Semiclassical model of light mesons

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The dominantly orbital state description is applied to the study of light mesons. The effective Hamiltonian is characterized by a relativistic kinematics supplemented by the usual funnel potential with a mixed scalar and vector confinement. The influence of two different finite quark masses and potential parameters on Regge and vibrational trajectories is discussed.

I. INTRODUCTION

Potential models, in particular semirelativistic ones, have been proved extremely successful for the description of mesons and baryons [1]. Several techniques have been developed in order to solve numerically the spinless Salpeter equation used in most works devoted to the study of light mesons. Variational calculations performed in well chosen bases [2,3], and the three-dimensional Fourier grid Hamiltonian method [4], for instance, can provide accurate numerical results. Nevertheless, it is interesting to obtain analytical results as well. The dominantly orbital state (DOS) description has been developed in this direction [5,6]. In this approach, the orbitally excited states are obtained as a classical result, while the radially excited states can be treated semiclassically. It is then possible to obtain information about family of states characterized by a high orbital angular momentum.

In Ref. [6], the DOS model has been applied to study light-light and heavy-light meson spectra with a confinement potential being a mixture of scalar and vector components. In the case of one or two quarks with vanishing masses, it was shown that linear Regge (orbital) and vibrational (radial) trajectories are obtained for an arbitrary scalar-vector mixture but that the ratio of radial to orbital energies is strongly dependent on the mixture. These results have been extended in Ref. [7] by considering the presence of a quark and an antiquark with finite identical masses and the introduction of a Coulomb-like interaction plus a constant potential in addition to the confinement. In the limit of small masses and small strength of the short-range potential, or in the limit of large angular momentum, it is shown that all calculations can be worked out analytically. One result is that the slopes of orbital and radial trajectories depend only on the string tension and on the vector-scalar mixture in the confinement potential. Moreover, small finite quark masses do not alter significantly the linearity, especially in the case of a dominant vector-type confining potential. As expected, the Coulomb-like potential has no effect on trajectories but its influence on the meson masses is determined in the approximation of the DOS description. Lastly, it is shown that the ratio of radial to orbital trajectory slopes depends on the scalar-vector confinement mixture only.

In this work, we consider the case of a quark and an antiquark with finite but different masses. The potential taken into account is the same as the one considered in our previous work, that is to say a linear scalar-vector mixed confinement potential supplemented by a Coulomb-like interaction. An expression for the square mass of a meson is obtained in the limit of small quark masses and small strength of the short-range potential, or in the limit of large angular momentum. The coefficients of the square mass formula cannot be obtained analytically but they are computed numerically. Nevertheless a coverage of all possible situations is performed. The square mass formula is established and discussed in Secs. II and III, respectively. Some concluding remarks are given in Sec. IV.

II. THE MODEL

A. The dominantly orbital state description

A detailed explanation of the technique of the DOS description is given in Ref. [6]. So we just recall the basic ideas and we only focus on differences between our work and previous work. In all our formulas, we use the natural units $\hbar = c = 1$.

We consider a system composed of two particles with masses m_1 and m_2 interacting via a scalar potential $S(r)$ and a vector potential $V(r)$ which depend only on the distance r between the particles. In the center-of-mass laboratory, the classical mass M of the system characterized by a total orbital angular momentum J is given by

$$M(r,p_r,J) = \sqrt{p_r^2 + \frac{J^2}{r^2} + [m_1 + \alpha_1 S(r)]^2}$$
$$+ \sqrt{p_r^2 + \frac{J^2}{r^2} + [m_2 + \alpha_2 S(r)]^2} + V(r),$$

(1)

where p_r is the radial internal momentum. The parameters α_1 and α_2 indicate how the scalar potential $S(r)$ is shared among the two masses. These quantities must satisfy the following conditions to ensure a good nonrelativistic limit:

$$\alpha_1 + \alpha_2 = 1 \quad \text{and} \quad \lim_{m_i \to \infty} \alpha_i = 0. \tag{2}$$

A natural choice is to take

$$\alpha_1 = \frac{m_2}{m_1 + m_2} \quad \text{and} \quad \alpha_2 = \frac{m_1}{m_1 + m_2}. \tag{3}$$

We will use this prescription in this work. In the following, it is assumed that $m_1 \leqslant m_2$. A parameter $\beta = m_1/m_2$ is introduced to measure the mass asymmetry, and the heaviest quark mass is denoted by m. Contrary to the philosophy adopted in our previous paper [7], it is necessary to work with the Hamiltonian formalism and to give up the Lagrangian formalism.

The idea of the DOS description is to make a classical approximation by considering uniquely the classical circular orbits, that is to say the lowest energy states with a given J. This state is defined by $r = r_0(J)$, and thus $dr/dt = 0$ and $p_r = 0$. Let us denote $M_0(J) = M(r = r_0, p_r = 0, J)$. In order to get the radial excitations, a harmonic approximation around a classical circular orbit is calculated. If the harmonic quantum energy is given by $\Omega(J)$, then the square mass of the system with orbital excitation J and radial excitation $n(0,1,\dots)$ is given by (see Ref. [6])

$$M^2(J,n) = M_0^2(J) + M_0(J)\Omega(J)(2n+1). \tag{4}$$

The harmonic approximation is relevant only if it can be assumed that $\Omega(J) \ll M_0(J)$. We will see in the next section that the last formula will be naturally obtained in the light meson case studied in this paper.

