Electromagnetic splitting for mesons and baryons using dressed constituent quarks

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Abstract

Electromagnetic splittings for mesons and baryons are calculated in a formalism where the constituent quarks are considered as dressed quasiparticles. The electromagnetic interaction, which contains coulomb, contact and hyperfine terms, is folded with the quark electrical density. Two different types of strong potentials are considered. Numerical treatment is done very carefully and several approximations are discussed in detail. Our model contains only one free parameter and the agreement with experimental data is reasonable although it seems very difficult to obtain a perfect description in any case.

1. Introduction

Quantum chromodynamics (QCD) is believed to be a good theory of strong interaction. It has met with numerous successes in many domains. However, in the low energy regime, it is extremely difficult to handle because of its nonperturbative character. Lattice calculations become more and more reliable but still remain very cumbersome, time consuming and not always transparent for the underlying physics. This explains why, in the meson and baryon sectors, a number of alternative simpler models were introduced. Among them, the nonrelativistic quark model (NRQM) is very appealing because of its high simplicity, ability to treat properly the centre of mass motion, and the large number of observables that can be described within its framework.

In NRQM formalism, the dynamical equation is the usual Schrödinger equation including a nonrelativistic kinetic energy term plus a potential term [1]. There exist a lot of different numerical algorithms to solve the two-body and three-body problems with a good accuracy (see for instance [2–4]).

Nowadays, it becomes more and more frequent to use a relativistic expression for the kinetic energy operator. The resulting dynamical equation is known as a spinless Salpeter

equation. It has several advantages as compared to the Schrödinger equation [5, 6] and the corresponding numerical algorithms are now well under control (see for instance [4, 6–9]). This kind of models and NRQM both use potentials, and are called potential models.

In potential models (and indeed in many other QCD inspired models), the quark degrees of freedom are no longer the bare quarks of the QCD Lagrangian, but are quasiparticles dressed by a gluon cloud and quark–antiquark virtual pairs. They are called constituent quarks and the most visible modification is the necessity to use in potential models a quark mass substantially larger than the bare mass. In principle the bare quark–quark potential should also be modified and folded with the quark colour density to give the final potential to be used in a Schrödinger or in a spinless Salpeter equation. In recent quark–quark potentials appearing on the market, this effect is taken into account [10–13]. Actually, this effect has been already considered since a rather long time (see i.e. [14, 15]). The resulting spectra of these models are in correct agreement with experimental data. However, it seems very difficult to get, in a unified scheme (same form of the potential and same set of parameters), a good description of both meson and baryon properties [16, 17]. For example in the well known works of Isgur [14, 15], the string tension is different in the mesonic and baryonic sectors. Recently, progresses in obtaining a unified description have been achieved [18].

The spectra are only a part of interesting observables, and the validity of a model should be tested on other observables, especially if they are very sensitive to the form of the wave function. Electromagnetic properties are best suited for such a study, because the basic QED formalism is very well known and precise, and thus the possible uncertainties coming from mechanisms or wave function (itself depending on the much less known strong interactions) are more conveniently identified.

In this paper, we focus our study on the electromagnetic splitting between charged hadrons, both in the meson and baryon sectors, within the framework of potential models. Since the earliest works on charmed mesons [19], and baryons [20], a number of similar studies were performed in the past [1, 12, 21–23]. Essentially three different sources for the splitting were identified: a small mass difference between up and down quarks, the coulomb interaction between charged quarks and their dipole–dipole interaction [23]. All of them seem to have an important effect and the final result is a very subtle interplay among them. This explains why a very proper and precise treatment must be invoked, and also why this observable is very interesting. The current mass ratio for the u and d quarks is probably comprised between 0.2 and 0.8 [24]. However their absolute values are presumably weak (a few MeV) with respect to the QCD scale parameter Λ. Thus the spontaneous symmetry breaking induces large constituent masses for the u and d quarks; the corresponding values are very close (making the $SU(2)$ isospin symmetry rather good) but nevertheless different. This small difference is a first source for the isospin splitting. Moreover, quarks being charged particles, the coulomb interaction is obviously present (very often, it has been treated as a perturbation). The dipole–dipole interaction (or hyperfine interaction) is a consequence of relativistic corrections to the coulomb potential.

The same problem was also undertaken using formalisms relying more basically on fundamental QCD. In [25], a heavy quark effective theory (first order in $1/m_Q$) is adopted to study the splitting in heavy mesons, using dispersion relations. In [26] a chiral field theory is employed to study several splittings in ordinary and strange sectors. In [27], the authors are interested in the charmed and bottom meson sectors with a formalism based on Cottingham formula. In [28] a tadpole term is introduced to deal with the $u - d$ mass difference in some hadrons. But in most of these studies, the authors limit themselves to very restricted samples, either in meson or in baryon sector. We think that so few states to test the validity of a model can be questionable.

In our paper we want to deal with *all the known splittings* both in mesons and in baryons in a consistent approach, and to push the potential model study further in several domains. First we want to perform a precise and complete treatment, avoiding perturbative expressions as much as possible. Second, we introduce the 'contact term', which arises on equal footing as the dipole–dipole relativistic correction, but which is neglected by most authors [1, 23].

Finally, our most important improvement to our point of view is the use of a dressed electromagnetic interaction between quarks. Since the constituent quarks are quasiparticles, the electromagnetic interaction should also be modified as compared to the bare one, in a very similar way to the quark–quark strong potential. However the electromagnetic Lagrangian is different from the QCD Lagrangian and the electromagnetic density for the quark, playing a role in the splitting, has no reason to be identical to the colour density occurring in the quark–quark strong potential. Such an approach has already been proposed in [21, 22] but with a different form for the electromagnetic density. Moreover, in these works, different sets of parameters have been used for mesonic and baryonic sectors separately.

