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Abstract

This article presents a framework for evaluation of the impact of languages in multilingual societies. We consider several ranking methods grounded on various principles, including *minimal disenfranchisement*, *communicative benefits*, *utilitarianism*, and the game-theoretical concept of the *Shapley Value*. We use data from the Special Barometer survey to apply these methods to languages within the European Union. Although the methods we consider are driven by distinct normative grounds, they generate quite similar results. Finally, we analyse the impact of *Brexit* on the rankings, especially if English forfeits its status as an *official* language in the Union.

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1 Introduction

Multilingualism is a pervasive phenomenon that can be traced back to ancient times. If people were members of small language communities, it was necessary for them to know two or more languages for trade or any other business and social interaction outside their own town or village (Holden, 2016). In more recent times, globalization and cultural openness foster multilingualism. A point in case is Europe, where there is no predominant language and English is often used as a language of communication. However, in multilingual countries such as Belgium (French, Dutch and German), Switzerland (German, French, Italian and Romanche), Spain (Spanish, Catalan, Basque and Galician) or Finland (Swedish and Finnish), it is usual to master two or even more languages. Some languages such as Danish, Swedish and Norwegian are close enough that it is generally more common for people to use their mother tongue rather than English in meetings.

In many situations, whether for practical or political reasons, it is necessary to select a language, or a group of languages. This could be the case for the choice of official languages, or for selecting the language which is used for business or social transactions between individuals with different mother tongues. In this paper, we present a stylized framework to rank languages in multilingual societies, such as the European Union (EU). Our methods allow for different commands of each language.

The first principle we examine, that of *Minimal Disenfranchisement*, focuses on the number of individuals who do not speak any of the chosen languages.[2] It is equivalent to Van Parijs' *Minimex* principle of minimal exclusion, which searches for a language that guarantees that the number of individuals who do not speak this language is the smallest possible. The notion of linguistic disenfranchisement, which may arise when the linguistic rights of parts of the society are denied or restricted, may have far-reaching consequences for economic growth and political stability and should be treated very carefully.

Another principle is that of *Communicative Benefits*, introduced by Selten and Pool (1991).[3] Note that, while the principle of Minimal Disenfranchisement does not distinguish between perfect and poor command of the language, as long as an individual speaks it, the communicative benefits approach is based on the perfect command of a language. It asserts that the utility of an individual is positively correlated with the number of others with whom he can perfectly communicate. Indeed, trade opportunities, earnings

[2]See Ginsburgh and Weber (2005) and Ginsburgh et al. (2005) for the discussion on linguistic disenfranchisement, with a special emphasis on the EU.

[3]See also Ginsburgh et al. (2007), Ginsburgh and Weber (2011), and Athanasiou et al. (2016).

and job prospects could be enhanced by the ability of individuals to communicate with each other. The opportunity to learn about different cultures becomes more important in the globalized world we live in. In our context, it implies that a language is "superior" to another if it allows more pairs of individuals to communicate perfectly. It is easy to see that the highest level of communicative benefits obtains by choosing the language with the largest number of individuals who know it perfectly.

The two principles focus on the extreme cases of language knowledge, where individuals either do not know it at all or speak it perfectly. The *Aggregate Knowledge* principle would allow to extend this dichotomy to intermediate cases of language knowledge and postulates that a language is preferred to another if its aggregate knowledge (weighing by individual degrees) is higher than that of another language. This is essentially the principle underlying *Utilitarianism*, a deeply rooted notion in economics and philosophy, which can be traced back to Jeremy Bentham and John Stuart Mill.

These three principles would trivially lead to the same ranking under the premise of dichotomous knowledge of languages, in which agents either speak or do not speak a certain language. In a more general setting, which allows for intermediate knowledge levels, we show that the principles may yield different rankings.

We also consider two game-theory-based rankings by using the *Shapley Value*, the well-known solution concept for cooperative games. In our setting, this concept amounts to the following. Suppose each agent speaks a set of languages and assigns, in some predetermined way, the weights to each of them. These weights could be either equal, or proportional to the levels of knowledge. Adding these weights across agents produces a score for each language and, thus, their ranking. As we show later, the two ensuing rankings based on the Shapley value can lead to rankings that are fundamentally different from the previous three.

The five methods are described in more detail in Section 2, where we also provide normative foundations for them. In Section 3, we use data from a Special Eurobarometer survey and apply these methods to rank languages spoken in the (pre-Brexit) EU. The results, somewhat surprisingly, indicate that, in spite of their different normative grounds, all rankings yield very similar results. This indicates that the ranking of languages within the EU is more robust than what might be thought. Section 4 uses *Minimal Disenfranchisement*, *Communicative Benefits* and *Aggregate Knowledge* to study the consequences of Brexit. Section 5 concludes.

2 Defining and analyzing rankings

We describe the linguistic landscape of a given society by a *language matrix*, A, whose rows refer to citizens (*agents*), and its columns to *languages*. Each entry a_{ij} of the matrix denotes the knowledge level of language j by agent i. We make the normalizing assumption that $a_{ij} = 0$ reflects no knowledge whatsoever, $a_{ij} = 1$ reflects perfect knowledge, and also assume that there exists a finite set of intermediate levels of knowledge ranging from 0 to 1. Here is an example of such a matrix.

