Non-linear satellite channel equalization based on a low complexity Echo State Network

M. Bauduin*, S. Massar†, F. Horlin*
mbauduin@ulb.ac.be
*Université Libre de Bruxelles (ULB) - OPERA department - CP 165/81, Avenue F. Roosevelt 50, 1050 Brussels
†Université Libre de Bruxelles (ULB) - LIQ department - CP 225, Boulevard du Triomphe, 1050 Brussels

***Abstract*—Satellite communications systems designers are continuously struggling to improve the link capacity. A critical challenge comes from the limited power available aboard the satellite. To ensure a sufficient signal-to-noise power ratio (SNR) at the terrestrial receiving side, the amplifier aboard the satellite is usually operated close to the amplifier saturation point which adds non-linear distortions to the communication channel. Several algorithms have been proposed to equalize the non-linear satellite channel. The Echo State Network (ESN) algorithm, coming from the field of artificial neural networks, has been shown to perform well in this task: it can achieve a similar bit error rate (BER) as the state-of-the-art Volterra equalizer. In the present paper we show that with an appropriate design, the complexity of the ESN can be significantly lower than that of the Volterra equalizer, while conserving the low BER.**

***Index Terms*—Satellite communications, Non-linear communication channel, Equalization, Volterra, Echo State Network**

I. Introduction

In most of the satellite communications systems, the satellite works as a relay between two terrestrial points: the satellite receives a signal transmitted from a first terrestrial location, amplifies it and retransmits it without digital signal processing to a second terrestrial location. This is for example the case in the DVB-S2 (Digital Video Broadcasting - Satellite - Second Generation) systems [1]. Communicating over the satellite channel is very challenging because of the constraints on the equipments aboard the satellite. The limited bandwidth of the filters aboard the satellite creates significant inter-symbol interferences (ISI) when the communication bandwidth is increased. To ensure a sufficiently high output power, the power amplifier generally works close to its saturation point. This operating point improves efficiency but also adds important non-linear distortions in the communication channel that further degrade the system reliability.

The resulting satellite non-linear communication channel with memory can fortunately be partially compensated at the ground stations with digital signal processing. The main contributions in the literature propose to use a baseband Volterra structure for the equalization of the satellite channel [2] [3] [4].

Other approaches based on artificial recurrent neural network (RNN) structures have been investigated. These solutions are however very complex to implement in practice due to the important number of parameters to be optimized before the RNN can work (training period). A paradigm, called Reservoir Computer, has been proposed to cope with this drawback [5]. One of its simplest forms is referred to as the Echo State Network (ESN) [6]. The main difference with classical RNNs is that an important part of the parameters of the ESN are randomly generated and fixed afterwards. Only a specific part of the RNN, the output layer, needs to be optimized which considerably reduces the training duration. Because of its structure, the ESN has also the advantage that it could potentially be implemented with analog components. High performance experimental implementations have been reported on optoelectronic [7] [8] and all-optical [9] [10] circuits.

Several papers have evaluated the performance of digital ESNs for the equalization of a non-linear communication channel. The comparison was however only made with a linear filter [11], or assuming a theoretical unrealistic channel model [12]. In [13], we applied the ESN to the satellite communication channel and compared its performance with the baseband Volterra equalizer. We demonstrated that the ESN is capable of performing equally well while incurring a similar computational complexity.

An important feature of the considered ESN is that some parameters are chosen randomly, meaning that their exact values have a small impact on the final performance. In this paper, we propose to select these parameters so as to reduce the complexity of the digital implementation without degrading the performance. Note that [14] proposes to modify the connections between the neurons in order to reduce the number of operations required by the ESN, but the other parameters of the ESN have not been investigated. We demonstrate that these parameters can also be chosen in order to reduce the overall complexity with a negligible impact on the performances. We mainly reduce the number of multiplications which is the most complex operation in digital implementations. This shows that ESN becomes a competitive solution for the equalization of the non-linear satellite communication channel in comparison to the baseband Volterra equalizer. Although the present work is restricted to the satellite channel, it can be anticipated that the conclusions are of wider applicability, and that the ESN will be a competitive solution for many other non linear communication channels.

