Receiver Design for Non-Linear Satellite Channels: Equalizer Training and Symbol Detection on the Compressed Constellation

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***Abstract*—Because of the small energy available aboard a satellite, the power amplifier must work with a restricted power supply which limits its maximum output power. To ensure a sufficient signal-to-noise power ratio (SNR) at the receiving side, the amplifier must work close to the saturation point. This is power efficient but, unfortunately, adds non-linear distortions in the communication channel. Several algorithms have been proposed to equalize this non-linear channel. The most widely used in the literature is the baseband Volterra filter. Recently, the Echo State Network (ESN), coming from the artificial neural network field, has been shown to perform equally well.**

To compensate for this channel, both equalizers adapt their coefficients with the help of a training sequence in order to recover the transmitted constellation points. We will show that, the usual detection, based on Euclidean distances, is no longer optimal. The aim of this paper is to first propose a new detection criterion which meets with the Maximum Likelihood (ML) criterion. Secondly, we will propose a modification of the training reference points to improve the performances of these equalizers and make the detection based on Euclidean distances optimal again. This last solution can offer a significant reduction of the Bit Error Rate (BER) without increasing the equalizers complexity. Only the new training reference points must be evaluated.

***Index Terms*—Echo State Network, Baseband Volterra Filter, Equalization, Non-linear Communication Channel**

I. Introduction

In satellite communication systems, like the DVB-S2 [1], the satellite works as a relay point between two terrestrial stations. A first terrestrial station transmits a signal to the satellite which amplifies it and retransmits it without digital signal processing to a second terrestrial station. Because of the constraints on the equipments aboard the satellite, these communications can be very challenging. To ensure a high output power, the amplifier in the satellite must work close to its saturation point. This operating point allows high efficiency but also adds important non-linear distortions in the communication channel. In addition, high bandwidth communications can be affected by inter-symbol interferences (ISI) due to the limited bandwidth of the filters aboard the satellite. This non-linear channel with memory can be compensated for at the receiver with equalization algorithms to ensure a low Bit Error Rate (BER).

An important part of the literature proposes non-linear filters to equalize the channel. The main contribution in the literature concerns the baseband Volterra filters [2][3][4][5]. Another part of the literature concerns the artificial recurrent neural networks (RNNs) [6]. But, because of the important number of parameters to optimize, the training task is very expensive. A family of RNNs has been proposed to avoid this drawback: the Reservoir Computer (RC) [7]. This paper will concern one of its most simplest form: the Echo State Network (ESN) [8] [9]. This algorithm became very popular these last years because it offers a good compromise between performance and complexity. It has been shown in [10] that it can offer performances similar to the state-of-the-art baseband Volterra equalizer for the equalization of the DVB-S2 communication channel.

Although both equalizers (Volterra and ESN) are different in their principle, the methods used to train them are similar. The standard approach in both cases is to minimize the mean square error (MSE) between the transmitted signals and the equalized ones [3][4][5]. In this way, they will compensate for the interferences and the compression of the signal due to the saturated behaviour of the power amplifier. However, we observe that the equalized constellation is affected by an important distortion [5]. It means that the noise and interferences at the output of the equalizers can no longer be approximated by a circular-symmetric Gaussian distribution. In this case, the detection criterion consisting of selecting the point on the transmitted constellation the closest in Euclidean distance to the equalized symbol is no longer optimal.

As a first contribution, we propose a new detection criterion which takes into account this distortion in order to meet with the Maximum Likelihood (ML) criterion. We observe a significant BER reduction.

