Tanguy Mertens

A new mapped infinite partition of unity method for convected acoustical radiation in infinite domains.

January 20, 2009

Université Libre de Bruxelles

Remerciements

Si tu donnes un poisson à un homme, il ne mangera qu'un jour. S'il apprend à pêcher, il mangera toute sa vie.

Proverbe de Confucius, repris plus tard par Dominique Pire dans le cadre de l'action Iles de Paix.

Je remercie Philippe Bouillard de m'avoir donné ma canne à pêche et conduit à l'étang. Je remercie Laurent Hazard et Guy Paulus pour tous leurs conseils avisés. Et tous les autres, famille, amis, qui ont gardé confiance en moi et m'ont encouragé à la persévérance.

C'est grâce à vous tous que je peux vous présenter avec fierté mon premier poisson.

Ceci dit, je voudrais remercier de manière plus académique tous les acteurs du projet.

Je remercie Philippe Bouillard qui est à la source du projet et qui a contribué à mon épanouissement scientifique durant cette aventure au sein du service BATir. Je remercie également pour leur encadrement: Guy Warzée pour sa gentillesse et sa patience ainsi que Jean-Louis Migeot pour son soutien industriel et ses remarques pertinentes.

Ce passage au service BATir a été une expérience riche sur le plan scientifique mais aussi humain. Je remercie mes collègues de l'ULB pour les relations que nous avons construites ensemble, pour les activités dans le cadre du travail ou hors des murs de l'université. Je pense évidemment à Laurent Hazard et à Guy Paulus, pour leur amitié mais aussi leur disponibilité et leurs nombreux conseils. Merci aussi à tous les membres du service des milieux continus qui ont ajouté une dimension humaine. La nature des contrats de recherche et d'assistanat est tel qu'ils furent nombreux a être passés par les murs de l'université. Je ne pourrai tous les citer mais je tiens quand même à remercier personellement Katy, Erik, Geneviève, Yannick, Louise, Dominique, Thierry, Sandrine, Bertha, David, Benoît, Peter et Kfir.

S'il est vrai qu'un travail de thèse tend à rendre le chercheur autonome, les collaborations et interactions restent importantes. Je remercie à cet égard toute l'équipe de l'Institute of Sound and Vibration Research de l'université de Southampton. Je remercie Jeremy Astley, chef du service de m'y avoir accueilli pendant un stage de six mois. Je le

remercie aussi pour l'environnement de travail de qualité qu'il m'a offert. Il est évident que la présence de Pablo Gamallo à l'I.S.V.R. a énormément contribué à l'aboutissement de ce travail de recherche. Je le remercie pour tout le temps qu'il m'a consacré, à Southampton et même par la suite lors de contacts e-mail ou lors de rencontres en conférences. Je le remercie pour l'intérêt qu'il porte à la présente étude, pour ses encouragements et pour tous ses enseignements. Mais ce séjour n'a pas été que professionellement enrichissant, je repense à l'accueil qui m'a été réservé ainsi que de nombreux excellents souvenirs qui font de ce séjour une partie inoubliable de ma vie. Je tiens donc à remercier chaleureusement pour leur amitié précieuse Vincent et Theresa, Luigi, Claire, Naoki, Rie, Sue, Lisa, Eugene, Isa, Daniel et Stéphanie, Alessandro et Emmet.

Ces quelques lignes indiquent à quel point les relations humaines ont une influence sur ma vie. Cela ne sera pas une surprise alors si je remercie les personnes pour qui je *travaille dans les avions* ou ceux qui me félicitent de mon travail, *les semaines durant lesquelles, ils n'ont pas trop entendu les avions*. Je pense bien entendu à ma famille, principalement ma maman, pour avoir fait de moi l'homme que je suis aujourd'hui et mes amis, plus particulièrement mes collocataires: Iyad, Mathieu, Quentin, Guerrick, Caroline et Florence. Merci à vous de partager mes valeurs et de colorier mon existence.

Enfin, je terminerai par remercier les deux petits rayons de soleil qui ont ensoleillé la fin de ce long parcours.

List of Symbols

Greek symbols

β	: $\sqrt{1-M_0^2}$	
γ	: Poisson ratio of specific heat capacities : c_p/c_v	
Γ	: interface separating the inner and the outer domains	
ε	: Error	
μ	: phase function	$[m]$
ρ	: mass density	$[kgm^{-3}]$
ρ_0	: steady mean density	$[kgm^{-3}]$
ρ_a	: acoustic density	$[kgm^{-3}]$
$\boldsymbol{\sigma}$	: stress tensor	$[Nm^{-2}]$
ϕ	: velocity potential	$[m^2s^{-1}]$
ϕ_0	: mean velocity potential	$[m^2s^{-1}]$
ϕ_a	: acoustic velocity potential	$[m^2s^{-1}]$
$\tilde{\phi}_a$	: amplitude of the harmonic acoustic velocity potential	$[m^2s^{-1}]$
$\tilde{\phi}^h$	: numerical approximation of $\tilde{\phi}_a$	$[m^2s^{-1}]$
$\tilde{\phi}_h^I$	: numerical approximation in the outer region Ω_o	$[m^2s^{-1}]$
Φ_α	: shape function for the α^{th} degree of freedom	
Φ_α^I	: infinite shape function for the α^{th} degree of freedom	
ω	: angular frequency	$[s^{-1}]$
Ω	: domain	
Ω_i	: inner region	
Ω_o	: outer region	

Arabic symbols

$\tilde{a}_n$	: normal acceleration of a vibrating wall	$[ms^{-2}]$
A_n	: normal acoustic admittance	$[m^2skg^{-1}]$
$A^{\pm}_{mn}$	: incident and reflected modal amplitude	$[m^2s^{-1}]$
c	: speed of sound	$[ms^{-1}]$
c_0	: steady mean part of the speed of sound	$[ms^{-1}]$
c_∞	: speed of sound at large distance from the source	$[ms^{-1}]$
c_p	: specific heat capacity at constant pressure	$[JK^{-1}]$
c_v	: specific heat capacity at constant volume	$[JK^{-1}]$
$dofs$	: number of unknowns of the approximation	
E	: energy flow out of a surface	$[J]$
$E^{\pm}_{mn}$	: incident and reflected modal patern	
f	: excitation frequency	$[s^{-1}]$
G	: geometric factor	
h	: mesh size	$[m]$
H	: Hilbert space	
i	: imaginary unit $= \sqrt{-1}$	
$\mathbf{I}$	: Sound intensity	$[Wm^{-2}]$
J'	: stagnation entropy	$[Jkg^{-1}]$
k	: wavenumber	$[m^{-1}]$
$k^{\pm}_{r,mn}$	: incident and reflected radial wavenumber	$[m^{-1}]$
k_B	: Boltzmann constant	$[JK^{-1}]$
$K^{\pm}_{z,mn}$	: incident and reflected axial wavenumber	$[m^{-1}]$
L^d_j	: Legendre polynomial of order d for node j	
L_s	: curve enclosing the boundary S_s	
L_v	: curve enclosing the boundary S_v	
m	: angular mode number	
$\mathbf{m}'$	: mass flux	$[kgm^-2s^{-1}]$
m_0	: radial order of the infinite element	
m_w	: mass of a molecule	$[kg]$
M_0	: mach number	
M_i	: Mapping function for node/point i	
$\mathbf{n}$	: outer normal to the domain	
n	: radial mode number	
n^I_d	: number of infinite degree of freedom	
$n\left(j\right)$	: size of the local approximation space at node j	
nni	: number of infinite nodes	
$nodes$	: number of nodes	
N_i	: Partition of Unity function of node i	
N_m	: number of angular modes	
N_n	: number of radial modes	
N_M	: number of reflected modes (unknown)	