In order to see the sensitivity of the results on the treatment of the strong interactions for the quark dynamics, we investigate the splitting produced with two types of wave function, one resulting from a phenomenological nonrelativistic Hamiltonian (AL1) [29, 30] and other with a semi-relativistic Hamiltonian (called here BSS) [18]. Moreover, since our aim is to consider mesons and baryons on equal footing, it is important to consider interquark potentials that lead to a correct description of both sectors. This is rather difficult to encounter. Both Hamiltonians considered here are suited for that. The first potential (AL1) relies on the so-called funnel or Cornell potential [31, 32]. It is completely phenomenological and one can consider that the dressing of the quarks is included and simulated in the value of the various parameters. The second one (BSS) starts with more fundamental QCD grounds and is based on instanton induced effects [16, 33]; the dressing is explicitly taken into account but there remain nevertheless some free parameters that are adjusted on the spectra.

The paper is organized as follows. Section 2 presents in more detail the strong potentials and the way to solve the two-body and three-body problems. Section 3 deals with the electromagnetic interaction responsible of the splitting. The results of our calculations are presented and compared to data in section 4. Conclusions are drawn in the last section.

2. Potentials and wave function

2.1. Strong potentials

As stated in the introduction, we use in this paper two kinds of interquark potentials: one that must be used in a Schrödinger equation (AL1) and the other in a spinless Salpeter equation (BSS). They both depend on the relative distance r between the interacting quarks and are able to describe in a satisfactory way both the meson and the baryon spectra. However the BSS potential is suited, because of its underlying QCD basis, only for the light quark sectors (u, d, s quarks). They differ by the type of kinematics and by the manner to deal with spin splitting; although they give spectra of similar good quality, the corresponding wave functions can differ appreciably. Their derivation and their parameters have been reported elsewhere, and here we just want to point out the essential features and stress their differences more explicitly. In both models, the u and d quarks are assumed to have the same mass. In the following, they will be noted by the symbol n (normal or nonstrange).

2.1.1. AL1 potential.
The AL1 potential [30], developed for a nonrelativistic kinematics, contains the minimum ingredients necessary to get an overall reasonable description of

hadronic resonances, a central part V_C and a hyperfine term V_H

$$V_{ij}(r) = -\frac{3}{16}\lambda_i \cdot \lambda_j \left[V_C^{(ij)}(r) + V_H^{(ij)}(r) \right]. \tag{1}$$

The central part is merely the Cornell potential composed of a short range coulombic part, simulating the one-gluon exchange mechanism, and a long-range linear term, responsible for the confinement (an additional constant is very important to get the good absolute values)

$$V_C^{(ij)}(r) = -\frac{\kappa}{r} + ar + C. \tag{2}$$

The colour dependence through the Gell–Mann matrices λ in relation (1) comes from one gluon exchange. There is no reason, except simplicity, that such a structure is kept for the confining and constant parts. Nevertheless, this ansatz works well for both meson and baryon sectors.

The hyperfine term has a short-range behaviour and is chosen as a Gaussian function

$$V_H^{(ij)}(r) = \frac{8\pi}{3m_i m_j}\kappa' \frac{\exp\left(-r^2/\sigma_{ij}^2\right)}{\pi^{3/2}\sigma_{ij}^3}\mathbf{s}_i \cdot \mathbf{s}_j. \tag{3}$$

The interesting property, as compared to Bhaduri's [32] or Cornell's [31] potentials, is that the range σ_{ij} of that force does depend on the flavour

$$\sigma_{ij} = A \left(\frac{2m_i m_j}{m_i + m_j} \right)^{-B}. \tag{4}$$

A kind of dressing is realized with the Gaussian function, since the theory predicts a delta contribution for the hyperfine potential. The various parameters, including constituent masses, have been determined on the spectra by a best fit procedure. Although very simple, this potential does a good job in hadronic spectroscopy.

The AL1 potential depends on quark masses in the strong hyperfine term. So a variation of the quark masses, as necessary for treating the electromagnetic splitting, has a strong effect on the meson masses. We have checked that this may induce wrong results in the case of a perturbative calculation.

2.1.2. BSS potential.   The BSS potential [18], developed for a semi-relativistic kinematics, contains the Cornell potential and an instanton induced interaction. The Cornell potential has the same form as in the AL1 potential, but the constant interaction is different for meson and baryon sectors. In contrast to the AL1 potential, the instanton formalism is based on a $SU(3)$ flavour symmetry and can hardly be generalized to the heavy quark sector.

The instanton induced interaction provides a suitable formalism to reproduce well the spectrum of the pseudoscalar mesons (and to explain the masses of η and η' mesons). The interaction between one quark and one antiquark in a meson is vanishing for $L \neq 0$ or $S \neq 0$ states. For $L = S = 0$, its form depends on the isospin of the $q\bar{q}$ pair

- For $I = 1$:

$$V_I(r) = -8g\delta(\vec{r}); \tag{5}$$

- For $I = 1/2$:

$$V_I(r) = -8g'\delta(\vec{r}); \tag{6}$$

- $I = 0$:

$$V_I(r) = 8 \begin{pmatrix} g & \sqrt{2}g' \\ \sqrt{2}g' & 0 \end{pmatrix} \delta(\vec{r}), \qquad (7)$$

in the flavour space $(1/\sqrt{2}(|u\bar{u}\rangle + |d\bar{d}\rangle), |s\bar{s}\rangle)$.

The parameters g and g' are two dimensioned coupling constants. Between two quarks in a baryon, this interaction is written [16, 34] as

$$V_I(r) = -4(g P^{[nn]} + g' P^{[ns]}) P^{S=0} \delta(\vec{r}), \qquad (8)$$

where $P^{S=0}$ is the projector on spin 0, and $P^{[qq']}$ is the projector on antisymmetrical flavour state qq'. We have shown in [18] that this model is able to describe correctly meson and baryon spectra in a consistent way if the following interaction is added for baryons only:

$$V_I^{\text{baryon}}(r) = C_I (P^{[nn]} + P^{[ns]}) P^{S=0} P^{L=0}, \qquad (9)$$

where C_I is a new constant.