As a second contribution, we propose to train these equalizers to only compensate for the interferences and not the compression. We show that the distortion on the equalized constellation is less important. In this case, our new detection criterion offers no significant BER reduction in comparison with a detection based on Euclidean distance. This result explains a part of the BER reduction observed in [10] where

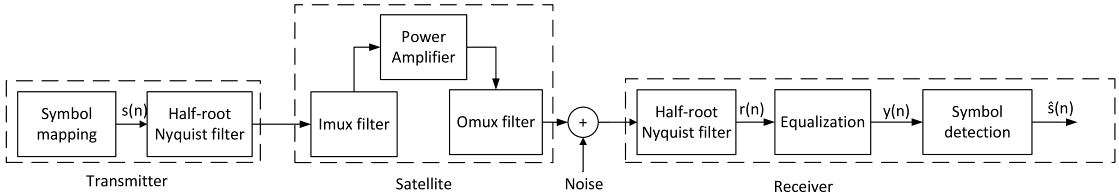

Fig. 1. Block diagram of a satellite communication channel.

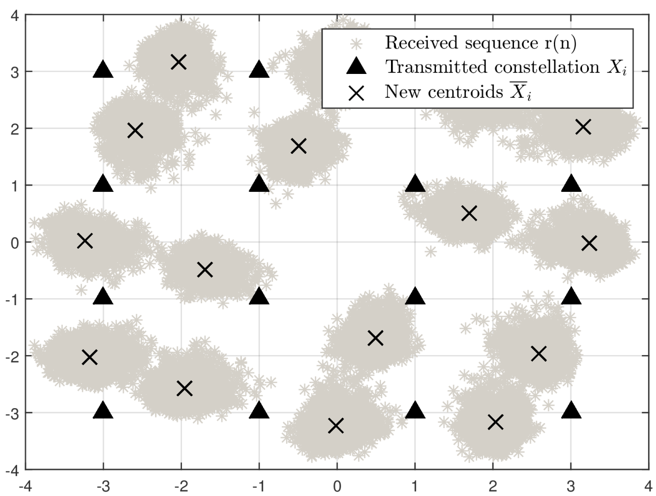

Fig. 2. 16-QAM modulation before and after a noiseless satellite communication channel with the parameters used in section 7 (OBO: -2 dB, roll-off of the two half-root Nyquist filters: 0.25).

this solution was applied on the ESN. In this paper, we show that we can also have a performance gain if we use this solution with a baseband Volterra filter.

The outline of this paper is the following. In Section 2, the satellite communication channel is described. A description of the baseband Volterra equalizer is given in Section 3. In the following, the term Volterra will always refer to the baseband Volterra model. A description of the ESN is given in Section 4. The new detection criterion is described in Section 5. The improvement of the training process will be proposed in Section 6. The performances of these two solutions are evaluated in term of BER in Section 7.

II. System model

Fig. 1 shows a block diagram of the baseband satellite communication channel [1]. It contains the power amplifier, which is the source of non-linearity. The memory comes from the filters in the satellite (imux and omux) and the half-root Nyquist filters in the terrestrial stations. As we have a line-of-sight propagation channel between the satellite and the terrestrial stations and because of the important directivity of the antennas, the propagation channel can be considered as memoryless [2].

The power amplifier is described by its baseband model which gives the amplitude modulation (AM-AM) and phase modulation (AM-PM) characteristics. If its input is defined by $u(n)$, the output of the amplifier $v(n)$ is:

$$v(n) \quad = \quad f_{\text{PA}}(|u(n)|)e^{j(\angle u(n)+g_{\text{PA}}(|u(n)|))}, \qquad (1)$$

where $f_{\text{PA}}(.)$ is the AM-AM relation, $g_{\text{PA}}(.)$ is the AM-PM relation, $\angle u(n)$ is the phase of $u(n)$ and $|u(n)|$ is its modulus. We use the power amplifier model proposed in [1].

The operating point is fixed by the output back off (OBO) defined as:

$$OBO = 20\log_{10}\frac{A_{\text{out}}}{A_{\text{sat}}}, \qquad (2)$$

where A_{sat} is the saturation amplitude of the amplifier and A_{out} is the root mean square (RMS) value of the signal $v(n)$ at the output of the power amplifier. A low OBO is required to work in the linear regime but reduces the efficiency of the power amplifier.

