A model updating technique based on the Constitutive Relation Error for in-situ identification of admittance coefficient of sound absorbing materials

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The development of new absorbing materials and the description of their acoustical properties take an important place in the current acoustical researches. This paper focuses on the identification of the admittance coefficient of sound absorbing material from in-situ measurements, using the CRE-based updating technique. This technique consists in a two-stages approach, allowing to regularize the inverse problem. Moreover, the technique allows the detection of faulty sensors and therefore the correction of the erroneous measurements before the updating process. The technique is developed, in a first part of this paper, for acoustical problems with generalized boundary conditions, and illustrated, in a second part, on a numerical and a physical two-dimensional test case.

1 Introduction

Nowadays, the improvement of noise control devices is of great interest to several industries such as automotive, aeronautics or building. Different types of material are used to reach this purpose. The most common ones are porous and fibrous materials (foams, glass-wool, rock-wool, ...) and have therefore long been developed to enhance their acoustical performances, especially at low frequencies [1]. More recently, for example, micro-perforated panels have been widely investigated as clean and healthy absorbing materials as an alternative to traditional fibrous and porous materials [2–5], and porous and fibrous metals are developed to be used in devices under extreme conditions [6, 7] because they combine the mechanical properties of metals (lightness, stiffness, conductivity, heat withstanding, ...) and acoustical properties of porous materials.

Corresponding acoustical calculations are however complex. Therefore industries generally use numerical tools to predict the influence of the absorbing materials on the sound propagation. In these ones, the acoustical properties of those materials are described by an admittance (or impedance) coefficient, which is simpler than a physical model. Different models have been developed to describe the admittance coefficients for many years [2, 8–12]. They may be roughly divided into three main categories [7, 9]: empirical, phenomenological and micro-structural models. Each of these models is frequency-dependent and requires a number of parameters, more or less important, to characterize the porous medium. Different methods are used to measure either the parameters of the coefficient [8, 13–15] or directly the frequency dependent coefficient. The two most common practical techniques to measure the admittance coefficient are the reverberant room method and the tube method [16, 17]. The first method does not provide phase information of the coefficient and the second method gives results only for normal incident waves which do not always represent the reality of practical problems. The limitations
of both methods have therefore led to the development of other methods of measurements [17–22].

In this paper, a model updating technique is used to identify the parameters required to describe admittance coefficients from sound pressure measurements inside a closed cavity. Updating techniques have been used for many years to improve numerical models, and consist in minimizing an error between the numerical solutions and a set of experimental results. An important number of methods have been developed in structural dynamics, as shown in the survey [23]. Updating techniques can be grouped into two major types [24]: the direct methods and the indirect or parametric methods. In the direct methods, the mass and the stiffness matrices are globally corrected. They are non-iterative methods, and therefore computationally very efficient. Despite the fact that the solutions of the new numerical model are close to the experimental data, the modified matrices have lost their physical meaning. This is why they have been progressively abandoned over the years. In the indirect methods, the matrices of the numerical model are assumed to be parameterizable and the updating technique consists in minimizing the error by changing some parameters of the numerical model. These methods are generally iterative and, as such, considerably more computationally expensive than the direct ones. However, the physical meaning of the updated parameters is preserved.

Indirect methods are inverse methods for which the problem is generally ill-posed, and the issue of the regularization is therefore very important [25–29]. The technique based on the Constitutive Relation Error (CRE), initially proposed by Ladevèze [30] for structural dynamics problems, is an indirect method. It consists in splitting the set of mathematical equations into two subsets of equations: the set of reliable equations and the set of less-reliable equations. The CRE, representing the error on the model, is constructed on this last set of equations. This error is an indicator based on an energy-norm and can consequently be locally evaluated [31]. It allows to define a two-steps method [31–34]. First, a step of localization consists in determining the regions in which the less-reliable equations are the most erroneous by checking the local estimators. Secondly, during a step of correction, only the parameters selected previously during the first step are updated. It reduces the number of parameters to be corrected at each iteration, which allows the regularization of the inverse problem.

In addition, this method takes into account the measurement errors. Similarly, the set of experimental quantities is divided into a set of reliable experimental quantities and a set of less-reliable experimental quantities, on which the error in measurements is constructed. The cost function is the sum of this error and the CRE, and is called the modified CRE. The error in measurements can be divided into local contributions from each sensor, and can be used as an indicator of defective sensors [31]. Then, erroneous measurements can either be corrected if the source of the error is found, or simply be removed from the set of experimental data, which improves the robustness of the updating process.

The CRE-based updating technique has already been applied to the acoustical problem in [35, 36]. The Robin-type boundary condition, which links the sound pressure to the normal component of the velocity through the admittance coefficient and therefore describes an absorbing boundary, is considered as the less-reliable equation. Concerning the measurements, amplitude of the measured pressures are considered as the less-reliable experimental quantity and led to the construction of the error in measurements. This paper proposes to improve the updating technique in three different ways. First, the generalized boundary condition, which allows to describe a boundary simultaneously absorbing (Robin-type boundary condition) and vibrating (Neumann-type boundary condition), is introduced and used to construct the CRE. The parameters to be updated are therefore the acoustical admittance coefficients of the absorbing boundaries and the normal component of the velocity of the vibrating boundaries (Section 2). Secondly, local estimators of the CRE and of the error in measurements are developed (Section 3) in order to define the two-steps method (Section 4). Thirdly, in [35, 36], the real and imaginary parts of the admittance coefficient are updated at each frequency. In this paper, a model to describe the admittance coefficient is a priori chosen, and the parameters of this one are updated on a frequency range, what allows to further regularized the inverse problem (see Section 3).

Section 5 presents the application of the updating process on two two-dimensional cases:

**A numerical test case:** a two dimensional domain, representing a car-cabin, is excited by a vibrating wall covered by an absorbing material. Some parts of the boundary of the domain are covered by absorbing materials. 32 sensors are placed near the boundaries and the measurements are simulated by the numerical model. Errors are artificially introduced on the parameters as well as on 3 sensors. The goal of this test case is to verify the method developed in this paper.

