



**Measuring Interconnectedness between  
Financial Institutions with Bayesian Time-Varying  
Vector Autoregressions**

Marco Valerio Geraci  
SBS-EM, ECARES, Université libre de Bruxelles

Jean-Yves Gnabo  
Université de Namur

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# Measuring interconnectedness between financial institutions with Bayesian time-varying vector autoregressions\*

Marco Valerio Geraci<sup>†,‡</sup> and Jean-Yves Gnabo<sup>‡</sup>

<sup>†</sup>*ECARES, Université libre de Bruxelles*

Email: mgeraci@ulb.ac.be

<sup>‡</sup>*CeReFiM, Université de Namur*

Email: jygnabo@fundp.ac.be

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## Abstract

We propose a market-based framework that exploits time-varying parameter vector autoregressions to estimate the dynamic network of financial spillover effects. We apply it to financials in the Standard & Poor's 500 index and estimate interconnectedness at the sector and institution level. At the sector level, we uncover two main events in terms of interconnectedness: the Long Term Capital Management crisis and the 2008 financial crisis. After these crisis events, we find a gradual decrease in interconnectedness, not observable using the classical rolling window approach. At the institution level, our framework delivers more stable interconnectedness rankings over time than other market-based measures of systemic risk.

**Keywords:** financial interconnectedness, time-varying parameter, systemic risk

*JEL Classification:* G01, G18, G32, C32, C51

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# I. Introduction

In this paper we develop a market-based statistical framework that uses Bayesian estimation of time-varying parameter vector autoregressions (TVP-VAR) to model the dynamic nature of connections between financial institutions. The framework allows connections to evolve gradually through time as opposed to the classical approach, which favours sudden, often unjustified, changes in interconnectedness. Paired with graph theory, our framework allows us to reconstruct a continuously evolving network of directed spillover effects. We use our framework to study the evolution of interconnectedness of the U.S. financial sector over the past two decades.

Market-based measures of interconnectedness apply statistical measures of association (e.g., correlation, Granger causality, tail dependence) to asset prices, in order to map and analyse the network of spillover effects between financial institutions (e.g., Diebold and Yilmaz (2009, 2014), Hautsch, Schaumburg, and Schienle (2014, 2015)). These standard statistical measures presuppose that the inferred relationships are time-invariant over the sample used for the estimation. To retrieve a dynamic measure of interconnectedness, the usual approach has been to divide the original sample period into multiple subsamples and calculate these statistical measures over rolling windows of data.

We argue that this is unsuitable if the system studied is time-varying. By relying on short subsamples, rolling windows lower the power of inference and induce dimensionality problems. Moreover, the rolling window approach is known to be susceptible to outliers because, in small subsamples, these have a larger impact on estimates (Zivot and Wang (2006)). On the other hand, choosing longer windows will lead to estimates that are less

reactive to change, biasing results towards time-invariant connections (Clark and McCracken (2009)). The rolling window approach will always involve a trade-off between the precision and reactivity of estimates of interconnectedness.

Our framework builds on the literature that uses *temporal* dependencies for measuring interconnectedness (Billio, Getmansky, Lo, and Pelizzon (2012), Barigozzi and Brownlees (2016), Barigozzi and Hallin (2015)). We construct a TVP-VAR model with features that allow for heteroskedasticity, skewness, and excess kurtosis of stock returns. The time-varying cross-autoregressive coefficients of the TVP-VAR capture, in every time period, the strength of directed connections between financial institutions. We determine connections by evaluating the posterior distribution of the parameters using Bayes factor. The framework is based on the whole sample of observed data and does not require setting any window size.

Although we focus on temporal dependencies, our framework can also be used to study interconnectedness based on *contemporaneous* dependencies (e.g., Acharya, Engle, and Richardson (2012), Acharya, Pedersen, Philippon, and Richardson (2010), Adrian and Brunnermeier (2016)). This is possible thanks to the time-varying covariance matrix included in our model, which effectively captures the contemporaneous (undirected) connections between returns.

We also contribute to the literature of Bayesian measures of interconnectedness. In this literature, Bernardi, Gayraud, and Petrella (2015) develop a Bayesian approach for measuring *CoVaR*. They focus on quantile effects, whereas we focus on conditional mean effects. Our framework could be extended with different distributional assumptions (Yu and Moyeed, 2001) to model quantile effects as in the VAR for VaR framework of White, Kim, and Manganelli (2015). Ciccarelli and Rebucci (2007) also adopt Bayesian TVP-VARs to measure contagion in the South American FX market. They assume asset returns follow a

multivariate  $t$ -distribution with constant scale matrix. Our approach is more flexible as we use a multivariate  $t$ -distribution with time-varying scale, which allows for heteroskedasticity and time-varying contemporaneous correlations between asset returns. Moreover, we include the possibility for returns to exhibit skewness by allowing for a so-called leverage effect between shocks to asset returns and shocks to volatilities.

Another related study by Adams, Füss, and Gropp (2014) examines interconnectedness between four U.S. financial sectors (commercial banks, investment banks, hedge funds and insurance companies) by proposing a state-dependent sensitivity Value-at-Risk (SDSVaR) model. The SDSVaR model is comparable to our proposed framework as it allows for three states of connections between sectors, according to whether financial markets are in a volatile, normal or tranquil condition. However, our TVP-VAR is more flexible as it does not restrict the number of states and allows connections to vary freely through time.

We apply our framework to analyse the U.S. network of financial spillovers. To do this, we use monthly stock price data for all financial institutions listed from 1990 to 2014 on the Standard & Poor's 500 index, including firms that have since gone defunct. We proceed in two steps. First, we estimate the time-varying network of spillovers at the sectorial level between the four sectors comprising our data: banks, broker-dealers, insurance companies and real estate companies. Second, we estimate the network at an individual level between the 20 most systemically important financial institutions. Our analysis yields four main results.

First, at the sectorial level, our framework identifies two main events in terms of interconnectedness: the 1998 Long Term Capital Management (LTCM) crisis and the 2008 financial crisis. After these crisis events, we observe a gradual decrease in interconnectedness, concurring with the credit freeze that occurred. On the other hand, the rolling window

approach yields a volatile measure of interconnectedness that anomalously rises after the crisis events. This result evidences the usefulness of the TVP-VAR framework for measuring interconnectedness.

Second, we examine interconnectedness between the four financial sectors included in our study and find that banks and broker-dealers were the largest contributors of financial spillovers. The real estate sector, composed primarily of real estate investment trusts, was the most influenced by these spillovers. This may be due to real estate investment trusts being less leveraged than banks and less exposed to non-agency mortgage-backed securities.

Third, at the individual institution level, American International Group, Goldman Sachs, and Merrill Lynch were found to be among the largest propagators of financial spillovers, highlighting their potential widespread influence. By contrast, Bear Stearns did not play a major role in the propagation of spillovers but rather was very receptive of incoming spillovers. These results were confirmed in an out-of-sample exercise using only data up to the end of 2007.

Fourth, we show that interconnectedness rankings, computed using the rolling window approach, are extremely volatile and unlikely to be useful for policy decisions. On the other hand, the TVP-VAR framework produces more stable rankings appropriate for monitoring purposes. We also find that our framework yields more stable rankings than other market-based measures (e.g., the marginal expected short fall of Acharya et al. (2012) and Brownlees and Engle (2016)) while having more reactive rankings than measures based on low-frequency book value data (e.g., leverage).

## II. The model

According to the classical approach for measuring interconnectedness by temporal dependencies, financial institution  $i$  has a directed spillover to financial institution  $j$ , if the stock return of  $i$  Granger causes the stock return of  $j$ . Effectively, the classical approach relies on an insample test of the cross-coefficients of a time-invariant VAR. If, during the sample period, the direction of causality were to change, estimation of the VAR and test inference could be affected.