Example 1

$$A = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \\ 1 & 0 & 1/2 \\ 1 & 0 & 1/2 \\ 0 & 1 & 0 \\ 0 & 1 & 1/2 \\ 1 & 1/2 & 1/2 \end{pmatrix}.$$

This matrix refers to the case in which there are eight agents and three languages. Agents 1, 2 and 3 only have intermediate knowledge of languages 2 and 3, which we model by the number 1/2, and no knowledge of language 1. Agents 4 and 5 have perfect knowledge of 1, intermediate knowledge of 3 and no knowledge of 2. Agent 6 only has perfect knowledge of 2, and does not know any other. Agent 7 has perfect knowledge of 2, intermediate knowledge of 3 and no knowledge of 1. Finally, agent 8 has perfect knowledge of 1, and intermediate knowledge of the other two languages.

2.1 Definitions

We now introduce our ranking methods. Formally, let N denote the set of agents, and L the set of spoken languages in the society.

Minimal Disenfranchisement (MD). Speakers of various languages are accounted for, whether they speak the language perfectly or not, as long as they speak it to some degree. Proficiency is thus disregarded, and all positive entries in matrix A are replaced by ones. We define the *Minimal Disenfranchisement* (MD) ranking as the one that ranks languages according to the number of disenfranchised individuals, i.e., those who do not know them.[4] The larger the number of disenfranchised agents, the lower the ranking of the language. Thus, according to MD, language j is ranked higher than language k if $\#\{i \in N | a_{ij} = 0\} < \#\{i \in N | a_{ik} = 0\}$. In Example 1, MD thus ranks language 3 first (7 speakers), then language 2 (6 speakers) and finally, language 1 (3 speakers).

Communicative Benefits (CB). The entries a_{ij} that represent the degree of knowledge of a language can be bounded from below by imposing $a_{ij} \geq \underline{a}_i$ and $a_{ik} \geq \underline{a}_k$, where both $\underline{a}_i$ and $\underline{a}_k$ take values between 0 and 1. If both bounds are set to 1, only perfect or native speakers are counted. Thus, according to CB, language j is ranked higher than language k if $\#\{i \in N | a_{ij} = 1\} > \#\{i \in N | a_{ik} = 1\}$. This is equivalent to ranking languages according to the number of pairs that could communicate perfectly in each language. In Example 1, CB ranks language 1 first (3 speakers), language 2 second (2 speakers), and finally, language 3 last (0 speakers).

Aggregate Knowledge (AK). If both $\underline{a}_i$ and $\underline{a}_k$ are set to $\varepsilon > 0$ sufficiently small, all agents who know the language, even if their knowledge is not perfect, are accounted for. But we count them weighing by their knowledge levels. In Example 1, language 1 scores 3, language 2 scores 4, and language 3 scores 3.5. Language 2 is thus first, language 3 is second, and language 1 is third. More generally, language j is ranked higher than k if $\sum_{i \in N} a_{ij} > \sum_{i \in N} a_{ik}$. AK ranks languages according to their aggregate knowledge level across society.

The first two rankings can be interpreted as translations to our context of *Approval Voting* (see Brams and Fishburn, 1978). This method allows each voter to cast his or her vote for as many candidates he or she wishes; each positive vote is counted in favour of the candidate. The votes are then added by candidate, and the winner is the one who gets the largest number of votes. All other candidates can also be ranked, according to the number of votes they obtain. *Minimal Disenfranchisement* obtains when an agent *approves* all languages with partial knowledge ($a_{ij} > 0$), whereas the *Communicative Benefits* obtains when an agent *approves* all languages with perfect knowledge only ($a_{ij} = 1$).

The third ranking can be thought of generalizing *Approval Voting*, to *Range Voting* (see Smith, 2004) or *Evaluative Voting* (see Hillinger, 2005).[5] Here, agents can express

[4]We may extend the notion of disenfranchisement to include those who speak languages only superficially.

[5]The difference between the methods actually lies on whether alternatives (knowledge levels in our

further evaluations of alternatives (for example numbers between 0 and 1) beyond just approving (or disapproving) them.

An alternative to Approval Voting is *Cumulative Voting* (Glasser, 1959; Sawyer and MacRae, 1962). It allows voters to distribute points among candidates in any arbitrary way. An interesting case is the one in which every agent is endowed with a fixed number of votes that are evenly divided among all candidates for whom he votes. This would translate into our context as the *Shapley ranking.*[6] Formally:

Shapley (S). Let $L_i(A) = \{j \in L : a_{ij} > 0\}$ denote the set of languages agent $i \in N$ has some knowledge of, and let $l_i(A)$ denote the number of such languages. Then, we say that language j is ranked higher than k if

$$\sum_{i \in N, j \in L_i(A)} \frac{1}{l_i(A)} > \sum_{i \in N, k \in L_i(A)} \frac{1}{l_i(A)}.$$

Consider the following matrix in which only the two extreme levels of knowledge are present.

Example 2

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}.$$

Matrix A describes a situation with three agents and four languages. Agent 1 only knows language 1, while the two other agents know languages 2, 3 and 4. Then, as stated above, the first three ranking methods discussed earlier would endorse the ranking in which languages 2, 3 and 4 come first, and language 1 comes last. The Shapley ranking inverts the ranking. Indeed, language 1 obtains a score of 1, while each of the three others obtains 2/3 (1/3 for each voter times 2 as there are two voters) and are thus ranked after language 1.

In order to interpret Shapley rankings, one could imagine that each agent is endowed with one vote which is shared among the candidates for whom he or she votes. Shares for each candidate are then added as above, and the candidate who gets the largest number

setting) can take any value in $[0, 1]$ or only a finite number of values.

[6] The name of the ranking comes from the rule introduced by Ginsburgh and Zang (2003) for the so-called museum pass game. Bergantiños and Moreno-Ternero (2015) extend the rule to more general museum pass problems. See also Ginsburgh and Zang (2013), who apply the rule for the ranking of wines.

of shares wins. All other candidates are ranked accordingly. As shown by Ginsburgh and Zang (2003), these numbers represent *Shapley Values* of candidates, which can also be interpreted as their *powers* resulting from the vote. This is different from Approval Voting where voters can cast as many votes as they wish.

