Spectrum Sensing Method Based on the Likelihood Ratio Goodness of Fit Test under Noise Uncertainty

Djamel Teguig, Bart Scheers, Vincent Le Nir
CISS Department
Royal Military Academy
Brussels, Belgium

François Horlin
OPERA-Wireless Communications Group
Université Libre de Bruxelles
Brussels, Belgium

***Abstract*—In cognitive radio, spectrum sensing is one of the most important tasks. In this article, a blind spectrum sensing method based on goodness-of-fit (GoF) test using likelihood ratio (LLR) is studied. In the proposed method, a chi-square distribution is used for GoF testing. The performance of the method is evaluated through Monte Carlo simulations. It is shown that the proposed spectrum sensing method outperforms the GoF test using Anderson Darling (AD) and the conventional energy detection (ED) in case of a limited number of received samples and low signal to noise ratio (SNR). We also evaluate the proposed method in case of a non-Gaussian noise and in case of noise uncertainty. It is shown that the GoF based spectrum sensing methods are less sensitive to both impairments, than the conventional ED. Finally, this paper investigates the influence of the number of samples on the detection performance. The performance difference between the GoF based sensing (LLR and AD) and ED increases with decreasing number of samples for sensing, which makes the proposed method very effective in CR systems with short sensing periods.**

Keywords—Cognitive Radio; Spectrum Sensing; Goodness of Fit test; Likelihood Ratio; Mixture Gaussian Noise.

I. Introduction

One of the most important task in cognitive radio (CR) is spectrum sensing. The main function of spectrum sensing is to detect the presence of other users utilizing the same frequencies, in order to access the channel without causing interference [1].

Spectrum sensing methods are classified into two categories, coherent sensing methods and blind sensing methods. In coherent sensing methods, such as Cyclostationary, matched filtering and waveform-based sensing [2] [3], the CR node uses a priori knowledge of the waveform of the considered signal. In case of blind sensing methods, the CR node does not require any prior knowledge of the transmitted waveform. Some examples are Energy Detection (ED) [4] and Goodness of Fit (GoF) tests [5]. Due to its low complexity, the ED is the most common method for spectrum sensing in CR. Nevertheless, the performance of the ED is deeply affected by noise uncertainty at low signal to noise ratio (SNR) [8].

The GoF test is a blind nonparametric hypothesis test problem which can be used to detect the presence of signals in noise by determining whether the received samples are (are not) drawn from a distribution with a Cumulative Distribution Function (CDF) F_0, representing the noise distribution. The hypothesis to be tested can be formulated as follows:

$$H_0: F_n(x) = F_0(x) \qquad (1)$$
$$H_1: F_n(x) \neq F_0(x)$$

where $F_n(x)$ is the empirical CDF of the received sample and can be calculated by:

$$F_n(x) = |\{i: x_i \leq x,\ 1 \leq i \leq n\}/n|, \qquad (2)$$

where $|\blacksquare|$ indicates cardinality, $x_1 \leq x_2 \leq \cdots \leq x_n$ are the samples under test and n represents the total number of samples.

A. Related Work

There have been many goodness of fit test based spectrum sensing proposed in literature. The most important ones are the Kolmogorov- Smirnov test [6], the Cramer-Von Mises test [9], the Anderson-Darling test [5] and order statistics [7]. All these tests are based on the hypothesis test as formulated in (1), but differ in the way the distance between the empirical cumulative distribution of the observations made locally at the CR user and the noise distribution is calculated. The calculated distance is compared with a threshold to decide whether the signal is present or not, given a certain probability of false alarm.

The GoF test based spectrum sensing was first presented in [5]. It is based on the Anderson-Darling GoF test to decide whether the received samples are drawn from the noise distribution F_0 (Gaussian distribution) or an alternative distribution. Authors in [5], show by simulations that AD-sensing outperforms the ED-sensing at low SNR. In [6], authors propose a new spectrum sensing method based on KS GoF test, and it is shown that the proposed method provides much higher sensitivity than the ED and it requires less samples of the received signal. Another blind spectrum sensing method using GoF is proposed in [7] based on Order Statistics (OS). It is shown that OS based sensing outperforms both AD and ED-sensing in AWGN channel and lower SNR. In [10], the authors reformulate the spectrum sensing into a Student's *t*-distribution testing problem and propose a blind spectrum sensing method which does not require any knowledge of the transmitted signal. The performance of the proposed method is better than ED-sensing but less than AD-sensing proposed in [5]. In [11], authors propose a spectrum sensing method based on KS two-sample test in which the sensing problem is formulated as a two-sample GoF test. Recently, detection methods based on Tietjen-Moore (TM) and Shapiro-Wilk (SW) tests are proposed to detect and suppress spectrum sensing data falsification (SSDF) attacks by malicious user in cooperative spectrum sensing [12].

