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A new mapped infinite partition of unity method for convected acoustical radiation in infinite domains.

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Remerciements

Si tu donnes un poisson à un homme, il ne mangera qu'un jour. S'il apprend à pêcher, il mangera toute sa vie.

Proverbe de Confucius, repris plus tard par Dominique Pire dans le cadre de l'action Iles de Paix.

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Enfin, je terminerai par remercier les deux petits rayons de soleil qui ont ensoleillé la fin de ce long parcours.

List of Symbols

Greek symbols

β	: $\sqrt{1-M_0^2}$	
γ	: Poisson ratio of specific heat capacities : c_p/c_v	
Γ	: interface separating the inner and the outer domains	
ε	: Error	
μ	: phase function	$[m]$
ρ	: mass density	$[kgm^{-3}]$
ρ_0	: steady mean density	$[kgm^{-3}]$
ρ_a	: acoustic density	$[kgm^{-3}]$
$\boldsymbol{\sigma}$	: stress tensor	$[Nm^{-2}]$
ϕ	: velocity potential	$[m^2s^{-1}]$
ϕ_0	: mean velocity potential	$[m^2s^{-1}]$
ϕ_a	: acoustic velocity potential	$[m^2s^{-1}]$
$\tilde{\phi}_a$	: amplitude of the harmonic acoustic velocity potential	$[m^2s^{-1}]$
$\tilde{\phi}^h$	: numerical approximation of $\tilde{\phi}_a$	$[m^2s^{-1}]$
$\tilde{\phi}_h^I$	: numerical approximation in the outer region Ω_o	$[m^2s^{-1}]$
Φ_α	: shape function for the α^{th} degree of freedom	
Φ_α^I	: infinite shape function for the α^{th} degree of freedom	
ω	: angular frequency	$[s^{-1}]$
Ω	: domain	
Ω_i	: inner region	
Ω_o	: outer region	

Arabic symbols

$\tilde{a}_n$	: normal acceleration of a vibrating wall	$[ms^{-2}]$
A_n	: normal acoustic admittance	$[m^2skg^{-1}]$
$A^{\pm}_{mn}$	: incident and reflected modal amplitude	$[m^2s^{-1}]$
c	: speed of sound	$[ms^{-1}]$
c_0	: steady mean part of the speed of sound	$[ms^{-1}]$
c_∞	: speed of sound at large distance from the source	$[ms^{-1}]$
c_p	: specific heat capacity at constant pressure	$[JK^{-1}]$
c_v	: specific heat capacity at constant volume	$[JK^{-1}]$
$dofs$	: number of unknowns of the approximation	
E	: energy flow out of a surface	$[J]$
$E^{\pm}_{mn}$	: incident and reflected modal patern	
f	: excitation frequency	$[s^{-1}]$
G	: geometric factor	
h	: mesh size	$[m]$
H	: Hilbert space	
i	: imaginary unit $= \sqrt{-1}$	
$\mathbf{I}$	: Sound intensity	$[Wm^{-2}]$
J'	: stagnation entropy	$[Jkg^{-1}]$
k	: wavenumber	$[m^{-1}]$
$k^{\pm}_{r,mn}$	: incident and reflected radial wavenumber	$[m^{-1}]$
k_B	: Boltzmann constant	$[JK^{-1}]$
$K^{\pm}_{z,mn}$	: incident and reflected axial wavenumber	$[m^{-1}]$
L^d_j	: Legendre polynomial of order d for node j	
L_s	: curve enclosing the boundary S_s	
L_v	: curve enclosing the boundary S_v	
m	: angular mode number	
$\mathbf{m}'$	: mass flux	$[kgm^-2s^{-1}]$
m_0	: radial order of the infinite element	
m_w	: mass of a molecule	$[kg]$
M_0	: mach number	
M_i	: Mapping function for node/point i	
$\mathbf{n}$	: outer normal to the domain	
n	: radial mode number	
n^I_d	: number of infinite degree of freedom	
$n(j)$	: size of the local approximation space at node j	
nni	: number of infinite nodes	
$nodes$	: number of nodes	
N_i	: Partition of Unity function of node i	
N_m	: number of angular modes	
N_n	: number of radial modes	
N_M	: number of reflected modes (unknown)	

