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A new mapped infinite partition of unity method for convected acoustical radiation in infinite domains.

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List of Symbols

Greek symbols

β	: $\sqrt{1-M_0^2}$	
γ	: Poisson ratio of specific heat capacities : c_p/c_v	
$\mathit{\Gamma}$	: interface separating the inner and the outer domains	
ε	: Error	
μ	: phase function	$[m]$
ρ	: mass density	$[kgm^{-3}]$
ρ_0	: steady mean density	$[kgm^{-3}]$
ρ_a	: acoustic density	$[kgm^{-3}]$
$\boldsymbol{\sigma}$	: stress tensor	$[Nm^{-2}]$
ϕ	: velocity potential	$[m^2s^{-1}]$
ϕ_0	: mean velocity potential	$[m^2s^{-1}]$
ϕ_a	: acoustic velocity potential	$[m^2s^{-1}]$
$\tilde{\phi}_a$	: amplitude of the harmonic acoustic velocity potential	$[m^2s^{-1}]$
$\tilde{\phi}^h$	: numerical approximation of $\tilde{\phi}_a$	$[m^2s^{-1}]$
$\tilde{\phi}_h^I$	: numerical approximation in the outer region $\mathit{\Omega}_o$	$[m^2s^{-1}]$
$\mathit{\Phi}_\alpha$	: shape function for the α^{th} degree of freedom	
$\mathit{\Phi}_\alpha^I$	: infinite shape function for the α^{th} degree of freedom	
ω	: angular frequency	$[s^{-1}]$
$\mathit{\Omega}$	: domain	
$\mathit{\Omega}_i$	: inner region	
$\mathit{\Omega}_o$	: outer region	

Arabic symbols

$\tilde{a}_n$	: normal acceleration of a vibrating wall	$[ms^{-2}]$
A_n	: normal acoustic admittance	$[m^2skg^{-1}]$
$A^{\pm}_{mn}$	: incident and reflected modal amplitude	$[m^2s^{-1}]$
c	: speed of sound	$[ms^{-1}]$
c_0	: steady mean part of the speed of sound	$[ms^{-1}]$
c_∞	: speed of sound at large distance from the source	$[ms^{-1}]$
c_p	: specific heat capacity at constant pressure	$[JK^{-1}]$
c_v	: specific heat capacity at constant volume	$[JK^{-1}]$
$dofs$	: number of unknowns of the approximation	
E	: energy flow out of a surface	$[J]$
$E^{\pm}_{mn}$	: incident and reflected modal patern	
f	: excitation frequency	$[s^{-1}]$
G	: geometric factor	
h	: mesh size	$[m]$
H	: Hilbert space	
i	: imaginary unit $= \sqrt{-1}$	
$\mathbf{I}$	: Sound intensity	$[Wm^{-2}]$
J'	: stagnation entropy	$[Jkg^{-1}]$
k	: wavenumber	$[m^{-1}]$
$k^{\pm}_{r,mn}$	: incident and reflected radial wavenumber	$[m^{-1}]$
k_B	: Boltzmann constant	$[JK^{-1}]$
$K^{\pm}_{z,mn}$	: incident and reflected axial wavenumber	$[m^{-1}]$
L^d_j	: Legendre polynomial of order d for node j	
L_s	: curve enclosing the boundary S_s	
L_v	: curve enclosing the boundary S_v	
m	: angular mode number	
$\mathbf{m}'$	: mass flux	$[kgm^-2s^{-1}]$
m_0	: radial order of the infinite element	
m_w	: mass of a molecule	$[kg]$
M_0	: mach number	
M_i	: Mapping function for node/point i	
$\mathbf{n}$	: outer normal to the domain	
n	: radial mode number	
n^I_d	: number of infinite degree of freedom	
$n\,(j)$	: size of the local approximation space at node j	
nni	: number of infinite nodes	
$nodes$	: number of nodes	
N_i	: Partition of Unity function of node i	
N_m	: number of angular modes	
N_n	: number of radial modes	
N_M	: number of reflected modes (unknown)	