All above mentioned methods take as noise CDF $\boldsymbol{F_0}$ for the GoF test, a CDF of a normal distribution, meaning that they all assume that the samples of the received signal are real valued. However, in CR spectrum sensing this is a limitation, as a radio receives complex valued IQ samples.

In this paper, we overcome this problem by considering the energy of the received samples and test them against a chi-square distribution under hypothesis H_0. Further, we will evaluate the performance of a more recent GoF test, i.e. the likelihood ratio (LLR) test, in the application of GOF based spectrum sensing. The simulation results illustrate that the proposed LLR-GoF method is performing better than the one based on AD test and ED spectrum sensing methods.

The main contributions of this work include:

- The proposition of a more realistic model based on the energy of the received samples, instead of the model of [5] in which they assume a static and real received signal.
- The study of a spectrum sensing method based on goodness of fit test using likelihood ratio test, and comparison of its performance with the AD-GoF test and the ED spectrum sensing.
- The evaluation of the GoF based spectrum sensing (LLR and AD) in non-Gaussian noise and in case of noise uncertainty. The Gaussian mixture (GM) noise is used to model a non Gaussian noise. To the best of our knowledge, there is no previous work on this topic.

The paper is organized as follows. Section II describes the powerful GoF test based on likelihood ratio (LLR). Section III presents the system model for spectrum sensing and the steps of the proposed LLR-GoF based sensing. Next, in section IV, the GoF based spectrum sensing is investigated under non Gaussian noise and the effect of noise uncertainty is studied. The impact of the number of samples on the detection performance is evaluated in section V. In parallel, simulation results and discussions are presented. Finally, we conclude the paper in section VI.

II. LIKELIHOOD BASED GOODNESS OF FIT TEST

In [13], the authors propose a new approach of parameterization to construct a general GoF test. With this approach, they could generate the traditional GoF tests including KS, CM and AD test. Moreover, they provided also a new, more powerful GoF test, based on likelihood ratio.

The authors in [13] formulated the hypothesis test as follows:

$$H_0:\ H_0(t): F_n(t) = F_0(t)\ \ for\ all\ \ t \in (-\infty, \infty) \qquad (3)$$

$$H_1:\ H_1(t): F_n(t) \neq F_0(t)\ \ for\ all\ \ t \in (-\infty, \infty)$$

Meaning that testing H_0 versus H_1 is equivalent to testing H_0(t) versus H_1(t) for every $t \in (-\infty, \infty)$.

Two types of statistic for testing H_0 versus H_1 were proposed:

$$Z = \int_{-\infty}^{\infty} Z_t\, dw(t) \qquad (4)$$

$$Z_{max} = \sup_{t \in (-\infty,\infty)} \{Z_t\, w(t)\} \qquad (5)$$

with Z_t a statistic for testing $H_0(t)$ versus $H_1(t)$ and $w(t)$ some weight function. Large values of Z or Z_{max} will reject a null hypothesis H_0. In [13], authors present two natural candidates for Z_t, the Pearson χ^2 test statistic and the likelihood ratio (LLR) test statistic. The LLR test statistic is given by:

$$G_t^2 = 2n[\, F_n(t) \log\left\{\frac{F_n(t)}{F_0(t)}\right\} + \left(1 - F_n(t)\right)\, \log\left\{\frac{1-F_n(t)}{1-F_0(t)}\right\}] \qquad (6)$$

where $F_n(t)$ is the empirical distribution function of the received samples.

