v)}{\Pi} \varrho$ is a particular solution of the Liouville equation (this can be verified by straightforward derivations). The sum (5.8) provides us with a complete set of solutions. The whole time-dependence in (5.8) is in the generators of motion $\overset{(v)}{\theta}$, which we call the "collision operators". Of special importance is the contribution for $v = 0$, the "vacuum of correlation": $\overset{(\sigma)}{\Pi} \to \overset{(\sigma)}{P}$ for $\lambda \to 0$ and corresponds then to the evolution of the diagonal element of ϱ. The usual kinetic description (e.g. Fokker-Planck equation or Pauli master equation) is limited to $\overset{(\sigma)}{\Pi}$ space. Moreover this space contains that asymptotic contribution to ϱ for time $t \to \infty$.

The expression (5.8) is the exact formal solution of the Liouville equation. To the various collision operators correspond different time scale. The superposition of these time scales leads precisely to the non-Markoffian behavior (including the memory terms) which are studied in non-equilibrium statistical mechanics.

Once we have derived the decomposition of ϱ into subdynamics, it is easy to go one step further and to obtain the spectral decomposition of L_H. The central theorem proved by T. Petrosky and the author [31] is that the eigenvalues of L_H are given by the eigenvalues of the collision operators $\overset{(v)}{\theta}$.

We can in this way solve the eigenvalue problem for LPS in the Liouville space. Let us briefly indicate the result. We obtain right eigenstates $|\overset{(v)}{F}_\alpha\rangle\rangle$ and left eigenstates $\langle\langle\overset{(v)}{\tilde{F}}_\alpha|$ which satisfy the equations

$$L_H |\overset{(v)}{F}_\alpha\rangle\rangle = \overset{(v)}{Z}_\alpha |\overset{(v)}{F}_\alpha\rangle\rangle, \quad \langle\langle\overset{(v)}{\tilde{F}}_\alpha| L_H = \overset{(v)}{Z}_\alpha \langle\langle\overset{(v)}{\tilde{F}}_\alpha|, \quad (5.10)$$

where we have (with a normalization constant N_α)

$$|\overset{(v)}{F}_\alpha\rangle\rangle = N_\alpha^{1/2} (\overset{(v)}{P} + \overset{(v)}{C}) |\overset{(v)}{u}_\alpha\rangle\rangle \quad (5.11)$$

with

$$\overset{(v)}{\theta} |\overset{(v)}{u}_\alpha\rangle\rangle = \overset{(v)}{Z}_\alpha |\overset{(v)}{u}_\alpha\rangle\rangle \quad (5.12)$$

and a similar expression for $\langle\langle\overset{(v)}{\tilde{F}}_\alpha|$ with $\overset{(v)}{P} + \overset{(v)}{D}$.

The relation with subdynamics is provided by the relations

$$\overset{(v)}{\Pi} = \sum_\alpha |\overset{(v)}{F}_\alpha\rangle\rangle\langle\langle\overset{(v)}{\tilde{F}}_\alpha|, \quad (5.13)$$

where $|\varrho\rangle\rangle$ denotes a "superstate" in the density matrix space.

The important point is that the eigenvalues and eigenstates are analytic in λ and can be obtained by a perturbation method exactly as in the Friedrichs model studied in section 4.

As the result we have therefore

$$\varrho(t) = e^{-iL_H t} \varrho(0) = \sum_{v,\alpha} e^{-i\overset{(v)}{Z}_\alpha t} |\overset{(v)}{F}_\alpha\rangle\rangle\langle\langle\overset{(v)}{\tilde{F}}_\alpha | \varrho(0)\rangle\rangle. \quad (5.14)$$

The index α refers to possible degeneracy in each subspace v.

Note that the eigenstates $|\overset{(v)}{F}_\alpha\rangle\rangle$ and $\langle\langle\overset{(v)}{\tilde{F}}_\alpha|$ are now density matrices and not wave functions; moreover, α is an index corresponding to possible degeneracy in subdynamics.

Formula (5.14) corresponds to our complex spectral theory in the Liouville space. As mentioned in section 1, it leads to an irreducible representation of the density matrix as the complex eigenvalue Z_α and the eigenfunctions cannot be expressed in terms of wave functions (then Z_α would be the difference between two eigenvalues and $|\overset{(v)}{F}_\alpha\rangle\rangle$ products of two wave functions). An example we mentioned is precisely the cross section. As in the case of the Friedrichs model, the eigenfunctions are a complex distribution, and (5.14) has to be used with suitable test functions.

Our spectral representation (5.14) describes the approach equilibrium and gives a microscopic meaning to entropy [31].