B. Application to mesons

As it is done in previous works, this formalism is applied to the study of mesons. The long-range part of the interaction between a quark and an antiquark is dominated by the confinement which is assumed to be a linear function of r. As its Lorentz structure is not yet determined, we assume that the confinement is partly scalar and partly vector. The importance of each part is fixed by a mixing parameter f whose value is 0 for a pure vector and 1 for a pure scalar. The short-range part of the interaction is assumed to be of vector-type and given by the usual Coulomb-like potential. Thus we have

$$S(r) = far, \tag{5}$$

in which a is the usual string tension, whose value should be around 0.2 GeV^2, and

$$V(r) = (1-f)ar - \frac{\kappa}{r}, \tag{6}$$

in which κ is proportional to the strong-coupling constant α_s. A reasonable value of κ should be in the range 0.1 to 0.6. The quark masses appearing in a spinless Salpeter equation are the constituent masses. Their values are model dependent, but generally we have $m_u = m_d \approx 0.2$ GeV and $m_s \approx 0.5$ GeV.

We are interested only in Regge trajectories which give the behavior of M^2 in terms of J for a large value of J. So to compute the square mass formula for the particular interactions (5) and (6), we will use the same technique as in our previous study. A detailed explanation is given in Ref. [7]. For a given set of parameters f and β, we have to compute the value of r_0 as a function of J, in the limit of great values of J. In the general case, this radius is given by a polynomial equation of the 20th degree. This equation is by no means obvious to obtain and cannot be considered as a simple extension of the two equal mass case. The software MATHEMATICA has been used to perform the longest calculations. The general formulas are very complicated and too long to be given in detail here, but they can be obtained on request.

When $\beta = 1$, the old formalism must be recovered. This is not trivial to demonstrate, but we have verified this point. In particular, when the two particles have the same mass, the degree of the general polynomial is reduced and the solution is one of the roots of a second degree equation. In this case, the determination of r_0 and all subsequent calculations can be performed analytically and are identical to the ones performed in Ref. [7].

When β is different from 1, a numerical solution for r_0 must be computed numerically for each value of parameters f and β. Note that when $f = 0$, only one relevant solution (r_0 real positive) exists (see Ref. [7]). To find the physically relevant value of r_0, we generate the solutions for increasing values of f from the known solution at $f = 0$, in order to be sure that the solution is a continuous function of f. This procedure is repeated for each value of β. Once the radius is calculated, its value is used to compute the square meson mass.

Finally, we obtain the square mass of the meson under the following form:

$$M^2 = aA(f,\beta)J + B(f,\beta)m\sqrt{aJ} + C(f,\beta)m^2 + aD(f,\beta)\kappa$$
$$+ aE(f,\beta)(2n+1) + O(J^{-1/2}), \tag{7}$$

where $0 \leqslant f \leqslant 1$, $0 \leqslant \beta \leqslant 1$, and m is the mass of the heaviest quark. The values of coefficients A and E as a function of the mixing parameter f and the asymmetry parameter β are given in Figs. 1. All coefficients are discussed in the next section. It is worth noting that the term in Ω^2 naturally does not appear as it is of higher order.

When $\beta = 1$, that is to say $m_1 = m_2 = m$, formula (7) reduces to formula (20) of Ref. [7], and coefficients A, B, C, D, and E are given by analytical expressions which can be found in this reference. We checked theoretically as well as numerically that our new expressions tend towards the previous ones at the limit $\beta \to 1$.

III. DISCUSSION OF THE MODEL

Equation (7) is the square mass of a meson for large value of J. Though coefficients of the formula cannot be obtained analytically, they are calculated for $0 \leqslant f \leqslant 1$ and for $0 \leqslant \beta \leqslant 1$, which covers all possible physical situations provided the mass m appearing in the formula is the mass of the heaviest quark. One can make the following comments.
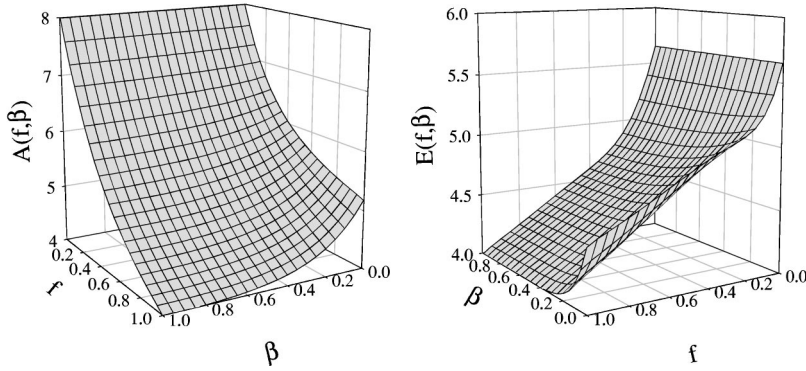
FIG. 1. Coefficient $A(f,\beta)$ and $E(f,\beta)$ of formula (7) as a function of the mixing parameter f and the mass asymmetry parameter β.