p	: fluid pressure	$[Pa]$
p_0	: steady mean fluid pressure	$[Pa]$
p_a	: acoustic pressure	$[Pa]$
$\tilde{p}_a$	: amplitude of the harmonic acoustic pressure	$[Pa]$
$\tilde{p}_{an}$	: analytic amplitude of the harmonic acoustic pressure	$[Pa]$
$\mathbf{q}$	: heat flux	$[Wm^{-2}]$
Q_w	: heat production	$[J]$
r_o	: distance to the source point	$[m]$
R	: specific gas constant	$[JK^{-1}mol^{-1}]$
R_j	: radial function for infinite node j	
R_j^d	: radial function of order d for node j	
s	: entropy	$[Jkg^{-1}K^{-1}]$
S	: boundary	
S_i	: mapping functions for the interface $\mathit{\Gamma}$	
S_M	: Modal boundary	
S_s	: soft wall	
S_v	: vibrating wall	
t	: time	$[s]$
T	: Temperature	$[K]$
T_j	: circumferential function for infinite node j	
$\tilde{u}_n$	: normal displacement of a vibrating wall	$[m]$
$\mathbf{v}$	: fluid velocity	$[ms^{-1}]$
$\mathbf{v}_0$	: steady mean fluid velocity	$[ms^{-1}]$
$\mathbf{v}_\infty$	: fluid velocity at large distance from the source	$[ms^{-1}]$
$\mathbf{v}_a$	: acoustic velocity	$[ms^{-1}]$
$\tilde{\mathbf{v}}_a$	: amplitude of the harmonic acoustic velocity	$[ms^{-1}]$
$\mathscr{V}$	: the Sobolev space $W^{1,2} = H^1 = \{f : f, \nabla f \in L^2\}$	
V_{jl}	: l^{th} local approximation function of node j	
$\tilde{w}_n$	: normal velocity of a vibrating wall	$[ms^{-1}]$
W_j	: weight function of node j	
W_j^I	: infinite weight function of the infinite node j	
$W_{M,nm}$	: modal weight function of the angular and radial mode (m, n)	

Operators

∇	: gradient operator
$\nabla\cdot$	: divergence operator
$\nabla\times$	: curl operator
$\mathit{\Delta}$	: Laplacian operator
$\frac{D}{Dt}$	: Total time derivative
$:$	: the double dot product of two tensors
$\langle\ \rangle$	: time average
$\Re$	: Real part

Contents

4

Three-dimensional formulation: Verification tests

Three-dimensional simulations have the advantage of being able to deal with any kind of geometry. Another advantage of three-dimensional models is that the excitation can be prescribed in one simulation. It does not need to be decomposed in several axisymmetric excitations and be computed for each azimuthal order separately.

However, the major drawback of three-dimensional computations consists in meshing the whole domain. This leads to high number of degrees of freedom, high computational time and high computer resources.

This chapter illustrates acoustic simulations with the Mapped Infinite Partition of Unity Method. This verifies the three-dimensional formulation.

4.1 Duct propagation

This section analyses acoustic propagation in a three-dimensional infinite duct. This is simulated by a duct of finite length. Modal boundary conditions are prescribed at both ends of the duct to prevent for reflections and then allow to simulate the duct of infinite length. We consider, in this work, acoustic modes traveling from the left end of the duct to the right one.

We illustrate the propagation of different modes in a cylindrical duct with and without mean flow. The enrichment is chosen to be constant at each node of the mesh: $V_{j1} = \{1\}$. The radius of the duct is $R_d = 0.5m$ and the length is $L = 1m$. Note that these dimensions correspond to those chosen for the axisymmetric duct applications, the results can then be compared to those of section 3.1. Figure 4.1 illustrates the geometry and a mesh generated with 18 elements along the axial direction and 6×6 elements within the cross-section.