To conclude with the inventory of rankings, one could consider a reasonable generalization of the Shapley ranking, in which agents distribute points among languages proportionally to their knowledge levels, which we call *Weighted Shapley* ranking. Formally,

Weighted Shapley (WS). Language j is ranked higher than k if

$$\sum_{i\in N, j\in L} \frac{a_{ij}}{\sum_{h\in L} a_{ih}} > \sum_{i\in N, k\in L} \frac{a_{ik}}{\sum_{h\in L} a_{ih}}.$$

Obviously, (S) and (WS) would coincide for cases in which knowledge is only maximal or minimal (such as in Example 2). Consider now the following matrix with four agents and three languages:

Example 3

$$A = \begin{pmatrix} 1 & 1/4 & 3/4 \\ 0 & 1/4 & 1 \\ 0 & 1 & 0 \\ 1/4 & 0 & 0 \end{pmatrix}.$$

Agent 1 speaks language 1 perfectly, and languages 2 and 3 partially (the last, better than the second). Agent 2 speaks language 2 partially and language 3 perfectly. Agent 3 only speaks language 2 (and she does so perfectly). Finally, agent 4 only speaks partially language 1.

The Shapley (S) scorings are as follows: Language 1 gets 1/3 from agent 1 and 1 from agent 4, This totals 4/3. Language 2 gets 1/3 from agent 1, 1/2 from agent 2, and 1 from agent 3 which totals 11/6. Language 3 gets 1/3 from agent 1 and 1/2 from agent 2 which totals 5/6. Therefore, language 2 would come first, followed by language 1 and then language 3.

The Weighted Shapley (WS) scorings would come as follows: Language 1 gets 1/2 from agent 1 and 1 from agent 4 which totals 3/2. Language 2 gets 1/8 from agent 1, 1/5 from agent 2, and 1 from agent 3 which totals 53/40. Language 3 gets 3/8 from agent 1 and 4/5 from agent 2 which totals 47/40. This produces the following ranking: language 1 would come first, followed by 2 and then 3.

2.2 A normative analysis

We now consider a number of reasonable assumptions (axioms). As we shall see different combinations of these axioms will characterise each of the five rankings presented above.

Axiom 1 Low Invariance. The ranking is not altered if non-perfect knowledge levels are replaced by no-knowledge levels, that is if every $a_{ij} < 1$ in matrix A is replaced by 0.

Axiom 2 High Invariance. The ranking is not altered if positive non-perfect knowledge levels are replaced by perfect knowledge level, that is if every $0 < a_{ij} < 1$ in matrix A is replaced by 1.

Axiom 3 Compensation. If all *but two* agents in a society have the same knowledge of two languages j and j', with the exception of agents i and i', and if i knows language j perfectly and has no knowledge of j', while the reverse holds for i', then both languages j and j' are ranked at the same level.[7]

Axiom 3' Generalized Compensation. Let the set of knowledge levels of every language and every agent be finite. If agent i increases her knowledge of language j to an immediate higher level, while agent i' decreases her knowledge of j to the immediate lower level, the ranking of languages does not change.[8]

Axiom 4 (Strong) Pareto Optimality. If all agents speak language j at least as well as language j', with at least one agent speaking j strictly better than j', then j is strictly ranked above j'.

We obtain the following characterizations for rankings MD, CB and AK.[9]

Proposition 1 *MD is the unique ranking that satisfies Axioms 1, 3 and 4.*

Proposition 2 *CB is the unique ranking that satisfies Axioms 2, 3 and 4.*

Proposition 3 *AK is the unique ranking that satisfies Axioms 3' and 4.*

We chose to focus on our specific case to provide new characterization results for our first three rankings. Note that some of the many existing characterizations of approval voting (see, for instance, Xu, 2010) can be used for the *Minimal Disenfranchisement*

[7]This is a weaker version of an axiom recently introduced by Macé (2015) who uses the same terminology, and will be considered next.

[8]This is the axiom recently introduced by Macé (2015) under the term *Compensation*.

[9]Appendix 1 contains more formal statements of the axioms, as well as the proofs of the propositions.

and *Communicative Benefits* rankings. Likewise, some of the many existing characterizations of *utilitarian* criteria (see, for instance, Blackorby et al., 2002) can be used for the *Aggregate Knowledge* ranking.

For the *Shapley* rankings, we consider a different scenario, previously used by Bergantiños and Moreno-Ternero (2015).[10] More precisely, suppose that each agent has a *vote* to be allocated among the languages he speaks. The ranking would then be generated from the overall number of votes allocated to each language. We consider the next three axioms for such a context.

Axiom 5 Equal Treatment of Equals. If two languages have the same number of speakers, then they should receive the same number of votes.
Axiom 6 Dummy. If no agent speaks a language, then it should get no votes.
Axiom 7 Additivity. Given two groups of speakers, it is equivalent to consider them separately, or as the same group.

This leads to Proposition 4, which can be stated as follows:[11]

Proposition 4 *S is the unique ranking that satisfies Axioms 5, 6 and 7.*

Such a result is thus equivalent to the seminal characterization of the *Shapley value* for TU-games (e.g., Shapley, 1953).[12] The Shapley value is the best-known solution concept for those games, which model the attempt to predict the allocation of resources in multi-person interactions (see Winter, 2002). It is remarkable not only for its attractive and intuitive definition but also for its unique characterization by a set of reasonable axioms. In addition, the value is also viewed as an index for measuring the power of players (here, languages) in a game (see Shapley and Shubik, 1954). The value uses averages (or weighted averages in some of its generalizations) to aggregate the power of players in their various cooperation opportunities.