Because of the demand for ever increasing bit rates, high symbol rates are used, leading to an increase in the spectral bandwidth. When the bandwidth of the signal becomes comparable with or larger than the bandwidth of the imux and omux filters, ISI will occur.

Before the power amplifier, the transmitted symbols are shaped with a half-root Nyquist filter and the imux filter. The signal at the output of the power amplifier is shaped with the omux filter. The channel is affected by an additive white Gaussian noise.

The receiver station carries out three operations. First the received signal is shaped with a half-root Nyquist filter. Second the channel is equalized. Third, a detection criteria is applied to decide which symbol was sent.

In order to minimise the BER, one can improve the equalizer, the detection criterion, or both. The main result of the present paper is to present novel detection criteria, and novel training methods for the equalizer, that decrease the BER.

If the channel was linear, ISI and noise would create clusters of points centred on the transmitted constellation points. The positions of the transmitted constellation points in the complex plan are defined by $(X_i)_{i=1}^{M}$ where M is the order of the constellation. But, in a non-linear channel, we can observe a compression of the received constellation which creates a displacement of the center of these clusters, called the

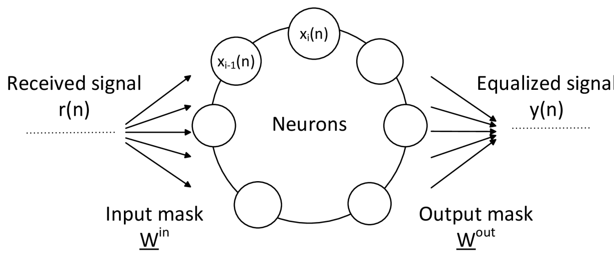

Fig. 3. Structure of the Echo State Network with a ring structure.

centroids (see Fig. 2) [11]. The effect is particularly marked for centroids that have large amplitude. The positions of the new centroids $(\overline{X}_i)_{i=1}^{M}$ in the received constellation are defined by:

$$\overline{X}_i = E[r(n)|s(n) = X_i] = \Gamma(X_i), \tag{3}$$

for $i = 1...M$. The function $\Gamma(X_i)$ defines the centroids displacement.

III. VOLTERRA EQUALIZER

The most common solution to equalize a non-linear communication channel is to use a Volterra filter. This filter is defined by the following equation which gives the relation between the received samples $r(n)$ and the equalized signal $y(n)$ [2]:

$$\begin{aligned} y(n) = \sum_{p=0}^{\frac{P-1}{2}} \sum_{n_1=-L'_{2p+1}}^{L_{2p+1}-1} \sum_{n_2=n_1}^{L_{2p+1}-1} \cdots \sum_{n_{p+1}=n_p}^{L_{2p+1}-1} \\ \sum_{n_{p+2}=-L'_{2p+1}}^{L_{2p+1}-1} \cdots \sum_{n_{2p+1}=n_{2p}}^{L_{2p+1}-1} k_{2p+1}(n_1, ..., n_{2p+1}) \\ \prod_{i=1}^{p+1} r(n-n_i) \prod_{j=p+2}^{2p+1} r^*(n-n_j), \end{aligned} \tag{4}$$

where $k_{2p+1}(n_1, ..., n_{2p+1})$ are the coefficients of the Volterra filter, $M_{2p+1} = L'_{2p+1} + L_{2p+1}$ is the length of the memory at order $2p+1$, P is the maximum order of non-linearity considered in the model and $(.)^*$ denotes the conjugate operator.

In this paper, we will use a Volterra equalizer with a linear memory M_1 equal to 10 and an order 3 non-linear memory M_3 equal to 5. The overall equalizer is composed by 85 coefficients which must be trained to minimize the MSE between the equalized sequence $y(n)$ and the objective.

IV. ECHO STATE NETWORK

This section gives a brief description of the ESN. More details can be found in [12]. We will use the same structure as used in [10]. A diagram of the algorithm is given in Fig. 3.