There are still many mathematical problems which deserve closer examinations. The collision operators $\overset{(v)}{\theta}$ are non-Hermitian operators and we do not know a priori which kind of spectral decomposition they admit. For simple examples (i.e. corresponding to dilute gases) the eigenvalue problem is studied in textbooks (see e.g. Balescu [33]). As Petrosky has shown, we may then use (5.11) to obtain the explicit form of the eigenfunctions of the Liouville equation [34]. It is interesting to note that the operator $\overset{(\nu)}{C}$ acting on non-equilibrium eigenfunctions of the collision operator $|\overset{(\nu)}{u_\alpha}\rangle\!\rangle$ leads to long-range correlations. This gives the dynamical basis for the coherent processes which are observed in non-equilibrium systems (see e.g. Nicolis and Prigogine [35]).

Let us summarize our conclusions: In spite of Poincaré's divergences, LPS become "integrable" through the introduction of a complex spectral representation in the Liouville space. As in the case of the Friedrichs model this involves non-unitary transformations and the introduction of a "rigged Liouville space".

We obtain in this way a description with broken time symmetry (again the solutions are less symmetric than the Liouville equation). But the main point is that our description is irreducible: it refers to the density matrix and cannot be expressed in terms of wave amplitude.

6. Quantum Chaos

Situations such as the Friedrichs model still remain in the frame of traditional quantum theory. We use a complex spectral theory bit the central quantity remains the wave function. The situation changes dramatically when we go to more general situations such as scattering or interactions between fields (in the language of kinetic theory we have then to consider both "loss" and "gain" terms). We have then to introduce *irreducible* representations in the Liouville space [31]. That means that the central quantity becomes then the *density matrix* and no more the wave amplitude. As the result, additional statistical features appear. We may speak here of "quantum chaos" by analogy with classical chaos. Classical chaos does not mean that Newton's equation become "wrong" but that trajectories loose their operational meaning. We expect that the situation would be similar in quantum theory: whenever we have an irreducible representation in the Liouville space this means that the wave function description looses its operational meaning. *This behaviour marks the limits of orthodox quantum theory.*

In classical dynamics chaos is associated with sensitively to initial conditions [36]. In quantum mechanics they prevent us from starting with a well-defined wave function. But whenever we have an irreducible representation in the Liouville space, the wave function is transformed into a density matrix.

The mechanism of this collapse of the wave function as the result of Poincaré's divergences has been studied by Petrosky and the author for scattering problems [37–39]. Let us summarize briefly the conclusions. The collapse appears for situations corresponding to *persistent* interactions, that means for problems which cannot be treated in the frame of the usual S-matrix theory. The basic assumption of the S-matrix theory is that we can introduce well-defined "in" and "out" states. A typical example is two-body scattering involving a localized wave packet. But in general this assumption is not satisfied. We may consider initial conditions corresponding to a plane wave and leading therefore to persistent interactions or three-body scattering (for free incident particles). Rescattering leads to divergences [40, 41].

In all these examples the wave function becomes ill-defined when we use a time-dependent description (they admit no well-defined limit for $t \to \infty$). This is due to resonances between bras and kets in the Hilbert space description. In contrast, our theory gives us an unambiguous expression for the density matrix. In our approach cross sections are given by the eigenvalues of the Liouville operators and are finite, while the usual quantum mechanical approach leads to divergences for the three-body scattering.

These are rather elementary examples. The main point is that free fields and interacting fields lead to dramatically different descriptions. Interacting fields belong to the category of LPS. We have already considered some simple examples such as the interactions of the conformal degree of freedom in general relativity with a scalar massive field. This leads to the instability of the quantum Minkowski vacuum and to irreversible processes leading both to the appearance of space-time curvature and matter [42].

7. Bohr and Einstein

We believe that our irreducible spectral representation in the Liouville space marks a noticeable advance over conventional quantum theory. The main reasons are the following:

1) it leads to solutions which have a broken time symmetry and therefore allow the introduction of irreversibility on the microscopic level;

2) it includes in the spectrum observables such as cross sections, life times associated to dissipative process;
3) it leads to a constructive perturbation approach for the solution of the spectral problem.

Finally irreducibility means that the central quantity is now the density matrix (and no more the wave functions). This marks the appearance of quantum chaos.

In the traditional perspective, adopted both in classical and quantum theory, stable systems were considered the rule and unstable the exception. We have now to reverse this perspective.

This brings us back to the epistemological problem of quantum mechanics and specially to the problem of measurement to which George Sudarshan has made important contributions [15]. As well known, the main epistemological difficulty associated with measurement is the transition from "potentiality" to "actuality". Is then the world to be understood as asking for an observer which would actualize its potentiality? This would lead to a subjective description of nature which was the main reason why Einstein refused to accept quantum mechanics.

