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On the Economics of Interpersonal Relationships: Three Essays on Social Capital, Social Norms, and Social Identity

Denis HERBAUX

Directeur:	Professeur Mathias DEWATRIPONT - Université Libre de Bruxelles
Co-directeur:	Professeur Georg KIRCHSTEIGER - Université Libre de Bruxelles
Membres du jury:	Professeur Micael CASTANHEIRA - Université Libre de Bruxelles
	Professeur Jean-Luc DE MEULEMEESTER - Université Libre de Bruxelles
	Professeur Jan POTTERS - Tilburg University

ACADÉMIE UNIVERSITAIRE WALLONIE-BRUXELLES

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Part I

Introduction

In 2004, M. Night Shyamalan directed a movie entitled "The Village" that tells the story of a community living outside of the real world without even knowing of the existence of any other life. The community works with very strong values and norms. All of its members live just as in the past, before electricity, running water or modern medecine. They all dress in an old fashioned way, red is a forbidden color, and "Those We Do Not Speak Of" cannot be disturbed or attracted.

The movie describes a situation in which a community works as a closed network, sharing norms, values, habits, in which inhabitants trust each other and deviants from the norms are easily punished. While this is a bit extreme, similar situations are nonetheless observed in everyday life. When trying to incorporate these facts into the neoclassical approach, one faces major difficulties, since interpersonal relationships are not really taken into account. These difficulties are mainly due to the differences between economics and sociology as outlined by Duesenberry (1960):

> "Economics is all about choice, while sociology is about why people have no choices."

The idea behind this is that for decades, economic theories have been mostly based on the rational choices made by selfish individuals to maximize their utility, while sociology spent a lot of effort describing the environment of individuals and explaining how this environment shapes their decisions. During the last thirty years, many new concepts have appeared in the economic literature. For example, behavioral economics has introduced features such as envy or altruism into traditional theories. Other notions such as social capital, social norms, trust, community or networks have become more and more present in economic research. The objective of this new strand of literature is to engage into a sort of socioeconomic approach and to shed some light on interpersonal relationships. As outlined by Akerlof (1997):

> "[...] the impact of my choices on my interactions with other members of my social network may be the primary determinant of my decision [...]"

The main objective of this thesis is to go further into this socioeconomic analysis, applying existing sociological concepts to classical economic models. In particular, we will use interpersonal relationships as a recurrent theme throughout the thesis, the final goal being to try to answer the following question:

Do interpersonal relationships matter?

This general question is investigated by way of three more specific ones, each of which examines a particular approach to the topic. A first step consists in evaluating the utility of interpersonal relationships. In particular, we argue that interpersonal relationships may be used to explain some phenomena that are observed in everyday life. Although similar investigations have already been conducted, we use a particular approach based on norms and communities, two concepts linked to interpersonal relationships, to study

How social norms constrain individual consumption.

In a second step, we explore the potential effect of interpersonal relationships based on the existence of *social identity*, i.e. the fact that individuals want others to know who they are. The question investigated is the following:

What is the impact of social identity on the market equilibrium?

In the last step, we describe interpersonal relationships through a highly popular concept in economics, *social capital*, which can be seen as a specific form of interpersonal relationships. We study what social capital is made of in Belgium, in terms of its level and composition. More precisely, we are interested in the following point:

Are there any regional differences in the levels of social capital in Belgium?

Together, these three parts should help us to gain a better understanding of *what interpersonal relationships are*, *what their utility is*, and *what their impact is on the results of classical economic models*. Answering the questions raised above is useful, notably in terms of policy implications. As already noted, interpersonal relationships are a reality, an aspect which cannot be neglected when designing policies to improve specific economic situations. Let us now present more precisely what each part of the thesis is made of.

One way of interpreting the introductory example is to say that it portrays a type of group behavior. Again, even if the example is quite extreme, there exist many other examples of group behavior: risky behavior by adolescents, religious behavior, dress code inside some communities, success or failure in education, consumption of a given product, are all examples of group behavior which seem quite evident to most people.

In the first paper, **The Tyranny of Social Norms on Individual Behavior**, we assume, based on social capital theories, that group behavior arises because of the existence of a norm inside a community and that negative outcomes may appear because of a potential high cost of moving. We use a particular definition of social capital, allowing us to introduce the concepts of communities and norms in a theory studying the dark side of interpersonal relationships. The idea is the following: by living in the same community, agents share interpersonal relationships, in particular under the form of social capital, and social assets such as values, codes, hobbies, etc. that are shared by all members. Suppose that a norm exists (for example a consumption norm) inside the community. If an agent does not want to consume the norm, given the possibilities of sanctions inside a community, he has to move to another community. But this has a cost, in the sense that the agent has to invest in new social assets to be accepted in the new community he wishes to enter. Obviously, the more different the two communities, the more costly it is to change communities.

Consider for example someone born in a very religious environment. Let us call him Christian. His parents are deep believers and the entire family goes to church four times a week. Christian goes to a religious school, so his friends are mostly believers. Let us consider that at the age of 15, Christian realizes that he does not believe in God as much as the others, and hence wants to change his "religious consumption". If he does it while staying in his original community, he will have to bear the social shame of his parents, his friends, and the whole community. Moreover, access to resources from the network will be harder (finding a job, getting a table at a restaurant, ...). Christian will have to consider different possibilities: either he is prepared to bear these costs, or he accepts to "stay with God", meaning that he keeps the same "religious consumption", although he does not get any utility from it (in fact, he gets a disutility). Of course, he may choose to change communities, but this has a cost to create new social capital, which can be seen as an investment in social assets: time to create new relationships, training in some activities practiced in the new group, buying new clothes to fit to the group, ... If the cost of moving is too high, and prevents him from running away, if the deviation cost is also high, social capital (a specific form of interpersonal relationships) may have a negative effect on his well being.

Our model investigates to what extent it is possible to replace social segregation, instrumented here by social assets, by a segregation in types, thereby allowing consumers to choose what they want. We use a model with two types of agents, two communities and an endogenous norm which is defined as the average of types inside a community. Because of social sanctions, agents are forced to consume something between what they would like to consume and what the norm tells them to consume. Hence, the norm exerts a burden on agents. We allow for the possibility of moving to another community, conditioned on investment in social assets. Depending on the size of this investment, we find that the different kinds of equilibria may arise. We may end up in an equilibrium in which there is only one type in each community, both types in each community, or one type in one community and both types in the other community. This means that there are cases in which consumers cannot break the social segregation on their own. Of course, these various equilibria have different effects in terms of welfare. We outline that some of these equilibria are suboptimal, in the sense that agents do not individually take the decision to move, though moving would be socially optimal, and that State intervention may help to reach an optimal equilibrium. Unfortunately, there are also cases in which it is too costly, even with State intervention, for people to change communities. In these situations, agents remain stuck in their community, forced to live with the burden of the norm.

As already said above, there are many examples of group behavior. Here is another: children living in a fancy neighborhood can often be recognized by the way they look: they usually wear the same expensive brand of clothes, Tommy Hilfiger or Scapa for example. They also share a series of specific behaviors and of social assets such as being capable of playing tennis, knowing the language to use and the behavior to have at a party, ... This example fits the previous setup. However, maybe a better way to explain this particular group behavior is to consider clothes not as a norm, but rather as a signal. The idea would then be that someone chooses a particular type of clothes in order to signal to others that he is part of a specific community. Individuals may do this for at least two reasons. First, they may do it to be accepted by a group, even if their type is in fact not the same as the type of the community. In that case, agents pretend to be something they are not in order to benefit from the community. Another reason is because agents like to be well understood by others, that is, they like others recognize their type by the signal they send. The second paper considers the latter by studying consumption behavior in the presence of social identity.

In this paper, **Social Identity, Advertising, and Market Competition**, we introduce the concept of social identity. Social identity is a sociological concept which basically states that consumption, besides its traditional role of satisfying needs, is used by people to signal their identity, i.e. type, to others. The underlying idea is that individuals tend to categorize others and to be caterogized. Following social identity, consumers want to be correctly categorized. As a consequence, agents get more utility if their type is understood correctly by others. For this, an agent will choose a good which will communicate his type to others. We introduce this theory into the classical Bertrand price competition framework, and analyze what the effects are on both consumer and firm behavior. Our model assumes a continuum of types and two firms, each producing an undifferentiated good (at least when there is no social identity and no advertising). Taking social identity into account has two consequences: first, if the number of goods is limited, it generates the formation of groups in the population, corresponding to a partitioning of types. Second, this creates market power for firms, leading to higher prices and profits. If there is coordination failure, then social identity generates multiple equilibria, each of which corresponds to a particular partition of agents.

One way this can be solved is by adding advertising to the model. Advertising is usually used by firms either to communicate the existence of the product (*informative advertising*) or to persuade consumers to buy the product (*persuasive advertising*). In our model, advertising is seen as in the *persuasive view* as well as in the *complementary view*, that is, the fact that consuming a good which is advertised provides utility. However, this utility decreases with the distance between a consumer's type and the advertising target, the idea being that someone does not like to consume a good which is targeted on a very different type than his own. With advertising, agents can now coordinate on specific equilibria. However, this also leads to more market power for firms, meaning higher prices and profits. Beyond the trivial equilibria in which both firms advertise, or neither firm does, depending on the advertising cost and the taste for advertising, our model also generates multiple equilibria, as well as asymmetric equilibria. Such asymmetric equilibria are characterized by one firm whose advertising target is the average type, while the other firm is seen as producing a "no brand" product. The equilibria that arise depend on the result of the combination of three effects. Since two firms compete for market shares, the first effect is what we called the competition effect, which basically leads firms to decrease their prices to win market shares. The way the continuum is divided into subsets has an impact on this effect since it more or less exposes firms to competition. The second effect is created by advertising, and the fact that consumers value it. It is called the market stealing effect which allows a firm that advertises to steal market shares from the other firm, thanks to the consumers' taste for advertising. Finally, the presence of social identity and advertising decreases price elasticity, leading to higher prices. This is the third effect that plays a role in which equilibria arise. We also show that in terms of aggregate consumer welfare, the optimal number of firms which advertise depends on the consumers' taste for advertising.

In the last paper, **Social Capital in Belgium**, we describe a specific kind of interpersonal interactions, i.e. *social capital*, using Belgium as a case study. More precisely, we characterize empirically what social capital is made of in Belgium and if there exist any regional differences in terms of levels of social capital. In the literature, all of the elements of the introductory example describing "The Village" are used (sometimes together, sometimes separately) to define the concept social capital. This concept was introduced almost thirty years ago, and has been used to explain a huge range of economic phenomena. In the paper, we start by underlining that there exist many definitions of social capital, from those considered to be the first (Loury 1977, and Coleman 1988) to the most recent ones (Durlauf and Fafchamps, 2005). In all of these definitions, elements such as norms, trust, networks, communities, civic behavior, etc. can be found.

Using the last wave of the European Social Survey, we develop an index of social capital from various questions available in the survey using principal component analysis. The questions are selected on the basis of the fact that they must instrument one or more elements proposed by the most frequently used definitions of social capital. The idea is to avoid restricting ourselves to a particular definition. On top of the created index of social capital, this methodology allows us to characterize three aspects which compose social capital: *Trust*, which is one of the most popular measures of social capital, *Social Activities*, which are interpreted as a way for individuals to regroup with others to achieve common goals, as well as a measure of civic behavior, and *Social Networks*, which is seen as a way to pursue purposive actions. Using two-stage least squares (2SLS) and some control variables, we show that there exist regional differences (Flanders has the highest level of social capital), and that education is an important variable in the formation of social capital. We also perform 2SLS on each aspect of social capital, and we find that education is still significant, and that regional differences are still present, except for *Social Activities*.

Finally, we extend the analysis to other European countries. We show that countries can be divided into groups on the basis of their social capital. Another result is that regional differences in social capital can be found in many Western European countries. Among these countries, Switzerland seems to have the highest regional differences in terms of social capital, while Ireland has no regional differences. Austria, The Netherlands and France have similar profiles. Concerning Belgium, the level of regional differences is higher than in Austria, The Netherlands and France, but lower than in Switzerland.

As stated at the beginning of this introduction, the main goal of the thesis is to show that interpersonal relationships do matter. Of course, interpersonal relationships mean many things, and it is not possible to cover everything in one go. We therefore restrict ourselves to three narrower topics, each one an aspect of interpersonal relationships. In each chapter, we try to answer a question that either has not been studied in the literature or to study an existing question in an original way.

The first paper examines at norms and communities, starting from the social capital literature. The goal here is to explain why some people in a community must consume a good that they do not like because the rest of the community does so. Although group behavior has been widely studied from role models to segregation models, we bring new aspects to the existing literature. First, we develop a framework using many concept drawn from the socioeconomic literature, trying to clarify how these aspects are related to each other and how they can be used together to explain how enforced consumption arises in a community. Second, we built a simple model describing such a situation, allowing us to determine under which conditions the State can intervene to resolve social segregation when agents cannot do it individually. Hence, this paper also has possible policy implications. It is therefore important to take into account such interpersonal relationships if policies are to be effective.