p	: fluid pressure	$[Pa]$
p_0	: steady mean fluid pressure	$[Pa]$
p_a	: acoustic pressure	$[Pa]$
$\tilde{p}_a$	: amplitude of the harmonic acoustic pressure	$[Pa]$
$\tilde{p}_{an}$	: analytic amplitude of the harmonic acoustic pressure	$[Pa]$
$\mathbf{q}$	: heat flux	$[Wm^{-2}]$
Q_w	: heat production	$[J]$
r_o	: distance to the source point	$[m]$
R	: specific gas constant	$[JK^{-1}mol^{-1}]$
R_j	: radial function for infinite node j	
R_j^d	: radial function of order d for node j	
s	: entropy	$[Jkg^{-1}K^{-1}]$
S	: boundary	
S_i	: mapping functions for the interface $\mathit{\Gamma}$	
S_M	: Modal boundary	
S_s	: soft wall	
S_v	: vibrating wall	
t	: time	$[s]$
T	: Temperature	$[K]$
T_j	: circumferential function for infinite node j	
$\tilde{u}_n$	: normal displacement of a vibrating wall	$[m]$
$\mathbf{v}$	: fluid velocity	$[ms^{-1}]$
$\mathbf{v}_0$	: steady mean fluid velocity	$[ms^{-1}]$
$\mathbf{v}_\infty$	: fluid velocity at large distance from the source	$[ms^{-1}]$
$\mathbf{v}_a$	: acoustic velocity	$[ms^{-1}]$
$\tilde{\mathbf{v}}_a$	: amplitude of the harmonic acoustic velocity	$[ms^{-1}]$
$\mathscr{V}$	: the Sobolev space $W^{1,2} = H^1 = \{f : f, \nabla f \in L^2\}$	
V_{jl}	: l^{th} local approximation function of node j	
$\tilde{w}_n$	: normal velocity of a vibrating wall	$[ms^{-1}]$
W_j	: weight function of node j	
W_j^I	: infinite weight function of the infinite node j	
$W_{M,nm}$	: modal weight function of the angular and radial mode (m, n)	

Operators

∇	: gradient operator
$\nabla\cdot$	: divergence operator
$\nabla\times$	: curl operator
Δ	: Laplacian operator
$\frac{D}{Dt}$	: Total time derivative
$:$	: the double dot product of two tensors
$\langle\ \rangle$	: time average
$\Re$	: Real part

Contents

8

Industrial application: Turbofan radiation

The previous chapters verify the Mapped Infinite Partition of Unity formulation and illustrate the performances of the method by analysing several academic applications. This chapter considers the radiation of a turbofan engine.

Aircraft noise is a major concern for approximately 100 square kilometers surrounding most major airports [81]. Stringent registrations tends to limit this environmental noise in spite of traffic growth which is planned to double in less than 15 years (India and China air traffic is expanding at 40% per year). There is then huge pressure on airplane manufacturers to significantly reduce aircraft noise.

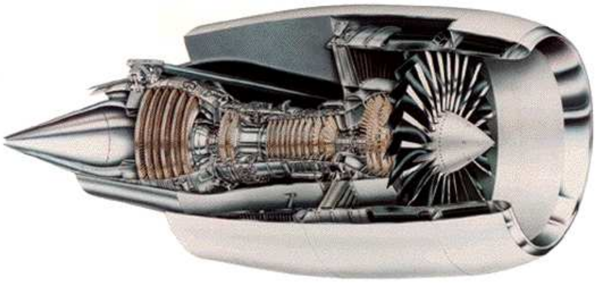
A lot of research has already been conducted to reduce aircraft noise. The european project SILENCE(R) [80], for instance, was launched in 2001 with the aim to reduce the noise by up to 6 decibels as of 2008. It involved 51 companies and was funded with a total budget which exceeding 110 M Euro.

Aircraft noise is composed of three principal sources: airframe, jet and fan. The airframe is aero-acoustic noise corresponding to turbulent flow. This airframe noise is generated by landing gear and high lift devices. Engine noise is composed of jet noise which is emitted at the back of the engine and fan noise which propagates towards the inlet before being radiated in front of the engine (figure 8.1).

We decided to focus on the noise radiated by the fan because high bypass ratio has led to significant reduction in jet noise. The fan noise is composed of tonal and broadband noise. The reduction in fan noise can be obtained by acting on the source (design of low noise compressors) or on the propagation through the inlet. The perceived noise can be reduced by attenuation through acoustic treatment in the nacelle (liner) or by diverting the noise away from the ground with scarf or scoop inlets (figure 8.2). Our goal is to simulate fan propagation through the inlet and compute the radiated pressure and evaluate the effect of acoustic treatment or the influence of the inlet geometry.

The noise radiated by the fan may be separated in several contributions. The tonal noise corresponds to rotor alone or rotor-stator interactions and the broadband noise is due to turbulence-fan interactions. The rotor alone noise has a frequency corresponding to

(a)



(b)

Fig. 8.1. General Electric GE90-115B high bypass turbofan [83].

