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A new mapped infinite partition of unity method for convected acoustical radiation in infinite domains.

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List of Symbols

Greek symbols

β	: $\sqrt{1-M_0^2}$	
γ	: Poisson ratio of specific heat capacities : c_p/c_v	
$\mathit{\Gamma}$	: interface separating the inner and the outer domains	
ε	: Error	
μ	: phase function	$[m]$
ρ	: mass density	$[kgm^{-3}]$
ρ_0	: steady mean density	$[kgm^{-3}]$
ρ_a	: acoustic density	$[kgm^{-3}]$
$\boldsymbol{\sigma}$	: stress tensor	$[Nm^{-2}]$
ϕ	: velocity potential	$[m^2s^{-1}]$
ϕ_0	: mean velocity potential	$[m^2s^{-1}]$
ϕ_a	: acoustic velocity potential	$[m^2s^{-1}]$
$\tilde{\phi}_a$	: amplitude of the harmonic acoustic velocity potential	$[m^2s^{-1}]$
$\tilde{\phi}^h$	: numerical approximation of $\tilde{\phi}_a$	$[m^2s^{-1}]$
$\tilde{\phi}_h^I$	: numerical approximation in the outer region $\mathit{\Omega}_o$	$[m^2s^{-1}]$
$\mathit{\Phi}_\alpha$	: shape function for the α^{th} degree of freedom	
$\mathit{\Phi}_\alpha^I$	: infinite shape function for the α^{th} degree of freedom	
ω	: angular frequency	$[s^{-1}]$
$\mathit{\Omega}$	: domain	
$\mathit{\Omega}_i$	: inner region	
$\mathit{\Omega}_o$	: outer region	

Arabic symbols

$\tilde{a}_n$	: normal acceleration of a vibrating wall	$[ms^{-2}]$
A_n	: normal acoustic admittance	$[m^2skg^{-1}]$
$A^{\pm}_{mn}$	: incident and reflected modal amplitude	$[m^2s^{-1}]$
c	: speed of sound	$[ms^{-1}]$
c_0	: steady mean part of the speed of sound	$[ms^{-1}]$
c_∞	: speed of sound at large distance from the source	$[ms^{-1}]$
c_p	: specific heat capacity at constant pressure	$[JK^{-1}]$
c_v	: specific heat capacity at constant volume	$[JK^{-1}]$
$dofs$	: number of unknowns of the approximation	
E	: energy flow out of a surface	$[J]$
$E^{\pm}_{mn}$	: incident and reflected modal patern	
f	: excitation frequency	$[s^{-1}]$
G	: geometric factor	
h	: mesh size	$[m]$
H	: Hilbert space	
i	: imaginary unit $= \sqrt{-1}$	
$\mathbf{I}$	: Sound intensity	$[Wm^{-2}]$
J'	: stagnation entropy	$[Jkg^{-1}]$
k	: wavenumber	$[m^{-1}]$
$k^{\pm}_{r,mn}$	: incident and reflected radial wavenumber	$[m^{-1}]$
k_B	: Boltzmann constant	$[JK^{-1}]$
$K^{\pm}_{z,mn}$	: incident and reflected axial wavenumber	$[m^{-1}]$
L^d_j	: Legendre polynomial of order d for node j	
L_s	: curve enclosing the boundary S_s	
L_v	: curve enclosing the boundary S_v	
m	: angular mode number	
$\mathbf{m}'$	: mass flux	$[kgm^-2s^{-1}]$
m_0	: radial order of the infinite element	
m_w	: mass of a molecule	$[kg]$
M_0	: mach number	
M_i	: Mapping function for node/point i	
$\mathbf{n}$	: outer normal to the domain	
n	: radial mode number	
n^I_d	: number of infinite degree of freedom	
$n(j)$	: size of the local approximation space at node j	
nni	: number of infinite nodes	
$nodes$	: number of nodes	
N_i	: Partition of Unity function of node i	
N_m	: number of angular modes	
N_n	: number of radial modes	
N_M	: number of reflected modes (unknown)	