In the second paper, we introduce social identity in the classical Bertrand price competition model. Our main contributions here are the following: first, the concept of social identity has almost never been used in economics. Second, we show that taking interpersonal relationships into account modifies classical results drastically, since it creates market power for firms. Third, we show that advertising, while allowing consumers to coordinate on specific equilibria, again increases market power for firms.

Finally, in the last paper, we describe a specific form of interpersonal relationships: social capital. Although such a description has already been carried out before, it is the first time that a study concentrates on Belgium, and more precisely on regional differences in the levels of social capital. Although this first paper is mainly descriptive, it presents interesting results in terms of the composition of social capital, as well as regional differences in Belgium and in other countries of Western Europe.

Throughout all three papers, we put forward what can be understood by interpersonal relationships and how important they are. By doing this, although there is still a great deal of work to be done, we hope to get a little closer to the full understanding of the socioeconomicus.

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Part II

The Tyranny of Social Norms on Individual Behavior

1 Introduction

Although this may sound like a cliché, people are not equal with respect to access to and success in higher education. Indeed, individual characteristics of successful students in universities tend to converge: students who come from some "good" neighborhoods and students whose parents are more educated tend to be more present and more successful in university populations. This suggets the existence of neighborhoods which do not invest in education. The question is then to find out what reasons, other than money, can explain such group behavior. One possible explanation lies in the theory of social capital: by growing and living in a given neighborhood, an agent builds a number of interpersonal relationships such as family of course, but also friends, colleagues, neighbors, etc. Because they live together, one can easily imagine that all of these people share similar characteristics, such as the music they listen to, the way they dress and speak, their hobbies and evening activities, etc. In fact, these similarities create the identity of the community, ensuring that the group is more or less homogenous with respect to these characteristics. However, this does not mean that an agent's preferences fit those of the group perfectly. In particular, it is possible that one member of the group might want to invest in education while nobody else in the group wants to do so. Should he decide to make this investment this person would turn into an outsider, implying in turn that the agent would have to bear a high psychological (social shame) as well as physical (aggression) or even monetary (since he is excluded from the community, he can no longer benefit from the resources of that community) cost. Moreover, to be able to enter into a new group (the educated one), it is necessary to invest in order to gain the same characteristics as the others. Hence, an individual facing this situation must make a choice between three possibilities: first, he may decide to follow the norm (not get educated) and then suffer from a loss in terms of utility. Second, he may change communities (which requires an investment). Finally, he may choose something between the norm and ideal consumption, i.e. to stay in his original community, but bearing the cost of deviation. Since the cost of deviation and the cost of investment can be quite high, this may end in a "no deviation from the equilibrium (no education)" dominant strategy. Note that we do not consider the peer group here as having an impact on some endogenous preferences but rather as exerting pressure leading to a constraint.

This example clearly shows that belonging to a given community may affect agents negatively (other examples could be risky behavior by adolescents, such as drinking alcohol, taking drugs or religious extremism). Even if this negative aspect of interpersonal relationships is somewhat intuitive, it is not often investigated in the social capital literature. Indeed, though the last thirty years have seen many new concepts appear in the economic literature (social capital, social norms, endogenous preferences, community and networks) with the objective of engaging into a sort of socioeconomic approach and to shed some light on interpersonal relationships, most of the time authors consider social relations as improving economic situations. Without denying this important aspect, in this paper we focus on the dark side of interpersonal relationships. We truly believe that filling this gap in the literature is relevant, in particular as far as economic policy is concerned: one cannot expect any policy to be fully efficient if all forces at play (both positive and negative) have not been fully investigated.

In order to study the negative side of interpersonal relationships, we develop a model of communities' behavior with an endogenous norm based on the social capital literature. We show that in order to see a "moving" behavior arise, the degree of segregation between the two communities must be sufficiently high with respect to the moving cost. Key results are that the type of equilibrium depends crucially on the distribution of types accross communities as well as on the size of the moving cost. Moreover, we stress that socially suboptimal equilibria may arise, and that these equilibria may be corrected by State intervention through transfers to finance the inter community mobility. Unfortunately, we also show that in some cases, the State cannot intervene, so people remain stuck in their original community, thus forced to consume something different than their idal consumption. In that case, social segregation cannot be replaced by a segregation by types.

The paper is organized as follows: section 2 presents a short review of the literature, shedding some light on the link between social capital and group behavior. In section 3, we propose our model of communities' behavior. Section 4 concludes.

2 From social capital to group behavior

One of the main goals of this paper is to understand how agents can be harmed by belonging to a group in which they are forced to consume the same good as other members, without having the possibility to move. One consequence of this is the existence of similar behavior. Similarities of behavior can be explained in many ways. The presence of externalities when consuming the same good, the presence of information channels, or a taste for conformity can be potential answers. Focusing on a specific community or group, Manski (2000) states that people in the same group tend to behave similarly. Moreover, Postlewaite (1998) argues that the individual's choice problem is affected by the social group. Among the social processes that create group behavior, let us mention "conspicuous consumption" as in the Veblen effect, which states that the intrinsic value of a product may be less important than its social meaning, and "keeping up with the Joneses", where utility of consumption depends on the absolute level of consumption but also on how much is consumed in comparison to others (Janssen and Jager, 2001). Many empirical studies have been carried out in order to study such behavior. For example, Lachance and al. (2003) study the role of three socialization agents, namely parents, peers, and TV on adolescents' brand sensitivity. They find that peers seem to be the most important. Grinblatt and al. (2004) analyze the automobile purchase behavior in two Finnish provinces. It seems that a consumer's purchases are strongly influenced by the neighbors, information sharing being the key determinant. More negative relationships have also been studied. Phenomena such as risky behavior (Case and Katz, 1991; Kling and al., 2005; Clark and Lohéac, 2007), problems in education (Benabou, 1996; Fryer and Levitt, 2004; Austen-Smith and Fryer, 2005) or criminal activities (Glaeser et al., 1996; Donohue and Levitt, 2001) have been investigated and various explanations have been proposed by the above-mentioned authors and some other, to explain the exitence of these phenomena inside groups. The contagion model, the role model and the peer effect model state that the environment of an agent influences or shapes his preferences. Hence, in these models, agents adopt a given behavior because of their peferences. For example, children who grow up in a family or a neighborhood in which crime or use of drugs is highly present are more likely to develop the same behavior. In institutional theories, agents' behavior depends on institutions such as school, social services, police, etc. For example, someone living in a neighbohood with only bad schools will rarely be able to reach a high education level. Hence, consumption choices are constrained by the environment. A third possibility is segregation and discrimination models, which explain why and how people with similar characteristics end up facing similar situations (Cutler and Glaeser, 1997; Bertrand and Mullainathan, 2004). An example of the latter is the fact that being named Emily or Greg in the United States gives you more chances of getting a job interview than being called Lakisha or Jamal.

Our paper also concentrates on the negative aspect of group behavior, through the existence of norms and moving costs. For this, we base our argument on the social capital literature.

The concept of social capital is almost thirty years old. There exist many definitions of social capital, some rather general, others much more precise. Some of the authors who use this concept are Loury (1977) and Coleman (1988), who are considered as being at the origin of social capital, and Putnam (2000), Bowles and Gintis (2002), Durlauf and Fafchamps (2005). In all of the definitions, elements such as norms, trust, network, purposive action, etc. can be found. To make our point as clear as possible, we take these various elements and develop a theory explaining how social capital can harm people. The general idea is the following: having social capital means sharing interpersonal relationships with others, which is of course the case in communities. Inside such communities, agents may behave similarly, one reason for this being the existence of a norm. If one individual does not want to follow that norm, he will have to invest in social assets, allowing him to develop new social capital and enter into a new community.

To develop our argument, we rely on a definition by Lin (1999) which states that:

> "social capital can be defined as resources embedded in a social structure which are accessed and/or mobilized in purposive actions"

In this view, trust and norms are not seen as being social capital, but rather as social assets used to create it. This narrower definition is coherent with that of Sobel (2002) in which individuals can use membership in groups and networks to secure benefits. For the remainder of the paper, we rely on a definition inspired by both Lin and Sobel. Before defining our vision of social capital, we need to clarify the notion of social asset.

Definition 1 *A social asset is an asset that gets its (additional) value because it is used in a social context.*

This definition follows that of Mailath and Postelwaite (2006). Let us illustrate it by some examples: learning the values of a group is an investment in a social asset, because this asset (knowledge of values) will generate returns once it is used in contact with others. Another example is "golf knowledge", which can be seen as a "normal" asset (playing as a hobby) or as a social asset (since I can play golf, I will be able to play with my boss and get promoted more easily). It is important to note that social assets may be acquired through investment (I want to have this asset, so I am investing to acquire it), inheritance (nobility for example) or willingness of the parents (education or values for example). We can now give our definition of social capital.

Definition 2 *Social Capital is an amount of interpersonal relationships, created using social assets, and which can be used in purposive actions.*

Using this more precise definition has two major advantages: first, it allows the reader to have a clear idea of what we mean when we use the term social capital. Second, using a separation between the investment and the result makes the modelling and the understanding of social phenomenum such as the one we describe in this paper easier.

In most cases, social capital is used to correct inefficiencies due to some coordination failure or imperfect information. For example, asymmetric information about the quality of a worker can be (partially) solved if the worker and the owner of the firm know each other through a common network. Another example is that it is easier and cheaper to trade with someone you trust than with someone unknown (because the explicit contract is partially replaced by an implicit one). Portes (1998) identifies three basic functions of social capital: family support, social control, and benefits through extrafamilial networks. The first function has to do with the role of parents in education achievements (see Coleman, 1988) or preventing risky behavior, while the second deals with maintaining discipline and promoting cooperation (as explained by Bowles and Gintis, 2002). The third one seems to be the most studied in the literature, and is the main argument of Granovetter (1973). Although Portes makes networks explicit only for the third function, we already underlined that all of them rely (at least partially) on the existence of networks. We then need to characterize more precisely what networks are about.

In fact, the concept of network is relatively close to that of community. More precisely, one can view a community as a network subject to more conditions than only interpersonal relationships. One way of viewing a community is to consider that being part of a community requires having some common characteristics, which can be instrumentalized by social assets. We can then define a community in the following way:

Definition 3 *A community is a network of individuals, with a common basis of social assets (including values and norms), who share interpersonal relationships.*

Since social capital is often linked to the concept of community, we will focus on it. In the literature, the term community is used to describe how people interact in their daily lives, in families, in the neighborhood, and in work groups, not just as buyers, sellers, and citizens (Bowles and Gintis, 2002). Following these authors, being part of a community can have numerous advantages and increase welfare: for example, associations of neighbors to prevent crime and promote education, or fishermen who share income, information and training. The main justification is that repeated interactions, which are consequences of being part of a community, may prevent from things like free riding, and social sanctions following a deviation may be much more harmful than legal sanctions. One feature often encountered in communities is social norms. Fehr and Fischbacher (2004) outlined that *"we still know little about how social norms are formed, the forces determining their content and [...] the requirements that enable a species to establish and enforce social norms"*. However, some research has been undertaken to determine what a norm is and why people obey norms. Elster (1989) describes different kinds of norms that may exist. Among them are, on the one hand, consumption norms which, besides the traditional consumption goods, include things like dress code and table manners, and on the other hand cultural behavior, such as syntax, vocabulary and pronunciation, and even movies, books, sports... People obey norms in order to avoid sanctions or disapproval of other people. For sociologists (see Elster, 1989), actions of agents can be influenced by rationality, social norms or a compromise between both. However, economists do not share this view. Indeed, they rather consider that agents maximize utility under constraints, following what they call rationality. In other words, conforming to the norm does not mean irrationality. Moreover, since there is a cost of deviation from the norm, in terms of loss in reputation involving a loss in utility, people may be happy to conform to the established norm (Akerlof, 1980). Even if we still do not know (exactly) how norms are formed (tradition or construction), we do know that possibility of punishment is a way to maintain the norm (this has been tested empirically, see Fehr and Fischbacher, 2004), and this is done best inside a community, thanks to repeated interactions[1].

This allows us to state our point: inside a community, agents share interpersonal relationships, in the form of social capital, and interact repeatedly. If a norm exists inside such a community, then deviants are easily recognized and punished, which leads to the sustainablity of a norm. If someone wants to escape from a norm, he may have to change communities, requiring then to invest in social assets in order to create new social capital. The negative effect of interpersonal relationships arises precisely there: social capital (or at least the non transferable part of it) imposes a cost on agents who want to move (through the investment in social assets needed to enter the new community), which may lead to the fact that agents do not move in the end, even if consuming the norm of a group is not their first choice.