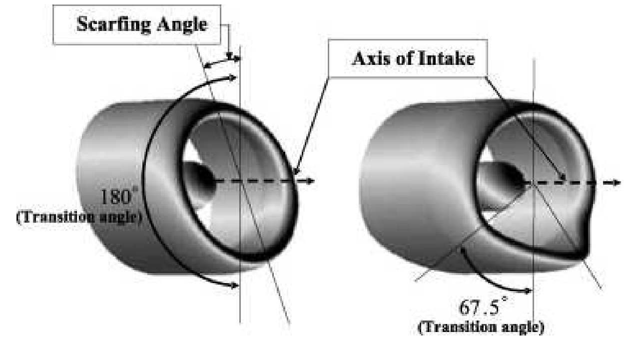


Fig. 8.2. Illustration of scarf (left) and scoop (right) inlet [82].

the blade passing frequency (BPF) and is represented by the first radial mode of the B^{th} azimuthal mode $(m, n) = (B, 1)$, where B is the number of blades. At subsonic fan speed, the rotor alone noise is cut-off. This means that it is an evanescent mode and it does not radiate noise. However, during takeoff, the fan speed at the tip of the blades is supersonic and the the rotor alone noise is cut-on, it has then to be taken into account in simulations. In the case of rotor-stator interactions, rotor wakes impinge on stator vanes. The dominant tones correspond to harmonics of the blade passing frequency. According to the theory of Sofrin and Tyler [78], the corresponding azimuthal mode is given by the following relation: $m = iB \pm jV$, where V is the number of stator vanes, i is the time harmonic index and j is an integer number. Note that tonal noise is prescribed with a small number of modes while broadband noise is modelled by a multi-mode source of fixed energy.

A full nacelle analysis requires three-dimensional modelling. Unfortunately such applications are computationally demanding. A first approach consists in simulating nacelle radiation by an axisymmetric model which can take into account admittance and non-uniform flow but only considers axisymmetric geometries (no scarf, no spliced acoustic liners). Note that Duta and Giles [79] described a spectral formulation allowing to decompose the circumferential geometry by a Fourier spectral representation. They reduced a three-dimensional simulation of an asymmetry inlet with a scarfing angle of about 5 $^\circ$ with approximately 3 million of unknowns to a total of 0.24 million of unknowns for the spectral approximation.

The Mapped Infinite Partition of Unity Method has been developed for axisymmetric geometry as well as three-dimensional applications. We illustrate in this chapter nacelle radiation with the axisymmetric formulation. The geometry is realistic in the sense that it approximates the dimensions and curvatures of modern turbofan excepted for non-axisymmetric features (azimuthally segmented liners, scarf or scoop angle). The geometry of the application is illustrated with a mesh of 1260 elements in figure 8.3. We analyse the radiation of the rotor alone tone of an engine of 26 fan blades operating close to its maximum speed corresponding to the $(26, 1)$ mode at blade passing frequency: nondimensional wave number $ka = 30$ ($f = 4000Hz$ and $a = 0.4m$). These characteristics aim to represent real applications and are inspired from an analysis performed by Astley [84].

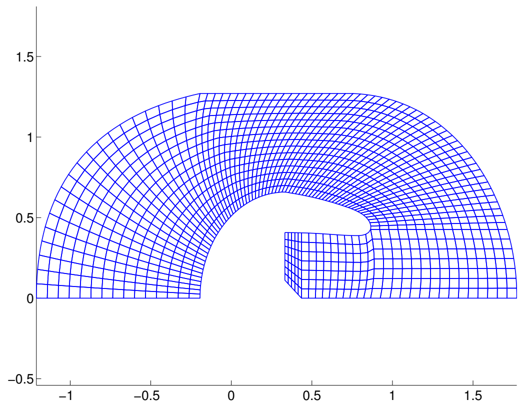


Fig. 8.3. Illustration of the geometry of the nacelle and an associated inner mesh of 1260 Q8 quadratic elements. Note that infinite elements (not represented) are attached to the interface Γ with the outer region.

The current analysis illustrates the radiation of mode $(26, 1)$ at $ka = 30$ for three different cases. We first consider no flow propagation, then take into account non-uniform flow and finally apply acoustic treatment within the convected simulation. The radiation of each case is illustrated with plots of the pressure within the computational domain. We also use directivity paterns to select appropriate discretizations and compare the constant enrichment, second order enrichment of the Mapped Infinite Partition of Unity Method and quadratic Finite Element simulations (ACTRAN^{TM}). The infinite radial order used for the computations is $m_0 = 15$.

8.1 Radiation without flow

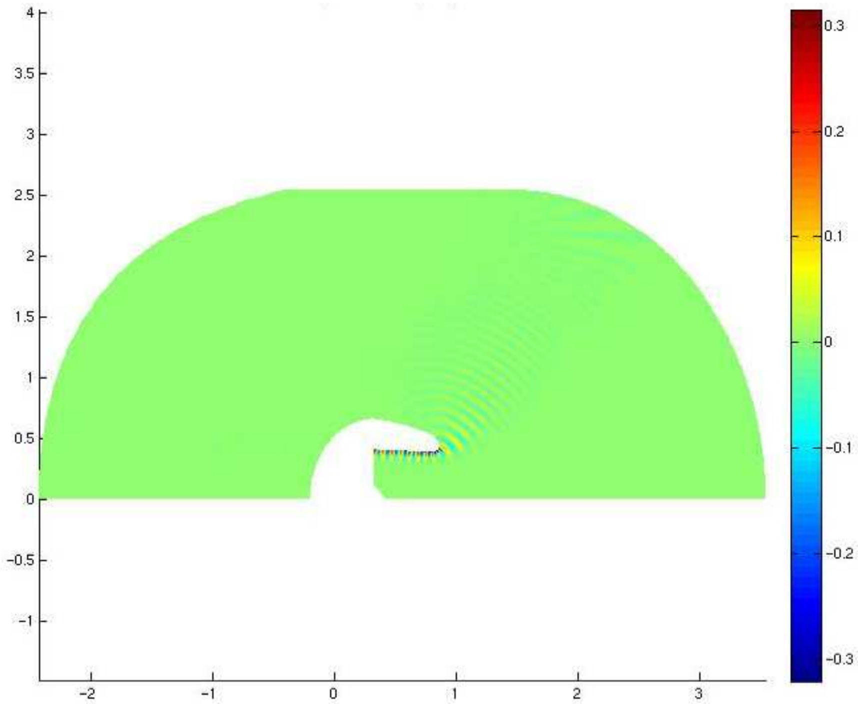
The distribution of the pressure is illustrated in figure 8.4 for the inner region and a partly in the outer region.

We compare directivity plots obtained with the Mapped Infinite Partition of Unity Method. The aim is to find the appropriate discretization for the application and compare the required number of degrees of freedom for the constant enrichment and the second order enrichment.