**A physical test case:** The goal of this test case is the validation of the method. The parameters of the admittance coefficient models, identified by the common Kundt’s tube method, are compared to those identified by the updating method. Two different materials are used for this purpose: a foam and a felt. The admittance coefficients are first measured with the Kundt’s tube setup and the parameters of the admittance coefficient models are identified. The duct is then modeled by a two dimensional rectangular domain. The boundary conditions of the numerical problems are chosen to describe as well as possible the real conditions of the experimental setup. The measurements used in the updating process are those from the microphones of the Kundt’s tube.
The initial parameters of the updating process are deliberately different from those identified with the Kundt’s tube method.

2 Formulation of the CRE-based updating problem

2.1 Reference problem

Let us consider an acoustical domain \( \Omega \) (Fig. 1), in which the sound propagation is described by the Helmholtz equation

\[
\Delta p + k^2 p = 0
\]

where \( p \) is the acoustical pressure and \( k = \frac{2\pi f}{c} \) is the wave number.

On the boundary \( \partial \Omega \) of the domain, two different boundary conditions are considered

1. a Dirichlet boundary condition

\[
p = \tilde{p} \tag{2}
\]

where \( \tilde{p} \) is the prescribed pressure on a part \( \partial_D \Omega \) of the boundary.

2. a generalized boundary condition

\[
v_n = \lambda A_n p + (1 - \lambda) \tilde{v}_n \tag{3}
\]

In Eqn. (3), defined on \( \partial_G \Omega = \partial \Omega \setminus \partial_D \Omega \), the parameter \( \lambda \) determines the nature of the boundary as follows

- \( \lambda = 0 \): the boundary is purely vibrating and described by a Neumann boundary condition, which prescribes the normal component of the velocity \( (v_n = \tilde{v}_n) \)
- \( \lambda = 1 \): the boundary is purely absorbing and described by a Robin boundary condition, which links the normal component of the velocity to the pressure through the admittance coefficient \( A_n \) \( (v_n = A_n p) \)
- \( 0 < \lambda < 1 \): the boundary is simultaneously vibrating and absorbing \( (v_n = A_n p + \tilde{v}_n) \), where \( A_n = \lambda A_n \) and \( \tilde{v}_n = (1 - \lambda) \tilde{v}_n \)

The reference problem consists in finding the pressure \( p \) which verifies the set of equations (1)-(3).

2.2 CRE-based updating technique: principles and application to the acoustical problem

In the CRE-based updating technique the set of the physical equations and the set of measurements are both split in two sets, the sets of equations and experimental quantities considered as reliable and the sets of equations and experimental quantities considered as less reliable. The CRE-based updating technique aims at minimizing an error, called the modified CRE, constructed on the less reliable equations and on less reliable experimental quantities. It can be expressed, at a given frequency \( \omega = 2\pi f \), as follows

\[
e^2_{\omega} = \frac{\varepsilon^2_{\omega}}{\bar{\varepsilon}^2_{\omega}} + \frac{r}{1 - r} \eta^2_{\omega} \tag{4}
\]

where \( \varepsilon^2_{\omega} \) is the relative CRE, at a given frequency \( \omega \), constructed on the less reliable equations and represents the error on the model, \( \eta^2_{\omega} \) represents the relative error on measurements, at a given frequency \( \omega \), constructed on the less reliable experimental quantities and \( r \) is a weighting factor which translates the degree of confidence to the measurements: the value of \( r \) is close to 1 if the measurements are considered as very reliable and close to 0 in the opposite case.

The reliable equations and quantities define the admissible set of solutions \( S_{ad}^a \). An admissible solution will therefore exactly satisfy the reliable equations and experimental quantities. The problem consists in finding, at a given frequency \( \omega \), the solution \( (\tilde{p}, \tilde{v}_n) \) which is admissible \( (\in S_{ad}^a) \) and which verifies the less reliable equations and experimental quantities as closely as possible.

Admissibility

Concerning the acoustical problem, Helmholtz Eqn. (1) is considered as reliable because it comes from the equations of conservation (mass and momentum). Let us define two Hilbert spaces \( V_1 \) and \( V_2 \) of square-integrable functions with their first derivatives in \( \Omega \)

\[
V_1 = H^1_D(\Omega) = \{ p \in H^1(\Omega) | p = \tilde{p} \text{ on } \partial_D \Omega \} \\
V_2 = H^1_0(\Omega) = \{ q \in H^1(\Omega) | q = 0 \text{ on } \partial_D \Omega \}
\]

A solution \( (\tilde{p}, \tilde{v}_n) \) is admissible if it satisfies Eqn. (1) and the problem is expressed, in a weak form, by

\[
\int_{\Omega} (\Delta \tilde{p} + k^2 \tilde{p}) q^* d\Omega = 0
\]

\[
\Rightarrow \int_{\Omega} \nabla \tilde{p} \nabla q^* d\Omega - \frac{\omega^2}{c^2} \int_{\Omega} \tilde{p} q^* d\Omega + j \omega \int_{\partial_D \Omega} \tilde{v}_n q^* d\Gamma = 0 \tag{5}
\]
using the following expression, which links the pressure to the normal component of the velocity

\[ \hat{v}_n = \frac{j}{\omega \rho} \frac{\partial \hat{p}}{\partial n} \tag{6} \]

Equation (6) is the condition on \( \hat{v}_n \), for the admissibility of \( (\hat{p}, \hat{v}_n) \).

**Error on the model**

The less reliable equation is the generalized boundary condition (3). Indeed, on one hand, the parameters of models describing the admittance coefficients \( A_n \) are often identified in conditions quite different from those in which they are really used and, on the other hand, the value of the normal component of the prescribed velocity \( \hat{v}_n \) often comes from previous computations and can thereby be unreliable. The relative CRE \( \xi_{\text{var}}^2 \) is then constructed on the generalized boundary conditions and expressed by

\[ \xi_{\text{var}}^2(\hat{p}, \hat{v}_n) = \frac{\omega^2 \rho^2}{D_{\text{in}}} \int_{\partial \Omega} |\hat{v}_n - \lambda A_n \hat{p} - (1 - \lambda) \hat{v}_n|^2 d\Gamma \]

where \(|.|^2\) denotes the modulus of a complexe number and \( D_{\text{in}} \) is a normalization factor

\[ D_{\text{in}} = \omega^2 \rho^2 \int_{\partial \Omega} \lambda^2 A_n A_n^* p_d p_d^* d\Gamma \]

where \( p_d \) is the solution the problem (10) with \( r \approx 1 \), giving a high confidence in the measurements.