The process of moving is to a certain extent similar to what is described by Tiebout (1956), but also to the literature on club theory and group formation. In the Tiebout model, agents move to the jurisdiction in which the tax-public good package is the closest to their preferences, thereby creating homogenous jurisdictions. Although the underlying mechanism of our model is clearly linked to that of Tiebout, there are many differences: the consump-

[1] A question which remains open is to find out whether the community imposes a norm to agents who belong to the group, or whether it is the fact that agents share a given norm that creates the community.

tion norm is endogenous, we include the moving cost in the analysis, described here as the investment in social assets needed to enter into a new community, we describe the social environment of the agents and the type of good is not a public good, but can rather be referred to as private consumption. Concerning club theory (Buchanan, 1965), this essentially says that agents form clubs on a voluntary basis in order to lower the cost of production of an unpure public good, while enjoying its consumption. In our case, agents inside a community do not get together for joint production, but are forced to consume a joint norm. Hence, the community (the club) does not *do something for me*, but rather, *prescribes or proscribes my consumption patterns*[2]. Finally, our model is close to the literature on group formation. In this literature, there are two main reasons for group formation: either because individuals prefer to associate with people similar to them (Milchtaich and Winter, 2002), or because individuals prefer to associate with people making the same choice (Karni and Schmeidler, 1990). Our model combines both aspects: agents try to get into the community in which the norm is the closest to their type, meaning that the majority of people is similar to them. However, this taste for similarity does not appear directly in the utility function but indirectly through the norm and how it is defined.

3 The model

This section is organized as follows: subsection 3.1 describes the setup of the model. Subsections 3.2 and 3.3 analyze two different games: the first game considers a case in which agents must consume the norm of the community they belong to, while the second game allows for a consumption located between the norm and the "ideal consumption", i.e. the consumption witout social constraints. In each game, we study the various possible equilibria as well as welfare. In particular, we investigate whether the individual decision is socially optimal or not. Finally, subsection 3.4 studies the possibility for the State to intervene in order to solve

[2] Note that a community can be seen as providing benefits to its members. In that setting, the norm could be seen as a necessary cost to enjoy these benefits. However, in this paper, we concentrate on the norm and on the moving cost, and not on the community's benefits. This can be justified by saying that all communities generate the same benefits, so agents do not take them into account.

the socially suboptimal cases.

3.1 The setup

We consider an economy with n agents who are either of low (l) or high (h) type. There are n_l ($n_l \neq 0$) agents of type l and n_h ($n_h \neq 0$) agents of type h, with $n_h + n_l = n$. The economy is divided into two groups: A and B. There are n_A agents in group A and n_B agents in group B, with $n_A + n_B = n$. We define n_{ik} as the number of type i in group k ($i \in \{l, h\}, k \in \{A, B\}$) with $n_k = \sum_i n_{ik}$ and $n_i = \sum_k n_{ik}$.

An agent of type l (h) has preferences such that his *ideal consumption* (that is, without social constraint) is given by y_l (y_h).

Inside each community k, a norm $\overline{y}_k$ prevails, this norm being defined as the average of the types belonging to this community. Mathematically, if π_{ik} is the proportion of i type in group k ($i \in \{l, h\}, k \in \{A, B\}$), $\overline{y}_k = \pi_{lk} y_l + \pi_{hk} y_h$, with $\pi_{lk} + \pi_{hk} = 1$. Note that this definition is based on individuals who all have the same weight in terms of influence. Of course, it may be the case that part of the group is more influent than the rest. In that case, a proportion would not be interpreted as the relative size, but as relative power (or eventually both). This would make no real difference in the analysis, so we do not take it into account for the rest of the paper. Using a norm which is defined as an average can be debatable. However, this seems to us to be quite natural, and to reflect some kind of implicit negotiation process having taken place inside the community. Another possibility would be to use the median as the norm. This would be interpreted more like a majority rule in an election. This specification appears to be just a special case of the one we use and creates no significative difference in the results.

Agents are initially randomly distributed between the two groups, both types being present in both communities. An agent who belongs to community k owns an amount φ_k of social assets. This amount is independent of the composition of the community, and is the same for all agents who initially belong to a given community. In other words, agents are initially segregated with respect to social assets and not by type (for example, someone could have been born in a given community in which he grows up and of which he accumulates the social assets, but his type may be different from the type of others in the community). An agent can then be defined by a vector composed of his social assets and his type: (φ_k, y_i) $i \in \{l, h\}, k \in \{A, B\}$. Each agent decides whether to move to the other group or not by comparing his utility in each group, taking into account a moving cost C. This cost is assumed to be symmetric, in the sense that an agent located in A has to bear the same cost to go to B as an agent located in B who wants to move to A. The moving cost in fact corresponds to social costs, that is the investment in social assets needed to be accepted in the new community. Hence, the moving cost is fixed, but only between two given communities. Indeed, if two communities are close in the sense that the social assets of their members are similar (vectors φ_A and φ_B are similar), then the cost of moving, that is the investment in social assets, is small. On the contrary, if the two vectors are very different, meaning that there are almost no similarities, there is a high moving cost. To be precise, we should then write the cost as a function of two vectors of social assets $C(\varphi_k, \varphi_l)$. However, since we consider only two communities here, for simplicity we write C to represent the cost of moving between the two communities. Without loss of generality, we assume that $y_l < y_h$ and $\pi_{lA} \geq \pi_{lB}$. These two last assumptions imply that $\overline{y}_A \leq \overline{y}_B$. The static game works as follows: agents compare the utility they have with the utility that they would obtain by moving (taking into account the moving cost) to another community, and decide whether to move or not. Hence, besides the consumption choice, there are two possible actions a_j for player j:

$$a_j \in \{move, not\ move\}.$$

We assume that players have a price taking behavior, they consider that their behavior has no impact on the norm. Each agent decides to move or not given the distribution of types. We then aim to find a Nash equilibrium of this game, i.e. a pair

$$(\pi_{lA}^*, \pi_{lB}^*) \in [0,1] \times [0,1]$$

such that

$$a_j^* = not\ move \quad \forall j$$

The goal is of course to find out if the equilibirum will be such that the segregation in terms of social assets is replaced by a segregation in terms of types. For example, is it possible that all agents who wants to get educated move to the same community, allowing them to reach the educational level they want, without being punished by those who do not want to get educated? Hence, in our setup, segregation of types is not seen as a negative result, but rather as welfare maximizing. It is important to keep in mind that there is no vertical differentiation between types, that is, being a high type agent is not better than being a low type agent. The point of the paper is to investigate under which conditions an agent can escape from the social pressure of his community and to acquire a consumption corresponding to his type.

3.2 The Enforced Consumption Game (ECG)

We start by analyzing agents' behavior in the most simple possible setting in which all agents who belong to a community are forced to consume the norm: each member of group k has to consume the norm of the group $\overline{y}_k$. Although this case is relatively unrealistic, it will help us to understand some features of the effect of social norms on individuals. This setting can be thought of as some kind of central authority imposing an infinite cost on agents who deviate from the norm (for example a life long prison sentence), or equivalently the authority making the level of the norm the only available quantity of an indivisible good (in team sports for example, in which the number of training sessions is fixed and compulsory to be part of the team).

In the ECG, the utility of an agent of type i who belongs to group k is given by

$$U_{ik} = -\left|y_i - \overline{y}_k\right|$$

where y_i is not chosen, but corresponds to the ideal consumption of a type i. This linear utility function means that an agent wants to minimize the distance between the norm and his own type. The utility of an agent is maximized when the norm is equal to his own type, that is when only individuals of the same type are part of the community. In this setting, having a full segregation by type is welfare maximizing. However, since the consumption choice is reduced to consuming the norm of his community, the level of an agent's utility is constant inside a group, which does not mean that he has no decision power. Indeed, what an agent can always choose to do is to move to another community, in order to be located somewhere where the norm is closer to his own type, taking into account the fact that there is a moving cost. Hence, when deciding to move or not, an agent i who belongs to group k must compare

$$-\left|y_i - \overline{y}_k\right| \text{ to } -\left|y_i - \overline{y}_{-k}\right| - C$$

Lemma 1 *In the Enforced Consumption Game, if no group is perfectly homogenous and if an agent of type i in k decides to move, then an agent of type $-i$ in $-k$ also decides to move.*

Proof. See appendix. ■

This result is due to the functional form of the utility function. Basically, it says that since utility is linear, agents are interested in the distance between the two norms and the moving cost, by assumption the same for all agents. Hence, agents will decide to move if the difference between the two norms (benefit of moving) is high enough compared to the cost of moving (C). It is easy to see that only the high types of A and the low types of B may have an incentive to move. Let us now define an equilibrium in this game.

Definition 4 *An equilibrium in the Enforced Consumption Game is a pair (π^*_{lA},π^*_{lB}) such that*

$$-\left|y_i - \pi^*_{ik} y_i - \pi^*_{-ik} y_{-i}\right| > -\left|y_i - \pi^*_{i-k} y_i - \pi^*_{-i-k} y_{-i}\right| - C$$

$\forall i \in \{l, h\} \, \forall k \in \{A, B\}$ *and i belongs to k*

This definition is quite intuitive. It simply states that an equilibrium is reached as soon as nobody wants to move anymore, i.e. when the cost of moving is higher than the benefit of moving. Once again, we want to stress that this definition relies on the assumption that when deciding whether to move or not, agents do not take into account the fact that if they decide to go to the other community, the norm may be modified.

From this definition, we expect different kinds of equilibria to arise. If the two communities are composed identically, then the norm is the same and there is no incentive to bear the moving cost. From the point of view of the norm, it is as if the two groups were to merge into a unique community. This is why we call this equilibrium a *merging equilibrium*. If, on the other hand, the distribution of types is not the same accross groups, then the norms are different and agents will move depending on the moving cost: if this cost is too high, nobody moves and we have a *mixed equilibrium*. Finally, if we have one type of agents in each community, then the utility of each agent is maximum and nobody wants to move: we are in a segregating equilibrium. This can be summarized as follows:

Proposition 1 *With moving costs, there are three possible types of equilibria:*

1) Merging equilibria when $\pi_{lA}^* = \pi_{lB}^*$, *with* $\pi_{lk}^* \in (0,1)\, \forall k \in \{A, B\}$

2) A segregating equilibrium when $\pi_{lA}^* = 1$ *and* $\pi_{lB}^* = 0$

3) Mixed equilibria when $\pi_{lA}^* - \pi_{lB}^* \leqslant -\dfrac{C}{y_l - y_h}$

Proof. See appendix. ■

As can be seen from Proposition 1, the equilibrium that prevails is directly dependent on the distribution and the moving cost. In the first case, the distribution is the same in the two communities, implying that the norm is the same hence *agents do not want to move*. In the third case, the moving cost is too high with respect to the benefit of moving, resulting in a situation in which no community is perfectly segregated by types. Finally, in the segregating equilibrium, there is only one type of agents in each community, implying that the norm in each community is equal to the unique type composing this community. Hence, the utility of each agent is maximum and nobody wants to move. In this case, the social segregation linked to the existence of social assets and communities is replaced by a segregation in types. Note that in a mixed equilibrium, as well as in the merging equilibrium case, social segregation does exist.

Two remarks have to be made at this point: first, if there is no moving cost ($C = 0$), then the mixed equilibrium disappears and either the distribution is the same, and we have a merging equilibrium, or the distribution is not the same, and we move towards the segregating equilibrium. Second, the ECG does not allow for any asymmetry in the decision of moving, i.e. if one side decides to move (let say the low types agents in B), then the other side (the high types agents in A) also moves. This is of course not very realistic, and the possibility of asymmetry will be introduced in the next game, the Peer Pressure Game.