The ESN is a framework for training RNNs. The algorithm is composed by N neurons connected to each other with a $N \times N$ interconnection matrix $\underline{\underline{W}} = (w_{ij})$. The connections with the input signal are defined by a $1 \times N$ vector $\underline{W}^{\text{in}} = (w_i^{\text{in}})$ which is the input mask. The output signal is created with a weighted summation of the neurons defined by the output weights ($1 \times N$ output mask $\underline{W} = (w_i^{\text{out}})$).

The equalized sequence $y(n)$ is found at the output of the ESN defined by [8]:

$$\begin{cases} x_i(n) = f_{\text{NL}}(\sum_{j=1}^{N} w_{ij} x_j(n-1) + w_i^{in} r(n)), & (5) \\ y(n) = \sum_{i=1}^{N} w_i^{out} x_i(n). & (6) \end{cases}$$

To reduce the complexity of the learning task, only the output weights w_i^{out} are trained to equalize the channel. The input weights w_i^{in} are real numbers randomly defined with a uniform distribution. The activation function $f_{\text{NL}}(.)$ creates the non-linear behaviour of the ESN. We will use the activation function proposed in [10]:

$$f_{\text{NL}}(a) = a.(c_1 + c_3|a|^2), \tag{7}$$

where $c_1 = 0.716$ and $c_3 = -0.0478$ [10]. In this paper, we use an ESN composed by 50 neurons.

We will use the circular interconnection matrix $\underline{\underline{W}}$ proposed in [13]:

$$\underline{\underline{W}} = \alpha \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{pmatrix}, \tag{8}$$

where α is the feedback gain. It has been shown in [13] that this matrix can offer similar results as the initially proposed random interconnection matrix. It has been shown in [10] that such a structure can equalize the DVB-S2 non-linear channel. With this structure, the evolution of the neurons defined by (5) can be replaced by [13]:

$$x_i(n) = f_{NL}(\alpha x_{i-1}(n-1) + w_i^{\text{in}} r(n)). \tag{9}$$

An important property of the ESN is the echo state property which specifies that each neuron has a fading memory. It means that they tend to forget their initial state. As each neuron is connected to only one other neuron, it has been shown in [14] that this property is met if the feedback gain α is lower than 1 for a linear ESN. In our case, the connection between the neurons also depends on the activation function $f_{NL}(.)$. If this function is Lipschitz continuous, the product of its Lipschitz constant and the feedback gain α must be lower than 1 to meet with the echo state property [15].

V. NEW DETECTION CRITERION

If the transmitted symbols are independent and transmitted with equal probability, the optimal detection criterion is the ML criterion. It consists in selecting the symbol $\widehat{s}(n)$ which maximizes the probability $p(y(n)|\widehat{s}(n) = X_i)$. This is equivalent in finding the symbol $\widehat{s}(n)$ which minimizes the Euclidean distance with the equalized symbol $y(n)$ if the noise and the

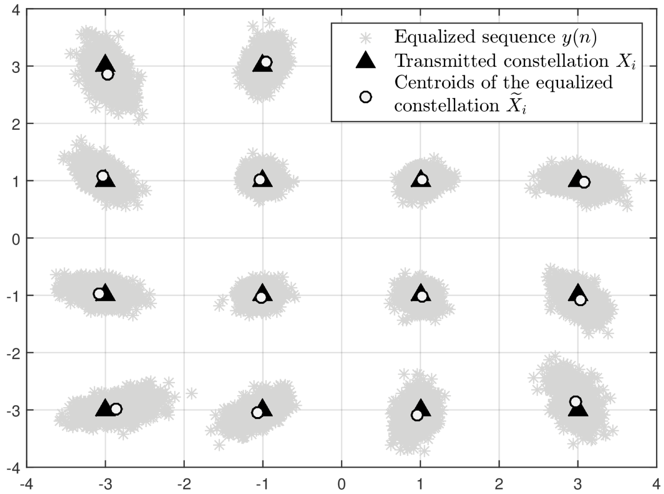

Fig. 4. Constellation equalized with an ESN trained to recover the transmitted constellation and composed by 50 neurons after a channel characterized by the parameters used in section 7 (OBO: -2 dB, roll-off of the two half-root Nyquist filters: 0.25) and affected by a SNR of 30 dB.