**Error on the measurements**

The experimental quantities considered as less reliable are the amplitudes of the measured pressures, and the error in measurements is given by the following expression

\[ \eta_{\text{var}}^2(\hat{p}) = \frac{|\hat{p} - \tilde{p}_{\text{in}}|^2}{|\hat{p}_{\text{in}}|^2} = \frac{\eta_{\text{var}}^2(\hat{p})}{|\hat{p}_{\text{in}}|^2} \tag{9} \]

where \( \hat{p}_{\text{in}} \) are the measured pressures at a given frequency \( \omega \), \( \pi \) is a projection operator which gives the value of the pressure \( p \) at the sensor location and \(|.|^2\) is an \( L^2 \)-norm.

**Problem**

The problem can be expressed as follow

\[ \text{find } s = (\hat{p}, \hat{v}_n) \left\{ \begin{array}{l} \text{s exactly satisfies Eqn. (5)} \\ \epsilon_{\text{var}}^2(s) \text{ is minimum} \end{array} \right. \tag{10} \]

with

\[ \epsilon_{\text{var}}^2(\hat{p}, \hat{v}_n) = \frac{\xi_{\text{var}}^2(\hat{p}, \hat{v}_n)}{D_{\text{in}}} + \frac{r}{1 - r} \frac{\eta_{\text{var}}^2(\hat{p})}{|\hat{p}_{\text{in}}|^2} \tag{11} \]

**2.3 Finite Element formulation**

Let us introduce nodal unknowns \( \{P\} \) and \( \{V\} \) used to approximate, respectively, the pressure \( \hat{p} \) and the normal component of the velocity \( \hat{v}_n \)

\[ \left\{ \begin{array}{l} \hat{p} = [N]^t \{P\} \\
\hat{v}_n = [N]^t \{V\} \end{array} \right. \tag{12} \]

where \([N]\) are the shape functions of the finite element formulation.

**Admissibility**

Equation (5) becomes

\[ ([K] - \omega^2 [M]) \{P\} + j \omega \rho [C_0] \{V\} = 0 \tag{13} \]

where

\[ [K] = \int_{\Omega} (\nabla [N]^t \nabla [N]) d\Omega \]
\[ [M] = \frac{1}{\pi} \int_{\Omega} [N]^t [N] d\Omega \]
\[ [C_0] = \int_{\partial \Omega} [N]^t [N] d\Gamma \]

**Error on the model**

The CRE \( \xi_{\text{var}}^2 \) becomes

\[ \xi_{\text{var}}^2(\{P\}, \{V\}) = \omega^2 \rho^2 \left[ [V]^t [C_0] \{V\} + \{P\}^t [C_2] \{P\} \right] - \omega^2 \rho^2 \left[ [P]^t [C_1]^* \{V\} + [V]^t [C_1] \{P\} \right] + \omega^2 \rho^2 \left[ \{B_1\}^* \{P\} + \{P\}^* \{B_1\} \right] - \omega^2 \rho^2 \left[ \{V\}^* \{B_0\} \right] \tag{14} \]

where

\[ [C_1] = \int_{\partial \Omega} \lambda A_n [N]^t [N] d\Gamma \]
\[ [C_2] = \int_{\partial \Omega} \lambda^2 A_n A_n^* [N]^t [N] d\Gamma \]
\[ \{B_0\} = \int_{\partial \Omega} \lambda (1 - \lambda) \hat{v}_n [N]^t d\Gamma \]
\[ \{B_1\} = \int_{\partial \Omega} \lambda (1 - \lambda) A_n \hat{v}_n [N]^t d\Gamma \]
\[ D = \int_{\partial \Omega} (1 - \lambda)^2 \hat{v}_n^2 d\Gamma \]

The normalization factor (8) is expressed by

\[ D_{\text{in}} = \omega^2 \rho^2 \{P_D\}^* [C_2] \{P_D\} \tag{15} \]

where \( \{P_D\} \) is the solution of problem (18) with \( r \approx 1 \). \( \{P_D\} \) can be seen as an extension of \( \hat{P} \) on the whole acoustical domain.
Error on the measurements

The error $\eta_{00}^2$ is expressed by

$$\eta_{00}^2(\{P\}) = \{|\Pi \cdot \hat{P}\|^2 - \{\Pi \cdot \hat{P}\}\| Z\}$$

where the projection operator $\Pi$ is a matrix whose elements $\Pi_{ij}$ are equal to 1 if the sensor number $i$ is localized at the node $j$, and equal to 0 otherwise. The normalization factor of the error on the measurements is expressed by

$$|\hat{P}|^2 = \{\hat{P}\| Z\}$$

Problem

The problem can be expressed as follow

$$\text{find } s = (\{P\}, \{V\}) \begin{cases} \text{s exactly satisfies Eqn. (13)} \\ \epsilon_{\omega r}^2(s) \text{ is minimum} \end{cases}$$

with

$$\epsilon_{\omega r}^2(\{P\}, \{V\}) = \frac{\bar{\eta}_{00}^2(\{P\}, \{V\})}{\bar{D}_{00}^2} + \frac{r}{1 - r} \frac{\eta_{00}^2(\{P\})}{|\bar{P}_{00}|^2}$$

This problem can be solved using the Lagrange multipliers and expressed by the following linear system of equations

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} V \\ P \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

where

$$A_{11} = \frac{\omega^2}{\bar{D}_{00}^2} (\{C_0\} - [\Theta] [C_1]^*)$$
$$A_{12} = \frac{\omega^2}{\bar{D}_{00}^2} ([C_1] - [\Theta] [C_2]) + \frac{r}{1 - r} \frac{1}{|\bar{P}_{00}|^2} [\Theta] [\Pi]^T [\Pi]$$
$$A_{21} = j \omega \bar{P}_{00} [C_0]$$
$$A_{22} = [K] - \omega^2 [M]$$
$$B_1 = \frac{\omega^2}{\bar{D}_{00}^2} ([B_0] - [\Theta] [B_1]) + \frac{r}{1 - r} \frac{1}{|\bar{P}_{00}|^2} [\Theta] [\Pi]^T \{\hat{P}\}$$
$$B_2 = [Z]$$

and

$$[Z] \text{ is a zero vector}$$
$$[\Theta] = j \omega \bar{P}_{00} [C_0] ([K] - \omega^2 [M])^{-1}$$

3 Regularization of the inverse problem

3.1 Local estimators

The inverse updating processes are generally ill-posed problems and it is therefore necessary to regularize the problem. Moreover, the number of parameters to update can be high. Indeed, the admittance coefficients $A_n$ and the normal component of the prescribed velocities $\nu_n$ are described by frequency dependent models, using a certain number of constant parameters, and the total number of parameters to update will depend on the number of absorbing materials and vibrating boundaries and on the number of parameters necessary to describe them.