Proposition 2 $\forall$ *(π^*_{lA},π^*_{lB})* $\in [0,1]\times[0,1]$*, in the ECG, the aggregate level of the consumption of y (Y) in the economy is always the same and is equal to $n_l y_l + n_h y_h$.*

Proof. See appendix. ■

From Proposition 2, the aggregate level depends only on the population of the economy, and not on how agents are distributed between groups. This means that there are only distributional effects, in the sense that if the State decides to intervene, such a policy would not affect the aggregate level, but may affect the utility of the population. Remember the example of education. This would mean that the aggregate level of education remains constant, but by making it possible for agents to segregate, being then able to do what they want, the utility of agents may increase. Let us now examine the different equilibria in terms of social welfare using the utilitarian definition of the term[3]. Obviously, given the utility function and the way the norm is defined, the segregating equilibrium is welfare maximizing. Hence, in the two other types of equilibrium, total welfare is lower. The question is then to find out whether these two equilibria could be replaced by a segregating one in such a way that, when taking the moving cost into account, the aggregate benefit of moving is higher than its aggregate cost. In other words, we need to investigate whether the individual decision to move or not is optimal with respect to the corresponding aggregate cost and aggregate benefit. If not, the aggregate benefit of moving towards another equilibrium would be higher than the cost, so there is room for State intervention. Let us define W as the total Social

[3] A Social Welfare Function $W(u)$ is (purely) utilitarian if it has the form $W(u)=\sum_i u_i$

Welfare in the economy

$$W = \sum_i \sum_k n_{ik} U_{ik} - \sum_{j=1}^{n} C_j \qquad \forall i \in \{l, h\} \, \forall k \in \{A, B\}$$

$$\text{with } C_j = \begin{cases} C & \text{if} \quad \text{agent } j \text{ moves} \\ 0 & \text{if} \quad \text{agent } j \text{ does not move} \end{cases}$$

Total Social Welfare in a mixed equilibrium or in a merging equilibrium is given by

$$W^0 = -2(y_h - y_l)\,(n_l - n_{lA}\pi_{lA} - n_{lB}\pi_{lB})$$

Total Social Welfare when agents move away from a mixed equilibrium or a merging equilibrium in such a way that groups are segregated is given by

$$W^{SE} = -(n_{lB} + n_{hA})C$$

To study welfare properties, we need the following definition:

Definition 5 (Social Welfare Improving) *Let (π^1_{lA}, π^1_{lB}) and (π^2_{lA}, π^2_{lB}) be two different equilibria. Going from (π^1_{lA}, π^1_{lB}) to (π^2_{lA}, π^2_{lB}) is Social Welfare Improving (SWI) iff*

$$\sum_{j=1}^{n} U^2_j - \sum_{j=1}^{n} U^1_j - \sum_{j=1}^{n} C^{1\to 2}_j > 0 \tag{1}$$

$$\textit{with } C^{1\to 2}_j = \begin{cases} C & \textit{if} \quad \textit{agent } j \textit{ moves between } 1 \textit{ and } 2 \\ 0 & \textit{if} \quad \textit{agent } j \textit{ does not move} \end{cases}$$

with U^i_j the utility of agent j in distribution i

We now investigate under which conditions it is Social Welfare Improving to move to segregated groups, and whether agents individually take the socially optimal decision of moving or not. Before going to the Proposition, we need the following two lemmas.

Lemma 2 *It is Social Welfare Improving (SWI) to move to segregated groups iff*

$$\frac{C}{y_h - y_l} < \frac{2\left(n_l - n_{lA}\pi_{lA} - n_{lB}\pi_{lB}\right)}{\left(n_{lB} + n_{hA}\right)}$$

Lemma 3 *The individual benefit of moving is always strictly smaller than the social benefit of moving, that is*

$$\pi_{lA} - \pi_{lB} < \frac{2\left(n_l - n_{lA}\pi_{lA} - n_{lB}\pi_{lB}\right)}{\left(n_{lB} + n_{hA}\right)}$$

Proof. See appendix. ■

This leads to our third proposition.

Proposition 3 *1) In a segregating equilibrium, welfare is maximized and the equilibrium is socially optimal*

2) In a merging equilibrium or in a mixed equilibrium, agents individually take the socially optimal decision of not moving if $\frac{C}{y_h - y_l} \geqslant \frac{2\left(n_l - n_{lA}\pi_{lA} - n_{lB}\pi_{lB}\right)}{\left(n_{lB} + n_{hA}\right)}$.

3) In a merging equilibrium or in a mixed equilibrium, agents individually take the socially suboptimal decision of not moving if $\pi_{lA} - \pi_{lB} \leqslant \frac{C}{y_h - y_l} < \frac{2\left(n_l - n_{lA}\pi_{lA} - n_{lB}\pi_{lB}\right)}{\left(n_{lB} + n_{hA}\right)}$. *In this case, moving to segregated groups would improve social welfare.*

Proof. From Proposition 1, Lemmas 2 and 3. ■

This proposition states that in a segregating equilibrium, the utility of each agent is maximum, implying that total welfare is maximized. Obviously, this equilibrium is then socially optimal. If, on the other hand, a mixed equilibirum or a merging equilibrium arises, these are socially optimal or not depending on the size of the moving cost. In 2), the cost is too high so that moving to a segregating equilibrium would decrease welfare. In the third case, the cost of moving is too high so agents do not take the decision of moving by themselves. However, it would be Social Welfare Improving to move to segregation. In that case, agents take a suboptimal decision, in the sense that they are not able to break social segregation on their own, while it would be optimal from an aggregate point of view. Hence, State intervention is needed. The possibility for the State to intervene in order to solve this inefficiency is explored in section 3.4. Note that Proposition 3 is based on a relative cost, that is, the moving cost divided by $y_h - y_l$. When the latter increases, everything else being equal, agents are further from their ideal consumption, meaning that the relative cost of moving decreases.

3.3 The Peer Pressure Game (PPG)

As outlined above, the ECG has two unrealistic properties. First, agents have to consume the norm of the group, and do not maximize utility by choosing a consumption level, but only by choosing a community. A model where both maximizations take place would be more appropriate, in particular in order to take into account the cost of deviation from the norm. Second, the linearity of the utility function of the ECG does not allow for asymmetry in the moving decision. A more general model where both symmetric and asymmetric moving decisions can exist would be more realistic. Hence, we use a new utility function in this Peer Pressure Game to take account of the above mentioned remarks.

$$U_{ik} = -(1-\alpha)(y_i - y_{c,ik})^2 - \alpha(\overline{y}_k - y_{c,ik})^2$$

with $\alpha \in (0,1)$. In this game, an agent i in group k does not have to consume the norm of the group, but instead, he will choose $y_{c,ik}$ in order to maximize his utility. The utility function is divided into two parts: the first one measures the cost of deviation that an agent has to bear when consuming something different from his ideal consumption. The second part measures the cost of deviation from the norm (social sanction). Thus, an agent tries to minimize both deviation costs when choosing his consumption (one may think of examples such as education or religion in some communities where agents choose something between what the community prescribes and their own preferences). In this function, α measures the strength of the norm on the individual. When α tends to one, the social pressure is so strong that the agent consumes exactly the norm. When α tends to 0, the social constraint plays no role, and the agent consumes his ideal level. Note that if the community is perfectly homogenous, then as in the ECG the norm is exactly equal to the type, which is equal to the ideal consumption of agents, meaning that utility will be maximized.

The remainder of section 3.3 is organized as follows: in subsection 3.3.1, we investigate individual behavior, showing that, in this specification, there is a possibility of asymmetry in the moving decision which may lead to a new type of equilibrium in which one community is composed of only one type of agents, while the other community is composed of both types. In subsection 3.3.2, we study how the previous result affects welfare and find that a non socially optimal individual decision remains possible. This subsection concentrates on the results; all technical developments are in the appendix. Finally, in 3.3.3, we discuss the results and the underlying assumptions.

3.3.1 Individual behavior

The consumption which maximizes the utility function given above is

$$y^*_{c,ik} = (1-\alpha)y_i + \alpha\overline{y}_k$$

With this level of consumption, the utility level is

$$U^*_{ik} = -\alpha(1-\alpha)\left[y_i - \overline{y}_k\right]^2$$

Proposition 4 $\forall$ $(\pi^*_{lA}, \pi^*_{lB}) \in [0,1] \times [0,1]$, *the aggregate level of y in the Peer Pressure Game is the same as in the Enforced Consumption Game*

Proof. See appendix ■

This result says that, in the same way as in the ECG, the aggregate level of y is always the same, whatever the distribution of types across communities. Hence, as in the ECG, there are only distributional effects, that is when modifying the distribution of types, Y does not change, but individual as well as total welfare may be modified. We now need to determine the individual moving decision rule.

Definition 6 $\forall i \in \{l,h\} \forall k \in \{A,B\}$,

1) The set of agents of type i in k is a minority iff $\pi_{ik} < \pi_{-ik}$

2) The set of agents of type i in k is a relative minority iff $\pi_{ik} < \pi_{i-k}$

What matters in the decision rule is the relative minority, that is how small the set of individuals of one type is with respect to the set of those of the same type in the other group. This means that a majority inside a group may want to move because it is a relative minority compared to the same type of individuals in the other group.

Lemma 4 *1) Among agents initially located in A, only those of high type may want to move to B, and they will do so if and only if*

$$(\pi_{lA} - \pi_{lB})(\pi_{lB} + \pi_{lA}) > \frac{C}{\alpha(1-\alpha)(y_l - y_h)^2}$$

2) Among agents initially located in B, only those of low type may want to move to A, and they will do so if and only if

$$(\pi_{lA} - \pi_{lB})\left[2 - (\pi_{lB} + \pi_{lA})\right] > \frac{C}{\alpha(1-\alpha)(y_l - y_h)^2}$$

Proof. See appendix ■

Three important features of this lemma should be outlined: first, as in the ECG, only the relative minority of each community (agents of low type in B and agents of high type in A) may want to move. This obviously comes from the assumption on the distribution of low types accross groups, and from the way the norm is defined. Second, the cost to which the benefit is compared can be seen as a relative cost, in the same way as in the previous game. The interpretation of this remains that the bigger the difference between the type, the worse off agents are (everything else being equal), the smaller the relative cost of moving (or equivalently, the larger the benefit of moving). Finally, and more importantly, there is now a possibility of asymmetry in the moving decision rule. Indeed, as can be seen from lemma 4, the benefit of moving may be different depending on which community agents belong to. Moreover, the community which enjoys the larger benefit depends on the distribution of types, since this has an impact on which proportion is relatively the smallest. This is summarized in the following lemma.

Lemma 5 *1) If $\pi_{lB} + \pi_{lA} = 1$, the distribution of types in each group is symmetric with* *respect to relative majority and relative minority, i.e.* $\pi_{lA} = \pi_{hB}$ *and* $\pi_{hA} = \pi_{lB}$.

2) If $\pi_{lB} + \pi_{lA} > 1$, *the distribution of types in each group is asymmetric and* π_{hA} *is the smallest relative minority (and also the smallest proportion if* $\pi_{lB} \neq \pi_{lA}$*).*

3) If $\pi_{lB} + \pi_{lA} < 1$, *the distribution of types in each group is asymmetric and* π_{lB} *is the smallest relative minority (and also the smallest proportion if* $\pi_{lB} \neq \pi_{lA}$*).*

Proof. See appendix ■

Having a symmetric distribution of types in each group means that the relative majority in A is equal to the relative majority in B and the same for both minorities. In this case, both moving decision rules are equivalent, meaning that we are brought back to the ECG in the sense that if one side decides to move, the other side will also move. If the distribution of types is not symmetric, relative minorities in each community are not equal, since one is smaller than the other. In that case, agents who belong to the group in which their type represents the smallest proportion in the economy are actually the furthest from their ideal consumption, meaning that those people will have more incentives to move than those who belong to the other relative minority. In other words, depending on the distribution of types and on the cost of moving, new equilibria may arise, since the moving decision is no longer symmetric.

In this game, an equilibrium is defined the following way

Definition 7 *An equilibrium in the Peer Pressure Game is a pair* (π^*_{lA},π^*_{lB}) *such that*

$$-\alpha\left(1-\alpha\right)\left[y_i - \pi^*_{ik}y_i - \pi^*_{-ik}y_{-i}\right]^2 > -\alpha\left(1-\alpha\right)\left[y_i - \pi^*_{i-k}y_i - \pi^*_{-i-k}y_{-i}\right]^2 - C$$

$\forall i \in \{l,h\}\, \forall k \in \{A,B\}$ *and* i *belongs to* k

On top of the three types of equilibria already described in the ECG, a new one exists in this game. It is such that one community is composed of a single type and the other one of both types. We call this a *semi-mixed equilibrium*. The proposition below details all of these possible equilibria. For simplicity, we define a new parameter representing the relative moving cost: $\delta = \dfrac{C}{\alpha(1-\alpha)(y_l - y_h)^2}$.

Proposition 5 *With moving costs, there are four possible types of equilibria:*

1) Merging equilibria when $\pi^*_{lA} = \pi^*_{lB}$*, with* $\pi^*_{lk} \in (0,1)\, \forall k \in \{A, B\}$

2) A segregating equilibrium when $\pi^*_{lA} = 1$ *and* $\pi^*_{lB} = 0$

3) Mixed equilibria when

a) $\pi^*_{lB} + \pi^*_{lA} = 1$ *and* $(\pi^*_{lA} - \pi^*_{lB}) \leqslant \delta$

b) $\pi^*_{lB} + \pi^*_{lA} < 1$ *and* $(\pi^*_{lA} - \pi^*_{lB})\,[2 - (\pi^*_{lB} + \pi^*_{lA})] \leqslant \delta$

c) $\pi^*_{lB} + \pi^*_{lA} > 1$ *and* $(\pi^*_{lA} - \pi^*_{lB})(\pi^*_{lB} + \pi^*_{lA}) \leqslant \delta$

4) Semi-mixed equilibria when $\pi^*_{lB} = 0$ *and* $\pi^*_{lA} \in (0,1)$

Proof. See appendix ■

Points 1), 2) and 3) of Proposition 5 are similar to the results of Proposition 1 in the ECG. There is a merging equilibrium when both norms are equal, a mixed equilibrium in which both types are present in each community when the moving cost is too high, and a segregating equilibrium. Point 4) deals with the asymmetric case in which one community is now composed of a unique type, but the other community is still formed by two types, the moving cost being such that it is too costly for the minority to move to the other community.