Figure 4.2 illustrates computations performed for a frequency of 800 Hz. We simulate the propagation of the second radial mode ($n = 2$) of the first azimuthal order ($m = 1$). We show MIPUM 0 computed results (absolute, real and imaginary parts of the pressure) for a mean flow at rest as well as for convected wave propagation: $\mathbf{v}_0 = -100m/s\ \mathbf{1}_z$.

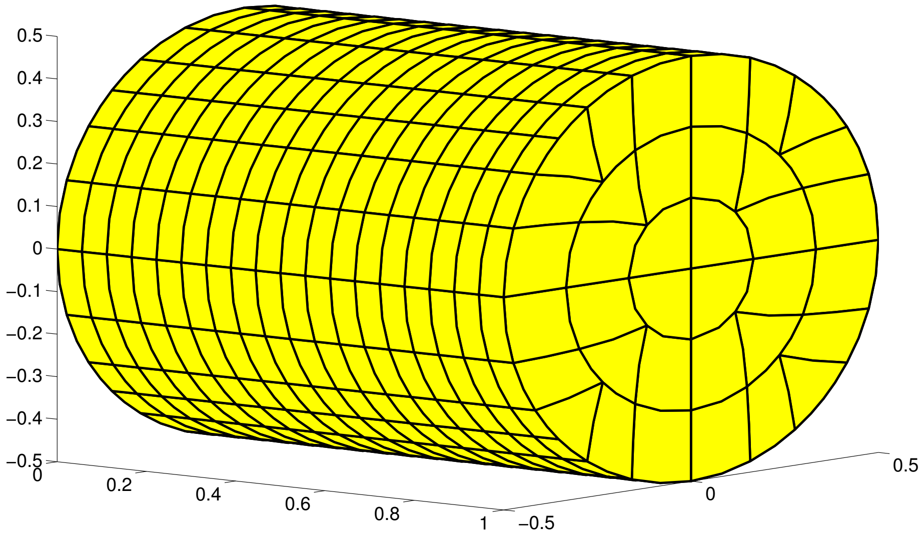


Fig. 4.1. Mesh of the cylindrical duct with a circular cross section (18 elements along the axial direction and 6 × 6 within the cross-section)

The mesh generated for the simulation has 50 elements in the axial direction and 20×20 within the section.

Figure 4.3 illustrates the same mode propagating in the duct. The parameters of the simulations are identical (mesh, frequency, etc.) but we incorporate a uniform lining on the walls. The acoustic treatment is inserted from $z = 0.14m$ to $z = 0.42m$. The acoustic properties of the liner are given by the normal admittance: $A_n = 0.001 + 0.002i$. As illustrated in section 3.1, we observe that the amplitude of the pressure has decreased thanks to the insertion of the liner.

Figure 4.4 illustrates other simulations: evanescent mode and high azimuthal order. The evanescent simulation consists in prescribing at 800 Hz, the fourth radial mode ($n = 4$) of the first azimuthal order ($m = 1$). The mesh has 50 elements in the axial direction and 20×20 within the cross-section. The second simulation represents the first radial mode ($n = 1$) of the fifth azimuthal order ($m = 5$). The frequency is increased to 1200 Hz to avoid this mode to be evanescent. The mesh generated for the simulation has 30 elements in the axial direction and 40×40 within the cross-section.

All these figures show that the three-dimensional formulation is able to simulate convected propagation in a cavity with acoustic treatment and the use of modal boundary conditions (propagating and evanescent modes).