The CRE is an energy-norm indicator and can be locally evaluated. The updating technique is therefore an iterative process consisting in two steps

1. the localization step: localization of parameters contributing most to the error on the model, by looking at the distribution of the local estimators $\xi_{E00}^2$ which gives the contribution to the global CRE $\xi_{E00}^2$ of each boundary conditions
2. the correction step: correction of the parameters chosen in the localization step

The number of parameters updated at each iteration is thereby reduced which helps to regularize the inverse problem.

On the other hand, the updating process aims at improving a numerical model by minimizing the error between its results and a set of experimental data. It is then very important to be able to detect defective sensors before carrying the updating process to ensure the reliability of its results. Defective sensors will then be localized by looking at the distribution of the local error indicator, giving the contribution $\eta_{n0}^2$ of each sensor $i$ to the error in measurements $\eta_{00}^2$. Thereby, it could be possible to correct the erroneous measurements if the source of the problem can be found, or simply to remove them from the set of experimental data in the opposite case.

3.1.1 Constitutive Relation Error

Let us divide the boundary $\partial \Omega$ into subboundaries $\partial \Omega \in \partial \Omega$, with $\bigcup_{E} \partial E \Omega = \partial \Omega$ and $\bigcap_{E} \partial E \Omega = \emptyset$. The CRE can be viewed as the sum of the contributions of each subboundary

$$\xi_{\omega r}^2(\hat{P}, \hat{v}_n) = \sum_{E=1}^{E_{\text{bound}}} \xi_{E00}^2(\hat{P}, \hat{v}_n)$$

where $E_{\text{bound}}$ is the number of subboundaries. The relative CRE for each subboundary at frequency $\omega$ is given by

$$\xi_{E00}^2(\hat{P}, \hat{v}_n) = \frac{\omega^2}{\bar{D}_{00}^2} \int_{\partial E \Omega} |\hat{v}_n - \lambda A_n \hat{P} - (1 - \lambda) \bar{v}_n|^2 d\Gamma$$

\footnotesize
\begin{enumerate}
\item for $i = 1, \ldots, N_{\text{sens}}$ and $j = 1, \ldots, N_{\text{tot}}$ ($N_{\text{sens}}$ is the number of sensors and $N_{\text{tot}}$ is the number of degrees of freedom)
\end{enumerate}
3.1.2 Error on measurements

Error on measurements (9) can be divided into contributions from each sensor at frequency $\omega$

$$
\eta^2_{\text{lor}}(\hat{p}) = \sum_{i=1}^{N_{\text{sens}}} \frac{\left|\pi_{\text{lor}} - \pi_0\right|^2}{|\hat{p}_{\omega,i}|^2} = \sum_{i=1}^{N_{\text{sens}}} \eta^2_{\text{lor}}(\hat{p})
$$

where $|.|^2$ is the norm obtained by setting all components of the vector to zero, except the one related to sensor $i$ and $N_{\text{sens}}$ is the number of sensors.

3.2 Updating on a frequency range

In order to further regularize the problem, the updating process is generally performed on a frequency range $[\omega_{\min}, \omega_{\max}]$ rather than for a single frequency $\omega$. Let us introduce a weighting function $z(\omega)$ such as

$$
\int_{\omega_{\min}}^{\omega_{\max}} z(\omega) d\omega = 1 ; z(\omega) \geq 0
$$

(24)

Because the measurements are performed at discrete frequencies ($\omega_1, \omega_2, ..., \omega_{N_{\text{meas}}}$), where $N_{\text{meas}}$ is the number of frequencies, the following weighting factor is used

$$
z(\omega) = \frac{1}{N_{\text{meas}}} \sum_{k=1}^{N_{\text{meas}}} \delta_{\omega_k}(\omega)
$$

(25)

where $\delta_{\omega_k}$ is the Dirac function associated with the frequency $\omega_k$. The expression of the relative CRE becomes

$$
\xi^2_{\text{ET}} = \int_{\omega_{\min}}^{\omega_{\max}} \eta^2_{\text{lor}} z(\omega) d\omega = \frac{1}{N_{\text{meas}}} \sum_{k=1}^{N_{\text{meas}}} \xi^2_{\text{lor},r}
$$

(26)

In the same way, we can define all the estimators on a frequency range

- $\xi^2_{\text{ET}}$ for the local contribution of the relative CRE
- $\eta^2_{\text{lor}}$ for the relative error in measurements
- $\eta^2_{\text{ET}}$ for the local contribution of the relative error in measurements
- $e^2_\text{ET}$ for the relative modified CRE

4 Implementation of the two-stages updating technique

4.1 Localization of defective sensors and correction of erroneous measurements

The solution of problem (20), for each experimental frequency $\omega$, allows to compute the estimators $\xi^2_{\text{ET}}, \xi^2_{\text{ET}}, e^2_\text{ET}, \eta^2_{\text{ET}}$ and $\eta^2_{\text{ET}}$. Defective sensors are detected by looking at the distribution of $\eta^2_{\text{ET}}$. It is therefore possible to correct the erroneous measurement or remove them from the set of measurements. Note that a correction is possible only if the sources of error are identified.

4.2 Two-stages updating technique

After correction of erroneous measurements, the model is updated with the new set of experimental results. The value of $\xi^2_{\text{ET}}$ allows to know if the model has to be updated or not. In practice, the updating process is necessary if $\xi^2_{\text{ET}} > \xi^2_{\text{ET}}$ where $\xi^2_{\text{ET}}$ is a prescribed value of the relative CRE in a frequency range, and represents the required quality level.