These various situations are summarized in Figure I.

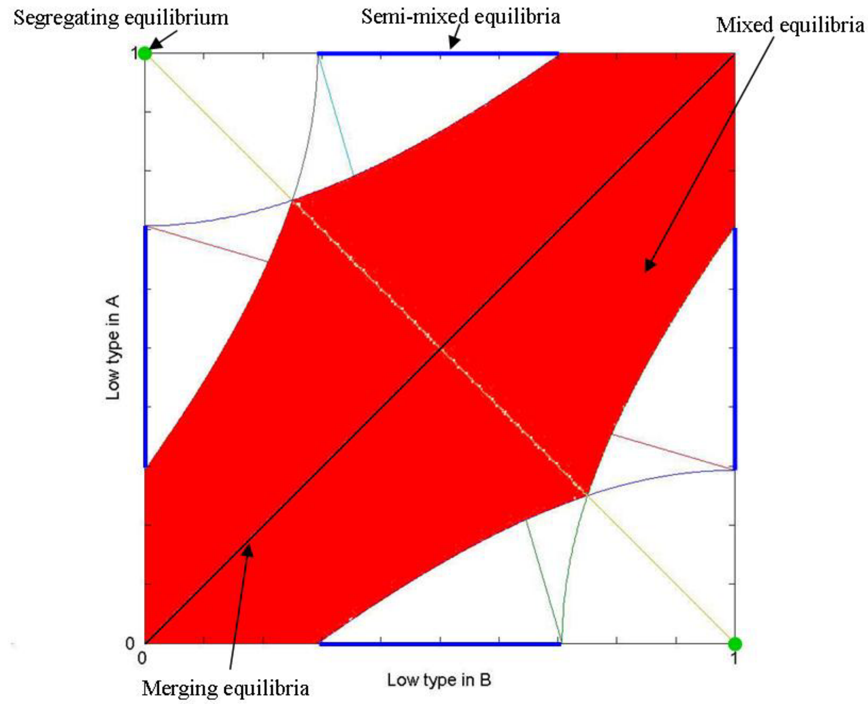

Figure I: Equilibria

Figure I represents all of the possible equilibria taking moving costs into account. This figure is designed under the assumption that $n_A = n_B = \dfrac{n}{2}$ and with an arbitrary moving cost $\delta = 0.5$. The vertical axis yields the proportion of low types in A, and the horizontal axis, the proportion of low types in B. The upward sloping diagonal line is the set of distributions of types such that a merging equilibrium arises, since proportions of low types are equal in the two communities. Note that our model is based on the assumption that $\pi_{lA} \geq \pi_{lB}$. Hence, our results are limited to the area above and on the upward sloping diagonal line. However, for reasons of symmetry, we can interpret the lower area as well. The green points correspond to the segregating equilibria, while the red area is the set of all mixed equilibria. The semi-mixed equilibria are located on the blue line. Finally, the downward sloping diagonal line is made of pairs corresponding to a symmetry in the distribution of types, so the moving decision is common to both relative minorities. In the segregating equilibrium, we end up with a segregation in types. On the contrary, in the mixed, semi-mixed, and merging equilibria, social segregation is still present (partially in the semi-mixed equilibrium).

3.3.2 Welfare and social optimality

In the previous subsection, we found that the equilibrium consumption will be located between the norm the "ideal" consumption, that there may be asymmetry in the moving decision and that this leads to a new type of equilibrium in which one community is composed of a single type of agents, while the other is composed of both types. What about welfare? As in the previous section, given the utility function we use and the way the norm is defined, the segregating equilibrium is still welfare maximizing. The question is then to find out if it is possible to make people move from one type of equilibrium to the segregating one, while increasing welfare, taking into account the moving cost. If so, then agents may take a decision that is individually optimal, while socially suboptimal. This possibility is analyzed in the following proposition.

Proposition 6 *1) In the segregating equilibrium, the distribution of types is welfare maximizing and socially optimal.*

2) In a mixed or a merging equilibrium, the distribution of types may either be socially optimal or socially suboptimal. In the suboptimal cases, either the segregating equilibrium or a semi-mixed equilibrium is socially optimal.

3) In a semi-mixed equilibrium, the distribution of types may either be socially optimal or socially suboptimal. In the suboptimal cases, the segregating equilibrium is socially optimal.

Proof. See appendix ■

This proposition deals with the optimality versus suboptimality of indivual decisions. It states that, with the exception of the segregating type equilibrium, all other types of equilibria allow for the possibility of suboptimality. This will happen when the cost is too high with respect to the benefit to be beared individually, while it would be socially a good thing to move to half or full segregation. It means that, as in the ECG, agents cannot themselves break the social segregation, while from an aggregate point of view, it would be better to have segregation in types. This can be seen in Figure II. Axes and diagonal lines have the same interpretation as in Figure I.

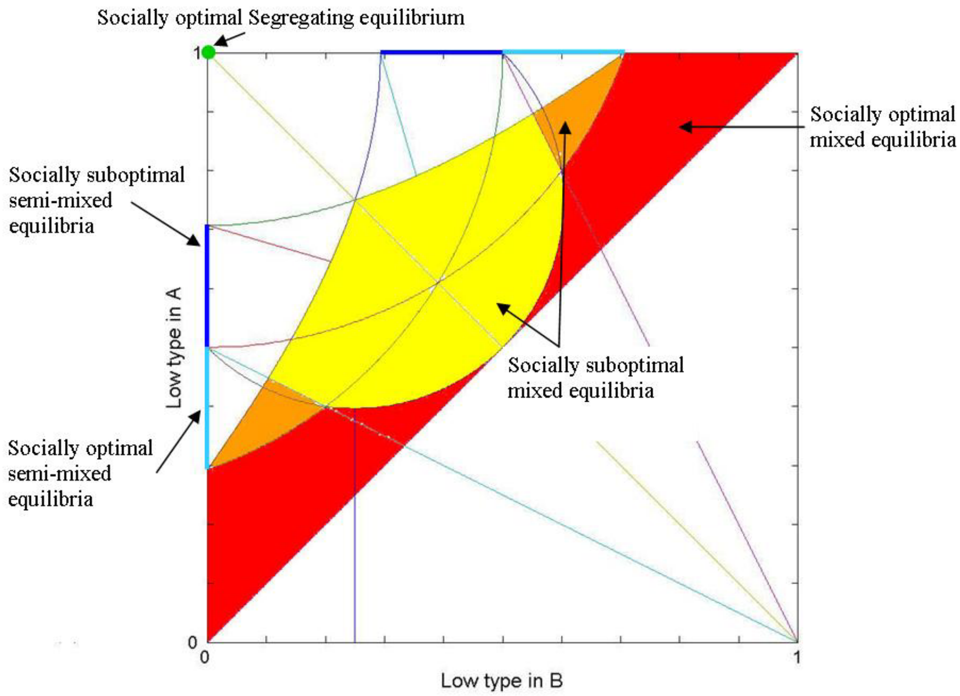

Figure II: Social optimality

The green point represents the segregating equilibrium in which agents individually take the optimal decision of moving to full segregation. The dark blue line represents the semi-mixed equilibria such that agents individually take the suboptimal decision of not moving to the segregating equilibrium. The light blue line represents the semi-mixed equilibria such that agents individually take the optimal decision not moving to the segregating equilibrium. The yellow area represents the mixed equilibria such that agents individually take the suboptimal decision of not moving to the segregating equilibrium. The orange area represents the mixed equilibria such that agents individually take the suboptimal decision of not moving to a semi-mixed equilibrium. The red area represents the mixed equilibria such that agents individually take the optimal decision of not moving.

3.3.3 Segregation as welfare maximizing

In our model, in the absence of moving costs, segregation by types is welfare maximizing. Indeed, in case of segregation by types, there is only one type of agent in each community, meaning that the norm in a community is exactly equal to the type composing the group, and hence that consumption is equal to the type. This means that agents value the homogeneity of the group (think of a religious community in which people prefer to interact only with believers). Of course, this assumption could be replaced by a taste of individuals for heterogeneity. This may be interpreted as some kind of complementarity between types, meaning that segregation by types would no longer be welfare maximizing. In our view, both assumptions are realistic, they simply describe different situations.

People often understand the term *segregation* as some kind of ordinal ranking between agents. It is important to underline that in our model, there exists no vertical differentiation between agents, which means that being a high type agent is not "better" than being a low type agent. Segregation by types is then not a bad thing.

As stated above, in the presence of moving costs modelled as social assets needed to enter into a new community, socially optimal equilibria may not be reached because the individual cost of moving is too high with respect to the private benefit. Moreover, if the moving cost is really high, then segregation by types may be not optimal at all. Analyzing these various situations is exactly the point of this paper: can social segregation be replaced by a segregation by types? In other words, the question is to find out to what extent, or under which circumstances, the social environment of an individual exerts a burden on him preventing him of choosing a consumption that fits his type or from joining another community where he could enjoy such a consumption. The results of our paper identify conditions under which these situations arise.

The results rely on a few assumptions. The first one is a symmetric cost of moving between the two communities. We do not consider this assumption to be unrealistic or too strong. Of course, it is possible that going from A to B is cheaper that going from B to A because, for example, B requests more general assets, that are shared by many communities, while A exhibits really specific assets. Modifying the assumption to take this into account would however not change the results qualitatively. The second assumption is that the type of a agent is independent of his social assets. More precisely, we assume that social assets are predetermined at the community level, independently of the composition of the community (a kind of historical inheritance), while the type of the agent is exogeneously given, independently of where he is born. Imposing this independence makes the problem easier to solve, and it is not completely unrealistic, especially in a static game. Of course, in a dynamic setting, it would be plausible that the social assets of a community evolve with the composition of the community, which would have an impact on the moving costs and thus on the arising equilibria. This type of evolution of some communities is observed in everyday life, but this phenomenon takes time, and would be better investigated in a dynamic setting.

3.4 State intervention and Pareto improvement

In the two previous sections, we showed that agents may take suboptimal decisions in the sense that they do not take a decision individually, while taking it would be social welfare improving. The reason for this is the moving cost which may be too high with respect to the private benefit, even if aggregate benefit is higher than aggregate cost. Equation (1) in section 3.2 defined the condition under which it is Social Welfare Improving to go from (π^1_{lA}, π^1_{lB}) to (π^2_{lA}, π^2_{lB}). Suppose that the SWI condition is satisfied between a given equilibrium and a segregating equilibrium, but no agents do move because of too high a cost. In this case, State intervention can correct this suboptimal situation by operating transfers between agents in order to decrease the individual cost.

The transfer scheme T $(\in \mathbb{R}^n)$ is the following: an agent j gets a transfer $t_j \in \mathbb{R}$, $j = 1, ..., n$ such that

$$t_j \begin{cases} > 0 & \text{if} \quad U^2_j - U^1_j - C^{1\to 2}_j \leq 0 \\ < 0 & \text{if} \quad U^2_j - U^1_j - C^{1\to 2}_j > 0 \end{cases} \tag{2}$$

$$\sum_{j=1}^{n} U^2_j - \sum_{j=1}^{n} U^1_j - \sum_{j=1}^{n} C^{1\to 2}_j + \sum_{j=1}^{n} t_j > 0 \tag{3}$$

$$U^2_j - U^1_j - C^{1\to 2}_j + t_j > 0 \tag{4}$$

$$\sum_{j=1}^{n} t_j \leq 0 \tag{5}$$

(3) ensures that it is still social welfare improving to go from (π^1_{lA}, π^1_{lB}) to (π^2_{lA}, π^2_{lB}) with

transfers, (5) says that the State cannot spend more than what it gets, and (4) means that, with transfers, agents take the decision of moving individually. (2) describes the structure of the transfers. This transfer scheme is possible since (1) means that the net aggregate benefit of going from (π^1_{lA}, π^1_{lB}) to (π^2_{lA}, π^2_{lB}) is strictly greater than the cost, meaning that this benefit is large enough to be taxed and used to cover the cost. It must be noted that (4) implies that this tranfer scheme not only solves the suboptimality problem, but also ensures that (π^2_{lA}, π^2_{lB}) is SWI as well as Pareto Improving (PI), which is outlined by the following lemma

Lemma 6 *If going from equilibrium* 1 *to equilibrium* 2 *is SWI, then there exists a transfer scheme* $T \in \mathbb{R}^n$ *such that it is also PI.*

Hence, the State can solve the problem of suboptimality of some equilibria, thereby killing the social segregation, and it can do it in such a way that the new equilibrium is also Pareto improving. This is stated in the next proposition.