The quark masses used in this model are the constituent masses and not the current ones. It is then natural to suppose that a quark is not a pure pointlike particle, but an effective degree of freedom which is dressed by the gluon and quark–antiquark pair clouds. The form that we retain for the colour charge density of a quark is a Gaussian function

$$\rho(\mathbf{r}) = \frac{1}{(\Gamma\sqrt{\pi})^{3/2}} \exp(-r^2/\Gamma^2). \qquad (10)$$

It is generally assumed that the quark size Γ depends on the flavour. So, we consider two size parameters Γ_n and Γ_s for n and s quarks respectively. More details are given in [12, 13, 18].

The instanton interaction acts differently on the symmetric $(ud + du)$ and antisymmetric $(ud - du)$ flavour states; in a framework where $m_u = m_d$, the isospin formalism can be introduced to classify the basis states and this property can be taken into account without problem. As soon as $m_u \neq m_d$, the up and down quark must be considered as different and the instanton becomes very difficult to handle. Thus, for baryons, the three-body treatment with BSS is done perturbatively. In this case, the potential has no mass dependence and the problem mentioned above for the AL1 potential does not appear.

2.2. Numerical techniques

There exist many numerical algorithms to compute the radial function of a meson. In this paper, we use the method based on Lagrange mesh, which is very simple, precise and fast, and for which a recent work has shown that relativistic kinematics can be handled without any problem [8]. Thus the same method can be applied for both AL1 and BSS potentials. To get a very high relative precision, around 10^{-10}, a typical number of basis states is $N = 60$. Technical details can be found in [8].

Our method to solve the three-body problem is based on an expansion of the space wave function in terms of harmonic oscillator functions with *two different sizes*. This method, which is an old one [37], was given up during a long time and renewed recently in [9] (many authors use a numerical treatment based on harmonic oscillators but with a *single* variational

parameter, for instance in [15]). In this paper, it was shown that the precision achieved was similar to the stochastic variational method [4]. This last method was also shown to be competitive with more conventional methods, such as Faddeev formalism [38]. The number of quanta is the relevant quantity for convergence; a very good relative precision of the order of 10^{-5}, necessary in our study, is achieved if we include all basis states up to 20 quanta. More details on the method can be found in [9].

3. Electromagnetic interaction

3.1. Bare potential

The electromagnetic interaction between two pointlike particles i and j of charges Q_i, Q_j and masses m_i, m_j is very well known. In addition to the usual coulomb potential U_{coul}, relativistic corrections (at lowest order) give rise to contact, hyperfine, tensor, symmetric and antisymmetric spin–orbit and Darwin terms. Tensor, spin–orbit, and Darwin terms are complicated and presumably their effects are weak, so that people neglect them. The hyperfine interaction U_{hyp} does play an important role and is included in all serious calculations. The contact term U_{cont} is usually discarded with no other justification than simplicity. In this study we keep it, so that our total bare electromagnetic potential can be written as

$$U_{ij}^{(b)}(\mathbf{r}) = (U_{\mathrm{coul}})_{ij}^{(b)}(\mathbf{r}) + (U_{\mathrm{cont}})_{ij}^{(b)}(\mathbf{r}) + (U_{\mathrm{hyp}})_{ij}^{(b)}(\mathbf{r}), \tag{11}$$

with

$$(U_{\mathrm{coul}})_{ij}^{(b)}(\mathbf{r}) = Q_i Q_j \frac{\alpha}{r}, \tag{12}$$

$$(U_{\mathrm{cont}})_{ij}^{(b)}(\mathbf{r}) = -\frac{\pi}{2} Q_i Q_j \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} \right) \alpha \delta(\mathbf{r}), \tag{13}$$

$$(U_{\mathrm{hyp}})_{ij}^{(b)}(\mathbf{r}) = -\frac{8\pi Q_i Q_j}{3m_i m_j} \alpha \delta(\mathbf{r}) \mathbf{s}_i \cdot \mathbf{s}_j, \tag{14}$$

where α is the fine structure constant. Let us stress a point that deserves further discussion. One can wonder why such a contact term is not kept as well in the strong potential AL1. This term comes from one gluon exchange and is proportional to α_s; but this parameter is essentially phenomenological and the Cornell form of the central potential is a kind of averaging of all unknown effects coming from QCD, including the contact term. Our philosophy is to maintain as far possible the form of the strong potential, which was shown to do a good job in hadronic physics. In contrast, the situation is completely different in the electromagnetic Hamiltonian (11) because the corresponding expression is the exact one and does not suffer from ambiguities or phenomenological uncertainties. This is why we decided to include the contact term, which is usually neglected, in order to grasp quantitatively its effect and to justify *a posteriori* the validity (or not) of neglecting it.

A perturbative treatment is compulsory for such a bare electromagnetic potential, owing to the presence of the delta function. It was shown that, in some cases, this is a very bad approximation [12].

3.2. Electromagnetic quark density

One way to introduce the very complicated mechanisms that transform the bare quarks (pointlike) to constituent quarks is to assume the existence of a phenomenological density

distribution for a quark. The electromagnetic density should in principle depend on the internal structure of the system and contain a lot of unknown processes (including probably interferences between QCD and QED) leading to isospin symmetry breaking.

This means that a constituent quark located at $\mathbf{r}$ is generated by a bare quark located in $\mathbf{r}'$ with a certain probability distribution (or density) $\rho(\mathbf{r} - \mathbf{r}')$. To have an appealing physical meaning this density must be a peaked function around zero with a certain size parameter, which depends in principle on the quark flavour. Moreover we require the natural property that, for a vanishing size, the constituent quark becomes pointlike and is identified to the bare quark. Mathematically, this means that the limit of the density $\rho(\mathbf{u})$ for a vanishing size is the delta function $\delta(\mathbf{u})$. Another natural property is that the density is isotropic. The last required property is that the integral of the density over the whole space is unity.