residual interferences at the output of the equalizer can be approximated by a circular-symmetric Gaussian distribution. This requires that the equalized constellation must be composed of circular clusters centred on the constellation points $(X_i)_{i=1}^{M}$.

If we observe the equalized constellation at the output of an ESN trained to recover the transmitted constellation (see Fig. 4), we see that the clusters are not exactly centred on the transmitted constellation points. The detection can be improved by using the center of these clusters defined by $\tilde{X}_i$ instead of X_i in the detection [11]. It means that we will minimize the Euclidean distance with constellation points defined on $(\tilde{X}_i)_{i=1}^{M}$ instead of $(X_i)_{i=1}^{M}$.

We also observe that the received clusters are not circular. In this way, the detection criterion based on Euclidean distances is no longer optimal. To take this into account, we propose to derive a new detection criterion from the ML criterion. Similar constellations are observed at the output of a Volterra equalizer.

To evaluate the expression of $p(y(n)|\widehat{s}(n) = X_i)$, we define two distributions:

$$\begin{cases} D_i(y(n)) = \mathrm{Re}\{y(n) - \tilde{X}_i\}, & (10) \\ D'_i(y(n)) = \mathrm{Im}\{y(n) - \tilde{X}_i\}. & (11) \end{cases}$$

Simulations show us that these distributions can be approximated by a Gaussian distribution. Their variance are respectively:

$$\begin{cases} \sigma_i^2 = E[D_i^2(y(n))|s(n) = X_i], & (12) \\ \sigma_i'^2 = E[D_i'^2(y(n))|s(n) = X_i], & (13) \end{cases}$$

and the correlation between the two distributions is defined by:

$$\rho_i = \frac{E[D_i(y(n))D'_i(y(n))|s(n) = X_i]}{\sigma_i \sigma'_i}. \quad (14)$$

The clusters can be approximated by a bivariate Gaussian distribution. The estimated symbol $\widehat{s}(n)$ is the one which maximizes the probability:

$$p(y(n)|\widehat{s}(n) = X_i) = \frac{1}{2\pi\sigma_i\sigma'_i\sqrt{1-\rho_i^2}} e^{-\frac{G_i(y(n))}{2(1-\rho_i^2)}}, \quad (15)$$

where

$$G_i(y(n)) = \frac{D_i^2(y(n))}{\sigma_i^2} - \frac{2\rho_i D_i(y(n))D'_i(y(n))}{\sigma_i\sigma'_i} + \frac{D_i'^2(y(n))}{\sigma_i'^2}. \quad (16)$$

This new criterion takes into account the shape of each cluster. We show in section 7 that we can achieve a significant reduction of the BER.

VI. New training method using centroids

In the state of the art, the two equalizers are trained to minimize the MSE between the equalized sequence $y(n)$ and the transmitted sequence $s(n)$. It means that part of the equalizer complexity is devoted to compensate for the displacement of the centroids defined by $\Gamma(s(n))$. However the new positions of the centroids can be taken into account during the detection operation. Furthermore, this correction during the equalization can create a noise and interference amplification as it depends on the amplitude of the received signal which is affected by noise and interferences.

We propose to train the equalizers to only compensate for the interferences. In this way, the equalizers will try to recover the constellation defined by the new centroids $\overline{X}_i$. We can see in Fig. 5 that the clusters of points are more circular which means that the interferences and noise at the output of the equalizers can be approximated by a symmetric-circular Gaussian distribution again. The detection based on Euclidean distance will be closer to the ML criterion.