If the model updating is considered as necessary, the starting point is the initial numerical solution, which depends on a number of uncertain parameters. These parameters are arranged into a vector $\{k\}$ and the corresponding space is denoted by $\mathbf{k}$. The first step consists in the selection of parameters contributing most to the global CRE and is based on the criterion

$$
\xi^2_{\text{ET}} \geq \delta \max E \xi^2_{\text{ET}}
$$

(27)

with, for example, $\delta = 0.8$. Let $\mathbf{Z}$ be the set of subboundaries which verify Eqn. (27). The parameters are then arranged into a vector $\{\hat{k}\}$ and the corresponding space, denoted by $\mathbf{K}$, is a subspace of $\mathbf{k}$. The second step consists in the correction of only the parameters selected at the localization step. The problem consists then in finding $\{\hat{k}\} \in \mathbf{K}$, which minimizes modified CRE (19).

After the correction step, the estimator $\xi^2_{\text{ET}}$ is reevaluated. If $\xi^2_{\text{ET}} < \xi^2_{\text{ET}}$, the updating process is ended. If not, a new iteration consisting in a localization step and a correction step is performed. This process continues until the required quality level is reached. Note that there could be situations in which the CRE $\xi^2_{\text{ET}}$ cannot be reduced below the required quality level $\xi^2_{\text{ET}}$ by acting on the parameters of the numerical model, indicating that errors are present in the model itself, and not only in its parameters.

5 2D applications

5.1 Introduction

This section presents the application of the process of acoustical model updating on two 2D cases. The first test case aims at verifying the strategy developed in this paper. It consists in a 2D acoustical domain representing a car cabin. The boundary of the domain is covered by three different absorbing materials, modeled by generalized boundary conditions with $\lambda = 1$. The domain is excited by a loudspeaker, itself covered by an absorbing material, modeled by a generalized boundary condition with $\lambda = 0.5$. To remain in a physical context, all the absorbing materials are described by a Delany-Bazley model (see subsection 5.3), with two parameters ($\sigma$ and $d$), modeling the acoustical properties of fibrous materials, and the normal component of the velocity of the loudspeaker membrane is given by one degree of freedom model (see subsection 5.2), with three parameters ($D$, $\omega_0$, and $\zeta$). There are therefore 11 parameters to update. 32 sensors are placed close to the
absorbing materials and to the loudspeaker. The measured pressures are numerically computed, using the exact value of the parameters. Errors on parameters and on three sensors are artificially introduced.

The second test case deals with a physical test case. A Kundt’s tube is used to measure the admittance coefficient of two different absorbing materials: foam, whose admittance coefficient is described by the Delany-Bazley model, and felt, whose admittance coefficient is described by the Delany-Bazley-Miki model (see subsection 5.3), which is slightly different from the Delany-Bazley one and allows to describe the acoustical properties of multilayer fibrous materials, especially at low frequencies. The parameters of these different models, for each material, are identified by using the least square method. The results are compared to those given by the updating method. The duct is then modeled by a two dimensional rectangular acoustical domain, excited by different models of the acoustic impedance are proposed in the literature [8], depending of the incidence of the acoustic wave and of the kind of the absorbing material. The laws of the Delany-Bazley model allow to replace a layer of fibrous material, with porosity close to 1, by a layer of equivalent material, whose admittance coefficient is described by the Delany-Bazley model.

5.2 The loudspeaker model

The loudspeaker is composed by a membrane, attached to a current-carrying coil, characterized by an electrical resistance $R$ and an inductance $L$. The displacement of the membrane is due to the movement of the coil in a magnetic field, created by a permanent magnet (see Fig. 2). The moving parts (membrane and coil), of mass $m_p$, are attached to the chassis by suspensions, described by an elastic constant $k_p$. The friction losses are described by a dissipation constant $c_p$. The transfer function $H_{V/X}$ between the complex displacement $X(\omega)$ and the complex voltage applied to the coil $V(\omega)$ is given by the expression [40]

$$H_{V/X}(\omega) = \frac{X(\omega)}{V(\omega)} = \frac{Bl}{Z_c(\omega)(-m_p\omega^2 + k_p + j\omega c_p) + j\omega(Bl)^2}$$

where

$$Z_c(\omega) = (R + j\omega L)$$

where $B$ is the magnetic field strength at the coil and $l$ is the length of wire in the coil. Typically, the value of the inductance $L$ of loudspeakers are about 0.1mH and the term $j\omega L$ can be neglected in Eqn. (28). The transfer function $H_{V/V}(\omega)$ between normal component of the velocity of the membrane of the loudspeaker $\bar{v}_n(\omega)$ ($\bar{v}_n(\omega) = j\omega X(\omega)$) and the voltage applied $V(\omega)$ is therefore given by a function $f_{HP}(\omega, a)$ of three parameters $a = (D, \omega_0, \zeta)$.

$$H_{V/V}(\omega) = \frac{\bar{v}_n(\omega)}{V(\omega)} = f_{HP}(\omega, a) = \frac{j\omega D}{\omega_0^2 - \omega^2 + 2j\omega \zeta \omega_0}$$

where $D = Bl/(R m_p)$, $\omega_0^2 = k_p/m_p$ is the natural frequency associated with the spring-mass system and $\zeta = (c_p R + (Bl)^2)/(2 R m_p \omega_0)$ is the damping ratio, including contributions from the structure and the back electromotive force.

5.3 Admittance models

The admittance coefficient $A_n(\omega)$ is linked to the acoustic impedance $Z_n(\omega)$ by the relation $A_n(\omega) = Z_n^{-1}(\omega)$. Different models of the acoustic impedance are proposed in the literature [8], depending of the incidence of the acoustic wave and of the kind of the absorbing material. The laws of the Delany-Bazley model allow to replace a layer of fibrous material, with porosity close to 1, by a layer of equivalent fluid. Therefore, in the case of normal incidence, the acoustic impedance of a layer of porous material layer, fixed on a rigid impervious wall, is given by

$$Z_n(\omega) = -jZ_c(\omega) \cot(k(\omega)d)$$

where $Z_c$ is the characteristic impedance of the material, $k$ is the complex wave number in the material and $d$ is thickness of the material. $Z_c(\omega)$ and $k(\omega)$ are given by the Delany-Bazley empirical laws

$$Z_c(\omega) = \rho c \left[ 1 + c_1 \left( \frac{f}{\sigma} \right)^{c_2} - j c_3 \left( \frac{f}{\sigma} \right)^{c_4} \right]$$

$$k(\omega) = \frac{\omega}{c} \left[ 1 + c_5 \left( \frac{f}{\sigma} \right)^{c_6} - j c_7 \left( \frac{f}{\sigma} \right)^{c_8} \right]$$

where $\sigma$ is the resistivity and the coefficients $c_i$ ($i = 1, \ldots, 8$) are given in the first row of Tab. 1 [16]. The Delany-Bazley
The model is valid in the frequency range

\[ 0.01 \leq \left( \frac{f}{\sigma} \right) \leq 1 \]  \hspace{1cm} (32)