We allow for time-varying spillover effects with the following TVP-VAR:

$$(1) \quad R_t = c_t + B_t R_{t-1} + u_t \equiv X_t' \theta_t + u_t,$$

where  $R_t \equiv [r_{1t}, \dots, r_{Nt}]'$  is the vector of  $N$  stock returns. We stack the time-varying intercepts  $c_t$  and matrix of time-varying coefficients  $B_t$  in  $\theta_t$  so that equation (1) can be interpreted as the measurement equation of a state space model.<sup>1</sup>

Paralleling the classical approach, we define a time-varying spillover at period  $t$  from  $i$  to  $j$ , denoted  $i \rightarrow_t j$ , if the  $ji$  element of  $B_t$ , denoted  $B_t^{(ji)}$ , is different from zero. In particular, the (absolute) value of  $B_t^{(ji)}$  represents the strength of the spillover from  $i$  to  $j$  at period  $t$ .

We let the vector of disturbances  $u_t$  have the following form:  $u_t = \sqrt{\lambda_t} \Sigma_t^{\frac{1}{2}} \varepsilon_t$ , where  $\nu/\lambda_t \sim \chi_\nu^2$  and  $\varepsilon_t$  is a vector of standard normal errors. This means that  $u_t$  is  $t$ -distributed with scale matrix  $\Sigma_t$  and degrees of freedom  $\nu$ .

The  $t$ -distributed errors allow for the returns in our TVP-VAR to exhibit fat tails. Moreover, the time-varying scale matrix  $\Sigma_t$  can capture heteroskedastic volatility and time-varying

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<sup>1</sup>The framework can be extended to account for higher order lags.

correlations between errors, which can be interpreted as the contemporaneous spillovers occurring between the financial institutions in the network. For the present study, we concentrate on  $B_t$ , which represents temporal spillovers and have the advantage of being interpretable as a directed network. Nonetheless, the framework is highly flexible and lends itself to the study of both temporal and contemporaneous spillovers.

As is done in the macroeconomic literature (e.g., Cogley and Sargent (2005), Primiceri (2005)), we let parameters evolve according to a driftless random walk. The state equation of the model is thus,

$$(2) \quad \theta_{t+1} = \theta_t + v_{t+1}, \quad v_t \sim \mathcal{N}(0, Q_t),$$

where we assume that  $\varepsilon_t$  and  $v_s$  are independent at all  $t$  and  $s$ .<sup>2</sup>

We allow the time-varying parameters to have heteroskedastic variance  $Q_t$ . This allows for additional flexibility for  $B_t$  and effectively means that the unconditional distribution of  $B_t$  is fat-tailed. We assume that  $Q_t$  is a diagonal matrix, with diagonal elements collected in  $q_t$  and evolving as a geometric random walk,  $\ln q_{t+1} = \ln q_t + \omega_{t+1}$ .

Regarding the time-varying scale matrix of the measurement equation  $\Sigma_t$ , we assume it takes the usual triangular form,  $\Sigma_t = A_t^{-1} H_t (A_t^{-1})'$ , where  $H_t$  is a diagonal matrix containing the stochastic volatilities and  $A_t$  is a lower triangular matrix with ones across its diagonal and the contemporaneous interactions as lower diagonal elements.

Let  $h_t$  denote the vector of diagonal elements of  $H_t$ , and  $\alpha_t$  the lower diagonal elements of  $A_t$  stacked by rows. Then, we assume the following laws of motion for the time-varying

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<sup>2</sup>This assumption is taken solely for reasons of convenience and could be relaxed.



parameters  $h_t$  and  $\alpha_t$ :

$$\ln h_t = \ln h_{t-1} + \eta_t$$

$$\alpha_t = \alpha_{t-1} + \tau_t$$

The vector of errors of the model  $[\varepsilon_t, \eta_t, \omega_t, \tau_t]'$  is jointly normal with mean zero and variance-covariance matrix  $V$ , defined as

$$V = \begin{bmatrix} I & \Omega & 0 & 0 \\ \Omega & Z_\eta & 0 & 0 \\ 0 & 0 & Z_\omega & 0 \\ 0 & 0 & 0 & S \end{bmatrix}$$

where,

$$\Omega = \begin{bmatrix} \rho_1 \sigma_1 & 0 & \cdots & 0 \\ 0 & \rho_2 \sigma_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \rho_n \sigma_N \end{bmatrix}, Z_\eta = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_N^2 \end{bmatrix}, \text{ and}$$

$$Z_\omega = \begin{bmatrix} \sigma_{\omega,1}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{\omega,2}^2 & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{\omega,N \cdot (1+N)}^2 \end{bmatrix}$$

The variance-covariance matrix  $V$  and its submatrix  $\Omega$  allow the error terms of the

measurement and volatility equations, namely  $\varepsilon_t$  and  $\eta_t$ , to be contemporaneously correlated row-by-row. In the case of stock returns this correlation is often negative and is known as the leverage effect (Nakajima and Omori, 2012). This assumption offers increased flexibility as it allows for the possibility of having skewness in the errors of our VAR equation in (1). Finally, following Primiceri (2005) and Baumeister and Benati (2013), we adopt a block-diagonal structure for  $S$ , which implies that the non-zero and non-unity elements of  $A_t$  belonging to different rows evolve independently.

### III. Estimation and inference

We estimate the model by standard Bayesian methods as described in Kim and Nelson (1999). An overview of the prior specification and the sampling algorithm are given, respectively, in Section A and Section B of the Internet Appendix.<sup>3</sup>

In order to determine the dynamic network of spillover effects, we evaluate, for every pair of institutions  $i \neq j$ , the time-varying null hypothesis,  $\mathcal{H}_{0,t}^{j,i} : B_t^{(j,i)} = 0$ , using Bayes factor. Bayes Factor gives the odds in favour of the null hypothesis,  $\mathcal{H}_{0,t}^{j,i}$ , against the alternative hypothesis,  $\mathcal{H}_{1,t}^{j,i} : B_t^{(j,i)} \neq 0$ . Unlike classical frequentist testing, Bayes factor weighs evidence in favour of the null and alternative hypothesis equally. We follow Koop, Leon-Gonzalez, and Strachan (2010) to estimate Bayes factor from the conditional posterior distribution of the parameters. An overview of the procedure is given in Section C of the Internet Appendix.

In order to determine the existence of a connection, we impose a hard threshold of 4 on the estimated Bayes factor. That is, if the estimated Bayes factor is below the threshold,

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<sup>3</sup>Available at: [www.dropbox.com/s/2icttdk4blll9ce/online\\_appendix.pdf?dl=0](http://www.dropbox.com/s/2icttdk4blll9ce/online_appendix.pdf?dl=0).

we draw a directed connection between  $i$  and  $j$  in period  $t$ , otherwise we draw no connection. Furthermore, if we draw a connection, we weigh it according to the absolute value of the estimated cross coefficient  $B_t^{(ji)}$  in order to capture the strength of that connection. Effectively, the threshold is a filtering mechanism and a higher threshold leads to a denser network with more links.<sup>4</sup>

We compared the proposed Bayesian TVP-VAR framework against the classical approach of Granger causality over rolling windows in a series of simulation exercises given in Section D of the Internet Appendix. We found that our proposed framework performs well, both in terms of mean-squared error of the estimated parameters and in terms of the precision and recall of the inferred connections.

## IV. Data

We selected financial institutions with Standard Industrial Classification (SIC) codes from 6000 to 6799 that were components of the S&P 500 from January 1993 to December 2014. For these companies we collected the monthly cum-dividend stock price from Thomson Reuters Eikon for the same time period. The monthly frequency makes it possible to reduce the amount of noise in the data. Data at the intra-daily or even daily frequency reveals a higher number of linkages, because stocks are more susceptible to market shocks that lead to a higher degree of co-movement.