To do this, the equalizers will be trained to minimize the MSE between the equalized sequence $y(n)$ and a new sequence defined by:

$$s_T^c(n) = \Gamma(s_T(n)). \quad (17)$$

In that way, we replace the symbols of $s_T(n)$ defined on $(X_i)_{i=1}^{M}$ by a new sequence of symbols $s_T^c(n)$ defined on $(\overline{X}_i)_{i=1}^{M}$. With such a training sequence, the training algorithm will only evaluate the weights that can compensate for the interferences. We will see in next section that this solution improves the performances of the equalizer without increase their complexity. We only need to evaluate the signal $s_T^c(n)$ from $s_T(n)$.

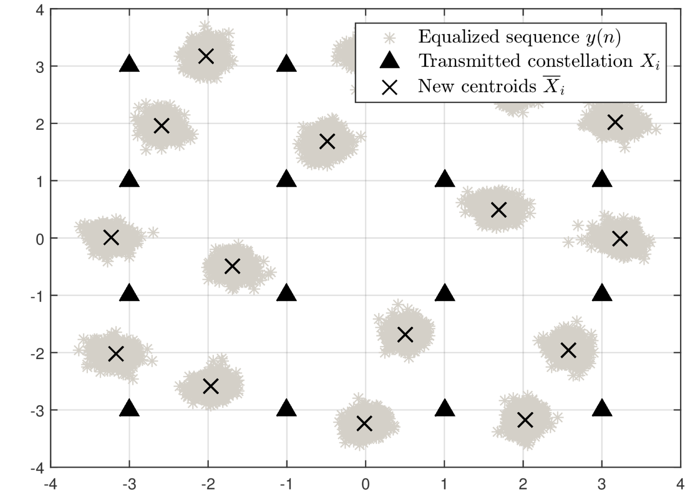

Fig. 5. Constellation equalized with an ESN trained on the centroids composed by 50 neurons after a channel characterized by the parameters used in section 7 (OBO: -2 dB, roll-off of the two half-root Nyquist filters: 0.25) and affected by a SNR of 30 dB.

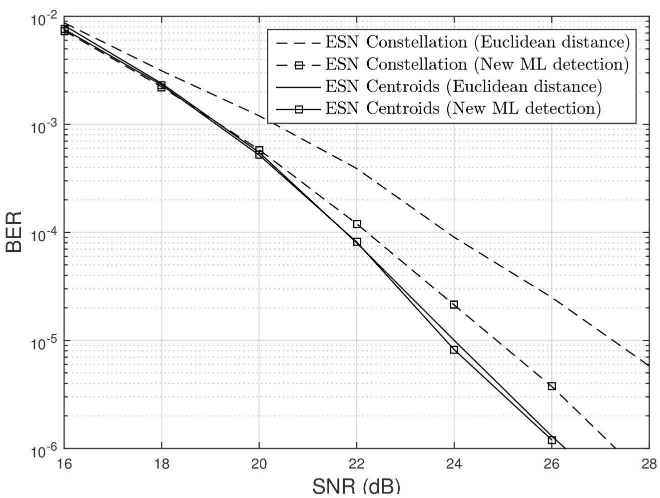

Fig. 6. Impact of a training on the centroids and the new detection criterion (denoted by New ML) for the ESN with 50 neurons.

VII. Numerical results

We will consider a 16-QAM modulation. The imux and omux filters have a bandwidth of 36 MHz. They can be modelled with a Butterworth response. The roll-off factor of the half-root Nyquist shaping filters on the terrestrial stations is fixed at 0.25. The symbol rate is 30 MHz. The operating point of the power amplifier will be defined by a -2 dB OBO. The functions $f_{PA}(.)$ and $g_{PA}(.)$ are described by a Ghorbani model [16]:

$$f_{\mathrm{PA}}(u(n)) = \frac{q_1|u(n)|^{q_2}}{1+q_3|u(n)|^{q_2}} + q_4|u(n)|, \tag{18}$$

$$g_{\mathrm{PA}}(u(n)) = \frac{q_5|u(n)|^{q_6}}{1+q_7|u(n)|^{q_6}}, \tag{19}$$

where $q_1 = 6$, $q_2 = 1.3$, $q_3 = 3.3$, $q_4 = -0.4$, $q_5 = 1.8$, $q_6 = 1.8$, $q_7 = 1.4$.