The most popular densities are of Lorentzian, Gaussian or Yukawa type. For instance, the authors of [21, 22] adopted a Gaussian form. In this study we choose an electromagnetic Yukawa density. There is a precise reason for that: this density is the leading ingredient of the meson charge form factor. It is an experimental fact that the data accommodate rather nicely to a Yukawa density, giving a form factor with an asymptotic behaviour Q^{-2} [35, 36], instead of a Gaussian density which leads to Gaussian asymptotic behaviour. Keeping in mind the previous remarks, the adopted density for quark of flavour i is

$$\rho_i(\mathbf{u}) = \frac{1}{4\pi \gamma_i^2} \frac{e^{-u/\gamma_i}}{u},$$ (15)

where γ_i is the electromagnetic size parameter.

The dressed potential U is obtained from the bare potential $U^{(b)}$ by a double convolution over the densities of each interacting quark

$$U_{ij}(\mathbf{r}) = \int d\mathbf{u} \, d\mathbf{v} \, \rho_i(\mathbf{u})\rho_j(\mathbf{v}) U_{ij}^{(b)}(\mathbf{r} + \mathbf{v} - \mathbf{u}).$$ (16)

With a trivial change of variable, it is quite easy to transform this double folding into a single one

$$U_{ij}(\mathbf{r}) = \int d\mathbf{r}' \, U_{ij}^{(b)}(\mathbf{r}')\rho_{ij}(\mathbf{r} - \mathbf{r}'),$$ (17)

with the definition of the new density

$$\rho_{ij}(\mathbf{u}) = \int d\mathbf{v} \, \rho_i(\mathbf{v})\rho_j(\mathbf{u} - \mathbf{v}).$$ (18)

Applying equation (18) to the Yukawa density (15), one gets

$$\rho_{ij}(\mathbf{u}) = \frac{1}{8\pi \gamma_i^3} e^{-u/\gamma_i} \qquad \text{if} \quad \gamma_i = \gamma_j$$ (19)

and

$$\rho_{ij}(\mathbf{u}) = \frac{1}{4\pi \left(\gamma_i^2 - \gamma_j^2\right)} \left(\frac{e^{-u/\gamma_i}}{u} - \frac{e^{-u/\gamma_j}}{u} \right)$$ (20)

$$= \frac{1}{\left(\gamma_i^2 - \gamma_j^2\right)} \left(\gamma_i^2 \rho_i(\mathbf{u}) - \gamma_j^2 \rho_j(\mathbf{u}) \right) \qquad \text{if} \quad \gamma_i \neq \gamma_j.$$ (21)

3.3. Dressed potential

Starting from equation (11) and using equations (17) and (19), it is easy to obtain analytically the dressed potential

$$U_{ij}(r) = (U_{\text{coul}})_{ij}(r) + (U_{\text{cont}})_{ij}(r) + (U_{\text{hyp}})_{ij}(r), \tag{22}$$

in the case of two interacting constituent quarks with the same size $\gamma_i = \gamma_j = \gamma$. It can be written as

$$(U_{\text{coul}})_{ij}(r) = \alpha Q_i Q_j \left[\frac{1}{r} \left(1 - e^{-r/\gamma} \right) - \frac{e^{-r/\gamma}}{2\gamma} \right], \tag{23}$$

$$(U_{\text{cont}})_{ij}(r) = -\frac{\alpha Q_i Q_j}{16\gamma^3} \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} \right) e^{-r/\gamma}, \tag{24}$$

$$(U_{\text{hyp}})_{ij}(r) = -\frac{\alpha Q_i Q_j}{3\gamma^3 m_i m_j} e^{-r/\gamma} \mathbf{s}_i \cdot \mathbf{s}_j. \tag{25}$$

Repeating the same thing in equation (21), one obtains the electromagnetic dressed potential in the case of different interacting quarks

$$(U_{\text{coul}})_{ij}(r) = \alpha Q_i Q_j \left(\frac{1}{r} - \frac{\gamma_i^2}{\gamma_i^2 - \gamma_j^2} \frac{e^{-r/\gamma_i}}{r} + \frac{\gamma_j^2}{\gamma_i^2 - \gamma_j^2} \frac{e^{-r/\gamma_j}}{r} \right), \tag{26}$$

$$(U_{\text{cont}})_{ij}(r) = -\frac{\alpha Q_i Q_j}{8(\gamma_i^2 - \gamma_j^2)} \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} \right) \frac{e^{-r/\gamma_i} - e^{-r/\gamma_j}}{r}, \tag{27}$$

$$(U_{\text{hyp}})_{ij}(r) = -\frac{2\alpha Q_i Q_j}{3 m_i m_j (\gamma_i^2 - \gamma_j^2)} \frac{e^{-r/\gamma_i} - e^{-r/\gamma_j}}{r} \mathbf{s}_i \cdot \mathbf{s}_j. \tag{28}$$

It is possible to treat the electromagnetic potential U as a perturbation. This procedure is only used for the baryons with the BSS potential (see section 2.1.2).

4. Results

4.1. Determination of the parameters

We do not want to introduce a lot of new parameters; here we restrict the number of free parameters to the minimum unavoidable. In particular the parameters of the strong potential are maintained without modification. Important information is contained in the electromagnetic size of the quarks γ_i. This size could depend on the flavour and on the electrical charge of the quark. If the dependence on charge were dominant, we could expect that sizes of quarks with the same charge are similar. Preliminary calculations have shown that the size must be strongly reduced for heavy quarks, showing that the dependence on mass must be dominant. Consequently, in order to restrict again the number of parameters, we assume that the sizes of the u and d quarks are the same. Thus we impose $\gamma_u = \gamma_d = \gamma_n$. For AL1 and BSS models, we have γ_n and γ_s as free parameters. An observable that is very sensitive to those parameters is the charge mean square radius.

In the mesonic sector, the bare charge square radius operator for pointlike quarks is defined by

$$(r^2)^{(b)} = \sum_{i=1}^{2} e_i (\mathbf{r}_i - \mathbf{R})^2, \tag{29}$$

Table 1. Electromagnetic sizes γ_f of the constituents quarks (in GeV^{-1}), for different flavours f, and for the two different strong potentials.