Proposition 7 *When a socially suboptimal equilibrium is reached, the State can operate transfers between agents in such a way that a socially optimal equilibrium is reached and that this new equilibrium is also Pareto improving.*

Nevertheless, there exists cases in which the State cannot operate transfers. This happens when the aggregate cost of moving is higher than the aggregate benefit. In these situations, indivuals are stuck in a community in which the social constraint forces them to consume something that does not fit their types. In these cases, the social segregation problem cannot be solved.

4 Conclusion

We started this paper with the example of undereducation in some neighborhoods to suggest that social capital, a specific form of interpersonal relationships, may have a negative impact on agents. The basic mechanism worked in the following way: a community, characterized by specific social assets, constrains its members with a strong norm and a high deviation cost. However, it sometimes seems to be quite difficult for the members to leave their communities. Our paper proposes an explanation for this phenomenon, both intuitively and analytically, relying on the existing literature as well as an original analysis.

The main argument is that the moving cost may prevent agents from changing communities, and high deviation costs constrain agents' consumption. Hence, people are forced to follow the norm, without a possibility of escape.

Using a simple model with two types of agents, two groups and an endogenous norm, we show that, depending on the distribution of types accross communities and on the moving cost, different kinds of equilibria arise. Among them, two are of particular interest because either the whole population or part of it cannot bear the cost of moving individually although their preferences are such that they would like to do so. We stress that in these cases, State intervention may sometimes overcome this problem by operating transfers between agents to lighten the cost, and hence moving from a social segregation to a segregation in types. This result is of primary importance concerning efficiency of public policies: it is possible to design policies in order to allow people who want to get educated to join a community in which they will be able to do so without suffering from any punishment by their peers, leading to social welfare improvement. Unfortunately, it is sometimes impossible for the State to intervene because the aggregate cost is too high, because, for example, the two communities are really different in terms of social assets. Individuals are then stuck in their original community, being forced to consume something they do not want to consume because of the norm. In these cases, the social segregation cannot be replaced by a segregation in types. It is important to note that we do not encourage at all any segregation between clever and less gifted pupils, but rather segregation between those who want to get educated and those who do not, without taking intelligence into account.

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Appendices

A Proofs

A.1 Lemma 1

Proof. If i in k decides to move, we have that

$$-\left|y_i - \overline{y}_k\right| < -\left|y_i - \overline{y}_{-k}\right| - C$$

Suppose $y_i - \overline{y}_k > 0$. This means that $y_i - \overline{y}_{-k} > 0$, and hence

$$\begin{aligned} -y_i + \overline{y}_k &< -y_i + \overline{y}_{-k} - C \\ \overline{y}_k - \overline{y}_{-k} &< -C \end{aligned}$$

Since $y_i - \overline{y}_k > 0$, $y_{-i} - \overline{y}_{-k} < 0$ and $y_{-i} - \overline{y}_k < 0$. If an agent $-i$ in $-k$ decides to move, the following must be true

$$\begin{aligned} -\left|y_{-i} - \overline{y}_{-k}\right| &< -\left|y_{-i} - \overline{y}_k\right| - C \\ y_{-i} - \overline{y}_{-k} &< y_{-i} - \overline{y}_k - C \\ \overline{y}_k - \overline{y}_{-k} &< -C \end{aligned}$$

Hence, the decision rule is the same for the two agent. ■

A.2 Proposition 1

Proof. By Lemma 1, we know that in this game, decision rule of both agents are the same. Hence, for moving to take place and since by assumptions $\overline{y}_A \leq \overline{y}_B$, we must have

$$\begin{aligned}
\overline{y}_A - \overline{y}_B &< -C \\
\pi_{lA}y_l + \pi_{hA}y_h - (\pi_{lB}y_l + \pi_{hB}y_h) &< -C \\
(\pi_{lA} - \pi_{lB})y_l + (\pi_{hA} - \pi_{hB})y_h &< -C \\
(\pi_{lA} - \pi_{lB})y_l + (1 - \pi_{lA} - 1 + \pi_{lB})y_h &< -C \\
\pi_{lA}(y_l - y_h) - \pi_{lB}(y_l - y_h) &< -C \\
\pi_{lA} - \pi_{lB} &> -\frac{C}{y_l - y_h}
\end{aligned}$$

If $\pi^0_{lA} = \pi^0_{lB}$, this condition is violated, and no moving takes place (and norms are equal in both communities). ■

A.3 Proposition 2

Proof. The aggregate level of y (Y) is given by

$$\begin{aligned}
Y &= n_A\overline{y}_A + n_B\overline{y}_B \\
&= n_A\left(\pi_{lA}y_l + \pi_{hA}y_h\right) + n_B(\pi_{lB}y_l + \pi_{hB}y_h) \\
&= n_A\left(\frac{n_{lA}}{n_A}y_l + \frac{n_{hA}}{n_A}y_h\right) + n_B(\frac{n_{lB}}{n_B}y_l + \frac{n_{hB}}{n_B}y_h) \\
&= n_{lA}y_l + n_{hA}y_h + n_{lB}y_l + n_{hB}y_h \\
&= (n_{lA} + n_{lB})y_l + (n_{hA} + n_{hB})y_h \\
&= n_l y_l + n_h y_h
\end{aligned}$$

■

A.4 Lemma 3

Proof. First, remember that the individual moving rule was given by

$$\pi_{lA} - \pi_{lB} \gtrless \frac{C}{y_h - y_l}$$

We must compare $\pi_{lA} - \pi_{lB}$ and $\dfrac{2\,(n_l - n_{lA}\pi_{lA} - n_{lB}\pi_{lB})}{(n_{lB} + n_{hA})}$. Suppose that $\pi_{lA} - \pi_{lB} \geqslant \dfrac{2\,(n_l - n_{lA}\pi_{lA} - n_{lB}\pi_{lB})}{(n_{lB} + n_{hA})}$.

1) if $\pi_{lA} = \pi_{lB}$, then we have $0 \geqslant \dfrac{2\,(n_l - n_{lA}\pi_{lA} - n_{lB}\pi_{lB})}{(n_{lB} + n_{hA})}$ which is not possible.

2) if $\pi_{lA} > \pi_{lB}$:

$(\pi_{lA} - \pi_{lB})(n_{lB} + n_A - n_{lA}) \geqslant 2n_l - 2n_A\pi_{lA}^2 - 2n_B\pi_{lB}^2$

$n_{lB}\pi_{lA} + n_A\pi_{lA} - n_{lA}\pi_{lA} - n_{lB}\pi_{lB} - n_A\pi_{lB} + n_{lA}\pi_{lB} \geqslant 2n_l - 2n_A\pi_{lA}^2 - 2n_B\pi_{lB}^2$

$n_B\pi_{lA}\pi_{lB} + n_A\pi_{lA} - n_A\pi_{lA}^2 - n_B\pi_{lB}^2 - n_A\pi_{lB} + n_A\pi_{lA}\pi_{lB} \geqslant 2n_l - 2n_A\pi_{lA}^2 - 2n_B\pi_{lB}^2$

$n_A\pi_{lA}^2 + n_B\pi_{lB}^2 + n\pi_{lA}\pi_{lB} + n_A(\pi_{lA} - \pi_{lB}) \geqslant 2(n_A\pi_{lA} + n_B\pi_{lB})$

$n_A\pi_{lA}^2 + n_B\pi_{lB}^2 + n\pi_{lA}\pi_{lB} - n_A\pi_{lA} - n_A\pi_{lB} - 2n_B\pi_{lB} \geqslant 0$

$n_A\left[\pi_{lA}^2 + \pi_{lA}\pi_{lB} - \pi_{lA} - \pi_{lB}\right] + n_B\left[\pi_{lB}^2 + \pi_{lA}\pi_{lB} - 2\pi_{lB}\right] \geqslant 0$

$n_A\left[\pi_{lA}(\pi_{lA} + \pi_{lB}) - (\pi_{lA} + \pi_{lB})\right] \geqslant n_B\pi_{lB}\left[2 - (\pi_{lA} + \pi_{lB})\right]$

$n_A(\pi_{lA} + \pi_{lB})\,(\pi_{lA} - 1) \geqslant n_B\pi_{lB}\left[2 - (\pi_{lA} + \pi_{lB})\right]$

$\dfrac{n_A}{n_B} \leqslant \dfrac{\pi_{lB}}{\pi_{lA} - 1}\dfrac{2 - (\pi_{lA} + \pi_{lB})}{\pi_{lA} + \pi_{lB}}$ which is not possible ∎

A.5 Proposition 4

Proof.

$$
\begin{aligned}
Y &= n_{lA}\left[(1-\alpha)y_l + \alpha\overline{y}_A\right] + n_{hA}\left[(1-\alpha)y_h + \alpha\overline{y}_A\right] + n_{lB}\left[(1-\alpha)y_l + \alpha\overline{y}_B\right] \\
&\quad + n_{hB}\left[(1-\alpha)y_h + \alpha\overline{y}_B\right] \\
&= \left[n_{lA}(1-\alpha) + n_{lB}(1-\alpha)\right] y_l + \left[n_{hA}(1-\alpha) + n_{hB}(1-\alpha)\right] y_h + \left[n_{lA}\alpha + n_{hA}\alpha\right]\overline{y}_A \\
&\quad + \left[n_{lB}\alpha + n_{hB}\alpha\right]\overline{y}_B \\
&= (1-\alpha)(n_{lA} + n_{lB})y_l + (1-\alpha)(n_{hA} + n_{hB})y_h + \alpha(n_{lA} + n_{hA})\overline{y}_A + \alpha(n_{lB} + n_{hB})\overline{y}_B \\
&= (1-\alpha)n_l y_l + (1-\alpha)n_h y_h + \alpha n_A\overline{y}_A + \alpha n_B\overline{y}_B \\
&= n_l y_l + n_h y_h - \alpha\left[n_l y_l + n_h y_h\right] + \alpha\left[n_A\overline{y}_A + n_B\overline{y}_B\right] \\
&= n_l y_l + n_h y_h
\end{aligned}
$$

■

A.6 Lemma 4

Proof. 1) Agents decide to move from A to B if

$$
\begin{aligned}
-\alpha(1-\alpha)(y_i - \overline{y}_A)^2 &< -\alpha(1-\alpha)(y_i - \overline{y}_B)^2 - C \\
-\alpha(1-\alpha)(y_i - \overline{y}_A)^2 + \alpha(1-\alpha)(y_i - \overline{y}_B)^2 &< -C \\
\alpha(1-\alpha)\left[(y_i - \overline{y}_B)^2 - (y_i - \overline{y}_A)^2\right] &< -C \\
\alpha(1-\alpha)\left[y_i^2 - 2y_i\overline{y}_B + \overline{y}_B^2 - y_i^2 + 2y_i\overline{y}_A - \overline{y}_A^2\right] &< -C \\
\alpha(1-\alpha)\left[2y_i(\overline{y}_A - \overline{y}_B) - (\overline{y}_A^2 - \overline{y}_B^2)\right] &< -C \\
\alpha(1-\alpha)(\overline{y}_A - \overline{y}_B)\left[2y_i - (\overline{y}_A + \overline{y}_B)\right] &< -C \\
\alpha(1-\alpha)(\pi_{lA} - \pi_{lB})(y_l - y_h)\left[2y_i - (\pi_{lB} + \pi_{lA})(y_l - y_h) - 2y_h\right] &< -C \\
\alpha(1-\alpha)(\pi_{lA} - \pi_{lB})(y_l - y_h)\left[2(y_i - y_h) - (\pi_{lB} + \pi_{lA})(y_l - y_h)\right] &< -C
\end{aligned}
$$

If $i = l$

$$\alpha(1-\alpha)(y_l - y_h)^2(\pi_{lA} - \pi_{lB})\left[2 - (\pi_{lB} + \pi_{lA})\right] < -C$$

which is impossible. If $i = h$

$$\begin{aligned}
-\alpha(1-\alpha)(y_l - y_h)^2(\pi_{lA} - \pi_{lB})(\pi_{lB} + \pi_{lA}) &< -C \\
\alpha(1-\alpha)(y_l - y_h)^2(\pi_{lA} - \pi_{lB})(\pi_{lB} + \pi_{lA}) &> C \\
(\pi_{lA} - \pi_{lB})(\pi_{lB} + \pi_{lA}) &> \frac{C}{\alpha(1-\alpha)(y_l - y_h)^2}
\end{aligned}$$