p	: fluid pressure	$[Pa]$
p_0	: steady mean fluid pressure	$[Pa]$
p_a	: acoustic pressure	$[Pa]$
$\tilde{p}_a$	: amplitude of the harmonic acoustic pressure	$[Pa]$
$\tilde{p}_{an}$	: analytic amplitude of the harmonic acoustic pressure	$[Pa]$
$\mathbf{q}$	: heat flux	$[Wm^{-2}]$
Q_w	: heat production	$[J]$
r_o	: distance to the source point	$[m]$
R	: specific gas constant	$[JK^{-1}mol^{-1}]$
R_j	: radial function for infinite node j	
R_j^d	: radial function of order d for node j	
s	: entropy	$[Jkg^{-1}K^{-1}]$
S	: boundary	
S_i	: mapping functions for the interface $\mathit{\Gamma}$	
S_M	: Modal boundary	
S_s	: soft wall	
S_v	: vibrating wall	
t	: time	$[s]$
T	: Temperature	$[K]$
T_j	: circumferential function for infinite node j	
$\tilde{u}_n$	: normal displacement of a vibrating wall	$[m]$
$\mathbf{v}$	: fluid velocity	$[ms^{-1}]$
$\mathbf{v}_0$	: steady mean fluid velocity	$[ms^{-1}]$
$\mathbf{v}_\infty$	: fluid velocity at large distance from the source	$[ms^{-1}]$
$\mathbf{v}_a$	: acoustic velocity	$[ms^{-1}]$
$\tilde{\mathbf{v}}_a$	: amplitude of the harmonic acoustic velocity	$[ms^{-1}]$
$\mathscr{V}$	: the Sobolev space $W^{1,2} = H^1 = \{f : f, \nabla f \in L^2\}$	
V_{jl}	: l^{th} local approximation function of node j	
$\tilde{w}_n$	: normal velocity of a vibrating wall	$[ms^{-1}]$
W_j	: weight function of node j	
W_j^I	: infinite weight function of the infinite node j	
$W_{M,nm}$	: modal weight function of the angular and radial mode (m, n)	

Operators

∇	: gradient operator
$\nabla\cdot$	: divergence operator
$\nabla\times$	: curl operator
$\mathit{\Delta}$	: Laplacian operator
$\frac{D}{Dt}$	: Total time derivative
$:$	: the double dot product of two tensors
$\langle\ \rangle$	: time average
$\Re$	: Real part

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Introduction

Environmental considerations are important in the design of many engineering systems and components. In particular, the environmental impact of noise is important over a very broad range of engineering applications and is increasingly perceived and regulated as an issue of occupational safety or health, or more simply as a public nuisance. The acoustic quality is then considered as a criterion in the product design process. Numerical prediction techniques allow to simulate vibro-acoustic responses. The use of such techniques reduces the development time and cost. These numerical methods analyse the influence of different parameters of the product without the need of generating prototypes. This dissertation focuses on acoustic propagation, assuming that fluid effect on the body can be neglected. We also assume that acoustic quantities are small harmonic variations which leads to analyses in the frequency domain.

Nowhere this is more apparent than in the air transportation sector where the accurate prediction and control of noise radiated from commercial aircraft during takeoff or approach is central to the design of new aircrafts. The methods presented in this dissertation, although quite broad in their application, have been developed specifically with this objective in mind, i.e. the prediction of noise radiating from turbofan engines. A characteristic of such predictions is that at frequencies of interest, the wavelength of the acoustic solution is generally small compared to a typical geometric lengthscale, such as the fan diameter D, though not small enough to garantee the use of asymptotic methods such as the ray approximation. Typically, values of the dimensionless wavenumber kD for a modern engine, where $k = \frac{2\pi f}{c_0}$, f is the frequency, and c_0 is the sound speed, lie in the range $0 \leq kD < 200$, with the first major engine harmonic, the blade passing frequency located in the vicinity of $kD \sim 40$. Hence many wavelengths of the solution must be captured within the computational domain, large systems of linear equations must be solved. Currently this places severe practical limitations on what can be achieved, particularly for a fully three-dimensional configuration where the number of degrees of freedom which must be used for an acoustic analysis of an engine nacelle is of the order of 10^6-10^7. Such calculations should ideally be able to be performed for multiple frequencies and for multiple acoustic impedances within a design optimization procedure. There are of course many other acoustic applications where the wavelength to geometry scale is somewhat similar.