Although these relations do not allow describing the exact acoustic behavior of all the porous materials in the frequency range defined by (32), they are widely used and can provide reasonable order of magnitude of \( Z_c \) and \( k \). Miki suggests slightly different empirical expressions of \( Z_c \) and \( k \) for multilayer fibrous materials, especially at low frequency. The coefficients \( c_i \) \( (i = 1, \ldots, 8) \) of the Delany-Bazley-Miki model are given in the second row of Tab. 1 [37] and its validity is extended to \( f/\sigma < 0.01 \).

### Table 1. Coefficients for Delany-Bazley and Delany-Bazley-Miki models

<table>
<thead>
<tr>
<th>Model</th>
<th>Delany-Bazley</th>
<th>Delany-Bazley-Miki</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_1 )</td>
<td>0.0497</td>
<td>0.0699</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>-0.7540</td>
<td>-0.6320</td>
</tr>
<tr>
<td>( c_3 )</td>
<td>0.0758</td>
<td>0.1070</td>
</tr>
<tr>
<td>( c_4 )</td>
<td>-0.7320</td>
<td>-0.6320</td>
</tr>
<tr>
<td>( c_5 )</td>
<td>0.0858</td>
<td>0.1090</td>
</tr>
<tr>
<td>( c_6 )</td>
<td>-0.7000</td>
<td>-0.6180</td>
</tr>
<tr>
<td>( c_7 )</td>
<td>0.1690</td>
<td>0.1600</td>
</tr>
<tr>
<td>( c_8 )</td>
<td>-0.5950</td>
<td>-0.6180</td>
</tr>
</tbody>
</table>

In the case of oblique incidence, if the velocity in the material is much smaller than that in the air, the acoustic impedance weakly depends on the angle of incidence and is close to its value for normal incidence.

The admittance coefficient is then given by a function \( f_{DB}(\omega, a) \) or \( f_{DBM}(\omega, a) \) (depending on the kind of material to describe) of two parameters \( a = (\sigma, d) \), given by Eqs. (30) and (31).

### 5.4 Numerical test case

#### 5.4.1 Reference problem

The two-dimensional acoustical domain, representing a car cabin [35], is meshed with 525 nodes and 450 elements, as shown in Fig. 3. The positions of absorbing materials and of the loudspeaker are also shown in Fig. 3. Let us call \( A_{n,1}, A_{n,2}, A_{n,3} \) and \( A_{n,4} \) the admittance coefficients of materials respectively covering the loudspeaker, the front seat, the roof and the rear seat and \( v_{n,1} \) the normal component of the velocity of the membrane of the loudspeaker.

The open squares in Fig. 3 represent the position of non-defective sensors while the solid ones represent the position of defective sensors, artificially affected by an error of 50% ². The measured pressures are computed using Structural Dynamics Tool for Finite Element Method and the value of parameters used to compute the measured pressure are given in the first column of Tab. 2. The updating process is performed in the frequency range [100-1600]Hz, with a step of 50Hz. The value of the parameters at the beginning of the process is given in the second column of Tab. 2.

#### 5.4.2 Localization of defective sensors

Figure 4 shows the contribution \( \eta_2^i \) (in percentage) of each sensor \( i \) to the error in measurements \( \eta_T^2 \). The sensors labeled 1 and 2 are placed close to the vibrating boundary, those labeled from 3 to 12 are placed close to the front seat, from the top to the bottom, those labeled from 13 to 22 are placed close to the roof, from the left to the right and, finally, those labeled from 23 to 32 are placed close to the rear seat, from the top to the bottom. The defective sensors are therefore the 6th, the 18th and the 28th sensors. Figure 4 clearly shows that the defective sensors are perfectly localized.

Let us assume in the following that the source of the errors has been found out and that the measurements have now been corrected.

#### 5.4.3 Updating process

The optimization problem is performed by a hybrid genetic algorithm [39]. The first step consists in using a genetic algorithm, to find the best set of parameters for the second step, during which the Matlab function \( \text{fmincon} \), solving constrained optimization problems, is used. The parameters are updated between two bounds

\[ 0.2 \times \text{Par}_0 \leq \text{Par} \leq 5 \times \text{Par}_0 \]  \hspace{1cm} (33)

²The value of the measurements is multiplied by \( (1 - \beta) \), with \( \beta = 50\% \), at each frequency
where $P_{0}$ is the initial value of the parameter, given in the second column of Tab. 2.

Table 2. Exact and initial values of the parameters - The exact values are used to produce the measurements.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Exact value</th>
<th>Initial value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{v}_{n,1}$</td>
<td>125 (A.s/kg.m)</td>
<td>175</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.60</td>
<td>0.50</td>
</tr>
<tr>
<td>$f_0$ (Hz)</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>$A_{n,1}$</td>
<td>$\sigma$ ($10^4$Ns/m$^4$)</td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td>$d$ (m)</td>
<td>0.040</td>
</tr>
<tr>
<td>$A_{n,2}$</td>
<td>$\sigma$ ($10^4$Ns/m$^4$)</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>$d$ (m)</td>
<td>0.020</td>
</tr>
<tr>
<td>$A_{n,3}$</td>
<td>$\sigma$ ($10^4$Ns/m$^4$)</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>$d$ (m)</td>
<td>0.015</td>
</tr>
<tr>
<td>$A_{n,4}$</td>
<td>$\sigma$ ($10^4$Ns/m$^4$)</td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td>$d$ (m)</td>
<td>0.032</td>
</tr>
</tbody>
</table>