Taking in (4) Z_t as G_t^2 and choosing an appropriate weight function $w(t)$, produces a powerful goodness of fit tests statistic Z_A, comparing to the traditional tests.

$$Z_A = -\sum_{i=1}^{n}\left[\frac{\log\{F_0(X_{(i)})\}}{n-i+\frac{1}{2}} + \frac{\log\{1-F_0(X_{(i)})\}}{i-\frac{1}{2}}\right] \qquad (7)$$

where $X_{(i)}$ are ordered samples: $X_{(1)} < X_{(2)} \ldots < X_{(n)}$

For the proposed spectrum sensing method in this paper, we will use the test statistic Z_A as LLR-GoF test and compare it with the traditional Anderson Darling test in which the GoF test statistic is given by:

$$A_n^2 = -n - \frac{\sum_{i=1}^{n}(2i-1)\left(\ln F_0(X_{(i)}) + \ln\left(1-F_0(X_{(n+1-i)})\right)\right)}{n} \qquad (8)$$

Ones the test Z_A is computed, it will be compared to a predefined threshold λ and the statistical test reduces to:

$$H_0: Z_A \leq \lambda \qquad (9)$$

$$H_1: Z_A > \lambda$$

III. SPECTRUM SENSING MODEL

As a starting point, we recall the model in [5] in which the authors consider an AWGN channel.

$$H_0: r_i = w_i \quad (10)$$

$$H_1: r_i = \sqrt{\rho}m + w_i$$

where H_0 and H_1 represent the hypothesis of absence and presence of a primary signal, respectively. m represents the transmitted signal, ρ is the signal to noise ratio (SNR), w_i is the real Gaussian noise with zero mean and unit variance and r_i are real valued. In [5], the sensing method is based on testing the GoF of the received samples compared to the Gaussian distribution.

The authors in [5] assumed that the transmitted signal m=1, in other words, the received signal is represented as $r_i = \sqrt{\rho} + w_i$. The model in (10) does not reflect a realistic scenario for spectrum sensing by a cognitive radio, as normally the received signal is complex and varies in time.

We have proposed in [14] to start from the more general model:

$$H_0: X_i = W_i \quad (11)$$

$$H_1: X_i = S_i + W_i$$

where S_i are the received complex samples of the transmitted signal and W_i is the complex Gaussian noise. We now consider the random variable $Y_i = |X_i|^2$ which corresponds to the received energy. It is known that, if the real and the imaginary part of X_i are normally distributed, which is the case under H_0 hypothesis, the variable $Y_i = |X_i|^2$ is chi-squared distributed with 2 degrees of freedom.

The spectrum sensing problem can now be reformulated as an hypothesis represented in (1) where we will test whether the received energy $Y_i = |X_i|^2$ are drawn from a chi-square distribution with 2 degrees of freedom or not. F_0, the CDF of the chi-square distribution is given by:

$$F_0(y) = 1 - e^{-y/2\sigma_n^2} \sum_{k=0}^{m-1} \frac{1}{k!} \left(\frac{y}{2\sigma_n^2}\right)^k, y > 0, \quad (12)$$

with m is the degree of freedom (in our case m=2) and σ_n^2 is the noise power.

A. *The proposed spectrum sensing*

The proposed spectrum sensing method can be summarized in the following steps:

- From the complex received samples X_i, calculate the energy samples $Y_i = |X_i|^2$.
- Sort the sequence Y_i in increasing order such as: $Y_1 \le Y_2 \dots \dots \le Y_n$
- Calculate the test Z_A according to (7), with F_0 given in (12).
- Find the threshold λ for a given probability of false alarm such that:
$$Pfa = P\{Z_A > \lambda | H_0\}$$
- Accept the null hypothesis H_0 if $Z_A \le \lambda$. Otherwise, reject H_0 in favour of the presence of the primary user signal

To find λ, it is worth to mention that the distribution of Z_A under H_0 is independent of the $F_0(y)$ [5] [16]. The value of λ is determined for a specific value of Pfa. A table listing values of λcorresponding to different false alarm probabilities Pfa is given in [13]. Otherwise, these values can be computed by Monte Carlo approach.

Figure 1 presents the detection probability as a function of the false alarm probability (ROC curves) of the proposed LLR-GoF based spectrum sensing method compared to the AD-GoF based sensing and the energy detection (ED).

The results are obtained by 10000 Monte-Carlo simulations. For the AD-GoF method, the same 5 steps as for the LLR-GoF are followed, except for step 3 in which we took as a test statistic A_n^2 as given in (8).

The simulations are performed using only 20 samples of the received signal with a signal to noise ratio (SNR) equal to *-6 dB*. It can be seen in figure 1 that the proposed LLR-GoF based sensing outperforms both AD-GoF based sensing and ED. For example, for *Pfa=0.2*, the probability of detection P_d for the ED sensing equals *0.392*, for AD based sensing P_d equals *0.695*. However, for the proposed LLR-GoF sensing, P_d equals *0.745*.

