The design of tensile surface structures
From a hand calculation in 1958 to a contemporary numerical simulation

This paper summarizes the differences in the design approaches for tensile surface structures between the earliest structures in the 1950s and today's practice. Current software tools allow more refined and advanced calculations. Nevertheless, a basic hand calculation can clarify the process in a few pages and provide the appropriate key data. A transparent setup allows the form-finding and structural analysis to be redone. The calculation of the cable net for the bandstand by André Paduart (1958) is analysed in this paper as a case study. Both the hand calculation (19 pages) and the numerical simulation are summarized and the design context of the initial and current calculations are described. The approximations made by Paduart resulted in a remarkably intelligible and coherent evaluation of the cable net structure. The historical approach can still be applied for a first verification of a pretensioned cable net or for a membrane structure as the simplified calculation method is similar.

1 The design of tensile surface structures

1.1 Form-finding

“Tensile surface structures” is a general term for form-active structures that are double-curvature pretensioned structural fabrics, foils or cable nets. They are either mechanically tensioned or inflated. Their form as well as their behaviour under load is different from conventional structural systems such as frames (bending-active) or truss systems (vector-active) [1]. The design process consists of calculating the equilibrium shape and verifying both the level of prestress and the deflection under load.

Tensile surface structures have a curvature that is either anticlastic (curved in opposite ways in two directions) or synclastic (curved towards the same side in all directions). Mechanically tensioned structures have an anticlastic shape, with both a hanging and an arching direction. In the case of fabric structures, the warp and weft directions of the textile are aligned with the principal curvatures. For cable nets, the hanging and arching cables follow these principal curvatures.

For a given set of boundary conditions, the shape of the surface is obtained by considering the equilibrium of the internal stresses and the external load (if applicable) at every point. The process of calculating this equilibrium shape is considered to be the form-finding process. The equilibrium form can be derived without the use of stiffness properties. By changing the tension in the elements and/or the geometry of the supports, the equilibrium shape can be adjusted [2], [3].

The geometric stiffness of a tensile surface structure is proportional to its prestress. The required level of prestress is adjusted to assure tension under all loading conditions (no wrinkles), to limit the deflection under load, to avoid ponding and to take into account the creep of the material over time. Curvature is an important criterion for reducing the tension in an element under the same external loading, as is expressed by the cable equation

\[ H = \frac{pl^2}{8f} \]

where:
- \( H \) horizontal component of force in hanging cable [kN]
- \( p \) distributed vertical load [kN/m]
- \( l \) span [m]
- \( f \) sag [m]

Recent structures happen to accept a lower double curvature, in which case a higher prestress may be required to avoid large deflections.

1.2 Structural analysis

Tensile surface structures are flexible. The strain in a structural membrane under load is much larger than in a steel plate, for instance. For that reason, the deflections under load are more important than for conventional structures. Moreover, as geometric changes under load cannot be neglected, the stresses do not rise linearly with the loading, so non-linear calculations need to be performed.

Real stiffness values for the membrane and/or cables are specified for the structural analysis. The membrane's stiffness properties have to be determined by biaxial testing.

As the surfaces are in double curvature, the wind pressure distribution and snow load accumulation are difficult to predict and not yet considered in the Eurocodes. Safe assumptions have to be made, typically resulting in the application of high safety factors. With respect to the effect of actions and combination of actions on membrane struc-
ticlastic cable net tensioned between an inclined mast at the front, 7.50 m high, and a curved wall with varying height (2.82–3.22 m) at the back. Nine (14 cables are shown on the plan, but the number was reduced to nine in the calculations) longitudinal cables were connected to the top of the inclined mast, kept in position by a vertical tie-down cable (Fig. 3). In the transverse direction, three main cables were tensioned by means of individual turnbuckles and attached between low, inclined supports on both sides of the structure. Two additional transverse cables with turnbuckles were fitted between steel columns integrated into the wall. Transverse and longitudinal cables were connected only by friction, as the longitudinal cables (negative curvature) lay below the transverse cables (positive curvature). Two ‘floating’ cables (visible above the cable net in Fig. 1) connected the top of the inclined mast with steel columns integrated into the wall. A canvas cover was placed between the longitudinal and transverse cables, folded over the curved wall at the back and attached by a rope. The


The bandstand was designed by the architect Oger Schomblood and the engineer André Paduart (1914–1985). Paduart was an independent consulting engineer, experienced in prestressed concrete and thin concrete shells.