[10] We are not aware of any axiomatic characterization of (Equal and Even) Cumulative Voting. Both Approval Voting and Cumulative Voting can, however, be seen as members of a family of voting procedures dubbed as *Size Approval Voting*, which are characterized by Alcalde-Unzu and Vorsatz (2009).

[11] See Appendix 1 for the formal proof.

[12] A coalitional game is cooperative if the players can make binding agreements about the distribution of payoffs or the choice of strategies, even if these agreements are not specified or implied by the rules of the game. Transferable-utility games (in short, TU-games) are one category of cooperative games in which one specifies a function that associates with each nonempty coalition a real number indicating the worth of the coalition. If a coalition forms, then it can divide its worth in any possible way among its members. A comprehensive analysis of TU-games can be found, for instance, in Peleg and Sudholter (2007).

One of the axioms characterizing the Shapley value is *Equal Treatment of Equals*. This is a compelling axiom when information is limited. For some specific applications, however, we might possess more information about the environment, which motivates the asymmetry of the solution concept.[13] This interpretation led to the concept of *Weighted Shapley Value*, which has also been characterized in the literature, replacing the axiom *Equal Treatment of Equals* by *Partnership Consistency*, which refers to the treatment of players who can only generate value together (see Kalai and Samet, 1987). We consider another related alternative to *Equal Treatment of Equals*, which leads to the characterization of WS.

Axiom 5' Weighted Treatment of Equals. If a speaker speaks two languages, then she should allocate vote shares among them that are proportional to their knowledge levels.

This leads to the characterisation of the WS ranking:

Proposition 5. *WS is the unique ranking that satisfies Axioms 5', 6 and 7.*[14]

3 Languages in the European Union

We now use the framework described in Section 2 to rank the main languages used in the EU. We distinguish official languages (see below) from other languages that are spoken, but are not official.

Official languages. In 1958, the Treaty of Rome and Regulation 1/1958 recognized four languages Dutch (NL), French (F), German (G) and Italian (I) as official languages.[15] Danish (DK), English (GB), Finnish (FIN), Greek (GR), Portuguese (P), Spanish (SP) and Swedish (SW) were added later. The 2004 enlargement to Eastern Europe resulted in adding Czech (CZ), Estonian (EST), Hungarian (H), Latvian (LV), Lithuanian (LT), Maltese (M), Polish (PL), Slovak (SL), and Slovenian (SLO). Irish (IRL) was given the same status in 2005 but it was agreed that the decision would be implemented only as of January 2007. We also included Bulgarian (BG) and Romanian (RO) which became

[13] In our context, this is so if we allow for different knowledge levels of languages.

[14] See Appendix 1 for the formal proof.

[15] Though the Treaty included six countries, the three languages that are spoken in Belgium (French, Dutch and German) are also spoken in other partner countries (France, the Netherlands and Germany); Luxembourg agreed not to include Luxembourgish (LX).

official in 2007 only, but were already included in the survey to be discussed below.[16] All these languages, listed as *Official* in Table 1 enjoy the same privileges as the original four.

Other languages. Table 1 also includes seven other languages that are used in EU countries, but are not official. Russian is spoken in many former Eastern Bloc countries; Basque, Catalan and Galician are spoken in Spain, and Luxembourgish in Luxembourg; Arabic and Turkish are languages mainly spoken by immigrants.[17]

To determine who speaks what, we use the Special Eurobarometer 243 (2006) survey carried out in November 2005 in all member countries of the EU, including Bulgaria and Romania (that were not yet members in 2005). In most countries, 1,000 citizens were interviewed, with the exception of Germany (1,500), the United Kingdom (1,300), Cyprus (500), Luxembourg (500) and Malta (500). The total number of usable interviews amounts to 26,700.[18]

The data that we use are taken from the answers to the following questions:

(a) D48a. What is your mother tongue? (do not probe – do not read out – multiple answers possible). Follows a list of 34 languages that include the 23 member states' official languages, as well as Arabic, Catalan, Chinese, Croatian, Luxembourgish, Russian, Turkish, Basque, Galician, Other regional languages, Other.

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[16] Croatia and Turkey were also candidates to accessing the EU, but this happened much later for Croatia, and did not happen for Turkey. Croatian is thus not included in our list. Turkish is, but only as a non-official language, spoken by migrants, and as a co-official language in Cyprus.

[17] Turkish is also, as mentioned above, a co-official language in Cyprus, which is a country included in our survey.

[18] It is worth mentioning the following technical specifications which ensure representativity:

"[...] the survey covers the national population of citizens [...] that are residents in those countries and have a sufficient command of one of the respective national language(s) to answer the questionnaire. The basic sample design applied in all states is a multi-stage, random (probability) one. In each country, a number of sampling points was drawn with probability proportional to population size (for a total coverage of the country) and to population density.

For each country a comparison between the sample and the universe was carried out. The universe description was derived from Eurostat population data or from national statistics offices. For all countries surveyed, a national weighting procedure, using marginal and intercellular weighting, was carried out based on this universe description. In all countries, gender, age, region and size of locality were introduced in the iteration procedure."

(b) D48b to D48d. Which languages do you speak well enough in order to be able to have a conversation, excluding your mother tongue? (do note probe – do not read out – multiple answers possible). Follows the same list of 34 languages. This question was asked for first, second and third foreign languages.

(c) D48f. Is your (language cited in 48b, 48c and 48d) very good, good, basic? (show card with scale).