In figure 2, the values of the detection probability versus *SNR* are plotted for the three sensing methods.
The *Pfa* is set to *0.05* and the *SNR* varies from *-20 dB* to *10 dB*, keeping the number of samples n to *20* samples. It can be seen that the proposed LLR-GoF based sensing has almost *1 dB* gain over AD based sensing and almost *5dB* over ED sensing with P_d *= 0.8* and *Pfa=0.05*, hence the performance of the proposed LLR based sensing is indeed better than that of AD based sensing and ED sensing.

IV. SPECTRUM SENSING UNDER NON GAUSSIAN NOISE AND NOISE UNCERTAINTY

A. *Non Gaussian noise (GM Model)*

It is worth to mention that the existing works on GoF for spectrum sensing [5] [6] [7] [10] is focusing on detecting a signal in white Gaussian noise. In our paper, we will also focus on detecting signals in white non-Gaussian noise. In literature, a lot of models are proposed to pattern a non Gaussian noise. The most used models are the Gaussian Mixture model (GM) and the generalized Gaussian model (GG). For our spectrum sensing model, we will work with the GM model [17], as it has been used in practical applications in [18] and in radio signal detection applications in [19]. To apply the GoF test for spectrum sensing, we need the Cumulative distributed function (CDF) of the non Gaussian noise (GM CDF). The pdf of GM noise has three parameters $\alpha, \beta,$ *and* σ and is defined as [19]:

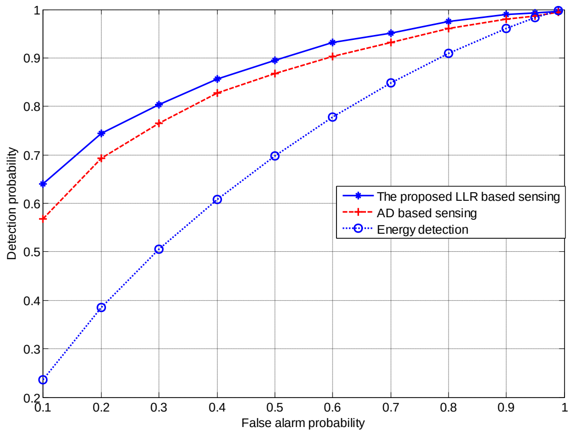

Figure 1. ROC curves over AWGN channel with *SNR*=-6dB and n=20 samples

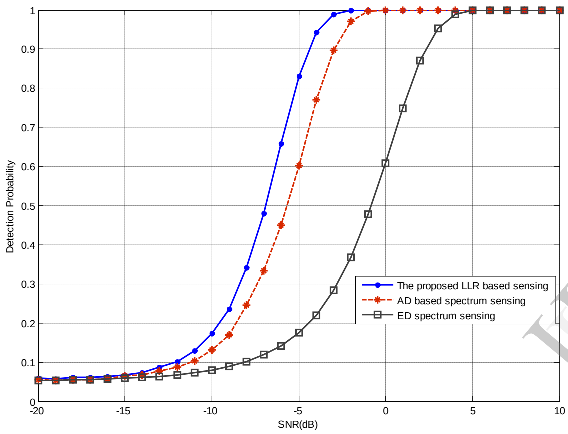

Figure 2. Detection probability versus *SNR* over AWGN channels with *Pfa*=0.05 and n=20 samples

$$f_w(w) = \frac{c}{\sigma\sqrt{2\pi}}\left[\alpha \exp\left(-\frac{c^2w^2}{2\sigma^2}\right) + \frac{1-\alpha}{\beta}\exp\left(-\frac{c^2w^2}{2\sigma^2\beta^2}\right)\right] \quad (14)$$

where $c = \sqrt{\alpha + (1-\alpha)\beta^2}$

Figure 3 depicts a probability distribution function (pdf) of a white non Gaussian noise (GM) with the following selected parameters α*=0.9*, β *=5* and σ*=1*.