	γ_n	γ_s	γ_c	γ_b
AL1	1.225	0.200	0.040	0.013
BSS	1.330	0.450	–	–

where e_i is the charge for quark i, $\mathbf{r}_i$ its position, and $\mathbf{R}$ the position of the centre of mass. For constituent quarks, this expression has to be folded with quark density and should be written instead

$$\langle r^2 \rangle = \sum_{i=1}^{2} e_i \int d\mathbf{r}'(\mathbf{r}' - \mathbf{R})^2 \rho_i(\mathbf{r}' - \mathbf{r}_i). \tag{30}$$

Averaging quantity (30) on the meson wave function provides us with the charge mean square radius of the meson. Performing the calculation, one finds that this observable is a sum of a term, that can be called the bare radius $\langle r^2 \rangle^{(b)}$ (which is essentially the mean value of quantity (29) on meson wave function), plus a term which is essentially the dipole moment of the density. With a Yukawa density, one has explicitly

$$\langle r^2 \rangle = \langle r^2 \rangle^{(b)} + 6 \sum_{i=1}^{2} e_i \gamma_i^2. \tag{31}$$

The dynamical contribution to the square radius is entirely contained in the bare quantity, whose expression is very well known and is not recalled here.

From relation (31), one sees that the pion radius depends only on γ_n and it is used to determine this parameter. The kaon radius depends on γ_n and γ_s; since γ_n has already been determined from the pion, the kaon radius is used to determine γ_s. In addition, the AL1 potential needs the further determination of γ_c and γ_b. Since the radii for D and B resonances are not known experimentally, we determine the corresponding sizes by requiring a smooth behaviour versus the mass. Moreover, the uncertainty on the kaon radius is rather large. Fortunately, we checked that the isospin splitting does not depend too much on the precise value of the quark size. Our accepted values for the electromagnetic sizes are summarized in table 1.

Other parameters that need to be changed are the quark masses. In fact, the only important ingredient for the splittings is the mass difference between the down and up quarks: $\Delta = \widetilde{m}_d - \widetilde{m}_u$. In view of this, we choose to maintain the s, c and b quark masses at their nonperturbative value $\widetilde{m}_i = m_i$, and to keep the average value of the isospin doublet at its nonperturbative value $(\widetilde{m}_d + \widetilde{m}_u)/2 = m_n$. The size parameters being determined once and for all on charge radii, we have only *one free parameter* Δ at our disposal to try to reproduce all the known electromagnetic splittings. To really see that this is a very big constraint, let us recall that for doing the same job, authors of [21, 22] used four free parameters and Genovese *et al* two free parameters. The π^0 and ρ^0 resonances are considered here as $n\bar{n}$ systems, composed of fictitious quark and antiquark of mass $\widetilde{m}_n = (\widetilde{m}_u + \widetilde{m}_d)/2 = m_n$. Doing this, we neglect mixing of $I = 0$ and $I = 1$ components; this is a good approximation up to second order in Δ/m_n. One can imagine several strategies to determine the parameter Δ. We first remarked that if we fit Δ on the mesons, the baryons were very badly reproduced, while if we fit it on the baryons, the mesons were spoiled less dramatically. Moreover, among the baryons, some splittings are more affected than others by a small change of Δ. Thus, we decided to fit this parameter on one of the most sensitive and well known splitting, namely $\Sigma^- - \Sigma^0 = 4.807 \pm 0.04$ MeV.

Table 2. Values of the parameter $\Delta = \widetilde{m}_d - \widetilde{m}_u$ (in MeV), for the two potentials, and for the four different approximations presented in the text.

	C	CC	CH	T
AL1	13.2	14.4	12.6	13.6
BSS	7.2	9.4	6.2	8.4

One aim of our study is to see the respective influence of each component of the electromagnetic potential (22). The coulomb part seems to us unavoidable, so we will consider four different approximations in the following:

- C: Electromagnetic potential restricted to the coulomb term (23) or (26) alone.
- CC: Electromagnetic potential restricted to coulomb and contact (24) or (27) terms alone.
- CH: Electromagnetic potential restricted to coulomb and hyperfine (25) or (28) terms alone.
- T: Total electromagnetic potential (22) taken into account.

Each approximation requires its own Δ parameter, but the γ_i parameters can be maintained to their values of table 1. The corresponding results are gathered in table 2. It is worth noting that the study of any of these approximations is a complete calculation by its own and needs a computational effort identical to the total treatment.

It is funny that for both potentials, increasing values of Δ are obtained with approximations CH, C, T, CC respectively. Is it a property independent of the strong potential (realistic enough to reproduce baryon and meson spectra)? We have no answer.

4.2. Experimental data

A number of experimental data, concerning the electromagnetic splittings, exist for both the mesonic and baryonic sectors, and for both light quark (u, d and s) systems and systems containing at least a heavy quark (c and b). The use of AL1 potential allows to study all data, while BSS is restricted to the light quark domain for the reasons mentioned in section 2.1.2. The values for the splittings are of order of some MeV and some of them are known with good accuracy.

From the splitting in the nucleon case and many others, the d quark mass is presumably larger than the u quark mass. From a naive argument based on rest masses only, the hierarchy of the splittings can be explained although the quantitative value needs much refined explanation. However, in contrast to this naive argument, a number of puzzling questions still arise and some of them are not solved in a satisfactory manner by the up to date theoretical studies. Let us list some of them:

- $n - p = 1.293$ MeV is much weaker than $\pi^+ - \pi^0 = 4.594$ MeV, despite the fact that a very naive quark model gives $n(udd) - p(duu) = \Delta$ and $\pi^+(u\bar{d}) - \pi^0([u\bar{u} - d\bar{d}]/\sqrt{2}) = 0$;
- $\pi^+ - \pi^0$ has a large positive value while $\rho^+ - \rho^0$ has a small negative value (a positive value is compatible with the error bar but with a very low value in any case);
- $D_2^+ - D_2^0 \approx 0$ (or even may be negative) while the values for other $c\bar{n}$ states are largely positive;
- Σ_c^{++} is the highest level of the multiplet, in contradiction to what one expects from naive arguments where it should be the lowest one;

Table 3. Meson electromagnetic splittings (in MeV) calculated for four different approximations C, CC, CH and T, as explained in the text. The wave functions result from the strong potential AL1 and the numerical treatment are same. The experimental values (Exp) are taken from [24].