In our calculations of the various rankings, we code as 1 the responses *mother tongue* (question D48a), and as 1/2 the responses to other languages known (questions D48b to D48d), combined with the self-evaluation of knowledge *very good* and *good* (question D48f). All other responses, that is *basic*, or *not knowing the language* are coded as 0.

Table 1 summarizes our findings. Note that we separate Russian, Arabic, Turkish, Catalan, Basque, Galician as well as Luxembourgish, which appear at the bottom of the table, as they are not official.[19] These results are obtained by considering the whole EU as a unique country. As the number of units surveyed are not proportional to the populations of the 27 countries i(Croatia is not included, see footnote 16) included, we had to weigh the numbers of each country by its population.[20]

[Table 1 approximately here]

In the table, column (1) contains the name of the language and column (2) the countries where it is official. Columns (3) to (7) contain the following rankings: *Minimal Disenfranchisement* (MD), *Communicative Benefits* (CB), *Aggregate Knowledge* (AK), as well as the *Shapley* ranking (S) and the *Weighted Shapley* ranking (WS):[21]

(a) column (3) (MD) contains the shares of the total EU population that does not know the language or whose knowledge is basic; for instance, the value 63 that appears in

[19] With the caveat made before about Turkish in Cyprus.

[20] More precisely, the population (in millions) we used for each country was the following: Austria 8.2; Belgium 10.4; Bulgaria 7.8; Cyprus 0.7; Czech Rep. 10.2; Denmark 5.4; Estonia 1.3; Finland 5.2; France 60.6; Germany 82.5; Great Britain 60; Greece 11.1; Hungaria 10.1; Ireland 4.1; Italy 58.5; Latvia 2.3; Lithuania 3.4; Luxembourg 0.5; Malta 0.4; Netherlands 16.3; Poland 38.2; Portugal 10.5; Romania 21.7; Slovakia 5.4; Slovenia 2; Spain, 43; Sweden 9.

[21] The numbers that appear in column (3) guided the ordering in which the languages appear in the table. This is of course arbitrary, as any other column could have been chosen as well.

the first row of column (3), means that 63 percent of the EU population does not know or only has a basic knowledge of English.[22]

(b) column (4) (CK): the numbers represent the shares of the total EU population which do not speak the language as mother tongue; the value 87, which appears in the first row of column (4), means that 87 percent of the EU population do not have English as mother tongue.

(c) column (5) (AK): the numbers are less obvious to be interpreted in a simple way, as they result from adding knowledge levels (a_{ij}) of which some are equal to 1, others are equal to 1/2. For consistency, we adopted the same convention as in the previous two rankings and express the numbers in terms of the percentages of the EU population. For example, the value 75, which appears in the first row of column (5), means that 25 percent of the EU population is obtained when we aggregate all individuals having English as mother tongue and one half of all individuals having good (or very good) knowledge of English.

(d) (6) and (7) (S) and (WS): the rankings using *Shapley Values* have no meaning other than giving a measure of the power of a language; what really counts here is the order, and to some extent, the differences between languages. One could surmise that English and Swedish are far away from each other, but there is no difference between Swedish and Bulgarian.

The main lesson to be drawn from our empirical analysis is that the rankings of official languages are extremely similar, in spite of the theoretical differences and distinct normative grounds of the ranking methods discussed above. The *Communicative Benefits* ranking, in column (4), points, nevertheless, to a couple of important differences with respect to the *Minimal Disenfranchisement* ranking in column (3). Most languages in the first half part of the ranking disenfranchise more citizens than in column (3), as they are not counted if not perfectly spoken. German is the most obvious change since it is the language with the highest number of native speakers, but it also is a language that less non-native individuals speak (even at intermediate levels), compared to English or French. English, French and Italian would generate identical levels of disenfranchisement.

In the *Shapley* ranking, the downgrading of Dutch is due to the fact that the number of Netherlanders and Flemish (in Belgium) who speak foreign languages (English,

[22]The 100 numbers appearing in this column, as well as the next ones, are a consequence of rounding, which we did to the closest integer percentages.

German and French essentially) is large, which decreases the Shapley Value of Dutch. The upgrading of Romanians is the consequence of the inverse phenomenon: Most of their population speaks only Romanian, which increases the Shapley Value. In the (WS) ranking, Romanian raises to rank 5, and comes before Spanish and Polish.

Russian is an interesting case as it belongs to the ten most important languages in the EU (with the exception of the CB ranking, as it is often the second language. This is, of course, due to the Russian influence in Eastern Europe after Word War II, where it is often a second language only (which explains why it is less well ranked in the CB, S and WS ranking). The forces in action nowadays may well decrease the knowledge of Russian among the younger generations, but there are also forces in the other direction, resulting from increased trade relations with Russia, that may foster the learning of Russian.

4 The consequences of Brexit

English is not only a so-called *official* language of the EU, but it is also the one that is most widely spoken as native (or well-known) by some 182 million out of 490 millions Europeans. It became de facto the *lingua franca* in Europe, though it disenfranchises large groups in many individual countries.[23] Following the pro-Brexit referendum held in the UK in June 2016, three main scenarios are possible.

First, the UK does not exercise the power it was given by the referendum to quit the EU, or it simply delays this power. This is increasingly discussed in the media. At the time of our writing, if Brexit happens, it will not be before 2019, as article 50 of the EU which triggers the starting point will not be invoked before 2017.[24]

Another scenario is that Brexit occurs, and that English loses its official status in the EU,[25] though it is an official language in Ireland (together with Gaelic) and in Malta (together with Maltese). It is indeed spoken by a very large majority of citizens in Ireland, which accessed the EU in 1973. Some 25 years later, it asked Gaelic to become its official language. This was accepted by the EU, and on January 1, 2007 Gaelic became one of the 24 official languages, with some derogations, that should however be brought to an

[23] See Fidrmuc et al. (2007).