2) Agents decide to move from B to A if

$$\begin{aligned}
-\alpha(1-\alpha)(y_i - \overline{y}_B)^2 &< -\alpha(1-\alpha)(y_i - \overline{y}_A)^2 - C \\
-\alpha(1-\alpha)(y_i - \overline{y}_B)^2 + \alpha(1-\alpha)(y_i - \overline{y}_A)^2 &< -C \\
\alpha(1-\alpha)\left[(y_i - \overline{y}_A)^2 - (y_i - \overline{y}_B)^2\right] &< -C \\
\alpha(1-\alpha)\left[y_i^2 - 2y_i\overline{y}_A + \overline{y}_A^2 - y_i^2 + 2y_i\overline{y}_B - \overline{y}_B^2\right] &< -C \\
\alpha(1-\alpha)\left[2y_i(\overline{y}_B - \overline{y}_A) - (\overline{y}_B^2 - \overline{y}_A^2)\right] &< -C \\
\alpha(1-\alpha)\left[2y_i(\overline{y}_B - \overline{y}_A) - (\overline{y}_B - \overline{y}_A)(\overline{y}_B + \overline{y}_A)\right] &< -C \\
\alpha(1-\alpha)(\overline{y}_B - \overline{y}_A)\left[2y_i - (\overline{y}_B + \overline{y}_A)\right] &< -C \\
\alpha(1-\alpha)(\pi_{lB} - \pi_{lA})(y_l - y_h)\left[2y_i - (\pi_{lB} + \pi_{lA})(y_l - y_h) - 2y_h\right] &< -C \\
\alpha(1-\alpha)(\pi_{lB} - \pi_{lA})(y_l - y_h)\left[2(y_i - y_h) - (\pi_{lB} + \pi_{lA})(y_l - y_h)\right] &< -C
\end{aligned}$$

if $i = h$

$$-\alpha(1-\alpha)(\pi_{lB} - \pi_{lA})(\pi_{lB} + \pi_{lA})(y_l - y_h)^2 < -C$$

which is impossible. If $i = l$

$$\begin{aligned}
\alpha(1-\alpha)(y_l - y_h)^2(\pi_{lB} - \pi_{lA})\left[2 - (\pi_{lB} + \pi_{lA})\right] &< -C \\
\alpha(1-\alpha)(y_l - y_h)^2(\pi_{lA} - \pi_{lB})\left[2 - (\pi_{lB} + \pi_{lA})\right] &> C \\
(\pi_{lA} - \pi_{lB})\left[2 - (\pi_{lB} + \pi_{lA})\right] &> \frac{C}{\alpha(1-\alpha)(y_l - y_h)^2}
\end{aligned}$$

■

A.7 Lemma 5

Proof.

$$\begin{aligned}
\pi_{lB} + \pi_{lA} &= 1 \\
\pi_{lB} &= 1 - \pi_{lA} = \pi_{hA} \\
\pi_{lA} &= 1 - \pi_{lB} = \pi_{hB}
\end{aligned}$$

If $\pi_{lB} + \pi_{lA} > 1$ and $\pi_{lA} \neq \pi_{lB}$, then π_{hA} is the smallest proportion. If $\pi_{lB} + \pi_{lA} < 1$ and $\pi_{lA} \neq \pi_{lB}$, then π_{lB} is the smallest proportion.

$$\begin{aligned}
\pi_{lA} &> \pi_{lB} \ (1) \\
1 - \pi_{hA} &> 1 - \pi_{hB} \\
\pi_{hB} &> \pi_{hA} \ (2) \\
\pi_{lB} + \pi_{lA} &> 1 \\
\pi_{lB} + 1 - \pi_{hA} &> 1 \\
\pi_{lB} &> \pi_{hA} \ (3) \\
\pi_{lB} + \pi_{lA} &< 1 \\
\pi_{lB} + 1 - \pi_{hA} &< 1 \\
\pi_{lB} &< \pi_{hA} \ (4)
\end{aligned}$$

$$\begin{aligned}
(1) + (3) &: \ \pi_{hA} < \pi_{lB} < \pi_{lA} \\
(2) &: \ \pi_{hA} < \pi_{hB}
\end{aligned}$$

$$
\begin{aligned}
(4)+(2) &: \quad \pi_{lB} < \pi_{hA} < \pi_{hB} \\
(1) &: \quad \pi_{lB} < \pi_{lA}
\end{aligned}
$$

■

A.8 Proposition 5

Proof. Starting from any distribution,

1) If $\pi_{lB} = \pi_{lA}$, then $\overline{y}_A = \overline{y}_B$ and no type has an incentive to bear the moving cost.

2) If $\pi_{lB} + \pi_{lA} = 1$, then the decision rule is the same for each type:

$if \quad \pi_{lA} - \pi_{lB} > \delta \quad then$ both relative minorities move

$if \quad \pi_{lA} - \pi_{lB} \leq \delta \quad then$ no one moves

3) If $\pi_{lB} + \pi_{lA} < 1$, then,

$$
(\pi_{lA} - \pi_{lB})\left[2 - (\pi_{lB} + \pi_{lA})\right] > (\pi_{lA} - \pi_{lB})(\pi_{lB} + \pi_{lA})
$$

a) if $\delta \geqslant (\pi_{lA} - \pi_{lB})\left[2 - (\pi_{lB} + \pi_{lA})\right] > (\pi_{lA} - \pi_{lB})(\pi_{lB} + \pi_{lA})$, then it is too expensive for both types to move, and no one moves

b) if $(\pi_{lA} - \pi_{lB})\left[2 - (\pi_{lB} + \pi_{lA})\right] > (\pi_{lA} - \pi_{lB})(\pi_{lB} + \pi_{lA}) > \delta$, then each type has an incentive to move, since cost is smaller than benefit

c) $\exists \delta > 0$:

$$
(\pi_{lA} - \pi_{lB})\left[2 - (\pi_{lB} + \pi_{lA})\right] > \delta \geqslant (\pi_{lA} - \pi_{lB})(\pi_{lB} + \pi_{lA})
$$

Hence, in a first step, all low types of B goes to A, which means that $\pi^1_{lB} = 0$ and π_{lA} increases. In a second step, high types of A will move to B iff

$$
\begin{aligned}
(\pi^1_{lA} - \pi^1_{lB})(\pi^1_{lB} + \pi^1_{lA}) &> \delta \\
\left(\pi^1_{lA}\right)^2 &> \delta
\end{aligned}
$$

$$\delta < \left(\frac{n_{lA}+n_{lB}}{n_A+n_{lB}}\right)^2$$

otherwise, high types in A stay there.

4) If $\pi_{lB}+\pi_{lA}>1$, we have

$$(\pi_{lA}-\pi_{lB})(\pi_{lB}+\pi_{lA}) > (\pi_{lA}-\pi_{lB})\left[2-(\pi_{lB}+\pi_{lA})\right]$$

a) if $\delta \geqslant (\pi_{lA}-\pi_{lB})(\pi_{lB}+\pi_{lA}) > (\pi_{lA}-\pi_{lB})\left[2-(\pi_{lB}+\pi_{lA})\right]$, then it is too expensive for both types to move, and no one moves

b) if $(\pi_{lA}-\pi_{lB})(\pi_{lB}+\pi_{lA}) > (\pi_{lA}-\pi_{lB})\left[2-(\pi_{lB}+\pi_{lA})\right] > \delta$, then each type has an incentive to move, since cost is smaller than benefit

c) $\exists\delta>0:$

$$(\pi_{lA}-\pi_{lB})(\pi_{lB}+\pi_{lA}) > \delta \geqslant (\pi_{lA}-\pi_{lB})\left[2-(\pi_{lB}+\pi_{lA})\right]$$

Hence, in a first step, all high types of A goes to be B, which means that $\pi^1_{hA}=0$ and $\pi^1_{lA}=1$. In a second step, low types of B will move to A iff

$$\begin{aligned}
\delta &< (\pi^1_{lA}-\pi^1_{lB})\left[2-(\pi^1_{lB}+\pi^1_{lA})\right]\\
\delta &< (1-\pi^1_{lB})\left[2-(\pi^1_{lB}+1)\right]\\
\delta &< (1-\pi^1_{lB})(1-\pi^1_{lB})\\
\delta &< \left(\frac{n_{hA}+n_{hB}}{n_B+n_{hA}}\right)^2
\end{aligned}$$

otherwise, low types in B stay there. ■

A.9 Proposition 6

Part 1

The total social welfare in a mixed equilibrium or in a merging equilibrium is given by

$$W^0 = -\alpha(1-\alpha)\left\{\left(n_l y_l^2 + n_h y_h^2\right) - \frac{\left(n_{lA} y_l + n_{hA} y_h\right)^2}{n_A} - \frac{\left(n_{lB} y_l + n_{hB} y_h\right)^2}{n_B}\right\}$$

while the total social welfare when everybody moves from a mixed equilibrium or a merging equilibrium to a segregating equilibrium is

$$W^{SE} = -(n_{hA} + n_{lB})C$$

Besides these two extreme cases, the PPG allows for semi-mixed equilibrium. For the rest of the proof, we use the term "full segregation" to refer to cases in which agents move from a mixed equilibrium or a merging equilibrium to a segregating equilibrium, and "half-segregation" for cases in which agents move from a mixed equilibrium or a merging equilibrium to a semi-mixed equilibrium. In the case of half-segregation, if $\pi_{lB} + \pi_{lA} > 1$, all high types in community A will move to community B and pay a moving cost equal to C. Hence, the norm in A will be $\overline{y}_A = y_l$, and the total social welfare amounts to

$$W^{half,h} = -\alpha(1-\alpha)\left\{\left(n_{lB} y_l^2 + n_h y_h^2\right) - \frac{\left(n_{lB} y_l + n_h y_h\right)^2}{n_{lB} + n_h}\right\} - n_{hA}C$$

On the other hand, if $\pi_{lB} + \pi_{lA} < 1$, all low types in community B will move to community A and pay a moving cost equal to C. Hence, the norm in B will be $\overline{y}_B = y_h$, and the total social welfare will be

$$W^{half,l} = -\alpha(1-\alpha)\left\{\left(n_l y_l^2 + n_{hA} y_h^2\right) - \frac{\left(n_l y_l + n_{hA} y_h\right)^2}{n_l + n_{hA}}\right\} - n_{lB}C$$

Finally, the total social welfare when agents move from a semi-mixed equilibrium to the segregating equilibrium is given by

$$W^{SE} = -n_{lB}C$$

when $\pi_{lB} + \pi_{lA} > 1$, and

$$W^{SE} = -n_{hA}C$$

when $\pi_{lB} + \pi_{lA} < 1$. Before going to the details of the welfare analysis, one more definition is necessary:

Definition 8 *Suppose moving from (π^1_{lA}, π^1_{lB}) to (π^v_{lA}, π^v_{lB}) is Social Welfare Improving (SWI) for all v such that (π^v_{lA}, π^v_{lB}) is an equilibrium. Then, moving from (π^1_{lA}, π^1_{lB}) to (π^2_{lA}, π^2_{lB}) is Socially Optimal (SO) iff*

$$\left(\sum_{j=1}^{n} U_j^2 - \sum_{j=1}^{n} C_j^{1\to 2}\right) - \left(\sum_{j=1}^{n} U_j^v - \sum_{j=1}^{n} C_j^{1\to v}\right) > 0$$

The difference between SWI and SO lies in the fact that SWI only requires total welfare to be higher at the new equilibrium, while SO imposes that social welfare is the highest among all the SWI equilibria. The three lemmas here under investigate the welfare property as well as the social optimality of half-segregation and full segregation.