Although several important cable net structures were erected at the World Fair in Brussels, the bandstand appears to be the only double-curvature cable-and-textile structure built for the event [6]. Further, different from most other structures, both shape and medium size are such that verification of the cover is feasible without requiring an excessively complex or time-consuming study.

The bandstand (Fig. 1) had a rounded wedge plan form (13.91 m long, 15.04 m wide) and consisted of an an-
cover was a half-open canopy, designed as a temporary structure (April to October 1958).

Photographs show a textile cover that was not perfectly tensioned, and some ponding was evident. During the World Fair, ponding was observed at the lower edge of the cable net, which was easily resolved by retensioning the wall attachment. During the event, the structure successfully withstood bad weather (summer storm).

The analysis in the following sections will focus on the structural analysis of the cable net.

3 Hand calculation

3.1 Form-finding

The calculation report of the design of the bandstand by Paduart is kept in the State Archive of Brussels [7]. It is a report containing the form-finding, the assessment of the global stability of the structure and the design of the lateral struts, the central column and the cables. It shows a clear and valuable analytical approach to determining the form and calculating the tensile forces [2], [9] (Note:
at a desired position, minor adjustments to the values of the forces were made by reiteration (not specified in the calculation report).

As an example, the calculation of the main longitudinal cable in the plane of symmetry (cable SR in the calculation report by Paduart, see Fig. 4, sr in Fig. 5 and 6) is included and commented.

The funicular curve of the main longitudinal cable (SR) is calculated under load vectors representing the interaction between the pretensioned longitudinal and transverse cables. The load vectors are obtained by multiplying the area of the appropriate quadrangle with the chosen interaction load of 25 kg/m².

The reactions at the supports are $R_R = 190.5$ kg and $R_S = 139.8$ kg. The depth (with respect to line sr) of the funicular curve is imposed at point 8 $f_8 = 1.02$ m. The vertical forces to the right of point 8 generate a moment of 777 kgm. As the bending moment needs to be zero, the effect of the horizontal reaction component (equal to 777 kgm/1.02 m = 761.7 kg) has to equilibrate the calculated moment. The depth of the funicular at the other nodes can also be derived from the expression that the bending moment has to be zero along the curve.

The magnitude of the axial forces can be obtained by drawing the force polygon.

The forces in the transverse cables were also determined by means of graphical analysis.

The position of the nodes where longitudinal and transverse cables intersect was checked according to the geometry of both cable systems, which revealed minor differences.

### 3.2 Structural analysis

Paduart considered the following load cases:

- **a.** Self-weight of construction: 6 kg/m²
- **b.** Wind: 50 kg/m² (upward, vertical) for transverse cables (45 kg/m² for verification of floating cables, see section 4.2.2)
- **c.** Accidental loading: 10 kg/m² (downward)

The self-weight was combined with the distributed loads (25 kg/m²) representing the action due to prestress of each cable direction on the other direction. A planar equilibrium calculation was performed for each individual cable. The curvature of the net was obtained by assembling the funicular lines of the longitudinal and transverse cables.

The force equilibrium was calculated for the longitudinal cables (negative curvature). The depth of a single point along the cable was ‘set’ to a plausible value from which the horizontal reaction force was derived. To arrive...
transverse cables) are obtained by applying factors of 33/25 and 47/25 respectively.

3.3 Material and sections

The central mast and the lateral oblique poles were both made of steel. Based on the load cases considered, the appropriate sections were selected taking into account the limit load for buckling.

Paduart did not calculate the diameter of the cables, he only specified the maximum force. A safety factor of 2.5 was taken into account to obtain the rupture load. With an assumed ultimate strength of 160 kg/mm², an appropriate section can be defined and the diameter estimated.