The calculation of Head Related Transfer Functions [86] in the audible range, for example, poses a slightly identical problem in terms of the required resolution. The Head Related Transfer Functions describe how a given sound wave input defined by its frequency and location is perceived by the ears. It has to be noticed that the reflection of noise on the head, pinnae and torso have a natural prefiltering effect, facilitating source location. These analyses allow to develop virtual acoustic imaging systems. The work reported here is directed towards the development of a more efficient computational approach for such problems.

A common approach for simulating turbofan noise radiation is to use the Finite Element Method, typically with linear or quadratic elements. This is effective for axisymmetric models at moderate frequencies and for three-dimensional models at low frequencies. The Finite Element Method is a deterministic approach in which meshes have to be generated such that the waves can be represented accurately by polynomial shape functions. This means that meshes are frequency dependent. A general rule of the thumb which is widely used advocates the use of 6 to 10 elements to approximate a wavelength in the solution. This rule has the virtue of simplicity, but has been shown to be invalid for short wavelength problems. Several methods have been proposed to reduce the numerical error for shortwave problems in acoustics, see Thompson [6] for a recent review.

In this dissertation, the Partition of Unity Method is explored. This method was proposed by Melenk and Babuška [8]. A variant of the method was applied to wave problems by Bettess and Laghrouche who enriched the numerical solution by using functions of the form Ae^{iks} where the phase s is obtained by an iterative approach [23]. The same authors also developed a Partition of Unity method by using discrete series of planar waves at each node [9] and a similar approach has been applied to the convected problem by Gamallo and Astley [28, 29]. **In the current thesis, the Partition of Unity method is implemented within a coupled Finite and Infinite element model**. In the Finite Element region, polynomial terms are used to enrich the solution at each node of the mesh. Within the Infinite Element region, outwardly propagating solutions of the homogeneous wave equation are used in addition to transverse polynomial terms. The current approach will be shown to give a significant benefit in terms of accuracy over conventional elements, while avoiding some of the problems - such as the use of complex integration schemes, high condition number and general lack of robustness - associated with the wave-based approach of Astley and Gamallo. Similar studies of the Partition of Unity Method in the field of plate vibration [24, 25] have demonstrated its effectiveness.

Since the focus of the current method is on flow acoustics in turbofan engines, the influence of non-uniform flow on acoustic propagation must be taken into account. This is achieved by solving the convected wave equation. This equation is obtained by assuming a potential mean flow. The mathematical statement of the problem is essentially the same as used by Astley and Gamallo [28, 29].

Since the current model must also be able to simulate exterior domains, a far field numerical treatment is required which is able to represent an anechoic termination in the presence of mean flow. There exists four families of techniques to simulate acoustic propagation in an unbounded domain (a good overview of recent techniques is given in [57] and its references): the Boundary Element Method (BEM), the Finite Element Method coupled to Infinite Elements or bounded by Non-Reflecting Boundary Conditions (NRBC) prescribed on a truncated domain. There exists two families of Non-Reflecting Boundary Conditions: the Perfectly Matched Layers (PML) and local/global Absorbing Boundary Conditions (ABC). The use of conventional Infinite Elements is well established as a termination for traditional Finite Element models with and without mean flow [30, 35]. **In the current instance Partition of Unity Infinite elements are used**.