p	: fluid pressure	$[Pa]$
p_0	: steady mean fluid pressure	$[Pa]$
p_a	: acoustic pressure	$[Pa]$
$\tilde{p}_a$	: amplitude of the harmonic acoustic pressure	$[Pa]$
$\tilde{p}_{an}$	: analytic amplitude of the harmonic acoustic pressure	$[Pa]$
$\mathbf{q}$	: heat flux	$[Wm^{-2}]$
Q_w	: heat production	$[J]$
r_o	: distance to the source point	$[m]$
R	: specific gas constant	$[JK^{-1}mol^{-1}]$
R_j	: radial function for infinite node j	
R_j^d	: radial function of order d for node j	
s	: entropy	$[Jkg^{-1}K^{-1}]$
S	: boundary	
S_i	: mapping functions for the interface Γ	
S_M	: Modal boundary	
S_s	: soft wall	
S_v	: vibrating wall	
t	: time	$[s]$
T	: Temperature	$[K]$
T_j	: circumferential function for infinite node j	
$\tilde{u}_n$	: normal displacement of a vibrating wall	$[m]$
$\mathbf{v}$	: fluid velocity	$[ms^{-1}]$
$\mathbf{v}_0$	: steady mean fluid velocity	$[ms^{-1}]$
$\mathbf{v}_\infty$	: fluid velocity at large distance from the source	$[ms^{-1}]$
$\mathbf{v}_a$	: acoustic velocity	$[ms^{-1}]$
$\tilde{\mathbf{v}}_a$	: amplitude of the harmonic acoustic velocity	$[ms^{-1}]$
$\mathscr{V}$	: the Sobolev space $W^{1,2} = H^1 = \{f : f, \nabla f \in L^2\}$	
V_{jl}	: l^{th} local approximation function of node j	
$\tilde{w}_n$	: normal velocity of a vibrating wall	$[ms^{-1}]$
W_j	: weight function of node j	
W_j^I	: infinite weight function of the infinite node j	
$W_{M,nm}$	: modal weight function of the angular and radial mode (m, n)	

Operators

∇	: gradient operator
$\nabla\cdot$	: divergence operator
$\nabla\times$	: curl operator
Δ	: Laplacian operator
$\frac{D}{Dt}$	: Total time derivative
$:$	: the double dot product of two tensors
$\langle\ \rangle$	: time average
$\Re$	: Real part

Contents

9

Conclusions

This dissertation focuses on the development of a deterministic prediction technique for acoustic radiation in a convected outer domain. The variational formulation is based on the convected wave equation and the steady mean flow is assumed to be known. **The major contribution is to couple the Partition of Unity Method to an original Mapped Wave Envelope Infinite Elements for convected wave propagation in an outer domain**. The final goal is a computationally efficient three-dimensional prediction technique. We decided to develop an axisymmetric and a three-dimensional toolbox. Axisymmetric predictions are used to assess the formulation and analyse the performances of the method. Numerous applications are presented in this dissertation to evaluate the influence of the parameters of the proposed method. We also simulate the radiation of an axisymmetric nacelle which shows that the Mapped Infinite Partition of Unity Method is convenient for such industrial application. Some three-dimensional simulations have been performed to confirm the axisymmetric observations.

Axisymmetric applications cover all the aspects of the model approach: cavities, outer domains, modal, admittance and vibrating wall prescriptions, uniform and non-uniform convected propagation. The convergence analyses show obviously that high order enrichment functions converge quicker than low order ones. We can draw the same conclusions by observing the performance curves. The CPU time required to solve the application within a constant accuracy varies exponentially with the excitation frequency. For each enrichment, there exists a frequency such that a small increase of frequency leads to enormous additional time to compute an accurate solution. This is due to the need of fine meshes for high frequencies related to pollution error. We show that high order enrichments allow us to reach higher frequencies before reaching high computer resources. As a conclusion the proposed method with high order enrichments is computationally efficient as it is less time demanding, more accurate and it allows to reach higher frequencies. Instabilities at high polynomial order may perturb the accuracy. This is the reason why we recommend to use polynomial functions up to order p=3.

The analysis of the parameters of the methods leads to some remarks and recommendations. We developed mapped finite and infinite elements where the corners are considered as nodes and mid-side nodes as geometrical points. The mid-side nodes describe the geometry, they do not support degrees of freedom as it is the case for the polynomial Finite Element Method. We observed that these mid-side points have a considerable effect on the accuracy of the method compared to elements with straight edges, especially with infinite elements (cf. radiation of multipole (section 5.2) and of the rigid piston (section 5.3)). We showed (section 5.2) that it is important to select the appropriate infinite radial interpolation order. This order depends on the complexity of acoustic field and then on the proximity of the interface Γ from the acoustic sources. The circumferential approximation of infinite elements depends on the inner enrichment for the first radial order (to ensure continuity). The circumferential approximation for higher radial orders is based on polynomials of order b_0. We showed (section 5.2) that it is not necessary to use high values of b_0 to obtain accurate solutions. This is an important result as it reduces the number of unknowns in the infinite elements and keeps an accurate solution (note that this analysis is based on L^2 relative errors computed in the inner region).

The Partition of Unity Method is known to be badly conditioned. We evaluated the condition number and observed that due to linear dependencies, the matrices are nearly singular. This drawback (as it prevents to use iterative solvers) may lead to the deterioration of the numerical solution. However, we used an appropriate solver (UMFPACK [45]) which ensures the accuracy of the solution even for high condition number and we did not observe a loss of convergence (for $p < 4$). For enrichement functions higher than $p = 4$, we observed instabilities leading to a loss of accuracy but the condition number can not be used as an indicator of instability. The condition number for an enrichment $p > 4$ is of the same order of magnitude for stable or unstable solution.