At the start of the updating process, the value of $r$ in Eqn. (19) is taken in order to have the same contribution of the errors on the model and on the measurements, ensuring a good sensitivity of both of them to a change of parameters. This value is equal to 0.76, representing in Fig. 5 by a dot, and $\eta_r$ and $\xi_r$ are equal to 51.75%$^3$. The initial value of the modified CRE $e$ is then equal to

$$e = \sqrt{\xi_r^2 + \frac{r}{1-r} \eta_r^2} = 105.63\%$$  \hspace{1cm} (34)
Figures 6 to 10 show the normal component of the velocity of the membrane loudspeaker \( \bar{v}_{n,1} \) and the admittance coefficients \( A_{n,i} \), with \( i = \{1,2,3,4\} \), for different sets of parameters: the solid curves for the exact value of parameters, the dotted curves for the initial value of parameters, the dashed curves for the value of the parameters from updating process without localization step and the dash-dot curves for the value of parameters from updating process with localization step. Figure 11 gives the pressure at the node 28, near the rear seat. The final value of the modified CRE \( \epsilon \) is about 0.86% without localization step and about 0.01% with localization step.

The results of this test case are very concluding. It is shown that the defective sensors can correctly be localized by looking at the distribution of the local estimator of the error in measurements. It is obvious that the larger the number of sensors, the more effective will be this step of localization. In this case, 10 sensors are placed near each of absorbing material, which makes it easy to localize a defective sensor. Wherever possible, it will be preferable to place a series of sensors, rather than a single sensor, so as to ensure the accuracy of measurements.

The number of parameters being quite large, the updating process can not reach their exact values without the localization step. However, the FRF calculated with the parameters of the updating process without localization step are extremely close to that used for the measurements. As expected, the step of localization of parameters contributing
most to the constitutive relation error significantly improves the results of the updating process.

5.5 Physical test case

5.5.1 Measurement of admittance coefficients with the Kundt’s tube setup

The Kundt’s tube, shown in Fig. 12, is a cylindrical cavity of 30.5cm long with a diameter of 10cm, in order to work on a frequency range from 0Hz to 1600Hz. The tube is excited by a loudspeaker on its left extremity and an absorbing material is placed on the other one. Two microphones are placed at 20cm and 10cm from the absorbing material. The norm NF EN ISO 10534-2 describes the use of this setup to directly compute the admittance coefficient of absorbing materials, from the transfer function of the pressures between the two microphones. The loudspeaker is excited by a white noise creating plane waves in the duct. The measured pressure at each microphone can be decomposed in an incident wave and a reflected wave

\[ p(x, \omega) = p_{inc}(\omega)e^{-jkx} + p_{refl}(\omega)e^{jkx} \]

where \( p_{inc} \) and \( p_{refl} \) are the amplitudes of the incident and the reflected waves, respectively, and \( R \) is the reflexion coefficient of the absorbing material. The transfer function of the pressures between microphones is given by

\[ H(\omega) = \frac{p(x_2, \omega)}{p(x_1, \omega)} \]

where \( x_1 = 10.5cm \) and \( x_2 = 20.5cm \). The reflexion coefficient \( R \) of the absorbing material can therefore be determined by

\[ R(\omega) = \left( \frac{e^{-jkx} - H(\omega)}{H(\omega) - e^{jkx}} \right) e^{-2jkx} \]

where \( \Delta x = 10cm \) is the distance between the two microphones. The admittance coefficient of the absorbing material is defined by (see Eqn. (3), with \( \lambda = 1 \))

\[ A_n = \frac{v_n(L, \omega)}{p(L, \omega)} = \frac{1}{\rho c} \frac{1 - R(\omega)e^{2jkL}}{1 + R(\omega)e^{2jkL}} \]

where \( L = 30.5cm \) and \( v_n \) is given by Eqn. (6). With this technique, only the transfer function between the microphones is necessary to determine the admittance coefficient of absorbing materials. The admittance coefficients of two different materials (foam and felt) were measured, using the Kundt’s tube, in the frequency range from 100Hz to 1600Hz. The results are presented in Fig. 13 for the foam and in Fig. 14 for the felt by the dashed curves.

Let us assume that the admittance coefficient of the foam can be described by the Delany-Bazley model and that of the felt by the Delany-Bazley-Miki model. The parameters of the models were identified using the least square method. The values are given in the first column of Tab. (4) and the solid curves of Fig. 13 and Fig. 14 give the admittance coefficient computed with Eqs. (30) and (31), using these values.

5.5.2 Identification of the parameters with the updating process

Modeling The Kundt’s tube is modeled by a two-dimensional rectangular domain size 0.305m \( \times \) 0.1m, meshed by 68 \( \times \) 2 rectangular elements (Fig. 15).

The loudspeaker and the absorbing material are modeled by a generalized boundary condition with \( \lambda = 0 \) and \( \lambda = 1 \),
respectively. Sensors are placed at the exact locations of the microphones in the Kundt's tube setup, i.e., $x = 0.105m$ and $x = 0.205m$ (represented by the squares in Fig. 15) and the measurements are the measured transfer function between the pressure at the microphones and the voltage applied to the loudspeaker (see Fig. 16 to Fig. 19).

$$H_{VP_i}(f) = \frac{P_i(f)}{V(f)}$$ (39)

where $i = 1, 2$. The frequency range in which the updating process will be performed is then $100Hz$ to $1600Hz$ with a step of $3.125Hz$.

The Kundt's tube setup can identify accurately the admittance coefficient. In this method, only the transfer function between the microphones is required which has several advantages. First, the excitation of the cavity is not

**Initial parameters for the updating process** The initial parameters for the updating process are chosen close to the reality. For the admittance coefficients it is the parameters identified by the standard test with the Kundt's tube. For the normal component of the velocity of the loudspeaker membrane, the "real" parameters are identified using a direct inverse method: the frequency dependence of the normal component of the velocity is calculated dividing the measured transfer function $H_{VP_i}(f)$ by the finite element solutions of the problem $p_i(f)$ (i = 1, 2) excited by a unitary velocity, $\bar{v}_n = 1$, and the parameters of the model (29) are then identified using the least-square method.

Figures 20 and 21 give the results of this calculation: the dashed curves are the results of the measurements divided by the numerical solution and the solid ones are the normal component of the velocity computed from the model (29) with the identified parameters.