The CDF F_0 of the energy of the non-Gaussian noise samples under H_0 hypothesis can be derived from the GM's pdf. For that we have: if $Y=X^2$ and X is GM noise with CDF $F_X(x)$,

$$F_0(y) = P(Y \leq y) = P(-\sqrt{y} \leq X \leq \sqrt{y}) \quad (15)$$
$$= F_X(\sqrt{y}) - F_X(-\sqrt{y})$$

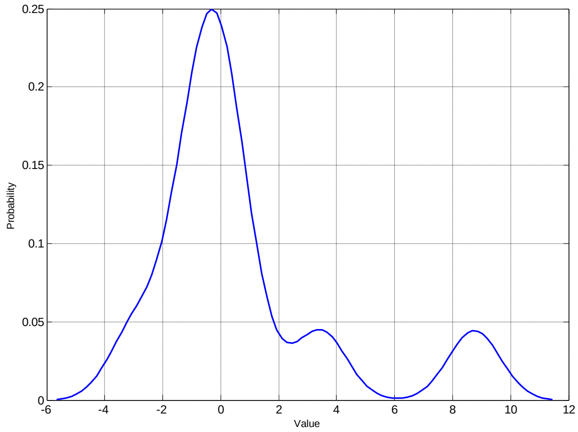

Figure 3. Probability distribution function (pdf) of GM noise α*=0.9*, β *=5* and σ*=1*

Once we get the CDF of the non Gaussian noise, we apply our proposed algorithm of subsection (III.A). Note that the knowledge of F_0 is required to apply the GoF test; therefore, if the parameters of the GM model are unknown, they must be estimated first.

To evaluate the effect of a non Gaussian noise on the sensing performance, we have performed simulations with the selected GM noise. We set the parameters of the non Gaussian noise as: α*=0.9*, β *=5* and σ*=1*. All other simulation parameters are the same as in section III. Figure 4 presents results of the AD-GoF based sensing under Gaussian noise and non Gaussian noise. It is shown that the effect of considering a non Gaussian noise decrease slightly the performance of the AD-GoF based sensing. Figure 5 shows the results of the LLR-GoF based sensing. Just as in the AD-GoF based sensing, our proposed method is slightly degraded under non Gaussian noise. However, it can be seen in figure 6 that the performance of the ED is significantly influenced by the considered non Gaussian noise. It has to be noted that the considered non Gaussian noise (α*=0.9*, β *=5* and σ*=1*) is very unfavorable for ED. In order to obtain a $P_{fa}=0.05$, the threshold λ in the binary hypothesis test needs to be shifted rightly at certain level. Anyway, GoF based spectrum sensing is less effected by the non Gaussian noise, as the test is performed on the mismatch between the measured CDF and the reference CDF F_0.

B. Noise uncertainty

One of the main issues with ED or with blind detection methods in general, is the impact of noise uncertainty on the detection performance. It is shown in [8] [21] that ED is very sensitive to noise uncertainty. The aim of this subsection it to study the effect of noise uncertainty on GoF based spectrum sensing compared to ED.

Through simulation, we have compared the impact of noise uncertainty on both methods, ED based spectrum sensing and GoF test based spectrum sensing.

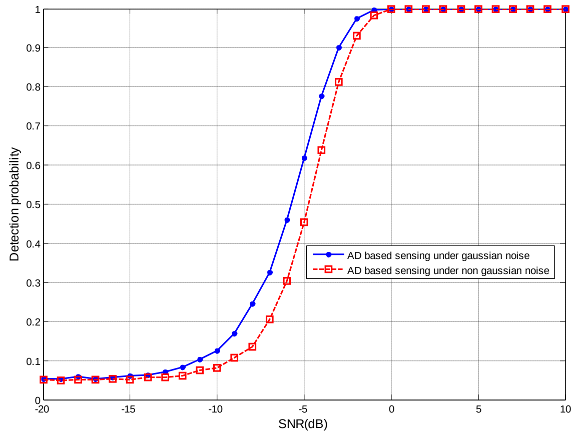

Figure 4. Detection probability versus *SNR* under Gaussian and non Gaussian noise for AD-GoF, with *Pfa*=0.05 and n=20 samples.

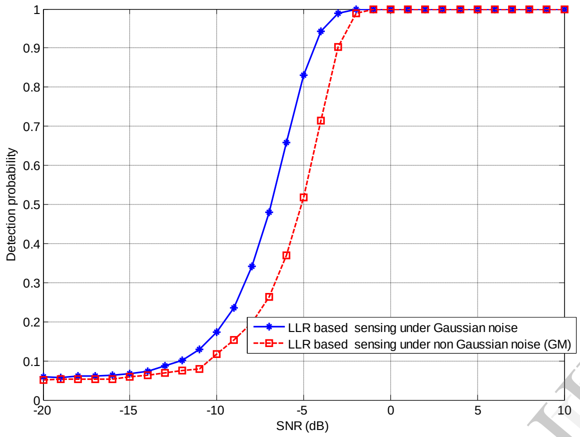

Figure 5. Detection probability versus *SNR* under Gaussian and non Gaussian noise for LLR-GoF, with *Pfa*=0.05 and n=20 samples.