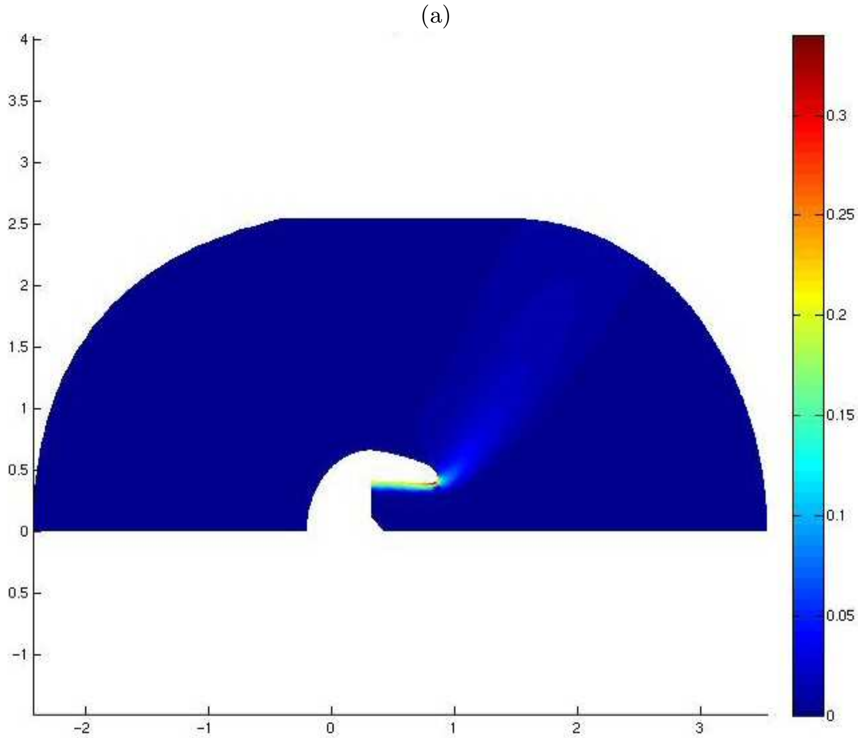
Directivity patterns (figure 8.5) represent the pressure in decibels[1] for a point located on a circle of radius $r = 2m$ and centered at the origin. Note that the maximal pressure in the nacelle is over $142dB$. We observe that the Mapped Infinite Partition of Unity Method with second order enrichment converges already with 9000 degrees of freedom while the constant enrichment requires 36000 degrees of freedom.

We also checked the validity of using an infinite radial order of $m_0 = 15$. Figure 8.6 illustrates the directivity pattern computed for three different radial orders: $m_0 = \{10, 15, 20\}$. The radial order $m_0 = 10$ gives wrong results. The two other radial orders show close results, excepted discrepancies at angles close to $80°$.

[1] $p_{dB} = \left(20log\left(\frac{p}{p_{ref}}\right)\right)$ with $p_{ref} = 0.00002Pa$

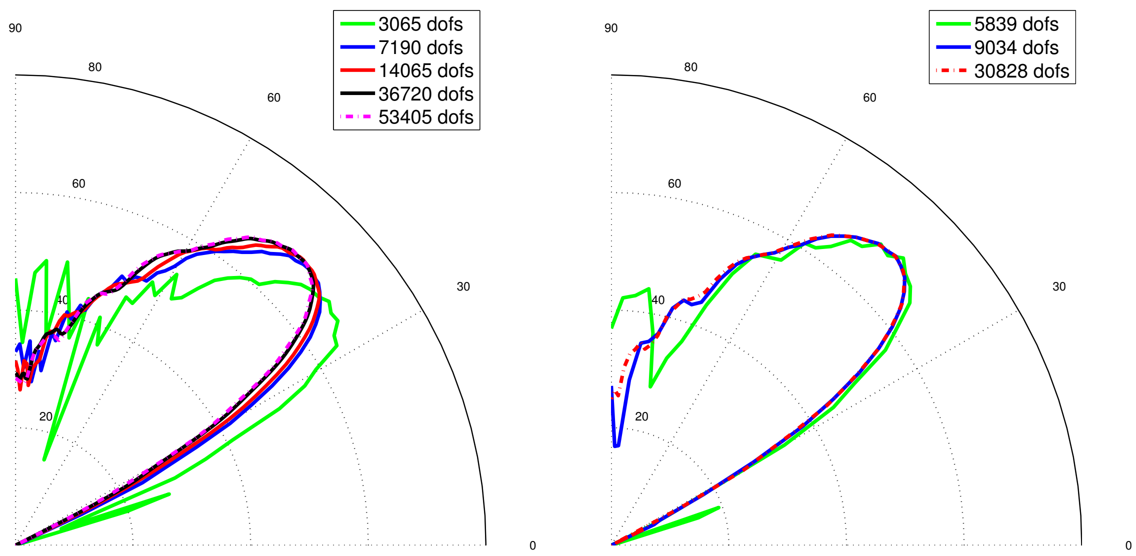


(a)



(b)

Fig. 8.4. MIPUM 0-15-1 (36720 degrees of freedom): Radiation of mode $(26, 1)$ at $ka = 30$ without flow: (a) real part of the pressure (b) absolute value of pressure.



(a) (b)

Fig. 8.5. Directivity pattern for the radiation of mode $(26, 1)$ at $ka = 30$ without flow: (a) MIPUM 0-15-1 (b) MIPUM 2-15-2.

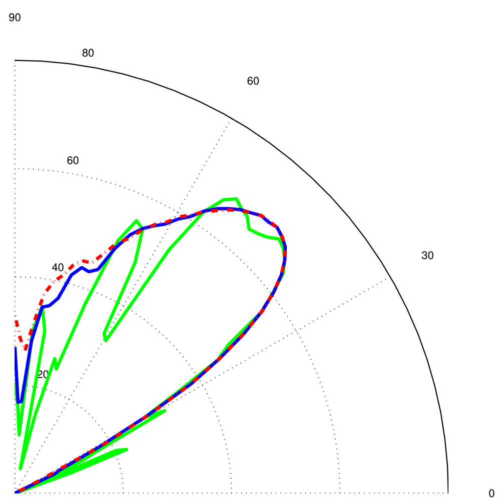


Fig. 8.6. Directivity pattern for the radiation of mode $(26, 1)$ at $ka = 30$ without flow: MIPUM 2-10-2 (-), MIPUM 2-15-2 (-) and MIPUM 2-20-2 (-).

8.2 Convected radiation

The convected radiation is illustrated in figure 8.8. The input steady mean flow is illustrated in figure 8.7 for the inner region. In the outer region, the flow is assumed to be uniform and oriented in the axial direction: $\mathbf{v}_0 = -50\ \mathbf{1}_z m/s$. The mean flow has been computed by the software ACTRAN^{TM}. Note that the mean flow is computed on a mesh of 2080 nodes and then interpolated on the nodes of the acoustic mesh.