The outline of this paper is the following. The system model of the satellite communication channel is described in section 2. The ESN is introduced in section 3. The different solutions to reduce the algorithm complexity are studied in section 4. The section 5 numerically compares the bit error rate (BER), the convergence speed, and the digital complexity of the ESN and the baseband Volterra equalizer.

II. System model

A block diagram of the baseband DVB-S2 communication channel is provided in Fig. 1 [1]. The non-linear behaviour of the channel comes from the power amplifier aboard the satellite. It is defined by a baseband model which describes the amplitude $f_{PA}(.)$ and phase $g_{PA}(.)$ distortion as a function of the amplitude of the input signal. If this signal is defined by $y(n)$, the output of the amplifier $z(n)$ is:

$$z(n) \quad = \quad f_{PA}(|y(n)|)e^{j(\angle y(n)+g_{PA}(|y(n)|))}, \qquad (1)$$

where $|y(n)|$ is the modulus and $\angle y(n)$ is the phase of $y(n)$. We use the power amplifier model proposed in [1].

The operating point is defined by the output back off (OBO) defined as:

$$\text{OBO} = 20 \log_{10} \frac{A_{\text{out}}}{A_{\text{sat}}}, \qquad (2)$$

where A_{out} is the Root Mean Square (RMS) value of $z(n)$ and A_{sat} is the saturation amplitude of the amplifier. An OBO close to 0 dB improves the efficiency of the power amplifier but also introduces important non-linear distortions.

The memory of the channel comes from the different filters in the satellite and on the ground stations. We have two half-root Nyquist filters, one at the transmitter side and one at the receiver side. The satellite contains an imux and an omux filter which are low-pass filters that can be modelled using a Butterworth model. As we have a line-of-sight propagation channel between the satellite and the ground stations and because of the important directivity of the antennas, the propagation channel can be considered as memoryless [2].

The sequence of transmitted symbols $s(n)$ is shaped with a half-root Nyquist filter and transmitted to the satellite. At the satellite, the signal is convolved with the imux filter, amplified, and convolved with the omux filter before it is transmitted back to the earth. At the receiver, the signal is filtered by the complementary half-root Nyquist filter and sampled at the symbol rate. Additive white Gaussian noise corrupts the received signal.

In a linear communication channel, the clouds of points which define the received samples are centred on the transmitted constellation. It is not the case in a non-linear channel because of the compression due to the non-linear power amplifier. We can observe a displacement of the center of the cloud of points, called centroids [15] (see Fig. 2).

III. Echo State Networks

The ESN is one of the simplest forms of Reservoir Computer which is a paradigm that has been proposed to reduce the complexity of the learning task of RNNs. An example of ESN is illustrated on Fig. 3. The algorithm is composed by N neurons connected to each other. The structure of these connections is defined by a $N \times N$ matrix $\underline{W} = (w_{ij})$. The neurons are connected to the input signal through an input mask defined by a $1 \times N$ vector $\underline{W}^{in} = (w_i^{in})$. In the first papers on ESN, $\underline{W}$ and $\underline{W}^{in}$ were generated randomly [6] [12]. Only the output weights (output mask $\underline{W}^{out} = (w_i^{out})$) are trained to equalize the non-linear channel. In this way, the number of connections to adapt is strongly reduced which accelerates the learning task.

The estimated symbols $\hat{s}(n)$ are found at the output of the ESN described by the three following equations [6]:

$$\begin{cases} a_i(n) = \sum_{j=1}^{N} w_{ij} x_j(n-1) + w_i^{in} r(n), & (3) \\ x_i(n) = f_{NL}(a_i(n)), & (4) \\ \widehat{s}(n+D) = \sum_{i=1}^{N} w_i^{out} x_i(n). & (5) \end{cases}$$

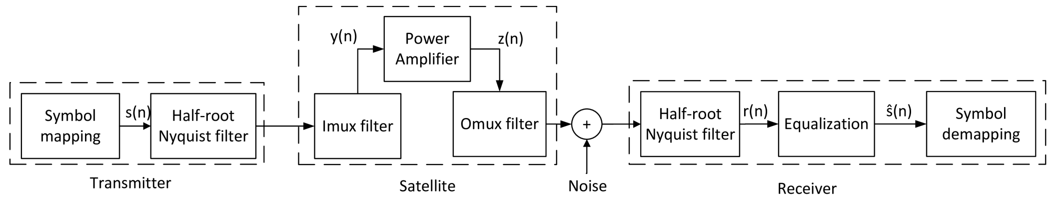

Fig. 1. Block diagram of a satellite communication channel.