[24] Reuters, Brexit may be delayed to allow government to prepare, *Newsweek*, August 14, 2016.

[25] Danuta Hbner, head of the European Parliament's Constitutional Affairs Committee (AFCO) warned on June 26 that English will lose its official status. See Hortense Goulard, English will not be an official EU language after Brexit, says senior MEP. See http://www.politico.eu/article/english-will-not-be-an-official-eu-language-after-brexit-senior-mep/, consulted on August 16, 2016.

end by December 31, 2021.[26] English is also spoken by a majority of Maltese. Accepting that Ireland (4.1 million inhabitants) and Malta (0.4 million inhabitants) each have two official languages (though English is common) may trigger other countries (or regions) to get their language accepted. Catalan that is spoken by 6.2 million people in Catalonia may be first in a row. Galician may follow suit with 3 million speakers, as well as Basque with 0.7 million people. All three languages are spoken by more people than Maltese and Gaelic, and Catalan is spoken by more people than the whole of Ireland. It may be that either Ireland or Malta accept to replace their official language by English, but it will be politically difficult for both of them to face their own population with such a proposal.

A third scenario is that the EU considers nevertheless keeping English as an official language. This may be rebuked by some countries, and if so, require a vote by the European Council. According to the EU's constitutional rules, votes on languages have to be unanimous. It is doubtful that Germany and France will cast a positive vote, as they both fight against English, which is indeed overused as a working language.[27]

We now explore the last two scenarios using three of the rankings discussed earlier (*Minimal Disenfranchisement*, *Communicative Benefits* and *Aggregate Knowledge*). We start with some intuitive numbers, and will refine this later using rankings. These numbers are not influenced by that fact that English may use its official status in the EU. They are just the consequence of the UK leaving the EU.

In 2005, the EU had some 490 million citizens. 182.5 of them were reasonably fluent in English. German (128.5 million), French (100.7 million), Italian (68.4 million), Spanish (57.2 million) and Polish (43.5 million) followed.[28] Brexit has two effects. The important one is the loss of some 60 million British native speakers, so that the 182.5 millions will drop to 122.5 million. The second effect, which is much less important, is concerned with the reduction of Britons and immigrants who live in the UK, and who know one of the other EU languages, in particular some 2 and 6 millions speakers of French and German.

[26]See Europa, Interinstitutional style guide, section 7.2.4, Rules governing the languages of the institutions.

[27]English is also heavily used in reports and rules set by EU bureaucrats. In 2008, 72.5 percent of the first versions of administrative and legislative documents were written in English, while 14 and 3 percent only were written in French and German, respectively. The situation has even become more skewed as English now makes for some 82.5 percent, while German dropped to 2 percent. To top things off, in 2014, 261.000 pages have been translated from other languages into English, 155.000 into French and 136.000 into German. This means that the EU essentially writes in English and translates into English.

[28]These numbers include native speakers, as well as non natives who acquired the language, and know it very well or well.

Other languages will also incur some losses, though these are smaller (Italian, -1 million, Spanish, -1.3 million).

[Table 2 approximately here]

This leads us to columns MD in Table 2, where we compare *Minimal Disenfranchisement* (which includes native speakers of a language as well as those who speak it very well and well) before (column B) and after (column A) Brexit, taking into account that the number of EU citizens dropped from 488 to 428 millions, as 60 million Britons will have left the EU. As can be seen, the number of speakers of German and English are equal after Brexit, which was far from being the case before. There is still a 'qualitative' difference, as those who speak English (121 million) are not living in the UK, while 91 million among the 121 million speakers of German are living in German speaking countries (Austria and Germany). Therefore, English could still keep its *lingua franca* status in the EU, whether or not it loses its official status.

The CB (*Communicative Benefits*) rankings in Table 2 are based on native speakers only. Here, the problem of English is getting very acute, as it will be spoken by some 6.5 million only, of which 4.1 live in Ireland. So English does not do better than any language that is spoken by more than 6.5 millions inhabitants, which include Bulgarian, Czech, Dutch, French, German, Greek, Hungarian, Italian, Polish, Portuguese, Romanian, Spanish and Swedish, that is, more than half of EU's official languages.

The *AK* (*Aggregate Knowledge*) ranking, in which native speakers are counted for one, while others, who speak it very well or well are counted for 1/2, produces a ranking in which German (76) and French (82) are better off than English (85), while Italian (86) and Spanish (89) are very close to English in terms of communication, though all of them, but English, are more local (in one or two countries) than English.

In conclusion, English remains a powerful language in Europe even after *Brexit*. The real question is whether it will remain an official language in the EU. If so, it will be interesting to see what kind of arguments will be used by the European Commission. Pushing English it as an official language in Ireland and Malta is quite unlikely to be accepted by the Germans and the French, and/or it opens the possibility of other languages wanting to become official (Catalan, in particular). If English loses its official status in the EU, it will no longer be used in the European Parliament nor will official documents be translated into English. The question is whether it will be able to maintain its status as working language.

5 Conclusions

In this paper, we provide a stylized framework for ranking languages in multilingual societies. We introduce five ranking methods. Three of these reflect appealing principles such as minimizing exclusion (MD), maximizing communication benefits (CB), or aggregating knowledge levels (AK). The two last, (S) and (WS), are inspired by game-theoretical concepts. We explore the normative foundations for each method and apply them for ranking languages in the EU. The results are remarkably similar in spite of the different nature (and normative grounds) of the methods. Some small differences do emerge but may be the consequence of sampling, with the exception of English and German in the CB ranking. The role of Russian, which is not a EU language is interesting. Nevertheless, the main lesson to be drawn from our analysis is that the ranking of official languages within the pre-Brexit EU is quite robust. The future role of English within the EU if Brexit is exercized will undoubtedly impact the linguistic reality and policies of the Union.