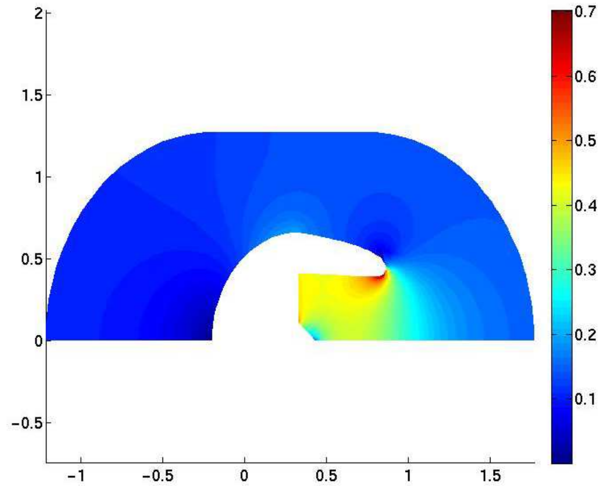
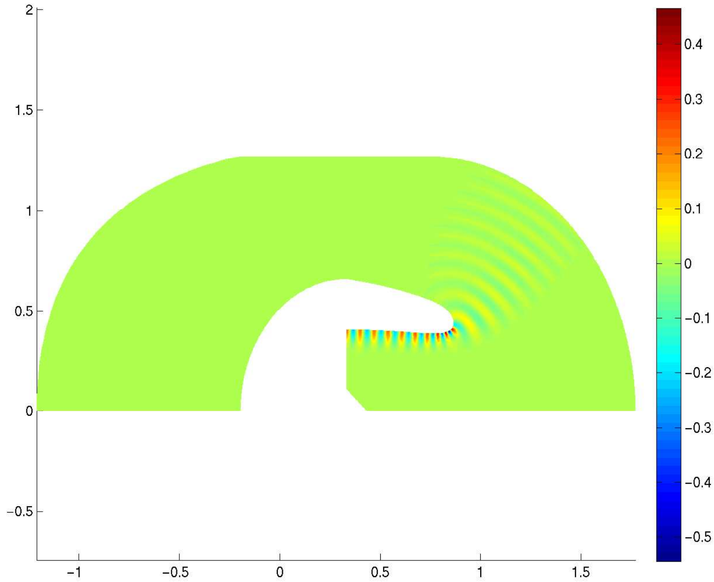
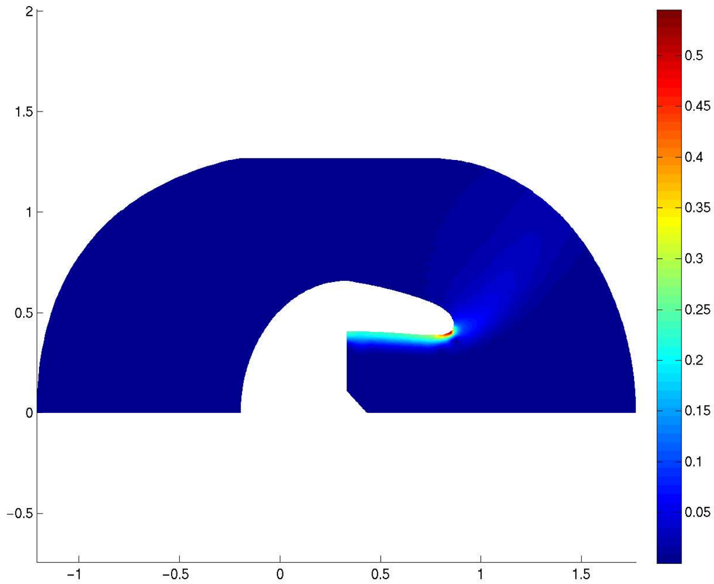


Fig. 8.7. Illustration of the steady mean flow distribution: Mach number

The directivity pattern at a distance of $2m$ for the convected radiation is shown at figure 8.9 for different discretizations. The maximal value of the pressure in the nacelle is about $138dB$. Figure 8.10 compares directivity for converged solutions. This shows that both MIPUM 2-15-2 and quadratic Finite Element Method converges towards the same solution.

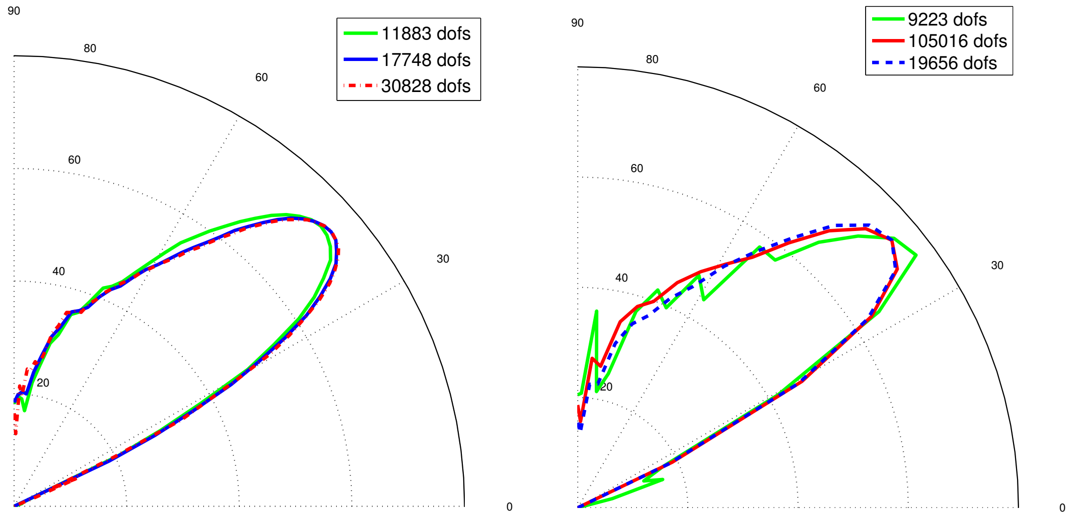


(a)



(b)

Fig. 8.8. Quadratic Finite Element Method (105016 degrees of freedom): Radiation of mode $(26, 1)$ at $ka = 30$ without acoustic treatment: (a) real part of the pressure (b) absolute value of pressure.



(a) (b)

Fig. 8.9. Directivity pattern for the convected radiation of mode $(26, 1)$ at $ka = 30$ without acoustic treatment: (a) MIPUM 2-15-2 (b) QFEM.