As the self-weight of the cables is only about 1 kg/m², it is negligible in comparison to the wind loading.

4 Numerical approach

4.1 Form-finding

Current software tools do not simplify the calculation to two-dimensional approximations, instead immediately calculate the three-dimensional equilibrium shape. Another advantage of numerical simulations is that the supporting structure (poles, tie-down cables, etc.) can be integrated into the model. (Note: forces resulting from the numerical simulation are expressed in kN.)

The forces considered by Paduart as well as the cable lengths have been used to calculate the force densities (axial force divided by length of cable segment) and are applied in the numerical three-dimensional form-finding [11].

The largest difference between Paduart’s analytical approach and the numerical model measures 48.6 cm (\(\Delta x = 32.8\) cm, \(\Delta y = -30.4\) cm, \(\Delta z = -18.5\) cm) and can be found in the connection between the cable net and the lateral poles introducing the lateral pretension.

Fig. 9 shows the axial forces (without external loading) in a plan view of the numerical model.

As verification, the axial forces in the cable net (without external loading) resulting from the numerical approach are compared with those of Paduart’s model. The forces in the three main cables (see Fig. 8) are given in Table 4.

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### Table 1. Axial forces along main longitudinal cable sr

<table>
<thead>
<tr>
<th>Element</th>
<th>(N_i) (kg)</th>
<th>(l_i) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>s_2</td>
<td>885.6</td>
<td>3.12</td>
</tr>
<tr>
<td>2_4</td>
<td>873.8</td>
<td>2.20</td>
</tr>
<tr>
<td>4_6</td>
<td>858.2</td>
<td>1.89</td>
</tr>
<tr>
<td>6_8</td>
<td>841.8</td>
<td>1.58</td>
</tr>
<tr>
<td>8_10</td>
<td>820.6</td>
<td>2.57</td>
</tr>
<tr>
<td>10_r</td>
<td>799.2</td>
<td>3.41</td>
</tr>
</tbody>
</table>

### Table 2. Maximum axial forces in compression elements

<table>
<thead>
<tr>
<th>Element</th>
<th>(N_i) (kg)</th>
<th>Diameter/thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central mast</td>
<td>18700</td>
<td>(\phi 159/6)</td>
</tr>
<tr>
<td>Oblique poles</td>
<td>1945</td>
<td>(\phi 51/3)</td>
</tr>
</tbody>
</table>

### Table 3. Breaking loads required for the cables

<table>
<thead>
<tr>
<th>Area (mm²)</th>
<th>(N_{\text{break}}) (kN)</th>
<th>(\Omega) (mm²)</th>
<th>Possible diameter using (N_{\text{break}}) (mm) [10]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal cables</td>
<td>16.98</td>
<td>10.82</td>
<td>–</td>
</tr>
<tr>
<td>Transverse cables</td>
<td>105.96</td>
<td>67.51</td>
<td>10.1</td>
</tr>
<tr>
<td>Vertical cable</td>
<td>384.85</td>
<td>245.19</td>
<td>20.1</td>
</tr>
<tr>
<td>Lateral cable 1</td>
<td>61.27</td>
<td>39.04</td>
<td>8.1</td>
</tr>
<tr>
<td>Lateral cable 2</td>
<td>28.46</td>
<td>18.13</td>
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</tr>
</tbody>
</table>

b. Load on transverse cables: \(22 \text{ kg/m}^2 + 25 \text{ kg/m}^2\) (wind uplift) = 47 kg/m²

The forces in the cables were calculated for a distributed load (representing the pretension) of 25 kg/m². This value can be understood as follows: the wind load of 50 kg/m² will be taken partly by the transverse cables (increasing the stress) and partly by the longitudinal cables (reducing the stress). To remain tensioned under load, the prestress is set to a distributed load of 25 kg/m².

The cable forces under the maximum loading of 33 kg/m² (in the longitudinal cables) and 47 kg/m² (in the transverse cables) are obtained by applying factors of 33/25 and 47/25 respectively.
4.2.2 Wind load in transverse direction

Although not visible in the photographs of the finished bandstand, two floating cables were considered in Paduart's numerical verification to ensure the stability of the high pole under a wind load of 45 kg/m² in the transverse direction (Figs. 12 and 13).