Bohr also was against any subjective interpretation of quantum mechanics. This is why he requested the apparatus to be described in classical terms. In our approach the answer is simple: The apparatus must be an unstable dynamical system leading to an irreducible representation in the Liouville space. As the result we deal directly with probabilities and not with wave functions. While wave functions have no classical analog, quantum probabilities have [37].

Our solution of the measurement paradox satisfies as well the second well-known Bohr's requirement: measurement must lead to irreversible effects. Of course these two requirements are not independent, as dissipation is the result of resonances which lead to instability.

Measurement is a way of communication with nature. Communication requires a common time concept; this common time concept arises from secular terms, from dissipation. In this way instability, and as the consequence quantum chaos, appears at the very roots of the possibility of our communication with the quantum world.

In addition it is not only the measurement apparatus which has to be described in terms of irreducible density matrix, but in general also the system which is measured. Schrödinger's cat is a living being, and life cannot be dissociated from irreversible processes. Therefore the microscopic description of Schrödinger's cat is from the start in terms of irreducible density matrices.

It is customary to refer to the famous debate between Bohr and Einstein which took place at the 5th Solvay conference in Brussels in 1927. Bohr was right, but at the time of formulation of quantum theory, dynamic instability and chaos were outside the perspective of normal physics. These concepts appear now as essential to insure the self-consistency of quantum theory. But Einstein was right also when he claimed that the quantum mechanics of his time could not be the final form of quantum theory.

Would it be too presumptuous to speculate that Einstein could adhere to this view? After all, the subjective features of orthodox quantum mechanics are now eliminated. Poincaré's divergences are a mathematical fact independent of any observer. They lead us to a new form of quantum theory and ultimately force us to accept a view of reality which incorporates instability and dissipation in our basic description of nature.

I believe that George Sudarshan would agree with the American philosopher Ivor Leclerc who in his monograph "The Nature of Physical Existence" has written [43].

> "Once again, as in the seventeenth century, the 'philosophy of nature' must not only be brought into the forefront, but the recognition of its intrinsic relevance to and need by the scientific enterprise must be restored. Then it will be seen that there are not two independent enterprises, science and philosophy, but one, the inquiry into nature, having two complementary and mutually dependent aspects."

As in the seventeenth century, our picture of the universe is undergoing a drastic change. In periods like that we have to go beyond the traditional fragmentation between science and philosophy. The scientific achievements of George Sudarshan provide an excellent illustration for the fruitfulness of such a global approach.

Acknowledgements

I want to thank the members of our group in Brussels and in Austin who contributed greatly to the results summarized in this paper. Special thanks to Dr. T. Petrosky for help in the preparation of the text and to Dr. Lee Jing-Yee for help in putting it into the final form. I also acknowledge the U.S. Department of Energy Grant N° FG05-88ER13897, the Robert A. Welch Foundation, the Belgian government and the European Communities Commission (contract n° PSS*0143/B) for support of this work.