Initially the sample contained 182 firms but was reduced to 154 after constraining our analysis to stocks with at least 36 monthly observations. The final sample of financial

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<sup>4</sup>Similar results are obtained using different thresholds and are available upon request.

institutions is given in Table A1 of the Internet Appendix.

The sample can be subdivided into four sectors based on the SIC code of the companies: banks (SIC codes 6000 to 6199), broker/dealers (SIC codes 6200 to 6299), insurers (SIC codes 6300 to 6499) and real estate companies (SIC codes 6500 and 6799). The final sample included 71 banks, 21 brokers/dealers, 40 insurers and 23 real estate companies.

We define monthly stock returns for company  $i$  at month  $t$  as  $r_{it} = \log [(p_{it} + d_{it})/p_{i,t-1}]$ , where  $p_{it}$  is the stock price of company  $i$  at the end of month  $t$  and  $d_{it}$  are dividends paid that month. Finally, we netted the risk free rate from the stock returns, as is often done in asset pricing. For this we used the monthly return on the three-month U.S. Treasury Bills.

We proceed by estimating interconnectedness for the U.S. financial system at two levels. In Section V, we estimate interconnectedness at the sectorial level. In section VI, we estimate interconnectedness at the individual financial institution level, between 20 systemically important financial institutions.

## V. Interconnectedness at the sectorial level

Similarly to Adams et al. (2014), we computed sectorial indices from the stock price returns described in Section IV. We then inferred the dynamic network of spillovers effects between banks, broker/dealers, insurers, and real estate companies using a four-variable TVP-VAR with one lag.

In order to summarize the evolution of interconnectedness between sectors, we computed the network density at every period. The network density is the average strength of a

connection in the network at a given time period. It can be computed as

$$(3) \quad \text{Density}_t = \frac{1}{N(N-1)} \sum_{i=1}^{N_t} \sum_{j \neq i} (i \rightarrow_t j) \cdot |B_t^{(j,i)}|,$$

with  $i, j \in \{\text{Banks, Brokers, Insurers, Real Estate}\}$  and  $i \neq j$ , where  $B_t^{(j,i)}$  is the cross coefficient connecting  $i$  to  $j$ , in period  $t$ , in the TVP-VAR, and where, in this case,  $N = 4$ .

Figure 1 shows, in bold solid, the evolution of the sectorial density together with some significant events for the U.S. economy. The sectorial density peaks at two main events for the U.S. financial sectors: the collapse of the LTCM fund in August 1998 and the financial crisis of September 2008. A smaller peak is also appears during 2004 when the SEC suspended the net capital rule.

The fall and subsequent bailout of LTCM was a major event in terms of contagion because the fund was highly leveraged and was contractually connected with an extensive number of counterparties (McDonough, 1998). Concerns for spillovers effects bringing down financial markets lead the Federal Reserve Bank of New York to coordinate a consortium of banks to bailout LTCM.

The 2007-2009 financial crisis represented an even more important event in terms of interconnectedness. The sectorial density measure, based on the TVP-VAR framework shows that interconnectedness between sectors gradually grew by more than twofold between the beginning of 2005 and September 2008. After the financial crisis, the density gradually decreased to below pre-crisis levels. This could reflect two phenomena. First, after the default of Lehman Brothers, in October 2008, there was an immediate market freeze generated by the high uncertainty among market agents. This caused a drastic decrease in interconnectedness.

However, the policies introduced to counter this sentiment, such as the Troubled Asset Relief Program and Dodd-Frank act could have slowed down the drop. Dungey, Luciani, and Veredas (2013) found a similar decrease using a realized volatility-based measure of systemic risk.

Figure 1 also depicts the sectorial density found using classical approach of Granger causality testing (at 10% significance rate) over rolling windows, with windows of 36 months and 24 months, with the light dashed lines. The rolling window estimates are substantially more volatile than the TVP-VAR estimates, exhibiting sudden short-lived peaks. The volatility of the sectorial density computed using the TVP-VAR framework was 0.06, whereas it was 0.13 using the rolling window approach with 36 months window size. Using a smaller window size of 24 months leads to more reactive density measure with an even higher volatility of 0.18.

There are several discrepancies in the two density measures estimated using the rolling window approach. The density computed using the 24-month rolling window is far noisier because it relies on fewer observations and therefore is more susceptible to extreme events. In particular, after the 2007-2009 financial crisis, the measure appears to jump strongly, whereas the same movement is not observable using 36 month rolling windows. We believe that this peak is generated by the crisis observations, relating to October 2008, exiting the rolling windows and creating an artificial jump in interconnectedness.

In order to further study the evolution of interconnectedness between sectors we analysed the time-varying cross-coefficients of the TVP-VAR model. Figure 2 gives the posterior density mean of the off-diagonal elements of  $B_t$ , i.e., the value of  $B_t^{(j,i)}$  for  $i \neq j$ . The (absolute) value of the time-varying cross-coefficient represents the strength of the directed

spillover effect from a given sector (indicated by the rows of the figure) to another sector (indicated by the columns of the figure). Moreover, the sign of the cross-coefficient can help us understand whether the spillover effect was positive or negative. We indicate the LTCM 1998 crisis and the Lehman default crisis in Figure 2 with two vertical dashed lines.

The coefficients evolved substantially through time, evidencing the usefulness of the TVP-VAR framework. Banks and broker-dealers had the largest increase in outgoing connections prior to the 2008 financial crisis. Both sectors played a crucial role during the crisis in propagating spillover effects. Large banks and broker-dealers were feared to be insolvent triggering high uncertainty in financial markets. In particular, from Figure 2 it seems that banks and broker-dealers were heavily influencing insurers and real estate companies.

The 2007-2009 financial crisis was very much tied to problems in the housing market but Figure 2 shows that the real estate sector was receiving spillovers to a greater extent than it was propagating them. We propose two explanations for this result.

First, 21 of the 23 real estate companies in our sample were real estate investment trusts (REITs), which primarily own income-producing real estate and in some cases finance real estate. Only in specific cases do REITs own mortgages and generally, if they do, these will be through agency mortgage backed securities (MBSs), which are guaranteed by agencies of the U.S. government. On the other hand, banks were directly hit by troubles in the real estate sector because of direct exposure to mortgages and non-agency or private label MBSs.

Second, banks were heavily leveraged prior and during the financial crisis, whereas REITs were far less leveraged.<sup>5</sup> Table 1 shows the average leverage ratios for the financial institutions in our sample. During the 2007-2009 financial crisis, the banks in our sample were six times

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<sup>5</sup>We thank the anonymous referee for this insight regarding REITs.

more leveraged than the publicly traded real estate institutions in our sample.

## **VI. Interconnectedness at the financial institution level**

To measure interconnectedness at the financial institution level, we inferred the dynamic network of spillovers effects between 20 systemically important financial institutions. We considered all banks and insurers from the FSB’s list of systemically important financial institutions and systemically important insurers. As done by Diebold and Yilmaz, 2014, we also considered systemically important financial institutions that were not part of the FSB’s list because they were acquired or went bankrupt during the 2007-2009 crisis. Thus, the subsample included: American Express, American International Group (AIG), Bank of America, Bank of New York Mellon, Bear Stearns, Citigroup, Fannie Mae, Freddie Mac, Goldman Sachs, JP Morgan, Lehman Brothers, Merrill Lynch, MetLife, Morgan Stanley, PNC Group, Prudential, State Street, U.S. Bancorp, Wachovia, and Wells Fargo.