Splitting	Exp	C	CC	CH	T
$\pi^+ - \pi^0$	4.594 ± 0.001	1.239	0.828	2.111	1.693
$\rho^+ - \rho^0$	-0.5 ± 0.7	1.009	0.857	0.870	0.713
$K^0 - K^+$	3.995 ± 0.034	8.096	9.290	7.054	8.109
$K^{0*} - K^{+*}$	6.7 ± 1.2	1.132	1.458	1.139	1.436
$K_2^0 - K_2^+$	6.8 ± 2.8	-0.794	-0.793	-0.763	-0.757
$D^+ - D^0$	4.78 ± 0.10	2.478	1.977	2.821	2.308
$D^{+*} - D^{0*}$	2.6 ± 1.8	1.513	1.120	1.428	1.037
$D_1^+ - D_1^0$	6.8 ± 5	-2.038	-2.409	-1.839	-2.161
$D_2^+ - D_2^0$	0.1 ± 4	-2.058	-2.411	-1.937	-2.235
$B^0 - B^+$	0.33 ± 0.28	-1.877	-1.715	-1.891	-1.713

Table 4. Same as table 3 for baryons. The theoretical uncertainty may affect only the last digit.

Splitting	Exp	C	CC	CH	T
$n - p$	1.293	0.89	1.15	0.91	1.15
$\Delta^0 - \Delta^{++}$	2.25 ± 0.68	2.68	3.68	2.79	3.72
$\Delta^+ - \Delta^{++}$	1.2 ± 0.6	0.57	1.22	0.73	1.35
$\Sigma^- - \Sigma^0$	4.807 ± 0.04	4.82	4.82	4.81	4.76
$\Sigma^- - \Sigma^+$	8.08 ± 0.08	7.87	8.41	8.27	8.55
$\Sigma^{-*} - \Sigma^{0*}$	2.0 ± 2.4	3.26	3.23	3.04	2.94
$\Sigma^{-*} - \Sigma^{+*}$	0 ± 4	1.71	1.99	1.68	1.96
$\Xi^- - \Xi^0$	6.48 ± 0.24	7.12	7.21	7.38	7.38
$\Xi^{-*} - \Xi^{0*}$	3.2 ± 0.6	3.01	2.91	2.80	2.66
$\Sigma_c^{++} - \Sigma_c^0$	0.35 ± 0.18	1.00	-0.02	1.35	0.37
$\Sigma_c^0 - \Sigma_c^+$	0.9 ± 0.4	0.32	0.64	0.02	0.33
$\Sigma_c^{++*} - \Sigma_c^{0*}$	1.9 ± 1.7	1.37	0.27	1.33	0.19
$\Xi_c^0 - \Xi_c^+$	5.5 ± 1.8	2.81	3.28	3.01	3.42
$\Xi_c^{0\prime} - \Xi_c^{+\prime}$	$\simeq 4.2 \pm 3.5$	0.20	0.49	-0.08	0.20
$\Xi_c^{+*} - \Xi_c^{0*}$	$\simeq 2.9 \pm 2.0$	-0.08	-0.31	-0.03	-0.25
$\Xi_c^{0**} - \Xi_c^{+**}$	$\simeq 4.1 \pm 2.5$	3.09	3.42	3.24	3.51

- All the members of the charmed Ξ verify $\Xi_c^0 > \Xi_c^+$ except the particular states $\Xi_c^{0*} < \Xi_c^{+*}$ which satisfy the opposite relation.

These remarks illustrate the fact that the mass difference between down and up quarks cannot be the only—or even leading—ingredient to explain the splitting; the internal dynamics also plays an important role. An important aim of this paper is devoted to examine this question.

4.3. Influence of various approximations

In this part we want to study the effect of using various approximations C, CC, CH and T of the electromagnetic Hamiltonian. An exact treatment is performed, except for the baryons with BSS potential, as explained previously.

Let us first present the results obtained with AL1 potential. They are gathered in table 3 for the mesons and in table 4 for the baryons. Few comments are in order:

Table 5. Meson electromagnetic splittings (in MeV) calculated for four different approximations C, CC, CH and T, as explained in the text. The wave functions result from the strong potential BSS and the numerical treatment is exact. The experimental values (Exp) are taken from [24].

Splitting	Exp	C	CC	CH	T
$\pi^+ - \pi^0$	4.594 ± 0.001	1.22	−0.21	4.11	2.97
$\rho^+ - \rho^0$	-0.5 ± 0.7	1.00	0.40	0.59	−0.01
$K^0 - K^+$	3.995 ± 0.034	1.54	3.68	−0.97	1.17
$K^{0*} - K^{+*}$	6.7 ± 1.2	2.88	4.50	2.63	4.25
$K_2^0 - K_2^+$	6.8 ± 2.8	2.41	3.49	2.07	2.83

Table 6. Same as table 5 for baryons. The theoretical uncertainty may affect only the last digit. Let us recall that, for technical reasons, the theoretical values were obtained with a perturbative treatment.