Paduart represents the wind action in the transverse direction by an action of 410 kg in the \( y \) direction at the top of the pole. It was considered that this load vector should be redistributed to the two floating cables – one increasing its axial force and the other decreasing it.

The values of the forces in the calculation report by Paduart correspond very well with the numerical model. It should be pointed out that the input for the form-finding was derived from the data (axial forces, cable lengths) listed in the calculation report by Paduart. The difference is due to the fact that 2D equilibrium information is now introduced into a 3D equilibrium calculation.

### 4.2 Analysis under load

#### 4.2.1 Wind load in longitudinal direction

The wind direction considered is from the high point (open side) towards the curved wall (closed side). The wind pressure considered in the numerical simulation is 0.44 kN/m² (the same wind pressure as used by Paduart to check the floating cables, see section 4.2.2).

The maximum vertical displacement is 15 cm. Paduart did not check deflections under wind load.

As is typical for an anticlastic cable net under upward wind loading, a decrease in the forces in the longitudinal hanging cables was noted (highest value 10.4 kN becomes 10.2 kN), while the forces in the transverse arching direction increase (highest value 22.5 kN becomes 45.7 kN, Fig. 11). Only two elements become slack, which indicates that the pretension is set to an appropriate level. Paduart installed a cable with a breaking load of 10,600 kg (104.0 kN) for the transverse cable with the highest axial force of 45.7 kN.

#### 4.2.2 Wind load in transverse direction

Table 4. Comparison of the axial force along the main cables

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440 kg (the floating cables had to remain tensioned, hence a pretension of 440 kg was needed).

The wind pressure considered in the numerical simulation was also 0.44 kN/m² (45 kg/m²).

Two numerical models were set up – with and without floating cables. The maximum vertical displacement in the model with floating cables is 40 cm, which is still acceptable for a ‘flexible’ structure (as long as no ponding occurs).

The highest tensile forces do not change too much (highest force in cable net was initially 22.5 kN, is now 28.7 kN, Fig. 15) and almost all elements remain tensioned.

The top of the high pole moves into the wind under wind load in the y direction. The pretension in the floating cable on that side is reduced to zero. The pretension in the other floating cable becomes 8.2 kN. This value is very close to Paduart’s prediction of 8.8 kN (pretension = 4.4 kN, additional value due to wind = 4.4 kN). On the windward side the highest axial forces in the longitudinal cables increase substantially (from 6.3 kN to 17.8 kN, Fig. 15). The floating cables do not appear in the as-built situation (Fig. 2). It is unclear as to whether they were installed during construction.

The highest force in the floating cables in the previous model is 8.2 kN, a value that could be taken by the cable net itself. To verify this, the same wind loading was applied to a model without floating cables. The forces in the longitudinal cables on the leeward side are higher (8.6 kN becomes 9.0 kN, 6.9 kN becomes 9.2 kN) and the maximum force on the windward side is still high at 16.9 kN. The breaking force derived by Paduart for this longitudinal cable is about the same – 16.98 kN. The legend for the cable sections mentions a breaking load of 23.5 kN for the longitudinal cable considered. It was not considered that this cable (with a prestress of 6.3 kN) should carry the transverse wind load.

**5 Final remarks**

The analysis report made in 1958 contained 19 pages, with easily verifiable statements and data. The approximations made by Paduart resulted in a remarkably intelligible and coherent evaluation of the cable net structure. Of course, approximations were made, but the method is valuable for any simple, temporary cable net. Analysis nowadays is much more precise, but the results are also more difficult to verify and retrace.

Compared with the initial design (based on symmetric uplift wind loading), the numerical approach considering different wind directions could result in a solution with heavier loadbearing members.
The hand calculation method can still be used for basic tensile surface structures made of structural fabrics. However, if curvatures vary substantially, if a more ‘stretchable’ material is used, if the systems are more complex (tensairity, bending-active components, etc.) or load distributions are irregular, the use of appropriate software tools is indispensable.

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References


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