The noise uncertainty is modeled by letting the actual noise variance be limited within a set given by a nominal noise variance and an uncertainty parameter ρ such that:

$$\sigma_n^2 \in [\frac{1}{\rho}\sigma^2, \rho\sigma^2] .$$

There is a fundamental difference between ED and GoF based sensing when it comes to noise uncertainty. The energy detector suffers under noise uncertainty because computing the threshold λ for the binary test requires knowledge of the underlying noise variance. In order to guarantee a given false alarm rate P_{fa}, the threshold λ will be calculated for the worst case, i.e. a noise variance of $\rho\sigma^2$ [21], leading to higher values of λ and hence to a decrease in detection probability.

In GoF based sensing, the distribution of the test statistic G^2_t or A^2_n under the H_0 hypothesis is independent of the noise distribution. As a consequence, the value of the threshold λ for the GOF binary test will not be influenced by the noise uncertainty. However, the calculation of the test statistic (G^2_t or A^2_n) requires the exact knowledge of the underlying

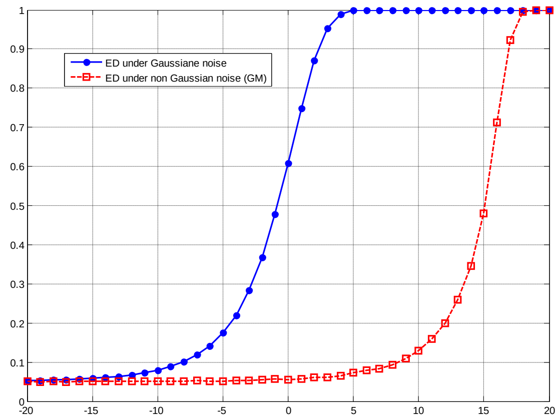

Figure 6. Detection probability versus *SNR* under Gaussian and non Gaussian noise for ED, with *Pfa*=0.05 and n=20 samples.

theoretical noise CDF F_0. In summary, for GOF sensing, noise uncertainty will, via F_0, indirectly affect the value of the test statistic, but not the detection threshold. For the simulation of the GoF based spectrum sensing under noise uncertainty, we will also follow a worst case approach, by considering a reference noise CDF F_0 given in (12) based on the highest noise variance $\rho\sigma^2$, which will eventually lead to a reduction of the detection probability.

In figure 7, we have plotted the detection probability versus *SNR* for several values of noise uncertainty (*0 dB, 0.5 dB, 2 dB, 4 dB*) in the case of the ED spectrum sensing method. It is shown that the performance of the ED is significantly decreasing when the noise uncertainty level is increasing. In similar way, in figure 8, we have plotted the detection probability as a function of *SNR* when considering a noise uncertainty for GoF based spectrum sensing. It can be seen that under uncertainty in the noise statistic of the CDF under hypothesis H_0 (F_0), the impact on the performance of the GoF based spectrum sensing is significantly less than the impact on energy detection. Intuitively, this can be explained by the fact that in ED, the value of Pfa and Pd are directly affected by the noise uncertainty. In case of GoF based sensing the test statistic Z_A (or A_n^2) is indirectly affected by the noise uncertainty via the CDF F_0 under hypothesis H_0.

Note also that, in figure 7, for high values of noise uncertainty the P_d drops to *0*. This effect is known as the *SNR* wall [20]. This effect is not observed in GoF based spectrum sensing for the given simulation parameters.

V. IMPACT OF SAMPLE SIZE ON SENSING PERFORMANCE

In this section we try to evaluate the influence of the number of sample on the detection performances of the considered spectrum sensing methods, GoF based sensing and ED based sensing. It is known from literature that the GoF tests have merit to perform well under small sample sizes [5].