Since our sample is unbalanced with several stock price time series substantially shorter than the complete sample period, we adopted a *pairwise* approach similar to that of Billio et al. (2012). For each pair of financial institutions, we estimated a bivariate TVP-VAR (with one lag), taking into account the longest common time period of available data between any given pair.

### **A. Financial institution centrality**

In order to assess the importance of each financial institution within the dynamic network, we analysed each financial institutions’ degree centrality, which measures the weighted sum



of the connections to and from a given financial institution. Since our network is directed, we can measure both the in-degree centrality as well as the out-degree centrality, respectively given by:

$$(4) \quad \text{In-Degree}_{i,t} = \frac{1}{(N_t - 1)} \sum_{j \neq i} (j \rightarrow_t i) \cdot |B_t^{(ij)}|,$$

$$(5) \quad \text{Out-Degree}_{i,t} = \frac{1}{(N_t - 1)} \sum_{j \neq i} (i \rightarrow_t j) \cdot |B_t^{(ji)}|,$$

where  $i, j \in \{1, \dots, 20\}$  and  $N_t$  is the number of financial institutions present at time  $t$ . Notice that, as for the density measure in equation (3), we chose to weigh connections by the absolute value of the underlying cross-coefficient in the bivariate VAR.

In-degree centrality measures the average strength of incoming connections to financial institution  $i$ . Effectively, it is an indicator of the extent to which other financial institutions influence the stock price of financial institution  $i$ . Therefore, it is a measure of vulnerability to financial spillover effects.

Out-degree centrality measures the strength of outgoing links from financial institution  $i$ . Thus, a financial institution with high out-degree is heavily influencing many of its neighbours, making it a propagator of spillovers. As suggested by Hautsch et al. (2014), such financial institutions should be monitored closely because they are highly interconnected.

Figures 3 and Figure 4 depict, respectively, the in- and out-degree for the financial institutions studied. The measures computed using our TVP-VAR framework are shown in bold solid; whereas the light dashed lines depict those computed from the rolling window approach with window size of 36 months. By visually inspecting each chart, we can im-

mediately notice that the two methodologies, the TVP-VAR framework and rolling window approach, produce very different pictures. For example, Figure 3 shows that in-degree Fannie Mae peaked at very different periods according to the methodology used.

Figure 3 shows that Bear Stearns was the financial institution to attain the highest level of in-degree, peaking at a value of 2.4 in March 2008 (the peak is not shown in the figure for visibility reasons). Notice that Bear Stearns' in-degree calculated using the classical rolling window approach jumps suddenly in March 2008, whereas the corresponding measure calculated with our time-varying parameters framework shows an increase that began long before this event. This means that it was receiving stronger spillovers from a growing number of firms, effectively increasing its fragility. Bear Stearns was the first large bank to collapse as a result of the subprime mortgage crisis of 2007. The data involved in calculating its in-degree did not contain the financial meltdown of September 2008. This is because the sample time-series of Bears Stearns stock price was quoted only until May 2008, when it was acquired by JP Morgan Chase. Therefore, the high in-degree of Bear Stearns cannot be attributable to any noise caused by the many events that occurred during September-October 2008.

Figure 4 shows that out-degree levels were more homogeneous between financial institutions compared to the in-degree levels showed in Figure 3. American International Group, Goldman Sachs, and Merrill Lynch were among the financial institutions with the highest out-degree. This concurs with the findings of Hautsch et al. (2014) and Dungey et al. (2013) who also identified these banks as very central. For Lehman Brothers, the level of out-degree found using our time-varying parameter framework was higher than that found using the rolling window approach. Nonetheless, it was not among the highest across the financial

institutions analysed. The result is consistent with the policy decision taken by the government and Federal Reserve to not bailout Lehman Brothers during the crisis because it was considered as not systemic (Committee on Oversight and Government Reform (2008)).

According to an asset and liability exposure analysis conducted by Scott (2012), Lehman's bankruptcy was not particularly destabilizing for its direct counterparties. However, without doubt Lehman's bankruptcy triggered a sentiment of fear and uncertainty in markets. Fear was primarily due to the realization that government rescue of large financial institutions was no longer guaranteed, whereas uncertainty surrounded the extent of losses incurred by other institutions due to Lehman's default. This led to a freeze in the short-term funding market and effectively a liquidity crisis that affected financial institutions indiscriminately of their contractual obligations with Lehman Brothers.

On the other hand, Lehman's reliance on short-term rather than long-term funding made it vulnerable to external shocks (Scott (2012)). This is detectable in the growing in-degree of Lehman, which according to our time-varying framework had began since 2006, as can be seen in Figure 3.

## **B. Stability of centrality rankings**

Using the approach developed by the Basel Committee on Banking Supervision (BCBS), the FSB ranks financial institutions according to their systemic importance and uses this ranking to determine their additional loss absorbency requirements.<sup>6</sup> In a similar spirit, we ranked the 20 systemically important financial institutions according to their in- and

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<sup>6</sup>Additional loss absorbency requirements have phased in starting in January 2016 with full implementation by January 2019 (see Financial Stability Board (2011)).

out-degree. These two rankings give only a partial view on systemic risk, one based only on interconnectedness, whereas the rankings drawn by the FSB are also based other determinants of systemic risk such as size and leverage. Nonetheless, the rankings can help us identify, at every point in time, the most exposed institutions to spillovers effects and the most important propagators of spillover effects.

As discussed by Daniélsson, James, Valenzuela, and Zer (2015) and Dungey et al. (2013), the usefulness of rankings for policy makers is severely limited if these are prone to frequent, drastic changes that lead to unmotivated excessive alarm. In an attempt to assess and quantify this aspect of interconnectedness rankings we developed a series of stability indicators.

Let  $Z_{it}^{in}$  be the ordinal ranking of institution  $i$  at time  $t$  in terms of in-degree. Similarly, let  $Z_{it}^{out}$  be the ordinal ranking of institution  $i$  at time  $t$  in terms of out-degree. We compute the *quadratic* ranking stability indicator as

$$SI_Q^{in} = \frac{1}{T-1} \sum_{t=2}^T \sqrt{\sum_{i=1}^{N_t} \frac{(Z_{it}^{in} - Z_{it-1}^{in})^2}{N_t}}, \quad SI_Q^{out} = \frac{1}{T-1} \sum_{t=2}^T \sqrt{\sum_{i=1}^{N_t} \frac{(Z_{it}^{out} - Z_{it-1}^{out})^2}{N_t}}.$$

Similarly, we construct the *absolute* stability indicator as

$$SI_A^{in} = \sum_{t=2}^T \sum_{i=1}^{N_t} \frac{|Z_{it}^{in} - Z_{it-1}^{in}|}{N_t(T-1)}, \quad SI_A^{out} = \sum_{t=2}^T \sum_{i=1}^{N_t} \frac{|Z_{it}^{out} - Z_{it-1}^{out}|}{N_t(T-1)}.$$

The *quadratic* stability indicators,  $SI_Q^{in}$  and  $SI_Q^{out}$ , measures the average change in the ranking between adjacent time periods. The quadratic term used in the calculation causes large deviations in the rankings to have a larger impact on the stability indicator compared

to smaller deviations. On the other hand, for the *absolute* stability indicator,  $SI_A^{in}$  and  $SI_A^{out}$ , the weight increases only linearly with the distance between ranking positions. As additional indicators of stability, we also computed the average percentage of financial institutions that kept their position in the ranking between adjacent time periods and the average change (in percentage terms) in the top 5 and top 10 rankings between adjacent time periods.