Splitting	Exp	C	CC	CH	T
$n - p$	1.293	2.78	4.28	2.62	4.08
$\Delta^0 - \Delta^{++}$	2.25 ± 0.68	4.24	8.21	4.37	8.34
$\Delta^+ - \Delta^{++}$	1.2 ± 0.6	1.18	3.69	1.59	4.09
$\Sigma^- - \Sigma^0$	4.807 ± 0.035	4.83	4.81	4.81	4.80
$\Sigma^- - \Sigma^+$	8.08 ± 0.08	7.68	8.90	8.47	9.69
$\Sigma^{-*} - \Sigma^{0*}$	2.0 ± 2.4	4.98	5.37	4.06	4.43
$\Sigma^{-*} - \Sigma^{+*}$	0 ± 4	3.12	4.54	2.88	4.33
$\Xi^- - \Xi^0$	6.48 ± 0.24	4.82	4.50	5.87	5.55
$\Xi^{-*} - \Xi^{0*}$	3.2 ± 0.6	5.09	5.38	4.17	4.46

- The various approximations differ significantly from other. This proves again that the electromagnetic splitting is an observable very sensitive to the physical content put in it. A more quantitative comparison is relegated later on.

- Although the experimental data cannot be reproduced with good precision, the calculated values have the good order of magnitude and respect more or less the hierarchy. We mean by this that the order of a given multiplet is generally the good one and that a large (small) theoretical splitting corresponds to a large (small) experimental one. Let us recall that we have only one free parameter Δ, which has been fitted on the $\Sigma^- - \Sigma^0$ value.

- From time to time the sign is wrong (order is opposite to the experimental one), but this effect occurs generally when compatibility is not excluded due to error bars, or at least when the experimental uncertainty is large.

- Let us comment on the puzzling questions raised in the introduction of this section. The splitting among the pions is larger than the splitting among the nucleons, but not enough to claim that the problem is solved. The $\rho^+ - \rho^0$ mass difference is still wrong but the value is lower than the naive expected one. The characteristic pattern for the D mesons is good but the quantitative values fail. The order in the Σ_c multiplet is a good one and the quantitative value is also very good; this case is really a success (let us mention that in [22] the theoretical value agrees well with the experimental data, but since then the data changed and the agreement is less good with the new value). The problem for the Ξ_c is not solved but the order is the good one for 3 resonances over 4.

Let us now have a look on the situation concerning the wave functions arising from the BSS potential. The mesonic sector is presented in table 5 and the baryonic sector in table 6.

Table 7. Chi-square values divided by the number of data in the sample. The meaning of approximations C, CC, CH, and T have been explained in the text. The meaning of the rows is related to the sub-samples taken into account: 'l', 'h', 'a' for light, heavy and all sectors respectively; 'M', 'B', 'H' for meson, baryon, hadron sectors respectively.

χ^2	AL1				BSS			
	C	CC	CH	T	C	CC	CH	T
lM	568	851	317	513	350	464	500	213
lB	3.3	3.0	3.8	5.0	34	127	23	129
lH	205	306	116	186	147	247	194	159
hM	119	169	90	134	–	–	–	–
hB	3.0	1.5	5.9	1.2	–	–	–	–
hH	51	73	41	56	–	–	–	–
aM	344	510	204	324	–	–	–	–
aB	3.2	2.4	4.7	3.3	–	–	–	–
aH	134	198	81	127	–	–	–	–

The same comments as for AL1 can be made for BSS generally, but in this case the puzzle concerning the ρ has found a solution. Unfortunately, the nucleon splitting is much too large.

Moreover the BSS results are quite different from AL1 results, indicating that, in order to describe the electromagnetic splitting, not only a good form for the electromagnetic part of the interaction is important, but also the strong one via the wave function. This conclusion is very important. In order to draw this conclusion, it was necessary to use very different strong potentials that lead equally good results on spectra both in the mesonic and in the baryonic sectors. These requirements are not easy to be met. AL1 and BSS allow such a fruitful comparison.

In order to grasp more quantitatively the influence of each approximation, as well as to compare the effect of the strong potential, we calculate a chi-square value on the experimental sample (in fact a chi-square divided by the number of data in the set). In order to avoid a too much important weight on very precise values, a minimum uncertainty at 0.1 MeV has been assigned arbitrarily to those values. The results are gathered in table 7. Let us note that this chi-square has no precise statistical relevance; it is just a convenient way to get a synthetic view of the comparison between the various approximations. To have a more refined analysis, we separate the sample for meson (denoted by 'M' in the table), for baryon ('B' in the table), or the entire sample of hadrons ('H' = 'M' + 'B' in the table). Moreover we also distinguish between light sector (u, d, s denoted by 'l' in the table), heavy sector (c, b denoted by 'h' in the table), or all sectors (denoted by 'a' = 'l' + 'h' in the table). For instance the line 'hB' means a chi-square calculated on heavy baryons, 'lM' calculated on light mesons, 'aH' on the entire sample and so on.

Interesting remarks can be emphasized:

- The baryonic sector is explained in a much more satisfactory way, whatever the chosen strong potential. This is the consequence of our arbitrary choice to adjust the free parameter Δ on a baryon resonance. Should we have chosen to fit Δ on a meson resonance, the mesonic sector would have been much best reproduced, but at the price of a dramatic spoiling of the baryonic sector, and with an overall worse description. It appears that a consistent description of electromagnetic splittings for both mesons and baryons is hardly feasible with a model containing only one free parameter. Let us stress

that our model probably does not contain all possible sources of splitting. For instance, the pion mass difference can be explained with the vector meson dominance assumption [39]. Nevertheless, this last model relies on pointlike pion while mesons are composite particles in our approach. So it is difficult to compare such so different phenomenological models of QCD.

- Concerning the BSS calculation, the exact formalism (T approximation) is the best one for mesons, but it is the CH approximation that is the best for baryons and the C approximation (presumably the crudest one) for the entire set. Anyhow, addition of the contact term deteriorates seriously the results.

- Concerning the AL1 calculation, several conclusions are drawn. For baryons, all approximations are roughly of the same quality. What is gained in data by one approximation versus the others is lost in another data. But curiously CC is the best approximation and T the worse one, so that the conclusion is rather different from BSS case. For mesons, CH is always the best approximation and CC the worse. For the totality of the sample this conclusion remains, while the exact treatment (T) is just a bit better than the crudest one (C). Here again the contact term has a very bad influence.