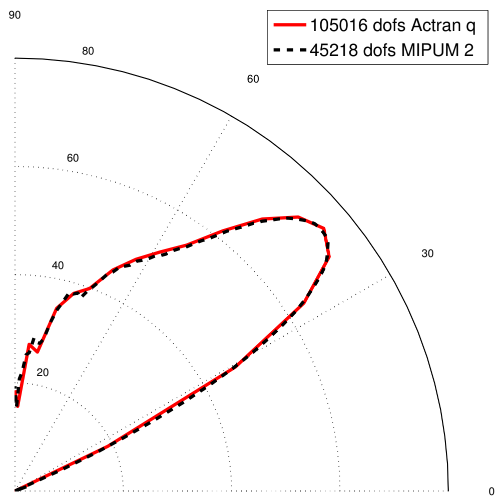
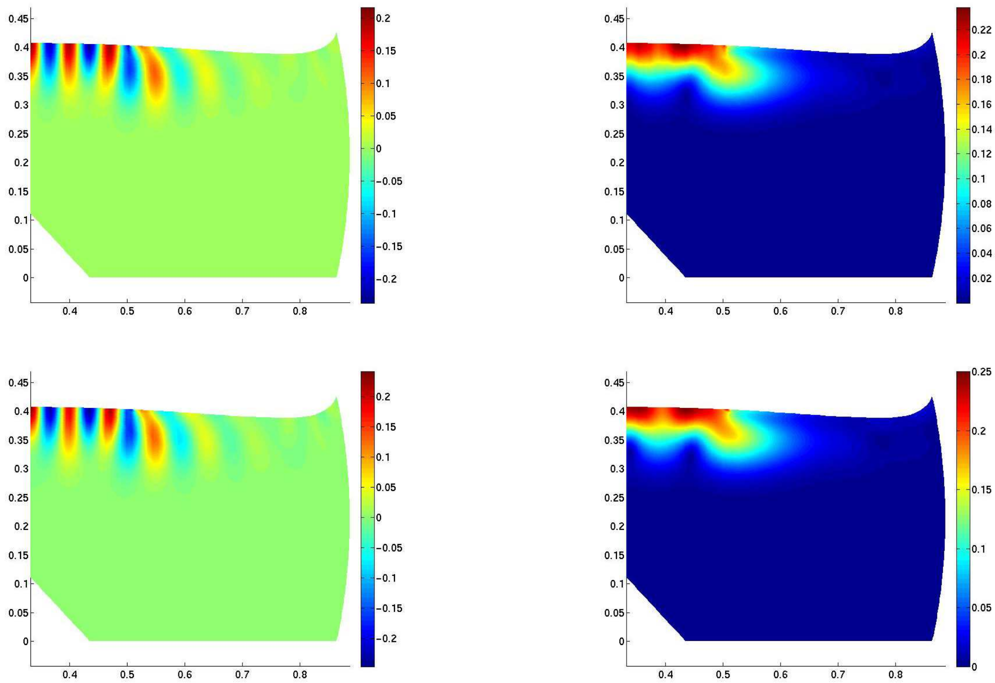


Fig. 8.10. Directivity pattern for the convected radiation of mode $(26, 1)$ at $ka = 30$ without acoustic treatment: comparison of converged directivity for QFEM and MIPUM 2-15-2.

8.3 Convected radiation and influence of liners

A liner is inserted in the nacelle from $x = 0.5m$ to $x = 0.75m$. It is simulated by prescribing an admittance boundary condition with an admittance of: $A_n = 0.001001 + 0.0005i$. Computed results from quadratic Finite Element method and second order enrichment Mapped Infinite Partition of Unity Method are compared in figure 8.11. The distribution of the pressure is illustrated from the fan plane to the lip of the nacelle because the noise is completely attenuated by the liner.



(a) (b)

Fig. 8.11. MIPUM 2-15-2 with 30828 degrees of freedom (top) and QFEM with 105016 degrees of freedom (bottom): Radiation of mode $(26, 1)$ at $ka = 30$ with admittance: (a) real part of the pressure (b) absolute value of pressure. The distribution of the pressure is illustrated from the fan plane to the lip of the nacelle.

References

1. Ph. Bouillard, F. Ihlenburg, Error estimation and adaptivity for the finite element solution in acoustics: 2D and 3D applications, Comput. Methods Appl. Mech. Engrg. 176 (1999) 147-163.
2. F. Ihlenburg, I. Babuška, Dispersion analysis and error estimation of Galerkin finite element methods for the Helmholtz equation, Int. J. Numer. Methods Eng. 38 (1995) 3745-3774.
3. J.T. Oden, S. Prudhomme, L. Demkowicz, A posteriori error estimation for acoustic wave propagation problems, Arch. Comput. Meth. Engng. 12 (2005) 343-389.
4. F. Ihlenburg, I. Babuška, Finite element solution of the Helmholtz equation with high wave number part II : the h-p version of the FEM, SIAM J. Numer. Anal. 34 (1997) 315-358.
5. O.C. Zienkiewicz, Achievements and some unsolved problems of the finite element method, Int. J. Numer. Methods Eng. 47 (2000) 9-28.
6. L.L. Thompson, A review of finite element methods for time-harmonic acoustics, J. Acoust. Soc. Am. 119 (2006) 1315-1330.
7. J.M. Melenk, I. Babuška, The partition of unity finite element method: basic theory and applications, Comput. Methods Appl. Mech. Engrg. 139 (1996) 289-314.
8. I. Babuška, J.M. Melenk, The Partition of Unity Method, Int. J. Numer. Methods Eng. 40 (1997) 727-758.
9. O. Laghrouche, P. Bettess, R. J. Astley, Modeling of short wave diffraction problems using approximating systems of plane waves, Int. J. Numer. Methods Eng. 54 (2002) 1501-1533.
10. Th. Strouboulis, I. Babuška, R. Hidajat, The generalized finite element method for Helmholtz equation: Theory, computation, and open problems, Comput. Methods Appl. Mech. Engrg. 195 (2006) 4711-4731.
11. Th. Strouboulis, R. Hidajat, I. Babuška, The generalized finite element method for Helmholtz equation Part II: Effect of choice of handbook functions, error due to absorbing boundary conditions and its assessment, Comput. Methods Appl. Mech. Engrg. 197 (2008) 364-380.
12. Th. Strouboulis, K. Copps, I. Babuška, The generalized finite element method, Comput. Methods Appl. Mech. Engrg. 190 (2001) 4081-4193.
13. Th. Strouboulis, I. Babuška, K. Copps, The design and analysis of the generalized finite element method, Comput. Methods Appl. Mech. Engrg. 181 (2000) 43-69.
14. C. Farhat, I. Harari, L.P. Franca, The discontinuous enrichment method, Comput. Methods Appl. Mech. Engrg. 190 (2001) 6455-6479.
15. B. Pluymers, W. Desmet, D. Vandepitte, P. Sas, Application of an efficient wave-based prediction technique for the analysis of vibro-acoustic radiation problems, J. Comput. Appl. Math. 168 (2004) 353-364.
16. P. Gamallo, R.J. Astley, A comparison of two Trefftz-type methods: The ultraweak variational formulation and the least-squares method, for solving shortwave 2-D Helmholtz problems, Int. J. Numer. Methods Eng. 71 (2007) 406-432.
17. P. Gamallo, R.J. Astley, The partition of unity finite element method for short wave acoustic propagation on non-uniform potential flows, Int. J. Numer. Methods Eng. 65 (2006) 425-444.
18. T. Huttunen, P. Gamallo, R.J. Astley, Comparison of two wave element methods for the Helmholtz problem, Commun. Numer. Meth. Engng. Article in Press (2008) doi:10.1002/cnm1102.
19. T. Huttunen, P. Monk, J.P. Kaipio, Computational aspects of the ultra weak variational formulation, J. Comput. Phys. 182 (2002) 27-46.
20. T. Huttunen, J.P. Kaipio, P. Monk, An ultra weak method for acoustic fluid-solid interaction, J. Comput. Appl Math. 213 (2008) 166-185.