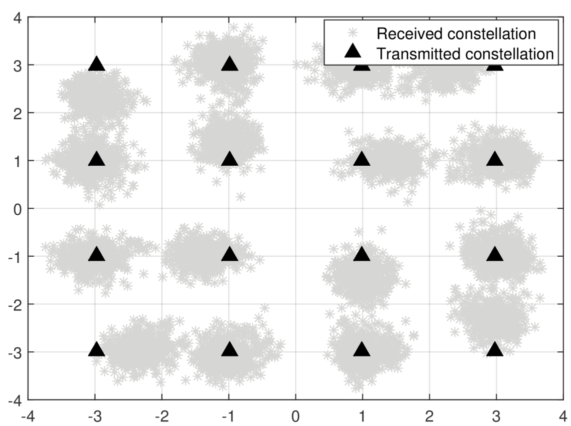

Fig. 2. 16-QAM constellation before and after a noiseless satellite communication channel with the parameters defined in section 5 (OBO: -2 dB, roll-off factor of the half-root Nyquist filters: 0.25).

The terms w_{ij} create the memory of the structure. The non-linear behaviour of the ESN comes from the activation function $f_{NL}(.)$. In general, the input weights w_i^{in} have a uniform distribution in amplitude [5] [6] [13]. The coefficients w_i^{out} are evaluated with a linear regression. The parameter D creates a delay between the received signal $r(n)$ and the output of the ESN to take into account the delay introduced by the channel.

The ESN requires the echo state property which specifies that the value of each neuron only depends on the past history of the input signal. In this way, the initial state of the ESN tends to be forgotten and will not affect the output signal $\widehat{s}(n)$. This property is respected if the product of the spectral radius of the interconnection matrix and the Lipschitz constant of the function $f_{NL}(.)$ (which has to be Lipschitz continuous) is lower than 1 [16] [17]. In this way, the network has a fading memory. In the other case, we could observe an exponential increase of the dependence of the past history corresponding to an unstable ESN.

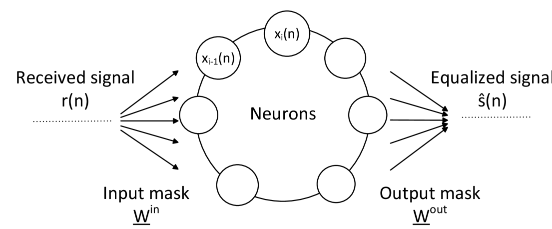

Fig. 3. Structure of the Echo State Network with a ring structure

In this paper, the interconnection matrix $\underline{\underline{W}}$ is defined by the circular matrix proposed in [14]:

$$\underline{\underline{W}} = \alpha \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 \end{pmatrix}, \tag{6}$$

where α is the feedback gain. The result is a ring structure as illustrated on Fig. 3. It has been shown in [14] that this matrix can offer the same performances as the initially proposed random matrix. It has the advantage to reduce number of operations by reducing the number of connections. The matrix has also been used in [13] to equalize the DVB-S2 communication channel.

It has been shown in [13] that the performances of the ESN can be improved, if it is trained to recover a constellation defined by the new centroids instead of the transmitted ones. In the case of a 16-QAM constellation, we have 16 centroids in the transmitted constellation defined by $(X_i)_{i=1}^{16}$. We can evaluate the position of the new centroids $\overline{X}_i$ by averaging the T_i received symbols corresponding to X_i over a finite learning sequence $s_T(n)$ of length T [15]:

$$\overline{X}_i = \frac{1}{T_i} \sum_{j=0}^{T-1} r(j | s_T(j) = X_i), \tag{7}$$

where

$$r(j|s_T(j) = X_i) = \begin{cases} 0, & s_T(j) \neq X_i \\ r(j), & s_T(j) = X_i \end{cases}. \quad (8)$$

A new training sequence $s_T^c(n)$ can be defined with a look-up-table which replaces the symbols of $s_T(n)$ defined on X_i by the new symbols defined on $\overline{X}_i$. The ESN and the Volterra equalizer will be trained to minimize the mean square error between the estimated sequence and $s_T^c(n)$.