In order to have a reference for comparison, and in a similar spirit to Nucera, Schwaab, Koopman, and Lucas (2016), we computed the stability indicators for four additional rankings based on the following measures of systemic risk: SRisk (Acharya et al., 2012, Brownlees and Engle, 2016), marginal expected shortfall (MES) (Acharya et al., 2010), leverage ratio (Engle, Jondeau, and Rockinger, 2015), and beta CAPM (Engle, 2012).<sup>7</sup>

Table 2 shows the stability indicators for rankings based on degree centrality measures found with our time-varying parameter framework and with the rolling window approach (top and bottom panel), as well as for rankings based on the four other systemic risk measures (middle panel).

The first four columns of Table 2 display the *quadratic* and *absolute* stability indicators. Both indicators, for both centrality measures (in- and out-degree), show that the rankings based on our time-varying parameter framework are far more stable than the rankings based on the classical rolling window approach. In fact, according to both stability indicators, the rolling window approach appears to produce rankings that are more than twice as unstable than those produced by the TVP-VAR framework.

The middle panel of Table 2 shows the stability of rankings obtained from other systemic

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<sup>7</sup>We obtain the monthly time-series of systemic risk measures from the Vlab website, <http://vlab.stern.nyu.edu>. The data covers periods 2000-2014.

risk measures. Notice that leverage provides the most stable ranking, according to the absolute and quadratic stability indicators. This is because leverage is based on the book value of assets, which is generally observed at low frequencies. On the other hand, MES and Beta, which are based on higher frequency market data, provide the most unstable rankings.

Comparing the middle panel to the top panel of Table 2, which shows the stability of the time-varying parameter and rolling window rankings for comparable periods of our sample, we see that the time-varying parameter framework produces rankings with similar stability to SRisk. SRisk uses a combination of book value and market data, whereas the our TVP-VAR framework uses exclusively market data.

The successive two columns headed “% Invariance”, denote the average percentage of financial institutions that kept the same position in the ranking between adjacent time periods. Between 2000 and 2014, on the average month, about 55% of financial institutions kept the same position they held in the previous month in the in- and out-degree rankings calculated with the TVP-VAR framework. For rankings computed using the rolling window approach, only about 37% of financial institutions kept the same position between adjacent months. The same difference was found when using the whole sample, from 1993 to 2014 (shown in the bottom panel of Table 2). This confirms the higher stability of rankings estimated with the TVP-VAR framework. The “% Invariance” of rankings based on other systemic risk measures seem to confirm the previous results, i.e., that the TVP-VAR yields more stable rankings than MES and market Beta, but appears to be similar, in terms of stability, to SRisk and less stable than Leverage.

The columns headed “ $\Delta$  Top 5” and “ $\Delta$  Top 10” show the average changes (in percentage terms) in the composition of, respectively, the top 5 and the top 10 financial institutions in

the rankings. For example, for the rankings computed using the rolling window approach, we can expect, on average, one firm to change in the top 5 (19% for in-degree, 20.9% for out-degree, see bottom panel of Table 2) every month. On the other hand, for the rankings computed using the TVP-VAR framework, only between 7.3% and 9.3% of the top 5 changed on average, so substantially less than one firm per month. Similar magnitudes of results were obtained for the 2000-2014 period (shown in the top panel of Table 2) and for the stability of the top 10 firms in the rankings.

All measures used to quantify stability indicate that the rolling window approach, with standard window size, provides less stable rankings compared to the time-varying parameter framework even though both approaches make use of the same data. The reasons for the higher stability offered by the time-varying parameter framework are to be found in the transition law imposed for time-varying connections. By allowing some degree of inertia between successive time periods, large exceptional observations have less influence on the estimated path of connections. On the other hand, with the rolling window approach, these observations have a larger weight in the estimation of connections.

The high instability of the rankings found using the rolling window approach would make these rankings difficult to use for policy purposes. It would be hard to justify policy decisions based on a ranking that changes, on average, one component in its top 5 most interconnected institutions every month. On the other hand, the time-varying parameter framework offers a generally stable ranking whilst allowing some degree of flexibility that can be useful to motivate policy intervention.

### C. Out-of-sample interconnectedness

In order to further illustrate the use of our framework for policy purposes, we conducted an out-of-sample exercise, which consisted in estimating interconnectedness using a smaller data sample restricted up to the end of 2007.<sup>8</sup> We then evaluated which institutions were the most interconnected, in terms of in-degree and out-degree, prior to the crisis at the end of December 2007.

Table 3 shows the top 5 financial institutions, in terms of in-degree and out-degree, found using our time-varying framework with the restricted data sample. Concurrent with previous results, Bear Stearns was in the top 5 ranking in terms of in-degree but was only ranked 18<sup>th</sup> (not shown) in terms of out-degree. Thus, the result that Bear Stearns was not an important propagator of spillover effects, whereas it was susceptible to receiving spillover effects from other institutions, was also detectable in December 2007 using a restricted data sample.

Similarly, the results regarding Lehman Brothers can be confirmed using the restricted data sample. Lehman Brothers resulted the fourth most interconnected financial institution in terms of in-degree in December 2007, whereas it was ranked 12<sup>th</sup> in terms of out-degree for the same period. On the other hand, AIG was ranked first in terms of out-degree in December 2007 using the restricted data sample. This highlights the potential widespread influence its default could have caused and concurs with the decision taken by the Federal Reserve to bailout AIG in September 2008.

Overall, the out-of-sample exercise shows that the TVP-VAR framework can identify highly interconnected financial institutions prior to the crisis, using a realistic data sample. Results are consistent with those obtained previously using the full data sample, both in

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<sup>8</sup>We thank the anonymous referee for suggesting this exercise.



terms of in-degree and in terms of out-degree.

## VII. Conclusion

We provide a framework for estimating interconnectedness between financial institutions that accounts for the dynamic nature of connections. We build our framework in a TVP-VAR setting and use Bayesian inference to evaluate, at every moment in time, the posterior probability of a connection existing between any two financial institutions. The framework surpasses several limitations of the classical approach for measuring interconnectedness by sequentially running Granger causality tests over rolling windows of data. Moreover, by modeling both temporal and contemporaneous dependencies, our framework contributes to both strands of the literature on interconnectedness market-based measures. Finally, we contribute to the TVP-VAR literature by proposing a model that accommodates many of the properties of asset returns, namely, heavy-tails, heteroskedasticity, and skewness.

We applied the proposed framework to estimate the time-varying network of spillover effects of the U.S. financial system. We showed that the TVP-VAR framework delivers sectorial density and financial institution centrality measures that are less noisy than the same measures delivered by the rolling window approach. At the sectorial level, we found that interconnectedness evolved gradually over time and peaked around two crucial events: the LTCM crisis in 1998, and the global financial crisis in 2008. At the financial institution level, rankings drawn using our TVP-VAR framework than rankings drawn using the classical rolling window approach. The TVP-VAR framework also delivered more stable rankings than other comparable market-based measures of systemic risk, such as the marginal expected

shortfall, but delivered more reactive rankings than measures based on book value data, such as leverage, which only evolve at very low frequencies.