21. P. Monk, D.-Q. Wang, A least squares method for the Helmholtz equation, Comput. Methods Appl. Mech. Engrg. 175 (1999) 121-136.
22. R. Sevilla, S. Fernández-Méndez, A. Huerta, NURBS-enhanced finite element method (NEFEM), Int. J. Numer. Methods Eng. Early ViewW (2008) doi: 10.1002/nme.2311
23. E. Chadwick, P. Bettess, O. Laghrouche, Diffraction of short waves modeled using new mapped wave envelope finite and infinite elements, Int. J. Numer. Methods Eng. 45 (1999) 335-354.
24. E. De Bel, P. Villon, Ph. Bouillard, Forced vibrations in the medium frequency range solved by a partition of unity method with local information, Int. J. Numer. Methods Eng. 162 (2004) 1105-1126.
25. L. Hazard, Ph. Bouillard, Structural dynamics of viscoelastic sandwich plates by the partition of unity finite element method, Comput. Methods Appl. Mech. Engrg. 196 (2007) 4101-4116.
26. T.J.R. Hughes, J.A. Cotrell, Y. Bazilevs, Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement, Comput. Methods Appl. Mech. Engrg. 194 (2005) 4135-4195.
27. B. Szabó, A. Düster, E. Rank, The p-version of the finite element method, in: E. Stein, R. de Borst, T.J.R. Hughes (Eds.), Fundamentals, Encyclopedia of Computational Mechanics, vol. 1, Wiley, New York, 2004 (Chapter 5).
28. R.J. Astley, P. Gamallo, Special short wave elements for flow acoustics, Comput. Methods Appl. Mech. Engrg. 194 (2005) 341-353.
29. R.J. Astley, P. Gamallo, The Partition of Unity Finite Element Method for Short Wave Acoustic Propagation on Non-uniform Potential Flows, Int. J. Numer. Methods Eng. 65 (2006) 425-444.
30. R.J. Astley, G.J. Macaulay, J.-P. Coyette, L. Cremers, Three-dimensional wave-envelope elements of variable order for acoustic radiation and scattering. Part I. Formulation in the frequency domain, J. Acoust. Soc. Am. 103 (1998) 49-63.
31. R.J. Astley, J.-P. Coyette, Conditioning of infinite element schemes for wave problems, Commun. Numer. Meth. Engng. 17 (2001) 31-41.
32. D. Dreyer, O. von Estorff, Improved conditioning of infinite elements for exterior acoustics, Int. J. Numer. Methods Eng. 58 (2003) 933-953.
33. J.J. Shirron, I. Babuška, A comparison of approximate boundary conditions and infinite element methods for exterior Helmholtz problems, Comput. Methods Appl. Mech. Engrg. 164 (1998) 121-139
34. D. Dreyer, Efficient infinite elements for exterior acoustics, PhD thesis, Shaker Verlag, Aachen, 2004.
35. W. Eversman, Mapped infinite wave envelope elements for acoustic radiation in a uniformly moving medium, J. Sound Vib. 224 (1999) 665-687.
36. S.J. Rienstra, W. Eversman, A numerical comparison between the multiple-scales and finite-element solution for sound propagation in lined flow ducts, J. Fluid Mech. 437 (2001) 367-384.
37. S.J. Rienstra, Sound transmission in slowly varying circular and annular lined ducts with flow, J. Fluid Mech. 380 (1999) 279-296.
38. S.J. Rienstra, The Webster equation revisited, 8th AIAA/CEAS Aeroacoustic conference (2002) AIAA 2002-2520.
39. M.K. Myers, On the acoustic boundary condition in the presence of flow, J. Sound Vib. 71 (1980) 429-434.
40. T. Mertens, Meshless modeling of acoustic radiation in infinite domains, Travail de spécialisation, Université Libre de Bruxelles (U.L.B.), 2005.
41. W. Eversman, The boundary condition at an impedance wall in a non-uniform duct with potential mean flow, J. Sound Vib. 246 (2001) 63-69.
42. B. Regan, J. Eaton, Modeling the influence of acoustic liner non-uniformities on duct modes, J. Sound Vib. 219 (1999) 859-879.
43. H.-S. Oh, J.G. Kim, W.-T. Hong, The Piecewise Polynomial Partition of Unity Functions for the Generalized Finite Element Methods, Comput. Methods Appl. Mech. Engrg. Accepted manuscript (2008), doi: 10.1016/j.cma.2008.02.035.
44. Free Field Technologies, Actran user's manual, http://fft.be.
45. University of Florida, Department of Computer and Information Science and Engineering, http://www.cise.ufl.edu/research/sparse/umfpack/
46. T.A. Davis, A column pre-ordering strategy for the unsymmetric-pattern multifrontal method, ACM Transactions on Mathematical Software 30 (2004) 165-195.
47. T.A. Davis, Algorithm 832: UMFPACK, an unsymmetric-pattern multifrontal method, ACM Transactions on Mathematical Software 30 (2004) 196-199.
48. T.A. Davis and I.S. Duff, A combined unifrontal/multifrontal method for unsymmetric sparse matrices, ACM Transactions on Mathematical Software 25 (1999) 1-19.