This dissertation presents the Mapped Infinite Partition of Unity Method as a computationally efficient method applicable to industrial applications such as turbofan radiation. The major part of the conclusions are based on axisymmetric results. Three-dimensional simulations have been performed to confirm the validity and the performances of the method observed with axisymmetric computations. We then think that all these conclusions stand for both axisymmetric and three-dimensional computations with the Mapped Infinite Partition of Unity Method.

This dissertation focuses on the properties and performances of the Mapped Infinite Partition of Unity Method. Turbofan radiation has been analysed to illustrate the applicability of the method to industrial simulations. A full detailed three-dimensional analysis of a nacelle has not been performed. This would consist in reducing noise radiated from the nacelle. This would require a multi-modal and multi-frequency analysis to simulate rotor alone, rotor-stator interactions and broadband noise sources; to select a proper geometry scarf or scoop nacelle. An optimisation process should be performed on the liner properties: location, values of the admittance (which may vary and be discontinuous in the axial direction), size of azimuthal splices. Note that the admittance of liners is frequency dependent and has to be interpreted in terms of physical parameters such as the honeycomb depth or the perforate fraction of open area. After this complete analysis, you can deduce the reduction achieved compared to current engines.

Recommendations for future developments

The dissertation illustrated the interest of using localised enrichments. The idea is to enrich nodes located close to complex distribution of pressure. We show (section 5.1) that this local enrichment improves the global accuracy of the solution. This also prevents of generating a finer mesh. We suggest to extend the idea to infinite elements and develop an automatic iterative method allowing to find appropriate enrichments for nodes and appropriate radial and circumferential orders for infinite elements. Of course, this is intended to applications requiring repetitive computations.

We also recommend to investigate the definition of enrichment functions in the parent element coordinates instead of the global ones (as it is presented here). The global enrichment functions allow to choose the orientation to enrich. For instance, in the case of the propagation of a plane wave in the x direction, we then may enrich the approximations with only polynomial terms in x. However, some preliminary analyses (not presented in this dissertation) motivate the use of parent enrichment functions. The first advantage is a lower computational time required. The evaluation of global enrichment functions need to be performed for each element as the functions depend on global coordinates. In the case of parent enrichments, the values at the Gauss points are the same for all elements. They need then to be evaluated once for all elements of the same topology. This would lead to a consequent reduction of time. Another advantage would consist in the prescription of boundary conditions. The value of the global enrichments (except for the constant term) at nodes is zero ($x - x_0 = 0 \textit{ for } x = x_0$). This is the same for normal derivatives of straight edges; e.g. the derivatives in the x-direction of a straight edge oriented in the y-direction lead to $\frac{\partial (x-x_0)^2}{\partial x}|_{x_0} = 2\,(x-x_0)\,|_{x_0} = 0$. The parent enrichment functions could easily be created such as nodal values and normal derivatives on edges do not vanish. This would improve the accuracy, as it has been observed during a first approach.

The accuracy of computed solutions has been evaluated in this dissertation with an inner indicator of error; i.e. the L^2 relative error computed in the inner region (outer indicators have not been used except for the nacelle radiation). Global indicators such as the L^2 relative error, are performant to analyse the properties of a given method. But it is not an engineering quantity of interest. In practical applications, engineers may be interested in less severe indicators such as directivity contours, or by evaluating the accuracy of field points in the inner/outer region.

The proposed method can be used to compute a broad range of applications in acoustics such as HVAC, engine block, tyre, submarine, etc. The method can also be used for updating techniques (requiring accurate simulations [85]). These techniques allow to improve numerical models by comparing computed results to experimental testings on a prototype and then adjust some parameters (such as normal admittance coefficients). The method has been presented in the field of acoustics but it can also be applied to predict other physical applications dealing with wave propagation (i.e. in the field of electromagnetism or earthquakes).

We present an efficient prediction technique for convected radiation. However, some remaining developments and analyses are required to compete with commercial softwares. This consists in using a performant programming language such as C or Fortran instead of Matlab. We also recommend to take care of the numerical integration. The order of the shape functions to integrate over a finite or infinite element may be very different. A special integration technique may then be developed to use the proper number of Gauss points for each integrand. These previous remarks correspond to the choice of the programming language and the care in writing effective functions. Another improvement would be to create a library of elements. This dissertation illustrates the performances of the method by considering (mapped) quadrangular elements for axisymmetric simulations and (mapped) hexahedral (6 faces, 6 vertices and 12 edges) elements for three-dimensional ones. The implementation of triangular elements for axisymmetric applications or tetrahedral and pentahedral elements for three-dimensional applications would greatly simplify mesh generation especially in the case of complex three-dimensional geometries.

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