All the "real" and the initial parameters are respectively given in the first and the second column of Tab. 4. Note that the fit between the model and the experiment is globally good, but there is a phase shift after $300Hz$ which is probably due to the presence of anti-aliasing filters in the acquisition system which are not represented in our model.

**Results of the updating process** The updating process is only performed without localization step, because of the small number of parameters to update. The results of the process is given in the third of Tab. 4.
involved in the transfer function and secondly the natural frequencies are canceled out while the anti-resonances emerge as shown in Fig. 22 and Fig. 23.

The parameters identified by the updating method do not match those identified with the Kundt’s tube. The differences can be explained as follows

1. The normal component of the velocity of the loudspeaker membrane must be known for the use of the method and the model may not be accurate.
2. The values of sound velocity $c$ in the Kundt’s tube and in the density $\rho$ of the air contained in the tube, involved in the models of the admittance coefficient and in the constitutive relation error, depend on the temperature $T$ within the tube. If the value of $T$ is poorly estimated, peaks of natural frequencies in the model will not match the experimental ones. These frequencies being in the frequency range of updating process, it could be difficult to update the parameters of the admittance coefficient around the peaks of natural frequencies.

Figures 24 and 25 show the admittance coefficients of foam and felt, respectively. Solid curves represent the coefficients calculated from the updated parameters. The imaginary part of the admittance coefficients is very close to that of the coefficients identified with the Kundt’s tube (dashed-dot curves) while the real part slightly deviates from the identified values. The real part of $A_n$ decreases the intensity of the pressure and the imaginary part shifts the natural frequencies. So if the natural frequencies are not well represented by the model, this will have an impact on the identified values of the admittance coefficients.

Figures 26 to 29 show the FRF calculated at the sensors, for both materials, from the parameters identified with the Kundt’s tube setup (dashed-dot curves) and with the updating technique (solid curves). Note that the fit is good at frequencies away from the natural frequencies, and in particular at the anti-resonances of both FRFs which correspond to the resonances and anti-resonances of the transfer function between the two pressure sensors, used as an input for the direct identification method. From that perspective, we see that the results obtained with the updating method are in good agreement with the results obtained with the direct inverse method.

To improve the results, an updating method in two steps could be performed. The first step would consist in the updating of the temperature $T$ and of the parameters of the normal component of the velocity of the loudspeaker membrane, from measurements without absorbing materials. The second step would then consist in updating the parameters of the admittance coefficient from measurements with the absorbing material.

It is clear that the updating method does not outperform the direct method for the simple case of the Kundt’s tube. The present application shows however that results of a comparable quality can be obtained with the proposed updating method, although the computational costs are much higher. The interest of the model updating method lies of course in its ability to deal with more complex problems where no analytical solution to the acoustical problem exists.

<table>
<thead>
<tr>
<th>Table 4. Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
</tr>
<tr>
<td>Foam</td>
</tr>
<tr>
<td>$\bar{v}_n$</td>
</tr>
<tr>
<td>$\zeta$</td>
</tr>
<tr>
<td>$f_0$ (Hz)</td>
</tr>
<tr>
<td>$A_n$ $\sigma$ (Ns/m$^4$)</td>
</tr>
<tr>
<td>$d$ (m)</td>
</tr>
<tr>
<td>Felt</td>
</tr>
<tr>
<td>$\bar{v}_n$</td>
</tr>
<tr>
<td>$\zeta$</td>
</tr>
<tr>
<td>$f_0$ (Hz)</td>
</tr>
<tr>
<td>$A_n$ $\sigma$ (Ns/m$^4$)</td>
</tr>
<tr>
<td>$d$ (m)</td>
</tr>
</tbody>
</table>
6 Conclusions

Nowadays, absorbing materials take an important place in acoustic research and the description of their acoustical properties still remains an issue highly developed by many authors. The main idea of this paper is to use an updating technique in order to identify the admittance coefficient of the absorbing materials from in-situ measurements. Updating techniques aim at minimizing the error between the results from a numerical model and a set of experimental data by acting on some parameters of the numerical model. In this paper, the CRE-based updating technique is applied to the acoustical problem, with generalized boundary conditions. The error to minimize is called the modified CRE and is constructed on equations of the continuous model and the quantities of the set of experimental data which are considered as less-reliable.

This technique is very interesting because the error, by its definition, is an energy-based indicator and therefore can be locally evaluated, which allows regularizing the problem. The process consists in an iterative process in which each iteration is divided into two steps: the localization step during which the parameters contributing most to the CRE are localized by comparing the local estimators of CRE, and the correction step, consisting in minimizing the global modified CRE by acting only on the parameters chosen during the localization step. The updating process ends when the global modified CRE reaches a prescribed value. On the other hand, defective sensors can be detected by checking the contribution of each sensor to the error in measurements. This allows either correcting the erroneous measurements or removing them from the set of experimental data.

The localization of defective sensors and the updating process, with and without localization of parameters contributing most to the constitutive relation error, are performed on a numerical two-dimensional test case, in order to verify the method developed in this paper. The results show the efficiency of the both steps of localization. A second test case aims at validating the method on a real two-dimensional cavity. The admittance coefficient of two different absorbing materials are first directly measured with the Kundt’s tube setup and then compared to the results of the updating process. The results show that the updating method is suitable for the identification of the parameters of the admittance coefficients, but also points out some practical difficulties, which will need to be further studied.

From the results of these test cases, some investigations are proposed to improve the method. The frequency range has to be optimally defined, models describing the admittance coefficient and the normal component of the velocity exciting the acoustical domain have to be correctly selected and the acoustical domain has to be meshed so as to provide a compromise between the size of the numerical prob-
lem and the quality of the results. In addition, the position of the sensors may affect the results of the updating process. Optimization of the sensors locations will be the topic of future research. Another important aspect is the application to a three dimensional experimental test setup which is more representative of real applications. A 3D cavity with four microphones is currently developed for that purpose.

Acknowledgments

This work is a part of an FRFC project in collaboration with the Université de Liège (ULg). The authors wish to thank the ULg for his contribution and the FNRS for financing this project. The measurements on the Kundt’s tube setup and on the 3D cavity have been performed in the Applied Mechanics Department of the FEMTO-ST institute (Besançon - France). The authors wish therefore to thank all the team of the lab and especially Professors M. Ouisse and E. Foltête for their collaboration.

References


