

# Finite Element Modeling of Shear in Thin Walled Beams with a Single Warping Function

```

    esclaves/ddl (Nme)      0
    types de chargement uniforme (Nch)  0
    types de chargement concentrique (Ncc) 1
    Nb noeuds principaux (Nnp) 21
    Nb segments rectilignes (Nsp) 20
    Nb types des sections transversales (Nsc) 1
    Nb d elements de maillage (Ne) 21
    Nb types de conditions aux limites/noeud 21
    Nb conditions maîtres-esclaves/ddl (Nme) 21
    Nb types de chargement uniforme (Nch) 21
    Nb types de chargement concentrique (Ncc) 21

```

```
****RELATIONS MAITRE-ESCLAVE((MME))
ID NODE1 DDL NODE2 DDL Value
```

## Katy Saadé

```

:q
[ksaade@airy]47> utilisation des multiplicateurs
calcul lineaire
Donner le nom du fichier data
wccm2
nb_eq bf5 20
fin rigidite k 26 0
fin reperes
Xc_zc 1.0441267936464221736185118708755567E17
-0.0250000000000000031891843727796077
debut calcul interne

```



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