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7 Appendix

We provide in this appendix a formal treatment to the contents of Section 2.2.

For each language matrix A, we construct two matrices associated to it: A^L and A^H. Formally, for each $(i,j) \in N \times L$,

$$a_{ij}^L = \begin{cases} 0 & \text{if } a_{ij} < 1 \\ 1 & \text{otherwise} \end{cases}$$

and

$$a_{ij}^H = \begin{cases} 1 & \text{if } a_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

Let $\succsim_A$ denote the ranking of languages associated to A. Our axioms are formally stated as follows:

- *Low Invariance*: $\succsim_A \equiv \succsim_{A^L}$.
- *High Invariance*: $\succsim_A \equiv \succsim_{A^H}$.
- *Compensation*: Let $l, k \in L$ be such that $a_{il} = 1 = a_{jk}$, $a_{jl} = 0 = a_{ik}$, and $a_{hl} = a_{hk}$, for all $h \in N \setminus \{i, j\}$. Then, $l \sim_A k$.
- *Strong Pareto*: Let $l, k \in L$ be such that $a_{il} \geq a_{ik}$, for each $i \in N$, with at least one strict inequality. Then, $l \succ_A k$.

Proposition 1 *MD is the unique ranking satisfying Low Invariance, Compensation and Strong Pareto.*

Proof. Let $A = (a_{ij})_{(i,j) \in N \times L}$ be a language matrix, and $l, k \in L$ be a pair of languages. By *Low Invariance*, $l \succsim_A k \iff l \succsim_{A^L} k$. By iterated application of *Compensation*, if necessary, we obtain that $l \sim_{A^L} k \iff \sum a_{il}^L = \sum a_{ik}^L$. By *Strong Pareto*, $l \succ_{A^L} k \iff \sum a_{il}^L > \sum a_{ik}^L$. Altogether, we have that $l \succsim_A k \iff MD(l) \geq MD(k)$. ■

Proposition 2 *CB is the unique ranking satisfying High Invariance, Compensation and Strong Pareto.*

Proof. Let $A = (a_{ij})_{(i,j) \in N \times L}$ be a language matrix, and $l, k \in L$ be a pair of languages. By *High Invariance*, $l \succsim_A k \iff l \succsim_{A^H} k$. By iterated application of *Compensation*, if necessary, we obtain that $l \sim_{A^H} k \iff \sum a_{il}^H = \sum a_{ik}^H$. By *Strong Pareto*, $l \succ_{A^H} k \iff \sum a_{il}^H > \sum a_{ik}^H$. Altogether, we have that $l \succsim_A k \iff CB(l) \geq CB(k)$ ■

In order to define the next axiom, suppose now that $a_{ij} \in \{a_1, a_2, \ldots, a_n\}$, where $a_1 < a_2 < \cdots < a_n$.

- *Generalized Compensation*: Let $l, k \in L$ be such that $a_{il} = a_m = a_{jk}$, $a_{jl} = a_{m-1} = a_{ik}$, for some $m \in \{2, \ldots, n\}$, and $a_{hl} = a_{hk}$, for all $h \in N \setminus \{i, j\}$. Then, $l \sim_A k$.

Proposition 3 *AK is the unique ranking satisfying Generalized Compensation and Strong Pareto.*

Proof. Let $A = (a_{ij})_{(i,j) \in N \times L}$ be a language matrix, and $l, k \in L$ be a pair of languages. By iterated application of *Generalized Compensation*, if necessary, we obtain that $l \sim_A k \iff \sum a_{il} = \sum a_{ik}$. By *Strong Pareto* and *Generalized Compensation*, $l \succ_A k \iff \sum a_{il} > \sum a_{ik}$. Altogether, $l \succsim_A k \iff AK(l) \geq AK(k)$.[29] ■

Suppose now that each agent has a *vote* to be allocated among the languages she speaks. In this context, a **rule** is a mapping R that associates with each language matrix A an allocation $(R_l(A))_{l \in L}$ indicating the amount of votes each language gets. The Shapley rule allocates for each language $l \in L$ the following votes:

$$R_l^S(A) = \sum_{i \in N, j \in L_i(A)} \frac{1}{l_i(A)},$$

where recall that $L_i(A) = \{j \in L : a_{ij} \neq 0\}$ and $l_i(A)$ denotes its cardinality.

This rule obviously leads to the Shapley ranking we introduced above. As shown by the next result, which replicates Theorem 1 in Bergantiños and Moreno-Ternero (2015), the Shapley rule is characterized by the following three axioms:

- *Equal treatment of equals*: Let $l, k \in L$ be such that $N_l(A) = \{i \in N : a_{il} \neq 0\} = \{i \in N : a_{ik} \neq 0\} = N_k(A)$. Then, $R_l(A) = R_k(A)$.

- *Dummy*: Let $l \in L$ be such that $N_l(A) = \emptyset$. Then, $R_l(A) = 0$.

- *Additivity*: For each pair of subsets of agents (N^1, N^2), and their corresponding language matrices A^1 and A^2, $R(A^1 \cup A^2) = R(A^1) + R(A^2)$, where $A^1 \cup A^2$ denotes the resulting matrix from merging the rows of both matrices.

Proposition 4 *A rule satisfies equal treatment of equals, dummy and additivity if and only if it is the Shapley rule.*

[29]This proof essentially mimicks the proof of Theorem 3 in Macé (2015).