## References

- Acharya, V. V.; R. Engle; and M. Richardson. “Capital Shortfall: A new approach to ranking and regulating systemic risk.” *American Economic Review*, 102 (2012), 59–64.
- Acharya, V. V.; L. H. Pedersen; T. Philippon; and M. Richardson. “Measuring Systemic Risk.” (2010). NYU Working Paper.
- Adams, Z.; R. Füss; and R. Gropp. “Spillover effects among financial institutions: A state-dependent sensitivity value-at-risk approach.” *Journal of Financial and Quantitative Analysis*, 49 (2014), 575–598.
- Adrian, T., and M. K. Brunnermeier. “CoVaR.” *American Economic Review*, 106 (2016), 1705–1741.
- Barigozzi, M., and C. Brownlees. “NETS: Network Estimation for Time Series.” (2016). Working Paper.
- Barigozzi, M., and M. Hallin. “Networks, Dynamic Factors, and the Volatility Analysis of High-Dimensional Financial Series.” (2015). ECARES Working Paper 2015-34.
- Baumeister, C., and L. Benati. “Unconventional monetary policy and the Great Recession: Estimating the macroeconomic effects of a spread compression at the zero lower bound.” *International Journal of Central Banking*, 9 (2013), 165–212.
- Bernardi, M.; G. Gayraud; and L. Petrella. “Bayesian tail risk interdependence using quantile regression.” *Bayesian Analysis*.
- Billio, M.; M. Getmansky; A. W. Lo; and L. Pelizzon. “Econometric Measures of Connectedness and Systemic Risk in the Finance and Insurance Sectors.” *Journal of Financial Economics*, 104 (2012), 535–559.
- Brownlees, C., and R. Engle. “SRISK: A Conditional Capital Shortfall Measure of Systemic Risk.” *The Review of Financial Studies*. Forthcoming.
- Ciccarelli, M., and A. Rebucci. “Measuring contagion and interdependence with a Bayesian time-varying coefficient model: An application to the Chilean FX market during the Argentine Crisis.” *Journal of Financial Econometrics*, 5 (2007), 285–320.
- Clark, T. E., and M. W. McCracken. “Improving forecast accuracy by combining recursive and rolling forecasts.” *International Economic Review*, 50 (2009), 363–395.
- Cogley, T., and T. Sargent. “Drifts and Volatilities: Monetary Policies and Outcomes in the Post WWII US.” *Review of Economic Dynamics*, 8 (2005), 262–302.

- Committee on Oversight and Government Reform. “Causes and Effects of the Lehman Brothers Bankruptcy.” Hearing (2008). Opening statement of Chairman Henry Waxman.
- Daniélsson, J.; K. James; M. Valenzuela; and I. Zer. “Model risk of risk models.” (2015). Working Paper.
- Diebold, F. X., and K. Yilmaz. “Measuring financial asset return and volatility spillovers, with application to the global equity markets.” *The Economic Journal*, 119 (2009), 158–171.
- Diebold, F. X., and K. Yilmaz. “On the Network Topology of Variance Decompositions: Measuring the Connectedness of Financial Firms.” *Journal of Econometrics*, 182 (2014), 119–134.
- Dungey, M.; M. Luciani; and D. Veredas. “Googling SIFIs.” (2013). ECARES Working Paper.
- Engle, R. F. “Dynamic Conditional Beta.” (2012). Social Science Research Network.
- Engle, R. F.; E. Jondeau; and M. Rockinger. “Systemic Risk in Europe.” *Review of Finance*, 19 (2015), 145–190.
- Financial Stability Board. “Policy Measures to Address Systemic Important Financial Institutions.” Tech. rep. (2011).
- Hautsch, N.; J. Schaumburg; and M. Schienle. “Forecasting systemic impact in financial networks.” *International Journal of Forecasting*, 30 (2014), 781–794.
- Hautsch, N.; J. Schaumburg; and M. Schienle. “Financial network systemic risk contributions.” *Review of Finance*, 19 (2015), 685–738.
- Kim, C.-J., and C. R. Nelson. *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*. Cambridge, MA: MIT Press (1999).
- Koop, G.; R. Leon-Gonzalez; and R. W. Strachan. “Dynamic Probabilities of Restrictions in State Space Models: An Application to the Phillips Curve.” *Journal of Business and Economic Statistics*, 28 (2010), 370–379.
- McDonough, M. J. “Statement by william j. mcdonough, president federal reserve bank of new york before the committee on banking and financial services u.s. house of representatives.” (1998).
- Nakajima, J., and Y. Omori. “Stochastic volatility model with leverage and asymmetrically heavy-tailed error using GH skew Student’s  $t$ -distribution.” *Computational Statistics & Data Analysis*, 56 (2012), 3690–3704.
- Nucera, F.; B. Schwaab; S. J. Koopman; and A. Lucas. “The information in systemic risk rankings.” *Journal of Empirical Finance*.

- Primiceri, G. “Time Varying Structural Vector Autoregressions and Monetary Policy.” *Review of Economic Studies*, 72 (2005), 821–852.
- Scott, H. S. “Interconnectedness and contagion.” (2012). Committee on Capital Markets Regulation Working Paper.
- White, H.; T.-H. Kim; and S. Manganelli. “VAR for VaR: Measuring tail dependence using multivariate regression quantiles.” *Journal of Econometrics*, 187 (2015), 169–188.
- Yu, K., and R. A. Moyeed. “Bayesian quantile regression.” *Statistics & Probability Letters*, 54 (2001), 437–447.
- Zivot, E., and J. Wang. *Modelling Financial Time Series with S-PLUS*, chap. 9. Springer (2006). 313–360.

## Tables

	<b>Banks</b>	<b>Brokers</b>	<b>Insurers</b>	<b>Real Estate</b>
<i>2007-2009</i>	32.9	12.6	8.3	4.8
<i>1993-2014</i>	24.0	14.6	7.5	6.9

**Table 1: Average leverage ratio for financial institutions in our sample.**

		Stability Indicators				% Invariance		$\Delta$ Top 5		$\Delta$ Top 10	
		<i>quadratic</i>		<i>absolute</i>		<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
		$SI_Q^{in}$	$SI_Q^{out}$	$SI_A^{in}$	$SI_A^{out}$	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
2000-2014	<b>RW-36M</b>	2.5	2.7	1.7	1.8	36.8	36.7	18.6	22.4	11.6	13.4
	<b>TVP-VAR</b>	1	1.1	0.6	0.7	56.3	54.4	4.9	8.5	4.5	4.8
		$SI_Q$		$SI_A$		% Invariance		$\Delta$ Top 5		$\Delta$ Top 10	
2000-2014	<b>SRisk</b>	1.3		0.8		57.7		17.3		10.9	
	<b>MES</b>	3.1		2.3		23.7		60		33.3	
	<b>Leverage</b>	0.8		0.5		69.2		24.2		12.8	
	<b>Beta</b>	3.1		2.3		23.5		60		33.5	
		$SI_Q^{in}$	$SI_Q^{out}$	$SI_A^{in}$	$SI_A^{out}$	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>	<i>in</i>	<i>out</i>
1993-2014	<b>RW-36M</b>	2.4	2.5	1.6	1.7	38.4	36.5	19	20.9	11.5	12.4
	<b>TVP-VAR</b>	1.2	1.2	0.8	0.8	52.1	50.8	7.3	9.2	5.5	6.3

**Table 2:** The stability indicators computed for rankings based on centrality computed with the TVP-VAR framework and with the rolling window approach (RW-36M), as well as for rankings based on SRisk, MES, leverage, and market beta. Stability Indicator expresses the extent of monthly changes in the rankings based on degree centrality. % Invariance measures the proportion of financial institutions that held the same position in the ranking between adjacent time periods.  $\Delta$  Top 5 and  $\Delta$  Top 10 measures the extent of monthly changes in the composition of, respectively, the top 5 and top 10 institutions of the rankings.

	<i>in-degree</i>	<i>out-degree</i>
# 1	Goldman Sachs Group Inc	American International Group
2	State Street Corp	Bank Of America Corp
3	Federal Home Loan Mortg Corp	Citigroup Inc
4	Lehman Brothers Holdings Inc	Wells Fargo & Co
5	Bear Stearns Companies Inc	Wachovia Corp

**Table 3:** Top 5 financial institutions in terms of in-degree and out-degree centrality on 31 December 2007 using a small sample (1994-2007).