IV. Complexity reduction

The most complex operation in the digital ESN algorithm are the multiplications. As we work with binary digits however, its complexity can significantly be reduced for numbers which are a power of 2. If we work with fixed point precision, this operation is equivalent to perform a shift in the register where the number is stored. In this paper, we will consider that the complex numbers are stored in Cartesian form. So each multiplication by a power of 2 requires two translations: one for the real part and one for the imaginary part.

In a first step, we will modify the parameters of the ESN in order to replace most of the multiplications by a translation in a register. We will start with the ESN proposed in [13]. In this paper, we proposed to use the following activation function instead of the hyperbolic tangent used in most of the papers [11] [12] [14]:

$$f_{NL}(a) = a.(c_1 + c_3|a|^2), \quad (9)$$

where the values $c_1 = 0.716$ and $c_3 = -0.0478$ have been considered in [13]. If we also use the interconnection matrix (6), the evolution of the ESN becomes:

$$\begin{cases} a_i(n) = \alpha x_{i-1}(n-1) + w_i^{in} r(n), & (10) \\ x_i(n) = a_i(n)(c_1 + c_3|a_i(n)|^2), & (11) \\ \widehat{s}(n+D) = \sum_{i=1}^{N} w_i^{out} x_i(n). & (12) \end{cases}$$

This algorithm needs $13N$ multiplications and $8N - 2$ summations.

We can reduce the number of multiplications if we use the following parameters:

- $\alpha = 2^{-1}$
- $c_1 = 0.75$
- $c_3 = -2^{-4}$
- $w_i^{in} = \pm 2^m$ where m is a random integer lying between -1 and 3.

It means that the multiplications with the feedback gain α, the input mask w_i^{in} and the coefficient c_3 become simple translations in a register .

The coefficients w_i^{in} are still random numbers, in order to reduce the correlation between the neurons, but their values are now limited to powers of 2. The value of α has been found by simulation to give a sufficient memory to the ESN. The values of c_1 and c_3 have been chosen by simulations to maximize the performance of the ESN.

As the ESN is non-linear, its behaviour depends on the amplitude of the received signal $r(n)$. We consider that, for a practical implementation, the gain controlled amplifier and/or the analog-to-digital converter of the receiver are able to deliver a digital signal $r(n)$ with a RMS value of 6. All these values have been found by simulations.

In summary, after the first step in complexity reduction, the ESN only needs $8N$ multiplications, $8N-2$ summations and $5N$ shifts. These parameters are independents from the channel so they can be implemented in hardware on the chip.

The second step in the complexity reduction concerns the activation function (see eq. (11)). This is an important source of complexity as the evaluation of the activation function requires 4 multiplications, 2 summations and 1 shift. In the literature, we can see that, for a non-linear ESN, all the neurons have an activation function [6] [13] [14]. This offers interesting performances but it also increases the complexity of the algorithm. In the present work we propose a modification of the ESN in which all the neurons do not have an activation function. More precisely, for N_L neurons, we remove the activation function. That is, for N_L indices i, eqs. (10) and (11) are replaced by

$$x_i(n) = \alpha x_{i-1}(n-1) + w_i^{in} r(n). \quad (13)$$

We denote by $N_A = N - N_L$ the number of neurons that have an activation function. With this simplification, the algorithm will require $8N - 4N_L$ multiplications, $8N - 2N_L - 2$ summations and $5N - N_L$ register shifts.

V. Numerical results

We consider the 16-QAM modulation. The imux and omux filters have a 36 MHz bandwidth. The roll-off factor of the half-root Nyquist shaping filters on the ground stations is fixed at 0.25. The symbol rate is 30 MHz.