Proof. It is obvious that the Shapley rule satisfies the axioms. Let R be a rule satisfying the three axioms in the statement. For each $i \in N$, let A_i denote the corresponding i-th row of A. By *dummy*, $R_l(A_i) = 0$ for each $l \notin L_i(A)$. By *equal treatment of equals*, $R_l(A_i) = R_k(A_i)$ for each pair $l, k \in L_i(A)$. As $\sum\limits_{l \in L} R_l(A_i) = 1$, we deduce that $R_l(A_i) = \frac{1}{l_i(A)}$ for each $l \in L_i(A)$. Consequently, it follows, by *additivity*, that $R_l(A) = \sum_{i \in N, j \in L_i(A)} \frac{1}{l_i(A)}$, for each $l \in L$, as desired. ■

To conclude, we consider the Weighted Shapley rule, which allocates for each language $l \in L$ the following votes:

$$R_l^{WS}(A) = \sum_{i \in N, l \in L} \frac{a_{il}}{\sum_{h \in L} a_{ih}}.$$

As shown in the last proposition, the following alternative axiom to *equal treatment of equals* leads to characterize the Weighted Shapley rule.

- *Weighted treatment of equals*: Let $i \in N$ and $l, k \in L_i(A)$. Let A_i denote the corresponding i-th row of A. Then, $\frac{R_l(A_i)}{R_k(A_i)} = \frac{a_{il}}{a_{ik}}$.

Proposition 5 *A rule satisfies weighted treatment of equals, dummy and additivity if and only if it is the Weighted Shapley rule.*

Proof. It is obvious that the Weighted Shapley rule satisfies the axioms. Let R be a rule satisfying the three axioms in the statement. For each $i \in N$, let A_i denote the corresponding i-th row of A. By *dummy*, $R_l(A_i) = 0$ for each $l \notin L_i(A)$. By *weighted treatment of equals* $\frac{R_l(A_i)}{R_k(A_i)} = \frac{a_{il}}{a_{ik}}$ for each pair $l, k \in L_i(A)$. As $\sum\limits_{l \in L} R_l(A_i) = 1$, we deduce that $R_l(A_i) = \frac{a_{il}}{\sum_{h \in L} a_{ih}}$ for each $l \in L_i(A)$. Consequently, it follows, by *additivity*, that $R_l(A) = \sum_{i \in N, l \in L} \frac{a_{il}}{\sum_{h \in L} a_{ih}}$, for each $l \in L$, as desired. ■

Table 1. Rankings of Languages in the EU Before Brexit

Languages		Rankings				
Name (1)	Native in (2)	MD (3)	CB (4)	AK (5)	S (6)	WS (7)
Official						
English	GB-IRL	63	87	75	106	96
German	G-A-B	75	82	79	78	54
French	F-B	80	87	84	64	43
Italian	I	87	88	87	49	35
Spanish	E	89	92	90	38	26
Polish	PL	92	92	92	32	22
Dutch	NL-B	95	96	95	12	10
Romanian	RO	95	96	96	19	28
Hungarian	H	97	98	97	11	8
Greek	GR-CY	97	98	97	10	7
Portuguese	P	98	98	98	10	7
Czech	CZ	98	98	98	8	6
Swedish	S	98	98	98	6	5
Bulgarian	BG	98	99	98	6	4
Slovak	SL	99	99	99	4	3
Danish	DK	99	99	99	3	3
Finnish	FIN	99	99	99	3	3
Lituanian	LT	99	99	99	2	2
Slovenian	SLO	99	100	99	1	1
Latvian	LV	100	100	100	1	1
Estonian	EST	100	100	100	<1	<1
Irish	IRL	100	100	100	<1	<1
Maltese	M	100	100	100	<1	<1
Others						
Russian	Non EU	95	99	97	10	6
Catalan	SP	99	99	99	2	2
Galician	SP	99	100	99	1	1
Arabic	Non EU	99	100	100	1	1
Turkish	CY	100	100	100	1	1
Basque	SP	100	100	100	<1	<1
Luxemb.	LX	100	100	100	<1	<1

Table 2. Rankings of Main Languages in the EU
Before (B) and After (A) Brexit

Languages		Rankings					
Name	Native in	MD		CB		AK	
(1)	(2)	B	A	B	A	B	A
English	GB-IRL	63	72	87	99	75	85
German	G-A-B	75	72	82	80	79	76
French	F-B	80	79	87	85	84	82
Italian	I	87	85	88	87	87	86
Spanish	E	89	88	92	91	90	89
Polish	PL	92	91	92	91	92	91
Dutch	NL-B	95	94	96	95	95	95
Romanian	RO	95	95	96	95	96	95
Hungarian	H	97	97	98	97	97	97
Greek	GR-CY	97	97	98	97	97	97
Portuguese	P	98	97	98	97	98	97
Czech	CZ	98	97	98	98	98	97
Swedish	S	98	98	98	98	98	98
Bulgarian	BG	98	98	99	98	98	98
Slovak	SL	99	98	99	99	99	99
Danish	DK	99	99	99	99	99	99
Finnish	FIN	99	99	99	99	99	99
Lituanian	LT	99	99	99	99	99	99
Slovenian	SLO	99	99	100	99	99	99
Latvian	LV	100	100	100	100	100	100
Estonian	EST	100	100	100	100	100	100
Irish	IRL	100	100	100	100	100	100
Maltese	M	100	100	100	100	100	100
Others							
Russian	Non EU	95	95	99	99	97	97
Catalan	SP	99	99	99	99	99	99
Galician	SP	99	99	100	100	99	99
Arabic	Non EU	99	100	100	100	100	100
Turkish	CY	100	99	100	100	100	100
Basque	SP	100	100	100	100	100	100
Luxemb.	LX	100	100	100	100	100	100