- AL1 and BSS calculations give results of comparable quality for the whole set, but BSS is much better for mesons, and AL1 much better for baryons. This may be explained by the fact that only a perturbative treatment (and not an exact one as in AL1) can be performed with BSS in the baryonic sector.

- It is interesting to compare our results with some previous studies treating both meson and baryon electromagnetic splittings:

  (i) In [21, 22], the results look at least as good as ours, but we want to address two comments. First, some data have changed since that time (for instance, the old value of 1.8 MeV for the $\Sigma_c^{++} - \Sigma_c^0$ splitting is now 0.35 MeV) and new ones are now available (for instance, their calculated value of 4.4 MeV for the $D_2^+ - D_2^0$ is now experimentally estimated at 0.1 MeV). Second, the value of the electromagnetic quark size is determined by a best fit on the splittings, while in our case, the same parameter is fixed by meson form factors and cannot be considered as a free parameter.

  (ii) In [23], the results look also rather good, but there exist several differences with the present work. First, the contact term is absent. Second, only the hyperfine term is dressed in order to avoid a collapse. Third, two free parameters were used, the u and d masses separately.

No doubt that if we have allowed more free parameters in our model (for example releasing the constraint $(\widetilde{m}_u + \widetilde{m}_d)/2 = m_n$), our results would have appeared better. But this was not the ultimate goal of our paper; we were interested in discovering what are the necessary ingredients to explain the splittings.

Another important point for the consistency of our approach is that, with the new values of the u and d masses, the absolute masses for the meson and baryon resonances are also well reproduced. This point is not often studied in previous works. Just to convince the reader that our formalism is able to provide a good description of meson and baryons simultaneously, we present below the absolute masses of one member of the multiplet (the other can be obtained using the values of the splittings given above) for both mesons (table 8) and baryons (table 9). The T approximation is used and the treatment is exact except for baryons with the BSS potential.

The small discrepancy for light baryons in the case of AL1 potential can be attributed to a three-body force [32], which is mass dependent. The instanton does not give rise to a three-body force for the baryon [16, 34], and the parameters have been fitted directly to

Table 8. Absolute masses for some mesons. Calculations are done with strong potentials AL1 and BSS, and with the total electromagnetic Hamiltonian. The experimental values (Exp) are taken from [24].

System	Exp	AL1	BSS
π^0	134.98	137.3	145.8
ρ^0	769.0	769.7	756.2
K^+	493.68	486.6	491.2
K^{+*}	891.66	902.7	888.7
K_2^+	1425.6	1333.4	1377.5
D^+	1869.3	1863.2	–
D^{+*}	2010.0	2016.4	–
D_1^+	2427	2419.6	–
D_2^+	2459	2451	–
B^+	5279	5295	–

Table 9. Same as table 8 for baryons.

System	Exp	AL1	BSS
p	938	994	935
Δ^0	1234	1308	1260
Λ	1116	1149	1105
Σ^-	1197	1233	1201
Σ^{-*}	1387	1439	1395
Ξ^-	1321	1343	1323
Ξ^{-*}	1535	1560	1522
Ω	1672	1675	1646
Λ_c^+	2285	2290	–
Σ_c^{++}	2453	2466	–
Ξ_c^+	2466	2467	–
$\Xi_c^{+\prime}$	2574	2572	–
Ξ_c^{+*}	2647	2650	–
Ξ_c^{+**}	2815	2788	–
Ω_c^0	2697	2675	–
Λ_b^0	5624	5635	–

the absolute masses of mesons and baryons. The agreement with experimental data is rather satisfactory.

5. Conclusions

In this paper, we calculated the electromagnetic splitting on hadronic systems. This observable is a very sensitive test of the formalism because it results from a very subtle and fine balance between several physical ingredients. In order to concentrate on the physical aspects of the problem, we were very cautious in the numerical treatment, both for the two-body and three-body problems. Thus we are very confident with our numerical results, and interesting conclusions can be emphasized.

As compared to previous works we considered the total electromagnetic Hamiltonian (excepted Darwin, spin–orbit and tensor forces that we believe to play a very minor role).

In particular we took into account the so-called contact term that is usually neglected. Moreover, we treated the quarks as constituent particles with an electromagnetic size that modifies the form of the electromagnetic interaction. Lastly, we based our calculations on wave functions resulting from two different strong potentials: the phenomenological AL1 potential to be used with a nonrelativistic kinematics energy operator and the more fundamental BSS potential, including instanton effects, to be used with a relativistic kinetic energy operator. The size of the quarks were determined in order to reproduce the pion and kaon charge form factors. Only one free parameter, the mass difference between down and up quarks, is left at our disposal; it was fitted on the $\Sigma^- - \Sigma^0$ splitting. We want to emphasize that in this paper, with only one parameter, we try to reproduce the totality of the experimental data, including mesons and baryons. To our opinion, this is a condition to pretend to some consistency in the formalism. This condition is rarely met in previous calculations, since authors very often restricted themselves either to mesons (or even to less restrictive domains) or to baryons.

We first showed that dressing the strong and electromagnetic interaction is a necessity to obtain in a consistent way both the hadron spectra, the meson form factors and the electromagnetic splitting.

By comparison of AL1 and BSS results, we stressed that the strong potential, via the wave function, is an important ingredient in the description of electromagnetic splittings. In our particular case, AL1 gives a better description of baryons and BSS a better description of mesons.

But we proved also that the electromagnetic Hamiltonian is equally important for explaining the splittings. Adding or removing a term (contact or hyperfine) has a nonnegligible influence. In particular taking into account the contact term spoils much of the results, and curiously it is the approximation based on coulomb + hyperfine (the usual ingredient for many people) which is globally the best.

In our formalism the splittings are described in a reasonable way, specially owing to the fact that we have only one free parameter to try to reproduce 26 experimental data; the order among multiplet masses is generally correct and the values have the right order of magnitude. But the agreement is far from being perfect; some suggested puzzles have been solved but some others are still open questions. New improvements must be done in future studies.

Acknowledgments

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