# Figures

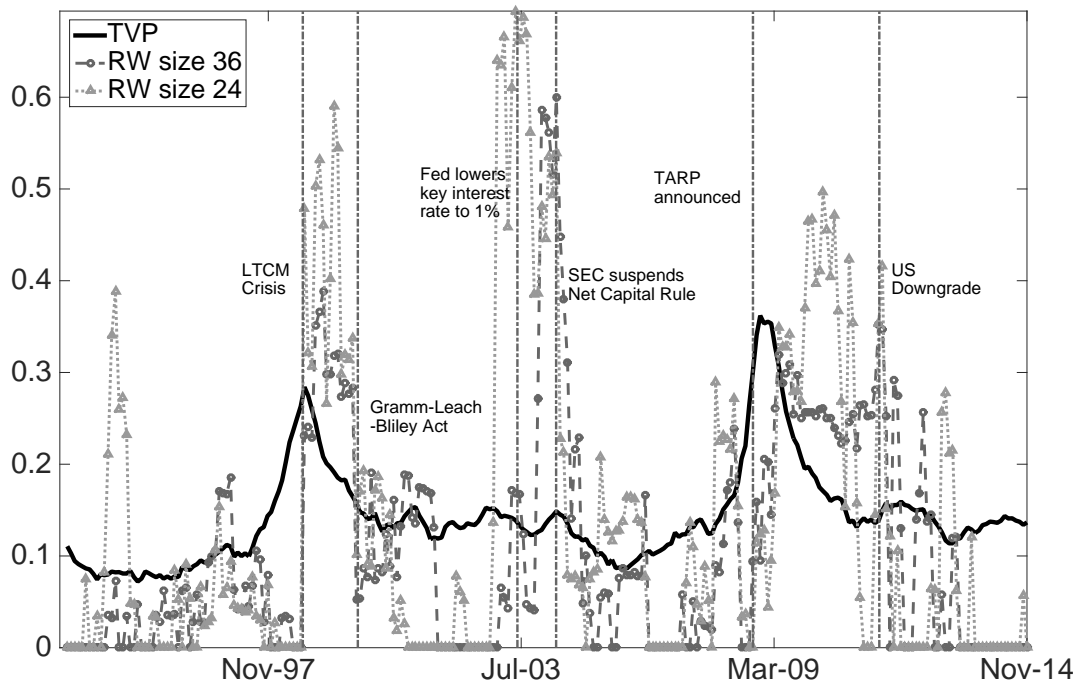
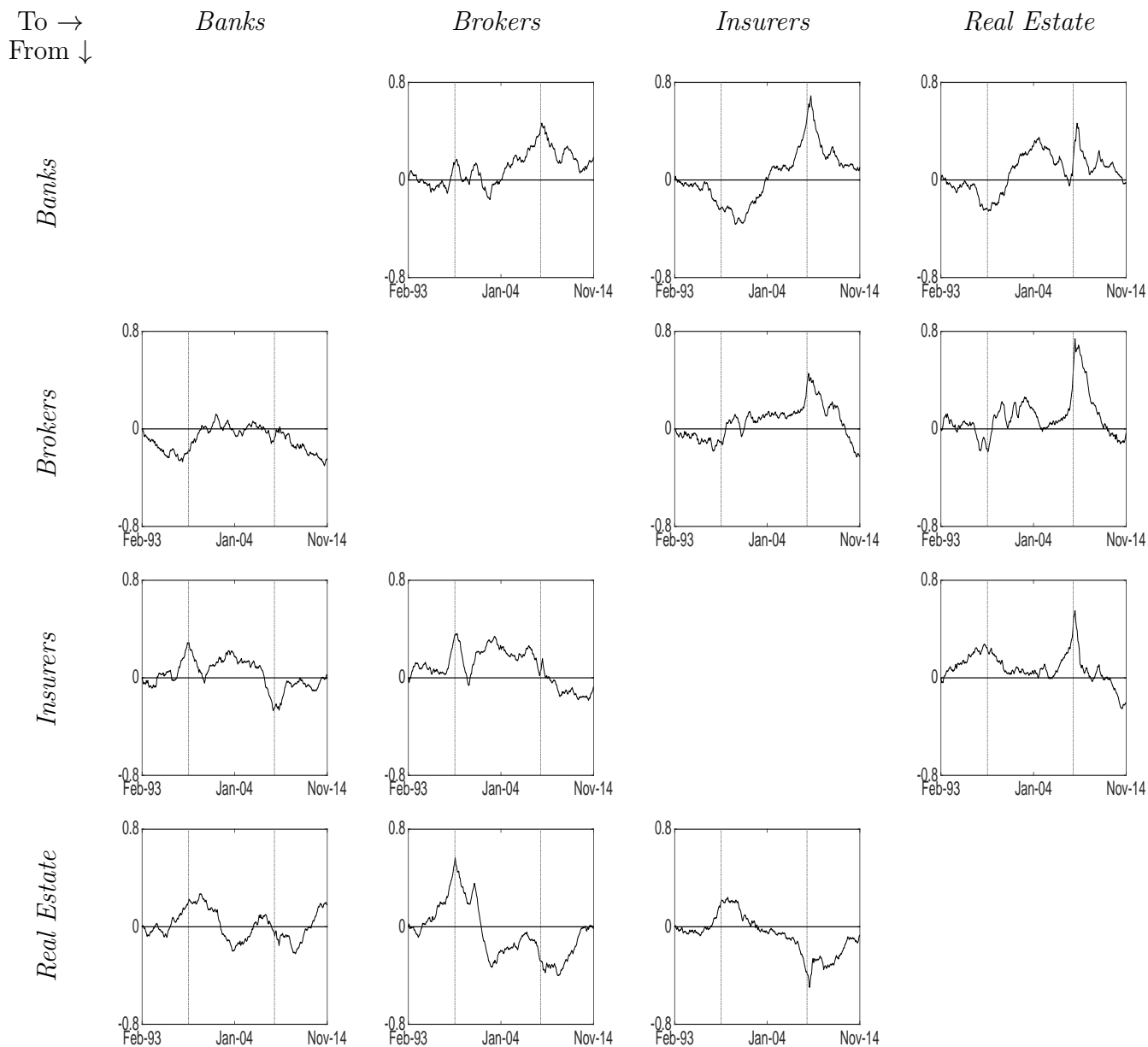
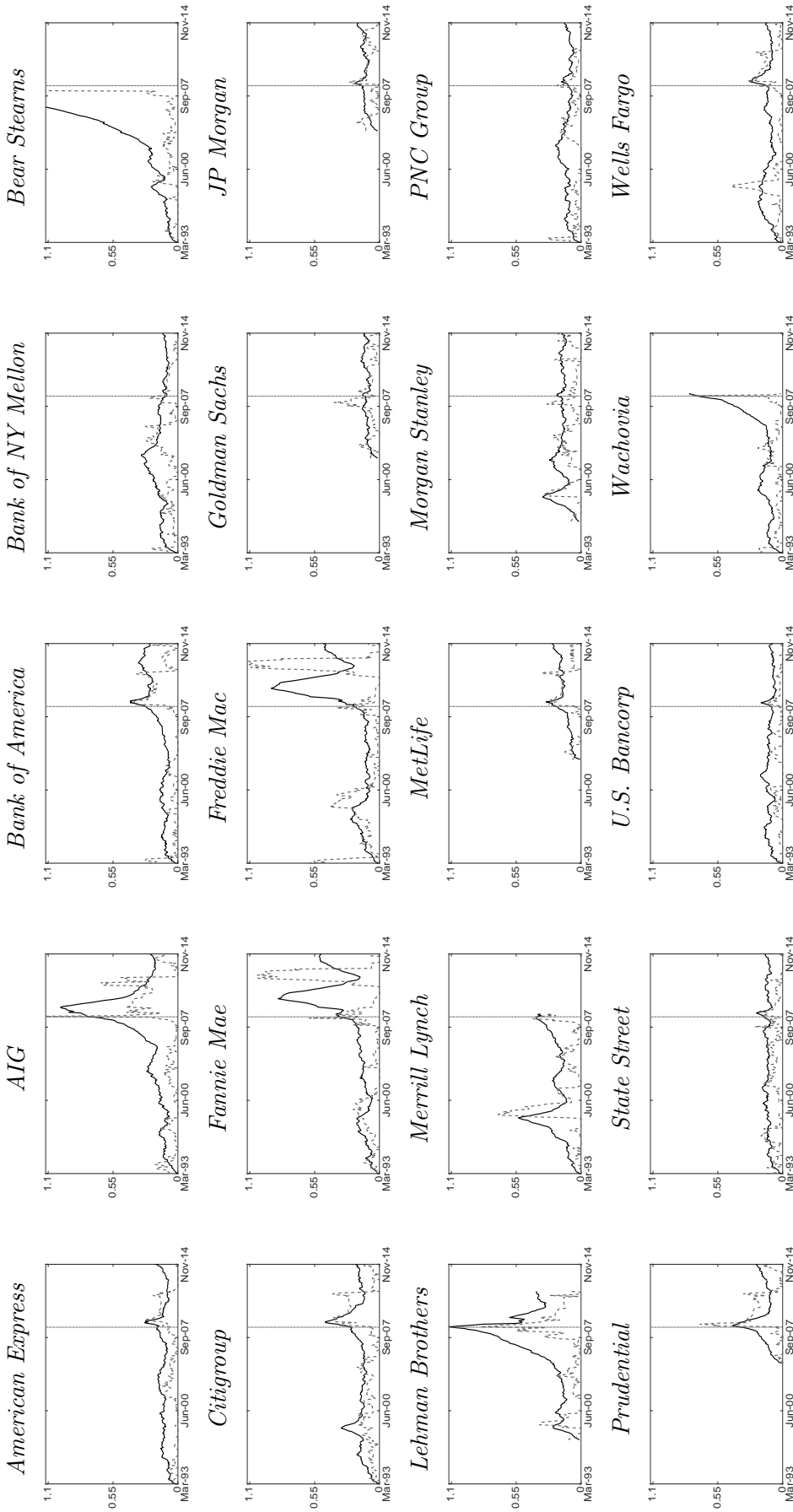