The operating point of the power amplifier is defined by a -2 dB OBO. The functions $f_{PA}(.)$ and $g_{PA}(.)$ are described using a Ghorbani model [18]:

$$f_{PA}(y(n)) = \frac{q_1|y(n)|^{q_2}}{1 + q_3|y(n)|^{q_2}} + q_4|y(n)|, \quad (14)$$

$$g_{PA}(y(n)) = \frac{q_5|y(n)|^{q_6}}{1 + q_7|y(n)|^{q_6}}, \quad (15)$$

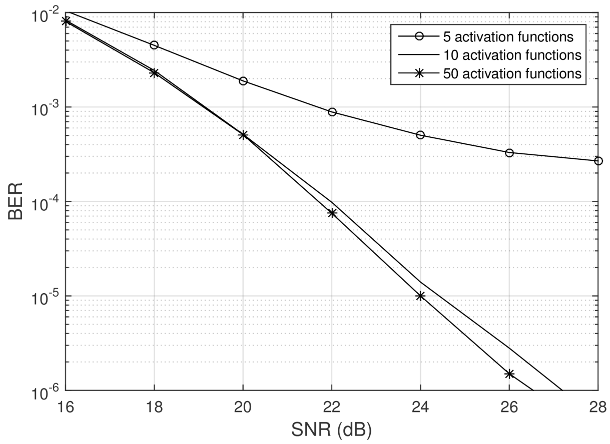

Fig. 4. Impact of the number of activation functions for an ESN composed of 50 neurons with $\alpha = 2^{-1}$, $c_1 = 0.75$, $c_3 = -2^{-4}$ and an input mask composed by powers of 2, as a function of the number N_A of neurons with activation function.

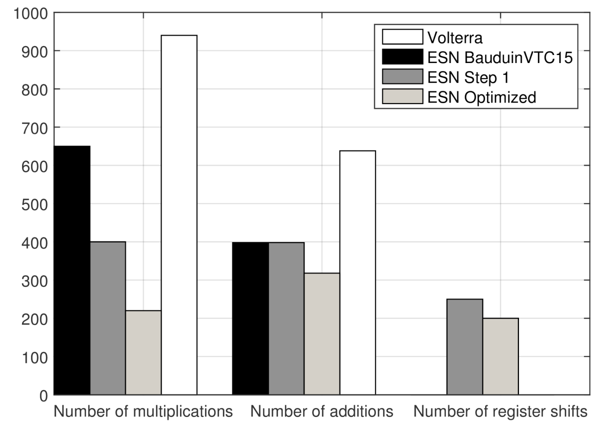

Fig. 5. Complexity comparison in term of number of operations (multiplications, additions, shifts) for an ESN with 50 neurons and a Volterra equalizer composed by 85 kernels.

where the following values are considered: $q_1 = 6$, $q_2 = 1.3$, $q_3 = 3.3$, $q_4 = -0.4$, $q_5 = 1.8$, $q_6 = 1.8$, $q_7 = 1.4$.

We used an ESN composed by 50 neurons. As basis for comparison we used the ESN of [13] with a feedback gain α of 0.35 (denoted in the figures by ESN Bauduin-VTC15). The value $\alpha = 0.35$ was found by simulation to minimize the BER. We compare this with the ESN obtained after step 1 (denoted by ESN Step 1), and with the fully optimised ESN (denoted ESN Optimised). In the optimized ESN, only ten neurons have an activation function, i.e. $N_A = 10$ and $N_L = 40$.

The ESN equalizers are compared with a Volterra equalizer. The Volterra equalizer has a linear memory L_1 equal to 10 and an order 3 non-linear memory L_3 equal to 5. So the Volterra equalizer is composed of 85 kernels (10 of order 1 and 75 of order 3) [2]. It requires $4L_1 + 12\frac{L_3^2(L_3+1)}{2}$ multiplications and $4L_1 + 8\frac{L_3^2(L_3+1)}{2} - 4$ summations.

The coefficients of the Volterra equalizer and of the ESN have been evaluated to minimize the MSE between the estimated sequence and the training sequence $s_c^T(n)$ defined on the new centroids. In all cases, we used a training sequence composed of 3000 symbols.