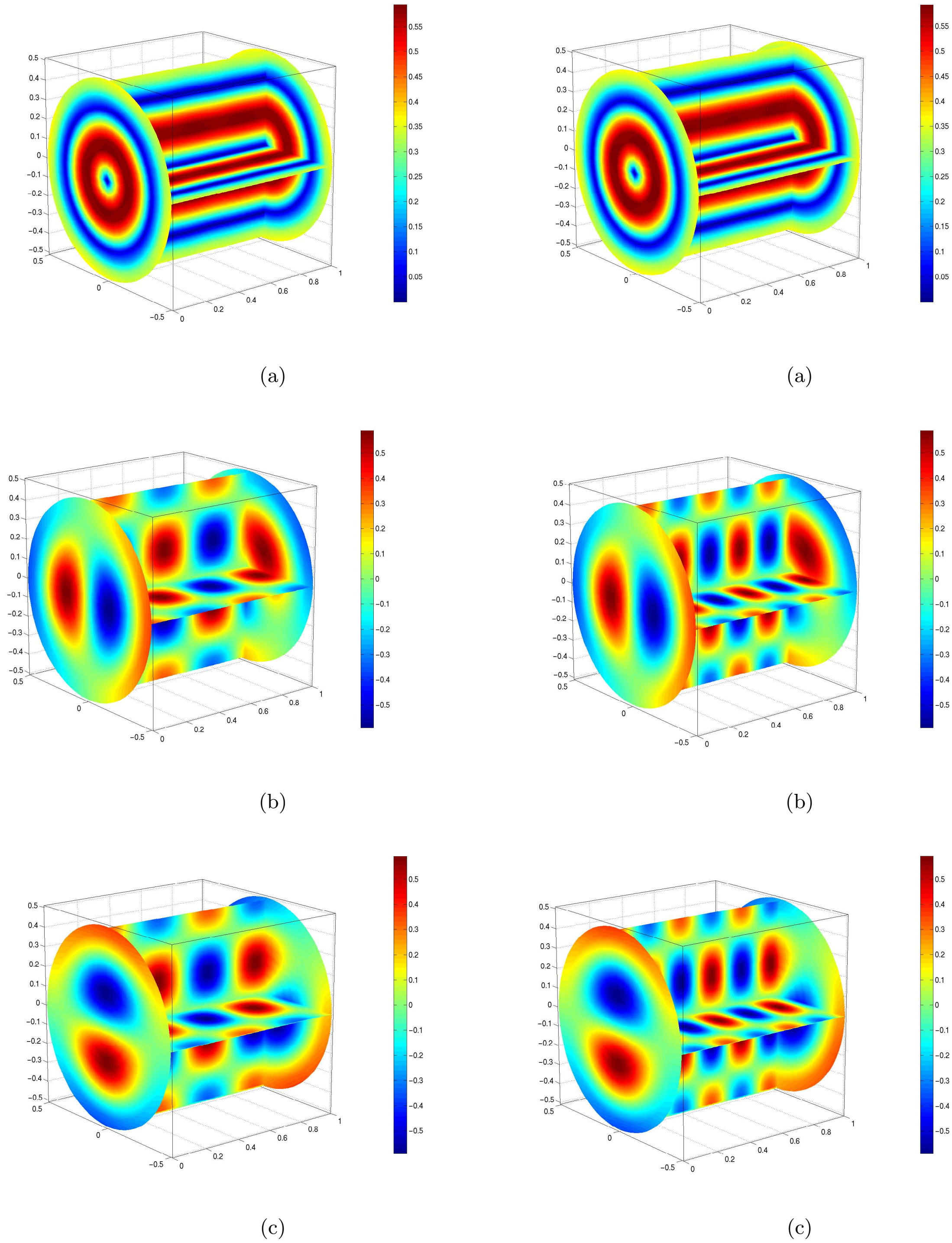
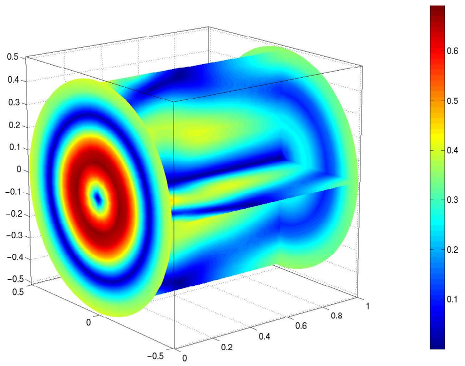
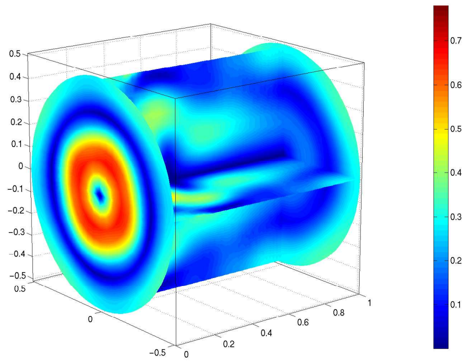