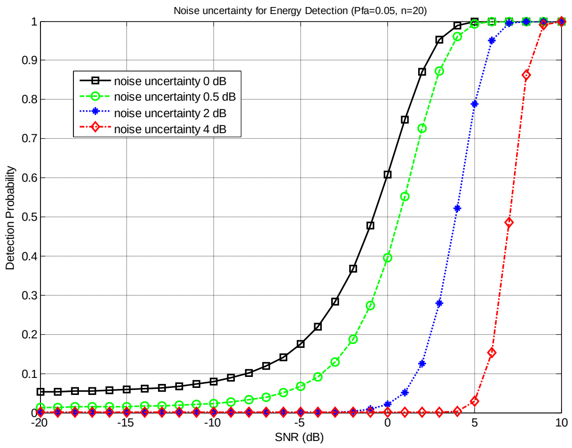

Figure 7. Impact of noise uncertainty on ED with *Pfa*=0.05 and n=20 samples.

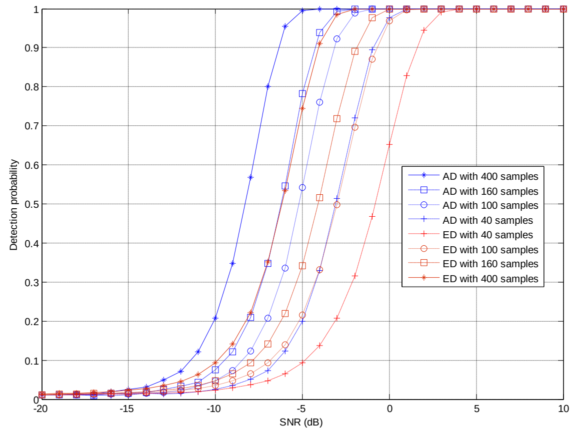

Figure 9. Detection probability versus *SNR* for different sample sizes (40 100 160 and 400 samples) with *Pfa*=0.01.

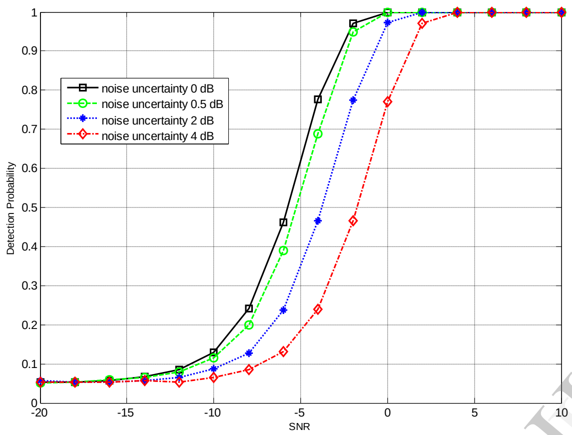

Figure 8. Impact of noise uncertainty on GoF test based sensing with *Pfa*=0.05 and n=20 samples.

The impact of the sample sizes on the GoF based sensing and ED sensing methods is presented in figure 9. The *Pfa* is set to *0.01* and the *SNR* varies from *-20 dB* to *10 dB* for different sample sizes (40 100 160 and 400 samples). It can be seen that the GoF based sensing outperforms ED sensing under a limited number of samples and that the ED based sensing yields the same performance as GoF based sensing in terms of detection probability if the sample size is approximately *2.5* times the sample size used for GoF based sensing. Therefore, GoF based sensing can be an appropriate sensing methods in applications where the sensing time is limited.

VI. Conclusion

In this paper, we have proposed a blind spectrum sensing method based on GoF test. The novelty in the proposed spectrum sensing method was to consider the energy of the received samples and test them against a chi-square distribution under hypothesis H_0 using the likelihood ratio test statistic. The LLR test statistic presents better performance compared to the other often used test statistics, like AD. It was shown by Monte-Carlo simulations that the proposed LLR-GoF sensing method outperforms both AD-GoF based sensing and ED based sensing, particularly for limited number of samples and low SNR values. We have also studied some typical impairment for spectrum sensing, i.e. the effect of a non Gaussian noise and noise uncertainty on the performance of GoF based sensing. As a model for the non Gaussian noise, we used the Gaussian mixture (GM). It was observed that a non Gaussian noise can affect noticeably the performance of ED, but has only a limited influence on the performance of the GoF based sensing methods. The same conclusion can be drawn for the noise uncertainty. This is mainly due to the fact that the test statistics in GoF testing is based on the difference of the measured CDF and the reference CDF and hence only indirectly influenced by noise parameters. Finally, it was shown that the LLR-GoF based sensing, AD-GoF based sensing and ED present similar performance in terms of detection probability for large number of samples. However, the proposed LLR-GoF sensing method, as well as AD based sensing method, perform well with a limited number of samples, hence they can be an appropriate methods in any cognitive radio system= with short sensing time.