For large value of J, the dominant term of the square mass formula is linear in J. This explains the existence of linear Regge trajectories. The slope of the Regge trajectories depend on three parameters: the string tension a, the mixing parameter f, and the mass asymmetry parameter β. The string tension is a constant and gives the energy scale of the slope. The slope depends strongly on the value of f, which is assumed to be also a universal quantity independent of the system. Unfortunately, as we have seen in our previous work [7], experimental data from symmetrical mesons (ρ and ϕ families) cannot provide reliable information about the value of f. The calculations favor slightly a dominant vector-type confining interaction ($f=0$). But this work shows that the slope depends also on the parameter β whose values are fixed by the system. On Fig. 1, one can see that the β dependence of the coefficient $A(f,\beta)$ is only marked when $f \gtrsim 0.5$ and $\beta \lesssim 0.5$. It is also shown in our previous work [7] that the conclusions we can obtain from our square mass formula can be applied to light mesons only, that is to say mesons containing u, d or s quarks. In this case, the only physical relevant value for β is given by the ratio m_u/m_s. For constituent masses, the value of this ratio must be around 0.5. This shows that the slope of the Regge trajectory for K^* mesons cannot be sensitively different from the ones for ρ and ϕ mesons. This is especially verified if the confinement potential is dominantly of the vector-type ($f \approx 0$) as it seems favored from experimental data.

One can see in Fig. 1 that the slope of the vibrational trajectories proportional to $E(f,\beta)$ presents the same characteristics as the coefficient $A(f,\beta)$. We can conclude that the slope of the vibrational trajectory for K^* mesons cannot be sensitively different from the ones for ρ and ϕ mesons. Obviously, the ratio $R(f,\beta)=2E(f,\beta)/A(f,\beta)$ is characterized by a similar behavior as a function of f and β.

A term proportional to $\sqrt{J}$ deforms the Regge trajectories for a low value of J. This term vanishes if $m=0$, that is to say the two quarks are massless, or if $\beta=0$, that is to say the lightest quark is massless. So, the linearity of the Regge trajectory is perfect if one quark is massless. This is also the case if the confinement potential is a pure vector-type interaction ($f=0$) whatever the masses are. The value of $B(f,\beta)$ decreases rapidly with f and β value parameters. This fact implies that the linearity of the Regge trajectory is only broken in the case of dominantly scalar-type confinement interaction. As it is mentioned above, this situation is not the one favored by experimental data. Note that the term propor-

tional to $\sqrt{J}$ is also independent of the strong-coupling constant κ.

The position of the trajectories with respect to the energy axe stems from three contributions; one $C(f,\beta)m^2$ is due to the finite mass of the quarks, another negative one $aD(f,\beta)\kappa$ is due to the strong-coupling constant, and the last one $aE(f,\beta)$ reflects the zero-point motion of the harmonic vibration. These terms are of minor importance since the zero-point energy of the orbital motion cannot obviously be calculated in our model. Nevertheless it is interesting to discuss a little bit the behavior of coefficients C and D. The coefficient $C(f,\beta)$ depends clearly on β whatever the value of f. When $f=0$, this coefficient can be calculated exactly: $B(0,\beta)=4(1+\beta^2)$. This term is linked to the contribution of the quark masses to the meson total mass. The presence of the Coulomb-like potential decreases the mass of the meson but has no influence on the Regge trajectories, as is expected for a potential which becomes very weak with respect to the confinement interaction as quark interdistance increases. Again the coefficient $D(f,\beta)$ is only affected by the mass asymmetry for $f \gtrsim 0.5$ and $\beta \lesssim 0.5$.

Finally, let us remark that there is no coupling between orbital and radial motion for large J values (absence of terms nJ). This is only a consequence of the Coulomb+linear nature of the quark-antiquark potential. This may not be true for other types of potentials.

IV. CONCLUDING REMARKS

We have shown that all light mesons exhibit linear orbital and radial trajectories in the dominantly orbital states (DOS) model. Slopes of both types of trajectories depend only on the string tension, the vector-scalar mixture in the confinement potential, and the mass asymmetry between the quark and the antiquark.

In the limit of small quark masses, it turns out that the mass asymmetry between the quark and the antiquark alters sensitively the slope of the trajectories only for large mass asymmetry and for dominant scalar-type confining potential. As the experimental data favor a vector-type confining potential (see our previous paper [7]) and as the mass ratio of nonstrange quark over the strange quark is not very small, the slope of the Regge and vibrational trajectory for K^* mesons cannot be sensitively different from the ones for ρ and ϕ mesons. This is in agreement with experimental meson spectra.

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