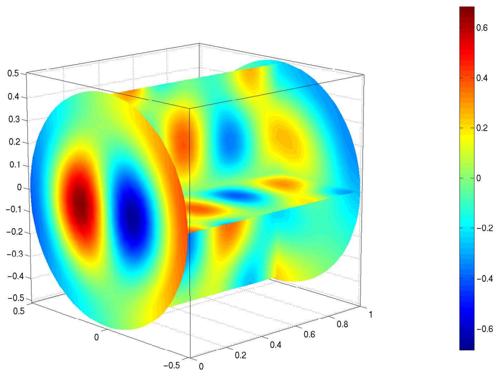
Fig. 4.2. MIPUM 0 computed results for the propagation at 800 Hz of the second mode of the first azimuthal order in a hard walled duct: Absolute (a), real (b) and imaginary (c) parts of the acoustic pressure. Left column: $\mathbf{v}_0 = 0m/s\ \mathbf{1}_z$. Right column: $\mathbf{v}_0 = -100m/s\ \mathbf{1}_z$



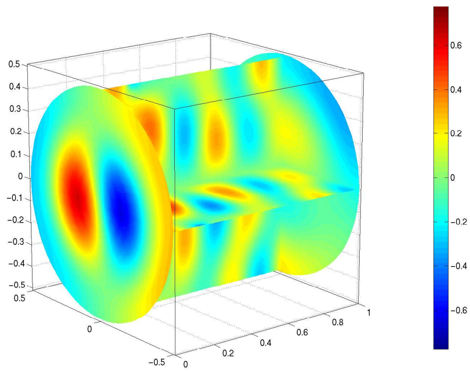
(a)



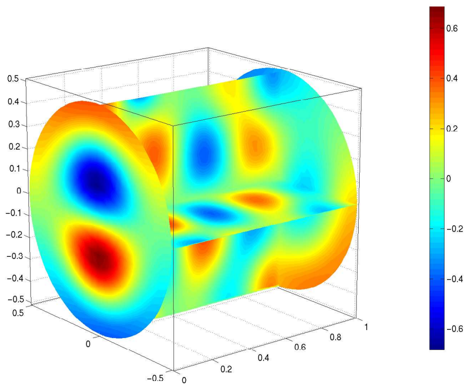
(a)



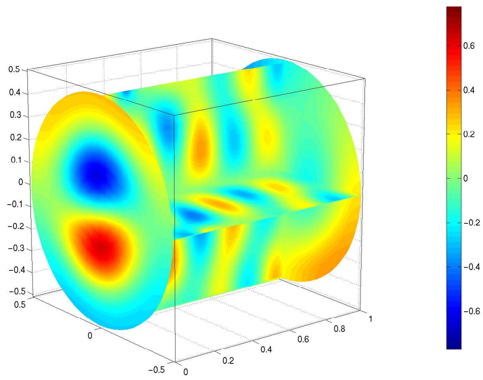
(b)



(b)



(c)



(c)

Fig. 4.3. MIPUM 0 computed results for the propagation at 800 Hz of the second mode of the first azimuthal order in a lined duct: Absolute (a), real (b) and imaginary (c) parts of the acoustic pressure. Left column: $\mathbf{v}_0 = 0m/s\ \mathbf{1}_z$. Right column: $\mathbf{v}_0 = -100m/s\ \mathbf{1}_z$