Objectives - Contributions of the thesis

To summarise, the current dissertation describes a **new formulation based on the Partition of Unity Method applied to Finite and Infinite Elements.** The objective is to develop a computationally efficient prediction technique to simulate acoustic propagation in three-dimensional domains. Three-dimensional and axisymmetric formulations are presented. The axisymmetric formulation considers special three-dimensional applications. This restricts the geometry of the application to be obtained by revolution and limits steady mean flow variable not to vary in the azimuthal direction. The computational domain is reduced then to a two-dimensional region but the computed acoustic solution is valid for the three-dimensional domain. Axisymmetric simulations then require less computational resources. We first assess the complete performances of this approach with the axisymmetric formulation and then confirm these results with some full three-dimensional simulations (required for industrial applications). The verification of the formulation, the influence of parameters on the accuracy and the analysis of the performances of the method are performed by comparing computed solutions to analytical solutions and to results obtained by using the conventional Finite and Infinite Elements available in the commercial software ACTRAN^{TM}.

Outline

Chapter 2 describes the three-dimensional formulation of the acoustic prediction tool developed. We derive the convected wave equation which is a scalar equation in terms of acoustic potential. This equation allows to compute acoustic propagation within a steady mean irrotational flow (assumed to be an input data). We detail the variational formulation and the boundary conditions required to simulate the applications presented in the dissertation. A literature review is then inserted to compare existing discretization methods for bounded applications and justify the choice of the Partition of Unity Method which is then described. We also list existing methods dealing with outer domains. The Mapped Infinite Partition of Unity elements are described. These elements are based on the Mapped Wave Envelope Infinite elements developed by Astley [30] but modified to be coupled with an inner Partition of Unity region. This defines the Mapped Infinite Partition of Unity Method for three-dimensional applications. The same approach is then described for axisymmetric applications.

The verification of the method is presented in chapters 3 and 4 for axisymmetric and three-dimensional applications, respectively. These chapters compare the distribution of pressure obtained by the proposed method with analytical solutions or computed results obtained with the Finite Element Method (ACTRAN^{TM}). The applications illustrated have been chosen to verify all the aspects of the method. Duct simulations validate propagations in inner regions with or without uniform mean flow but also modal and admittance boundary conditions. The non-uniform duct illustrate the propagation in a non-uniform flow with modal boundary conditions. Multipole and rigid piston radiations illustrate the concept of infinite domains in the no-flow case. These applications use the vibrating wall boundary condition. The infinitesimal cylinder radiation takes into account infinite elements and uniform mean flow. An application combining non-uniform flow, admittance and modal boundary conditions in an unbounded domain is illustrated in chapter 8 for the radiation of a turbofan.

The performances of the Mapped Partition of Unity Method are analysed in chapters 5 and 6 for axisymmetric and three-dimensional applications, respectively. The applications considered are the propagation in an infinite duct, multipole radiation and the radiation of a rigid piston. The performances are anlysed with respect to two kind of figures. The first one plots the L^2 relative error of an application (for a fixed excitation frequency) with respect to the number of degrees of freedom. The results compare the rate of convergence of the compared methods. This reveals a notion of accuracy. The second figure shown is about efficiency. We know that the number of degrees of freedom is not a good indicator of performance. It does not give any indication about the global computer resources required (CPU time, bandwidth, number of nonzero terms, condition number, numerical integration). Furthermore, convergence analyses only focus on one frequency. For these reasons, we compute performance curves which plots the CPU time required to compute the solution within an accuracy under 5% with respect to the non-dimensionnal wavenumber (ka). This illustrates the effective computational effort over a frequency range. Note that the effect of parameters such as the radial and circumferential orders of infinite elements and the topology of elements are analysed for the axisymmetric formulation. We also illustrate the influence of the condition number on the Mapped Infinite Partition of Unity computations.

Chapter 7 describes the effect of flow on the accuracy of the method. This chapter has been influenced by a previous communication of Gabard et al. [72, 73] who observed for the Finite Element Method an interaction between the flow, the frequency and the mesh leading to inaccurate results. We first introduce the concept of aliasing error and then illustrate that aliasing also appears for the Mapped Infinite Partition of Unity Method.

The last chapter consists of numerical results analysing turbofan radiation. We consider an axisymmetric nacelle and illustrate the radiation for a typical excitation. We plot the distribution of the pressure and directivity patterns to illustrate the influence of a mean flow and the presence of a liner. Results from the Mapped Partition of Unity Method are compared to Finite Element results. The aim is to show that the method is also applicable to industrial-like applications.

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