The simulations show us that the new input mask creates no visible degradation on BER curves. We can see in Fig.4 that using only ten neurons with activation function is a good compromise between performances and complexity. By simulations, we have seen that a uniform repartition of the non-linear neurons was the most efficient configuration.

We observe in Fig. 5 that the number of multiplications of the ESN has been reduced by a factor approximatively equal to 1.5 when the multiplications can be implemented with a shift in the register (ESN Step 1). Furthermore, we observe that reducing the number of activation signals gives us a gain equal to 3 compared to the ESN proposed in [13].

Fig. 6 shows that the performance degradation is negligible while the complexity gain is significant. It is also interesting to observe that the convergence speed is not affected as shown on Fig. 7.

If we compare our new ESN with the Volterra equalizer, we can see that the number of multiplications is reduced by a factor close to 5. The cost to pay is a loss of approximately 0.5 dB for a BER of 10^{-5}. The convergence speed of the ESN is much higher than the convergence speed of the Volterra equalizer. This result was expected as we have 85 weights to train for Volterra and only 50 for the ESN. We can see that the Volterra equalizer needs more that 1000 symbols to converge to the asymptotic BER. It is interesting to see that, for a training sequence lower than 1000 symbols, the ESN offers a better BER than the Volterra equalizer.

VI. Conclusion

We have shown how to reduce significantly the complexity of an ESN. To this end we used a ring topology, we chose the coefficients of the ESN in such that they could be implemented with simple shift registers, and we limited the number of neurons that have an activation function. In fact, after these simplifications, most of the

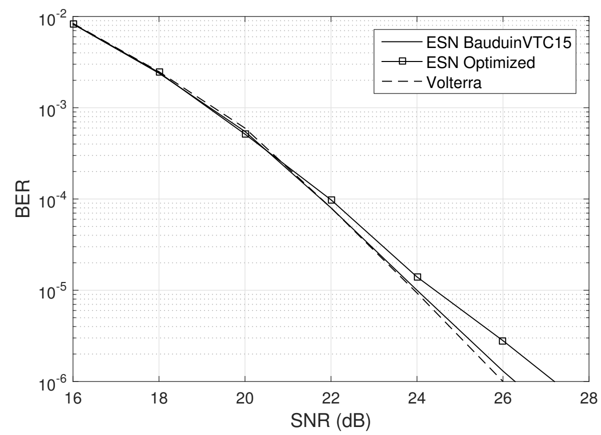

Fig. 6. Performances of the different solutions in terms of BER for different SNR.

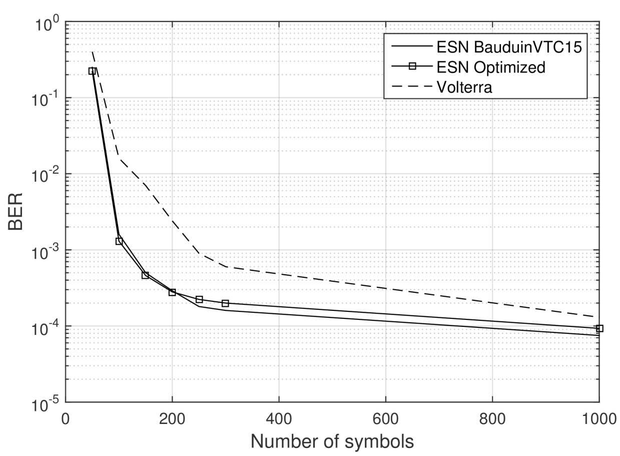

Fig. 7. Performance each solutions as a function of training sequence length with a SNR of 22 dB

complexity of the ESN lies in the output layer, eq. (12). We demonstrated that this reduction in complexity is accompanied by negligible performance degradation for the problem of equalisation of a non linear satellite channel. We compared these ESNs with a Volterra equalizer. The Volterra equalizer has a slightly better BER for high SNR. But the Volterra equalizer requires a longer training sequence, and because it does not contain multiplication by constants, it is not possible to reduce its complexity.

We expect that the complexity reduction presented here could find applications for the equalization of other non linear channels, and for other applications of ESNs.

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