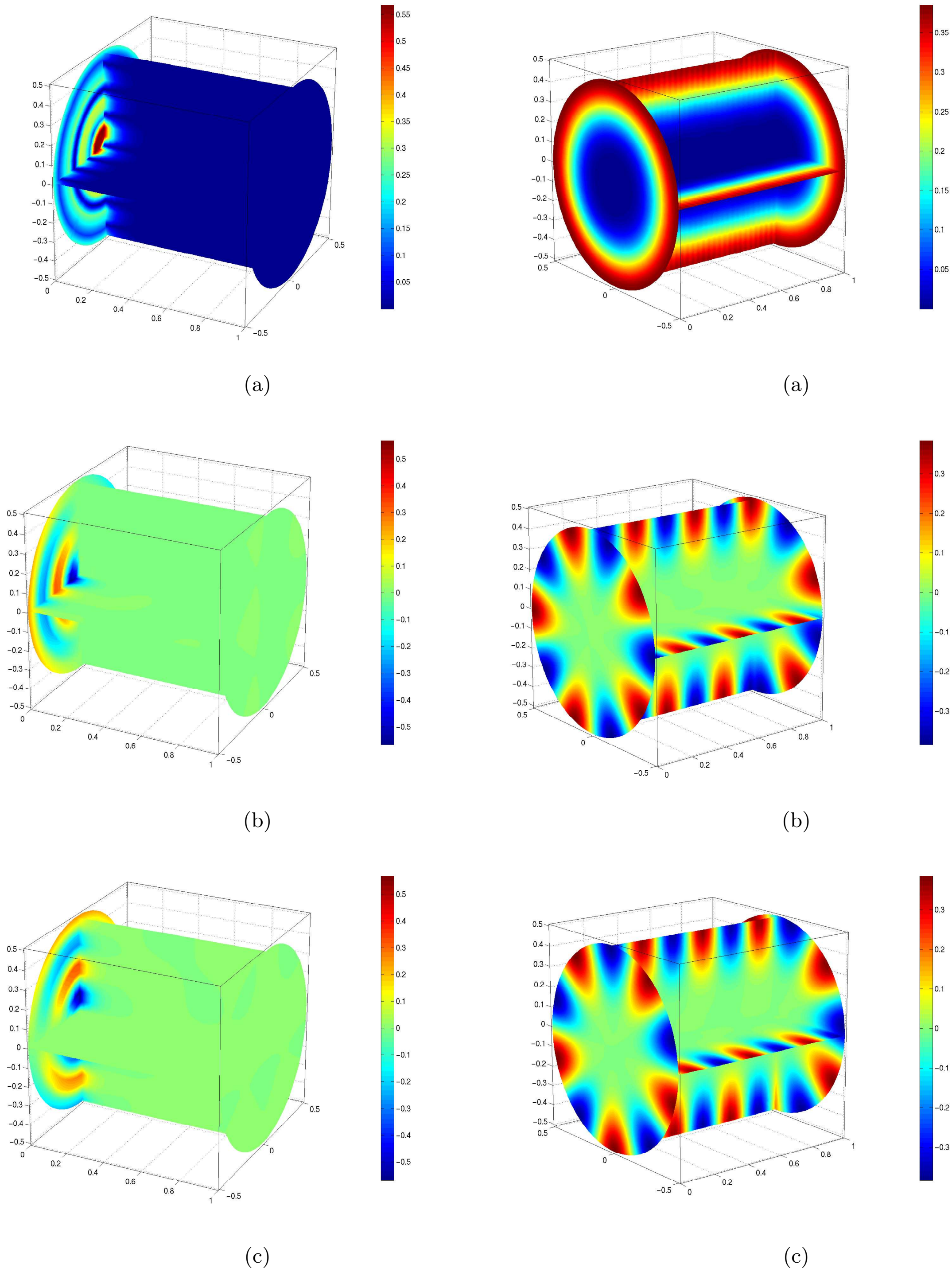


Fig. 4.4. MIPUM 0 computed results for the no-flow propagation in a hard walled duct: Absolute (a), real (b) and imaginary (c) parts of the acoustic pressure. Left column: fourth mode of the first azimuthal order at 800 Hz. Right column: first mode of the fifth azimuthal order at 1200 Hz.