49. T.A. Davis and I.S. Duff, An unsymmetric-pattern multifrontal method for sparse LU factorization, SIAM Journal on Matrix Analysis and Applications 18 (1997) 140-158.
50. A. Hirschberg, S.W. Rienstra, An introduction to aeroacoustics, Eindhoven University of Technology (2004).
51. G. Warzée, Mécanique des solides et des fluides, Université Libre de Bruxelles (2005).
52. M.J. Crocker, Handbook of acoustics, Wiley-Interscience, New York (1998).
53. R. Sugimoto, P. Bettess, J. Trevelyan, A numerical integration scheme for special quadrilateral finite elements for the helmholtz equation, Commun. Numer. Meth. Engng. 19 (2003) 233-245.
54. G. Gabard, R.J. Astley, A computational mode-matching approach for sound propagation in three-dimensional ducts with flow, J. Sound Vib. Article in Press (2008) doi:10.1016/j.jsv.2008.02.015.
55. C. Farhat, I. Harari, U. Hetmaniuk, A discontinuous Galerkin method with Lagrange multipliers for the solution of Helmholtz problems in the mid-frequency regime, Comput. Methods Appl. Mech. Engrg. 192 (2003) 1389-1419.
56. C.L. Morfey, Acoustic energy in non-uniform flows, J. Sound Vib. 14 (1971) 159-170.
57. F. Magoules, I. Harari (editors), Special issue on Absorbing Boundary Conditions, Comput. Methods Appl. Mech. Engrg. 195 (2006) 3354-3902.
58. M. Fischer, U. Gauger, L. Gaul, A multipole Galerkin boundary element method for acoustics, Engineering Analysis with Boundary Elements 28 (2004) 155-162.
59. J.-P. Bérenger, Three-dimensional Perfectly Matched Layer for the absorption of electromagnetic waves, J. Comput. Phys. 127 (1996) 363-379.
60. C. Michler, L. Demkowicz, J. Kurtz, D. Pardo, Improving the performance of Perfectly Matched Layers by means of hp-adaptivity, Numerical Methods for Partial Differential Equations 23 (2007) 832-858.
61. A. Bayliss, E. Turkel, Radiation boundary conditions for wave-like equations, Comm. Pure Appl. Math. 33 (1980) 707-725.
62. B. Engquist, A. Majda, Radiation boundary conditions for acoustic and elastic wave calculations, Comm. Pure Appl. Math. 32 (1979) 313-357.
63. K. Feng, Finite element method and natural boundary reduction, Proceedings of the International Congress of Mathematicians, Warsaw (1983) 1439-1453.
64. D. Givoli, B. Neta, High-order non-reflecting boundary scheme for time-dependent waves, J. Comput. Phys. 186 (2003) 24-46.
65. T. Hagstrom, A. Mar-Or, D. Givoli, High-order local absorbing conditions for the wave equation: Extensions and improvements, J. Comput. Phys. 227 (2008) 3322-3357.
66. T. Hagstrom, T. Warburton, A new auxiliary variable formulation of high-order local radiation boundary conditions: corner compatibility conditions and extensions to first order systems, Wave Motion 39 (2004) 327-338.
67. A. Hirschberg, S.W. Rienstra, An introduction to aeroacoustics, Eindhoven university of technology (2004).
68. V. Lacroix, Ph. Bouillard, P. Villon, An iterative defect-correction type meshless method for acoustics, Int. J. Numer. Meth. Engng 57 (2003) 2131-2146.
69. T. Mertens, P. Gamallo, R. J. Astley, Ph. Bouillard, A mapped finite and infinite partition of unity method for convected acoustic radiation in axisymmetric domains, Comput. Methods Appl. Mech. Engrg. 197 (2008) 4273-4283.
70. G. Gabard, Discontinuous Galerkin methods with plane waves for time harmonic problems, J. Comput. Phys. 225 (2007) 1961-1984.
71. L. Hazard, Design of viscoelastic damping for vibration and noise control: modeling, experiments and optimisation, PhD thesis, Univesité Libre de Bruxelles (2007).
72. G. Gabard, R.J. Astley, M. Ben Tahar, Stability and accuracy of finite element methods for flow acoustics. I: general theory and application to one-dimensional propagation, Int. J. Numer. Meth. Eng. 63, (2005) 947-973.
73. G. Gabard, R.J. Astley, M. Ben Tahar, Stability and accuracy of finite element methods for flow acoustics. II: Two-dimensional effects, Int. J. Numer. Meth. Eng. 63 (2005) 974-987.
74. A. Goldstein, Steady state unfocused circular aperture beam patterns in non attenuating and attenuating fluids, J. Acoust. Soc. Am. 115 (2004) 99-110.
75. T. Douglas Mast, F. Yu, Simplified expansions for radiation from baffled circular piston, J. Acoust. Soc. Am. 118 (2005) 3457-3464.
76. T. Hasegawa, N. Inoue, K Matsuzawa, A new rigorous expansion for the velocity potential of a circular piston source, J. Acoust. Soc. Am. 74 (1983) 1044-1047.
77. R.J. Astley, A finite element, wave envelope formulation for acoustical radiation in moving flows, J. Sound Vib. 103 (1985) 471-485.
78. J.M. Tyler, T.G. Sofrin, Axial flow compressor noise studies, SAE Transactions 70 (1962) 309-332.

79. M.C. Duta, M.B. Giles, A three-dimensional hybrid finite element/spectral analysis of noise radiation from turbofan inlets, J. Sound Vib. 296 (2006) 623-642.
80. H.H. Brouwer, S.W. Rienstra, Aeroacoustics research in Europe: The CEAS-ASC report on 2007 highlights, J. Sound Vib. Article in Press (2008) doi:10.1016/j.jsv.2008.07.020.
81. http://en.wikipedia.org/wiki/Aircraft_noise, 4th September 2008.
82. Y. Park, S. Kim, S. Lee, C. Cheong, Numerical investigation on radiation characteristics of discrete-frequency noise from scarf and scoop aero-intakes, Appl. Acoust. Article in press (2008) doi:10.1016/j.apacoust.2007.09.005.
83. General Electric Company, http://www.ge.com, http://www.turbokart.com/about_ge90.htm, 4th September 2008.
84. R.J. Astley, J.A. Hamilton, Modeling tone propagation from turbofan inlets - The effect of extended lip liner, AIAA paper 2002-2449 (2002).
85. V. Decouvreur, Updating acoustic models: a constitutive relation error approach, PhD thesis, Université Libre de Bruxelles (2008).
86. P.A. Nelson, O. Kirkeby, T. Takeuchi, and H. Hamada, Sound fields for the production of virtual acoustic images, Letters to the editor, J. Sound Vib. 204 (1999) 386-396.
87. Y. Reymen, W. De Roeck , G. Rubio, M. Bealmans, W. Desmet, A 3D Discontinuous Galerkin Method for aeroacoustic propagation, Proceedings of the 12th International Congress on Sound and Vibration 2005.
88. G. Gabard, Exact integration of polynomial-exponential products with an application to wave based numerical methods, Commun. Numer. Meth. Engng (2008) doi: 10.1002/cnm.1123.
89. New York University, http://math.nyu.edu/faculty/greengar/shortcourse_fmm.pdf, 1st december 2008.