REFERENCES

[1] J. Mitola and G. Q. Maguire: 'Cognitive radio: making software more personel', IEEE Personal Commun Mag., vol. 6, no. 4, pp. 13-18, Aug.1999.
[2] T. Yucek and H. Arslan: 'A survey of spectrum sensing algorithms for cognitive radio applications', IEEE Commun. Surveys and Tutorials, vol. 11, no. 1, pp. 116-130, Quat. 2009.
[3] E. Axell, G. Leus, E. G. Larsson and H. V. Poor: 'Spectrum sensing for cognitive Radio : State-of-the-art and recent advances', IEEE Signal Processing Mag., vol. 29, no. 3, pp. 101-116, May. 2012.
[4] H. Urkowitz: 'Energy detection of unknown deterministic signals', Proc. IEEE, vol. 55, no. 4, pp. 523-531, Apr. 1967.
[5] H. Wang, E. H. Yang, Z. Zhao and W. Zhang: 'Spectrum sensing in cognitive radio using goodness of fit testing', IEEE Trans. Wireless Commun., vol. 8, no. 11, pp. 5427-5430, Nov. 2009.
[6] G. Zhang, X. Wang, Y. C. Liang and J. Liu: 'Fast and robust spectrum sensing via kolmogorov-smirnov test', IEEE Trans. Commun. vol. 58, no. 12, pp. 3410-3416, Dec. 2010.
[7] S. Rostami, K. Arshad and K. Moessner: 'Order-Statistic Based Spectrum Sensing for Cognitive Radio', IEEE Commun. Lett.,vol. 16, no. 5, pp.592 595, May. 2012.

[8] R. Tandra, A. Sahai: 'Fundamental limits on detection in low SNR under noise uncetainty', International Conference on Wireless Networks, Communication and Mobile Computing, pp. 464-469, 2005.
[9] T. Kieu-Xuan, I. Koo: 'Cramer-von Mises test spectrum sensing for cognitive radio systems', Wireless Telecommunication Symposium (WTS), 2011.
[10] L. Shen, H. Wang, W.Zhang and Z. Zhao: 'Blind Spectrum Sensing for Cognitive Radio Channels with NOise Uncertainty', IEEE Commun. Lett.,vol. 10, no. 6, pp. 1721-1724, June. 2011.
[11] H. Wang, D. Du, Q.Li and H. Xiong: 'A Spectrum Sensing Algorithm based on Kolmogorov-smrinov Two-sample Test',Journal of Computational Information Systems,vol. 8, no. 6, pp. 2601-2607, 2012.
[12] S. S. Kalamkar, P. K. Singh, and A. Banerjee: 'Block Outlier Methods for Malicious User Detection in Cooperative Spectrum Sensing',IEEE Vehicular Technology Conference-Spring, February 2014.
[13] J. Zhang, Powerful goodness-of-fit tests based on the likelihood ratio, J.R.Statist. Soc.B 64, Part 2, pp. 281-294 (2002)
[14] D. Teguig, V. Le Nir and B. Scheers : 'Spectrum sensing method based on goodness of fit test using chi-square distribution', Electronics Letters, Volume 50, Issue 9, 24 April 2014, p. 713 715.
[15] M. A. Stephens: 'EDF Statistics for Goodness of Fit and Some Comparisons', Journal of the American Statistic Association, vol.69, No.347, pp.730-737, Sep. 1974.
[16] R. D.Agostino and M. Stephens: 'Goodness of Fit Techniques' ser. Statistics: Textbooks and Mono-graphs, M. Dekker, 1986.
[17] S. Bengio: 'Statistical Machine Learning from Data: Gaussian Mixture Models. http://bengio.abracadoudou.com/lectures/gmm.pdf
[18] A.A. Saha, and G.V. Anand: 'Design of detectors based on stochastic resonance' Signal Processing 83 (6) (2003), 1193-1212.
[19] A.A. Saha, and G.V. Anand: 'A robust detector of known signal in non Gaussian noise using threshold systems', Signal Processing 92 (2012), 2676-2688.
[20] R. Tandra, and A. Sahai: 'SNR walls for signal detection', IEEE Journal of Selected Topics in Signal Processing. March, 2008.
[21] W. Jouini: 'Energy Detection Limits Under Log-Normal Approximated Noise Uncertainty'. IEEE Signal Process. Lett. 18(7): 423-426 (2011