The coefficients of the Volterra equalizer and the ESN have been evaluated to minimize the MSE between the estimated sequence and the training sequence defined on the transmitted constellation X_i or the new centroids $\overline{X}_i$. In both cases, we consider that the training sequence was long enough to converge to the optimal weight to minimize the MSE.

We can see in Fig. 6 and Fig. 7 that our new detection criterion offers better results than the classical detection if the ESN is trained to recover the transmitted constellation. It is interesting to see that this improvement is achieved without modifying the number of coefficients of the equalizer. But we need a longer training sequence to evaluate the parameters σ_i, σ_i' and ρ_i of each cluster.

As shown in [10], a training based on the new centroids improves the performances of the ESN (see Fig. 6). A similar behaviour is observed with the Volterra equalizer (Fig. 7). When the equalizers are trained with the new centroids, we can observe that our new detection criterion offers no significant performance gain in comparison with the Euclidean detection. It confirms that the equalized clusters are closer to a symmetric-circular Gaussian distribution as we can observe on Fig. 5. The training based on the new centroids followed by a detection based on Euclidean distance offers the most interesting compromise between performances and complexity as this detection criterion is less complex to evaluate.

VIII. Conclusion

In this article, we showed that, if we keep a training sequence based on the transmitted constellation, the equalized clusters can no longer be approximated by a circular-symmetric Gaussian distribution. For this reason, the detection criterion based on Euclidean distance is no longer optimal. We proposed a new detection criterion which considers that the clusters follow a bivariate Gaussian distribution. In this way, one is closer to the ML criterion. This improved detection criterion offers a significant BER reduction for highly non-linear channels.

The importance of the centroids in the construction of the training sequence for an equalizer has been illustrated. We showed that for both equalizers, we can achieve an important performance gain without modifying them. As the clusters of the equalized sequence are more circular, we showed that the new detection criterion offer similar performances as the Euclidean distance. In this way, the BER reduction does not lead to an increase in complexity of the detection and the equalization algorithm. Only the new constellation points $(\overline{X}_i)_{i=1}^{16}$ must be evaluated to train the equalizers.

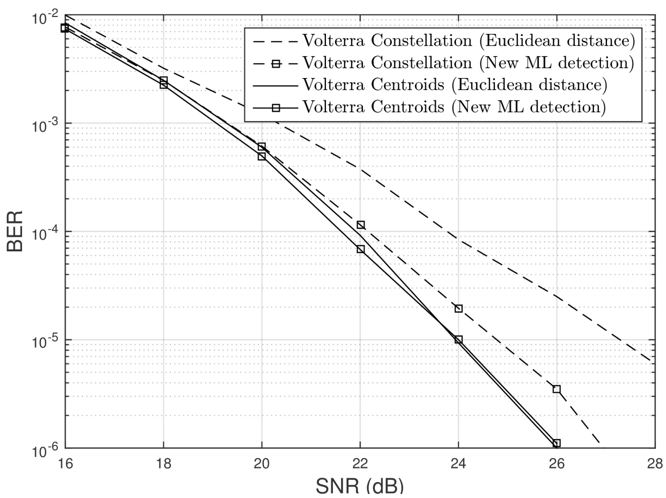

Fig. 7. Impact of a training on the centroids and the new detection criterion (denoted by New ML) for the baseband Volterra equalizer ($M_1 = 10$, $M_3 = 5$).

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