Figure 1: Sectorial density estimated by the TVP-VAR approach (bold solid) and by Granger causality testing with rolling windows of 36 months (light dashed) and of 24 months (triangles). Significant events are indicated by the dashed vertical lines.



**Figure 2:** The time-varying cross-autoregressive coefficients of the TVP-VAR with one lag at the sectorial level. Each chart displays the cross-autoregressive coefficient representing the temporal spillover effect from a given sector (indicated by the rows of the figure) to another sector (indicated by the columns of the figure). The two dashed vertical lines indicate the day of the day Russian default, 17 August 1998, which marked the beginning of the LTCM crisis, and the day of the bankruptcy of Lehman Brothers, 15 September 2008.



**Figure 3: In-degree for U.S. banks identified as global SIFs by the FSB. The bold solid lines indicate in-degree estimated using the TVP-VAR framework. The lighter dashed lines indicate in-degree estimated using the classical approach of Granger causality testing over rolling windows of 36 months. The dashed vertical lines indicate the day of Lehman Brothers bankruptcy, 15 September 2008.**



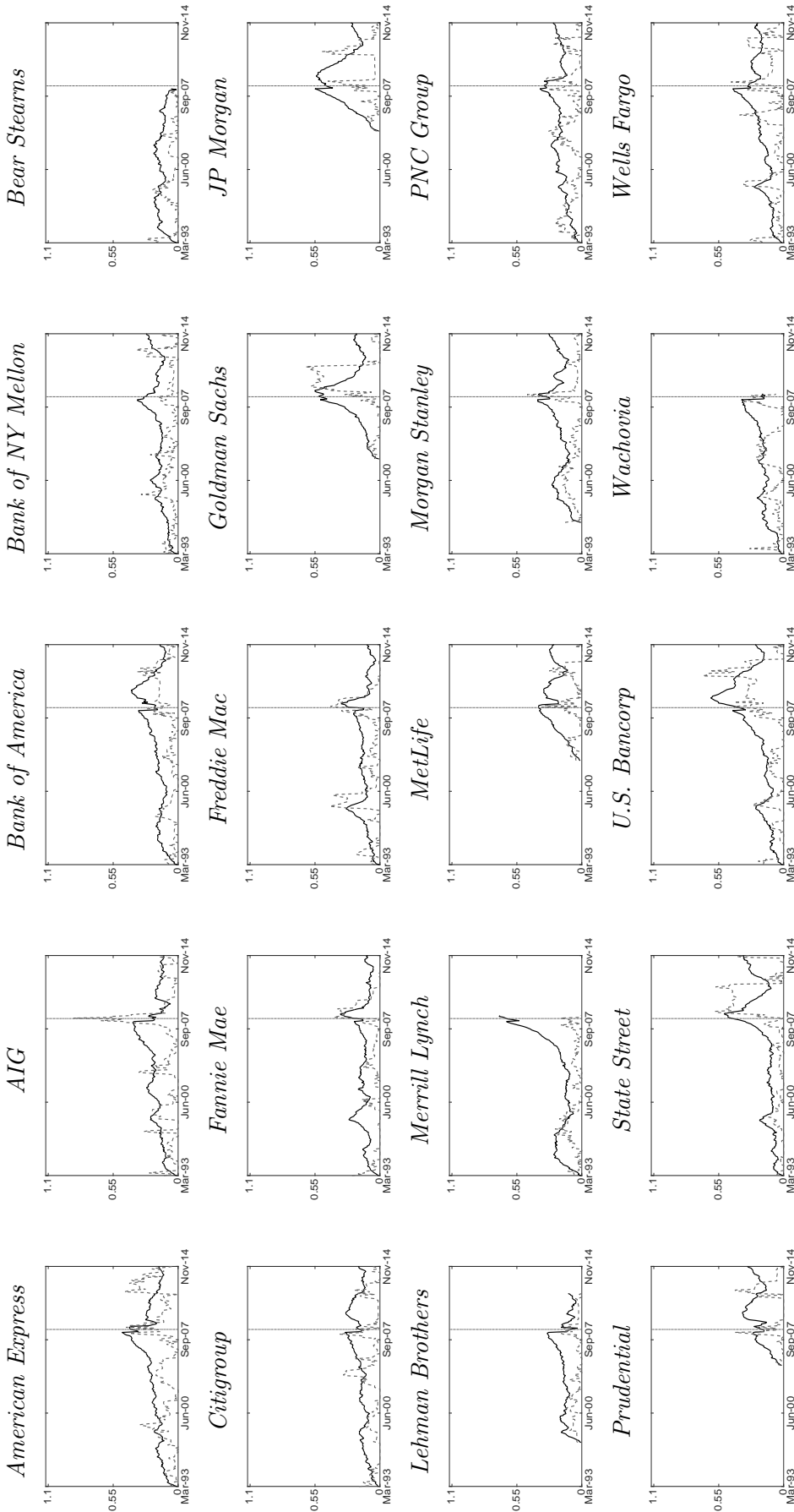


Figure 4: Out-degree for U.S. banks identified as global SIFs by the FSB. The bold solid lines indicate out-degree estimated using the TVP-VAR framework. The lighter dashed lines indicate out-degree estimated using the classical approach of Granger causality testing over rolling windows of 36 months. The dashed vertical lines indicate the day of Lehman Brothers bankruptcy, 15 September 2008.