[1] E. C. G. Sudarshan, Epistemologia XI – Science et Métaphysique (1988) p. 153.
[2] E. Schrödinger, The British Journal for the Philosophy of Science, Vol. **3**, 109 (1952).
[3] E. C. G. Sudarshan, paper presented at the seventeenth ICUS meeting Absolute Values and the Reassessment of the Contemporary World, Los Angeles, Nov. 24–27, 1988.
[4] I. Berlin, Against the Current, The Viking Press, New York 1980; p. 109.
[5] H. G. Schuster, Deterministic Chaos: An Introduction, VCH Verlagsgesellschaft mbH, Weinheim 1988.
[6] V. I. Arnold and A. Avez, Ergodic Problems of Classical Mechanics, Benjamin, New York 1968.
[7] H. H. Hasegawa and W. C. Saphir, Phys. Rev. **A 46,** 7401 (1992).
[8] P. Gaspard, J. Phys. **A 25,** L 483 (1992).
[9] I. Antoniou and S. Tasaki, J. Phys. **A 26,** 73 (1992).
[10] H. Poincaré, Méthodes nouvelles de la mécanique céleste, Vol. **1**, (1982), Dover, New York 1957.
[11] T. Petrosky and I. Prigogine, Physica **A 147,** 439 (1988).
[12] I. Antoniou and I. Prigogine, Physica **A 192,** 443 (1993).
[13] N. C. A. da Costa and F. A. Doria, Int. J. Theor. Phys. **30,** 1041 (1991).
[14] K. Friedrichs, Commun. Pure Appl. Math. **1,** 361 (1948).
[15] B. Misra and E. C. G. Sudarshan, J. Math. Phys. **18,** 756 (1977).
[16] Misra and Sudarshan have associated the Zeno time to the effect of quantum measurement [15]. This seems, however, implausible as a similar pre-exponential period appears also in classical physics (see [17, 18]). Also experimental effects associated to measurement [19] can be understood in terms of conventional theory without the use of repeated collapse of the wave function [20–23].
[17] T. Petrosky and H. Hasegawa, Physica **A 160,** 351 (1989).
[18] R. Willox, I. Antoniou, and J. Levitan, in Nonlinear Phenomena in Complex Systems, eds. V. Kuvsinov and G. Krylov, Institute of Physics, Minsk 1995.
[19] W. M. Itano, D. J. Heizen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. **A 41,** 2295 (1990).
[20] T. Petrosky, S. Tasaki, and I. Prigogine, Physica **A 170,** 306 (1991); T. Petrosky, S. Tasaki, and I. Prigogine, Phys. Lett. **A 151,** 109 (1990).
[21] A. Peres and A. Ron, Phys. Rev. **A 42,** 5720 (1990).
[22] E. Block and P. R. Berman, Phys. Rev. **A 44,** 1466 (1991).
[23] V. Frerichs and A. Schenzle, Phys. Rev. **A 44,** 1962 (1991).
[24] T. Petrosky, I. Prigogine, and S. Tasaki, Physica **A 173,** 175 (1991).
[25] Cl. George, Physica **65,** 277 (1973).
[26] This can be generalized to situations where the Hamiltonian is CPT invariant while the solutions are not. This is of basic importance for the kaon problem, see S. Tasaki and I. Prigogine, unpublished.
[27] A. Böhm, J. Math. Phys. **22,** 2813 (1981).
[28] A. Böhm and M. Gadella, Dirac kets, Gamow vectors and the Gel'fand triplets, Springer L.N. Phys. **348,** Berlin 1989.
[29] E. C. G. Sudarshan, C. B. Chiu, and V. Gorini, Phys. Rev. **D 18,** 2914 (1978).
[30] M. de Haan, 1969, unpublished.
[31] T. Petrosky and I. Prigogine, Physica **A 175,** 146 (1991).
[32] I. Prigogine, T. Petrosky, H. H. Hasegawa, and S. Tasaki, Chaos, Solitons and Fractals, **1,** 3 (1991).
[33] R. Balescu, Statistical Mechanics of Charged Particles, Wiley, New York 1964.
[34] T. Petrosky and I. Prigogine, Chaos, Solitons, and Fractals **4,** 311 (1994).
[35] G. Nicolis and I. Prigogine, Exploring Complexity, Freeman, New York 1989.
[36] M. Tabor, Chaos and Integrability in Nonlinear Dynamics, Addison-Wesley, New York 1989.
[37] I. Prigogine and T. Petrosky, in Proceedings of the Second Wigner Conference (1991).
[38] T. Petrosky and I. Prigogine, Proc. Nat'l. Acad. Sci. USA **90,** 9393 (1993).
[39] I. Prigogine, in Proceedings of the XXth Solvay Conference (1991).
[40] C. J. Joachain, Quantum Collision Theory, North Holland, Amsterdam 1983.
[41] R. G. Newton, Scattering Theory of Waves and Particles, Springer-Verlag, New York 1982.
[42] S. Tasaki, P. Nardone, and I. Prigogine, Vistas in Astronomy **37,** 231 (1993).
[43] I. Leclerc, The Nature of Physical Existence, George Allen and Unwin, London 1972, p. 351.

Addendum

Since the presentation of this paper in 1991, important progress has been realized. Here is a short list of the main recent publications.

Chaos

1. I. Antoniou and S. Tasaki, Physica **A 190,** 303 (1992).
2. H. H. Hasegawa and D. J. Driebe, Phys. Rev. **E 50,** 1781 (1994).
3. S. Tasaki, I. Antoniou, and Z. Suchanecki, Chaos, Solitons, and Fractals **4,** 227 (1994).
4. D. J. Driebe and G. E. Ordonez, Phys. Lett. **A 211,** 204 (1996).

Classical Dynamics

1. T. Petrosky and I. Prigogine, Chaos, Solitons, and Fractals **7,** 441 (1996).
2. I. Prigogine and T. Petrosky, in Gravity, Particles and Spacetime, eds. P. Pronin and G. Sardanashvily, World Scientific, Singapore 1996.

Quantum Dynamics

1. T. Petrosky, G. Ordonez, and T. Miyasaka, Phys. Rev. **A 53,** 4075 (1996).
2. T. Petrosky and I. Prigogine, in Adv. Chem. Phys., eds. I. Prigogine and S. Rice, **99,** John Wiley and Sons, New York 1997.