4.2 Multipole radiation

This section illustrates the radiation of a multipole in an infinite three-dimensional domain. The analytical expression of the multipole is given in section 3.3. We simulate the multipole by prescribing normal velocity on a sphere S_N of radius $r_1 = 1m$ (Neumann boundary condition). As the application is of infinite extent, we subdivide the domain in two regions with a spherical interface Γ of radius $r_2 = 2m$. The inner region is located between the vibrating sphere and the interface. This inner region is meshed with mapped tetrahedral elements. The outer domain is the volume outside the interface and is meshed with infinite elements. For practical reasons (we use symmetry properties to reduce the number of elements), the domain is composed of a quarter of space and the surfaces are quarter of sphere. Figure 4.5 illustrates the geometry and a mesh generated with 10 elements along the radial direction and $6\,(4 \times 4)$ elements within the vibrating quarter of sphere. The bases (face lying on the interface Γ) of infinite elements are represented in green.

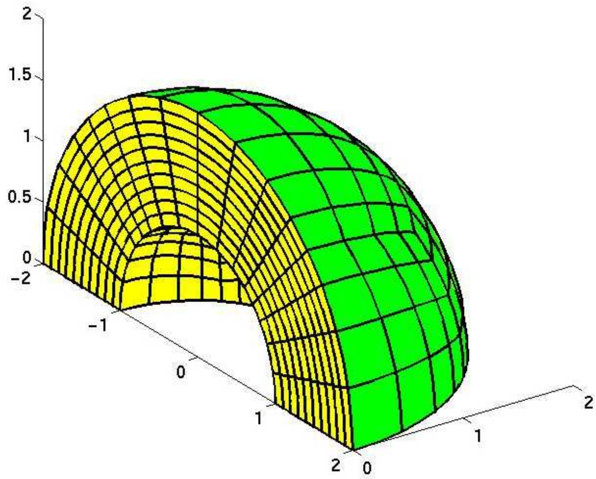
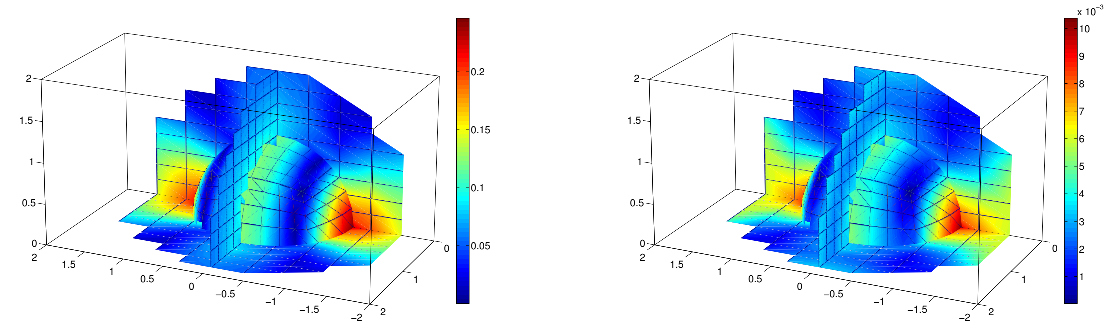


Fig. 4.5. Mesh of the quarter of sphere: 10 elements along the radial direction and $6\,(4 \times 4)$ within the vibrating quarter of sphere.

Figure 4.6 illustrates the radiation at 300 Hz of a multipole of order $N = 2$ (azimuthal order: $m = 0$). The mesh generated to compute the solution is illustrated in figure 4.5. MIPUM 0-N+1-1 (constant enrichment in the inner region, $(N+1)^{th}$ infinite radial order and linear functions for infinite circumferential functions) computational results are compared to analytical ones (we show MIPUM 0-3-1 computed pressure and the discrete L^2 norm in the inner region).

These results illustrate multipole radiation and verify the three-dimensional infinite formulation.



(a) (b)

Fig. 4.6. MIPUM 0-3-1 computed results for the propagation at 300 Hz of a multipole of order $N = 3$ and azimuthal order $m = 0$; $r_1 = 1m$ and $r_2 = 2m$: (a) Absolute part of the acoustic pressure, (b) discrete L^2 norm .

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