Lemma 7 *It is SWI to move to half-segregation iff*

1) when $\pi_{lB} + \pi_{lA} > 1$, $\delta < \pi_{lA} - \pi_{lB}\widetilde{\pi}_{lB}$

2) when $\pi_{lB} + \pi_{lA} < 1$, $\delta < \pi_{hB} - \pi_{hA}\widetilde{\pi}_{hA}$

where $\widetilde{\pi}_{lB} = \dfrac{n_{lB}}{n_B + n_{hA}}$ and $\widetilde{\pi}_{hA} = \dfrac{n_{hA}}{n_A + n_{lB}}$

Proof. Social optimality of half segregation if $\pi_{lB} + \pi_{lA} > 1:$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{(n_{lB}y_l^2 + n_h y_h^2) - \frac{(n_{lB}y_l + n_h y_h)^2}{n_{lB} + n_h}\right\} - n_{hA}C$$

$$> -\alpha(1-\alpha)\left\{(n_l y_l^2 + n_h y_h^2) - \frac{(n_{lA}y_l + n_{hA}y_h)^2}{n_A} - \frac{(n_{lB}y_l + n_{hB}y_h)^2}{n_B}\right\}$$

$$\Leftrightarrow (n_{lB}y_l^2 + n_h y_h^2) - \frac{(n_{lB}y_l + n_h y_h)^2}{n_{lB} + n_h} + n_{hA}\frac{C}{\theta} - (n_l y_l^2 + n_h y_h^2) + \frac{(n_{lA}y_l + n_{hA}y_h)^2}{n_A}$$

$$+\frac{(n_{lB}y_l + n_{hB}y_h)^2}{n_B} < 0$$

$$\Leftrightarrow (n_{lB} - n_l)y_l^2 - \frac{(n_{lB}y_l + n_h y_h)^2}{n_{lB} + n_h} + \frac{(n_{lA}y_l + n_{hA}y_h)^2}{n_A} + \frac{(n_{lB}y_l + n_{hB}y_h)^2}{n_B} + n_{hA}\frac{C}{\theta} < 0$$

$$\Leftrightarrow n_{hA}\left(\frac{C}{\theta} - \frac{(y_l - y_h)^2\left(n_{hA}\left(n_{lA} - n_{lB}\right)n_{lB} + n_{lA}\left(n_{hA} + 2n_{lB}\right)n_{hB} + n_{lA}n_{hB}^2\right)}{\left(n_{lA} + n_{hA}\right)\left(n_{lB} + n_{hB}\right)\left(n_{hA} + n_{lB} + n_{hB}\right)}\right) < 0$$

$$\Leftrightarrow \frac{C}{\theta} < \frac{(y_l - y_h)^2 \left(n_{hA}\left(n_{lA} - n_{lB}\right) n_{lB} + n_{lA}\left(n_{hA} + 2n_{lB}\right) n_{hB} + n_{lA} n_{hB}^2\right)}{\left(n_{lA} + n_{hA}\right)\left(n_{lB} + n_{hB}\right)\left(n_{hA} + n_{lB} + n_{hB}\right)}$$
$$\Leftrightarrow \frac{C}{\theta\left(y_l - y_h\right)^2} < \frac{n_{lA} n_{hA} n_{lB} - n_{hA} n_{lB}^2 + n_{lA} n_{hA} n_{hB} + 2 n_{lA} n_{lB} n_{hB} + n_{lA} n_{hB}^2}{\left(n_{lA} + n_{hA}\right)\left(n_{lB} + n_{hB}\right)\left(n_{hA} + n_{lB} + n_{hB}\right)}$$
$$\Leftrightarrow \delta < \frac{n_{lA}}{n_A} - \frac{n_{lB}^2}{n_B\left(n_A + n_B - n_{lA}\right)} = \frac{n_{lA}}{n_A} - \frac{n_{lB}}{n_B}\frac{n_{lB}}{\left(n_{hA} + n_{lB} + n_{hB}\right)}$$
$$\Leftrightarrow \delta < \pi_{lA} - \pi_{lB}\widetilde{\pi}_{lB}$$

Social optimality of half segregation if $\pi_{lB} + \pi_{lA} < 1$:

$$\Leftrightarrow -\alpha(1-\alpha)\left\{(n_l y_l^2 + n_{hA} y_h^2) - \frac{\left(n_l y_l + n_{hA} y_h\right)^2}{n_l + n_{hA}}\right\} - n_{lB} C$$
$$> -\alpha(1-\alpha)\left\{(n_l y_l^2 + n_h y_h^2) - \frac{\left(n_{lA} y_l + n_{hA} y_h\right)^2}{n_A} - \frac{\left(n_{lB} y_l + n_{hB} y_h\right)^2}{n_B}\right\}$$
$$\Leftrightarrow (n_l y_l^2 + n_{hA} y_h^2) - \frac{\left(n_l y_l + n_{hA} y_h\right)^2}{n_l + n_{hA}} + n_{lB}\frac{C}{\theta} - (n_l y_l^2 + n_h y_h^2) + \frac{\left(n_{lA} y_l + n_{hA} y_h\right)^2}{n_A}$$
$$+\frac{\left(n_{lB} y_l + n_{hB} y_h\right)^2}{n_B} < 0$$
$$\Leftrightarrow (n_{hA} - n_h) y_h^2 - \frac{\left(n_l y_l + n_{hA} y_h\right)^2}{n_l + n_{hA}} + \frac{\left(n_{lA} y_l + n_{hA} y_h\right)^2}{n_A} + \frac{\left(n_{lB} y_l + n_{hB} y_h\right)^2}{n_B} + n_{lB}\frac{C}{\theta} < 0$$
$$\Leftrightarrow \delta < 1 - \pi_{lB} - \frac{n_{hA}}{n_A}\frac{n_{hA}}{\left(n_A + n_{lB}\right)}$$
$$\Longleftrightarrow \delta < \pi_{hB} - \pi_{hA}\widetilde{\pi}_{hA} \quad \blacksquare$$

Lemma 8 *It is SWI to move to segregated groups iff*

$$\delta < \pi_{lA}\left(\frac{n_{hA}}{n_{hA} + n_{lB}}\right) + \pi_{hB}\left(\frac{n_{lB}}{n_{hA} + n_{lB}}\right)$$

Proof. $\Leftrightarrow -\left(n_{hA} + n_{lB}\right) C > -\alpha(1-\alpha)\left\{(n_l y_l^2 + n_h y_h^2) - \frac{\left(n_{lA} y_l + n_{hA} y_h\right)^2}{n_A} - \frac{\left(n_{lB} y_l + n_{hB} y_h\right)^2}{n_B}\right\}$

$$\Leftrightarrow -\left(n_{hA} + n_{lB}\right) C$$
$$+\alpha(1-\alpha)\left\{(n_l y_l^2 + n_h y_h^2) - \frac{\left(n_{lA} y_l + n_{hA} y_h\right)^2}{n_A} - \frac{\left(n_{lB} y_l + n_{hB} y_h\right)^2}{n_B}\right\} > 0$$
$$\Leftrightarrow -\left(n_{hA} + n_{lB}\right)\frac{C}{\theta} + (n_l y_l^2 + n_h y_h^2) - \frac{\left(n_{lA} y_l + n_{hA} y_h\right)^2}{n_A} - \frac{\left(n_{lB} y_l + n_{hB} y_h\right)^2}{n_B} > 0$$
$$\Leftrightarrow \frac{\left(n_{lA} y_l + n_{hA} y_h\right)^2}{n_{lA} + n_{hA}} + \frac{\left(n_{lB} y_l + n_{hB} y_h\right)^2}{n_{lB} + n_{hB}} - (n_{lA} + n_{lB}) y_l^2 - (n_{hA} + n_{hB}) y_h^2 + \left(n_{hA} + n_{lB}\right)\frac{C}{\theta} < 0$$
$$\Leftrightarrow \frac{C}{\theta} < \frac{1}{n_{hA} + n_{lB}}\left\{(n_{lA} + n_{lB}) y_l^2 + (n_{hA} + n_{hB}) y_h^2 - \frac{\left(n_{lA} y_l + n_{hA} y_h\right)^2}{n_{lA} + n_{hA}} - \frac{\left(n_{lB} y_l + n_{hB} y_h\right)^2}{n_{lB} + n_{hB}}\right\}$$
$$\Leftrightarrow \frac{C}{\theta\left(y_l - y_h\right)^2} < \frac{n_{lA} n_{hA} n_{lB} + n_{hA} n_{lB} n_{hB} + n_{lA}(n_{hA} + n_{lB}) n_{hB}}{(n_{lA} + n_{hA})(n_{lB} + n_{hB})\left(n_{hA} + n_{lB}\right)}$$
$$\Leftrightarrow \delta < \frac{n_B\left(n_A - n_{lA}\right) n_{lA} + n_A n_B n_{lB} - n_A n_{lB}^2}{n_A n_B (n_A - n_{lA} + n_{lB})}$$

$$\Leftrightarrow \delta < \frac{n_{lA}}{n_A}\frac{(n_A - n_{lA})}{(n_A - n_{lA} + n_{lB})} + \frac{n_{lB}}{(n_A - n_{lA} + n_{lB})} - \frac{n_{lB}}{n_B}\frac{n_{lB}}{(n_A - n_{lA} + n_{lB})}$$
$$\Leftrightarrow \delta < \pi_{lA}\frac{n_{hA}}{n_{hA} + n_{lB}} + \frac{n_{lB}}{n_{hA} + n_{lB}}(1 - \frac{n_{lB}}{n_B})$$
$$\Leftrightarrow \delta < \pi_{lA}\frac{n_{hA}}{n_{hA} + n_{lB}} + \pi_{hB}\frac{n_{lB}}{n_{hA} + n_{lB}} \quad \blacksquare$$

Lemma 9 *It is SWI to move from semi-mixed equilibrium to segregating equilibrium iff*

$$\begin{array}{ll} \delta < \dfrac{n_{lA} + n_{lB}}{n_A + n_{lB}} & if \quad \pi^0_{lB} + \pi^0_{lA} < 1 \\ \delta < \dfrac{n_{hA} + n_{hB}}{n_{hA} + n_B} & if \quad \pi^0_{lB} + \pi^0_{lA} > 1 \end{array}$$

Proof. It is better to have full segregation rather than half segregation ($\pi^0_{lB} + \pi^0_{lA} < 1$) iff

$$\Leftrightarrow -(n_{hA} + n_{lB})\,C > -\alpha(1-\alpha)\left\{(n_l y_l^2 + n_{hA} y_h^2) - \frac{(n_l y_l + n_{hA} y_h)^2}{n_l + n_{hA}}\right\} - n_{lB}C$$
$$\Leftrightarrow -n_{hA}C > -\alpha(1-\alpha)\left\{(n_l y_l^2 + n_{hA} y_h^2) - \frac{(n_l y_l + n_{hA} y_h)^2}{n_l + n_{hA}}\right\}$$
$$\Leftrightarrow \frac{C}{\alpha(1-\alpha)} < \frac{1}{n_{hA}}\left\{(n_l y_l^2 + n_{hA} y_h^2) - \frac{(n_l y_l + n_{hA} y_h)^2}{n_l + n_{hA}}\right\}$$
$$\Leftrightarrow \frac{C}{\alpha(1-\alpha)} < \frac{1}{n_{hA}}\left\{(y_l - y_h)^2\left(\frac{n_{hA}\,(n_{lA} + n_{lB})}{n_l + n_{hA}}\right)\right\}$$
$$\Leftrightarrow \frac{C}{\alpha(1-\alpha)\,(y_l - y_h)^2} < \frac{(n_{lA} + n_{lB})}{n_A + n_{lB}}$$

By symmetry, if $\pi^0_{lB} + \pi^0_{lA} < 1$

$$\Leftrightarrow \frac{C}{\alpha(1-\alpha)\,(y_l - y_h)^2} < \frac{(n_{hA} + n_{hB})}{n_{hA} + n_B} \quad \blacksquare$$

The question is to know whether in terms of welfare, full segregation or half-segregation is better. To analyse this, we must take a closer look at the three lemmas given above. For the simplicity of the reminder of the proof, we restrict ourselves to the following subset S of initial conditions:

$$S = \{(\pi_{lA}, \pi_{lB}) \in (0,1) \times (0,1) : \pi_{lA} \geqslant \pi_{lB} \text{ and } \pi_{lB} + \pi_{lA} \leq 1\}$$

This is done without loss of generality since all other cases can be deduced by symmetry. Hence, we restrict the analysis to the interval of initial distribution $[\pi_{lB} = 0, \pi_{lB} + \pi_{lA} = 1]$. Let us first define three social functions. The social benefit function of full segregation Ω

with

$$\Omega = \pi_{lA}\left(\frac{n_{hA}}{n_{hA}+n_{lB}}\right) + \pi_{hB}\left(\frac{n_{lB}}{n_{hA}+n_{lB}}\right)$$

the social benefit function of half segregation Φ when $\pi_{lB}+\pi_{lA}<1$, with

$$\Phi = \pi_{hB} - \pi_{hA}\widetilde{\pi}_{hA}$$

and the social benefit of full segregation rather than half segregation Λ with

$$\Lambda = \frac{n_{lA}+n_{lB}}{n_A+n_{lB}}$$

Proposition 8 *For given n, n_A, n_B, and a given $\delta \in (0,1)$, $\exists\, \overline{\pi}_{lB}$: $\forall\, \pi_{lB} \in [0, \overline{\pi}_{lB})$, $\exists$ values of $\pi_{lA}, (\pi_{lA}, \pi_{lB}) \in S$, such that*

$$\Phi > \delta > \Omega$$

$\forall\, \pi_{lB} \in (\overline{\pi}_{lB}, \pi_{lB} : \pi_{lB} + \pi_{lA} = 1]$, $\exists$ *values of* $\pi_{lA}, (\pi_{lA}, \pi_{lB}) \in S, s.t.$

$$\Omega > \delta > \Phi$$

Proof. *Step 1.* When $\pi_{lB} = 0$ (meaning that $n_{lB} = 0$), $\Omega = \pi_{lA}$ and $\Phi = 1 - \pi_{hA}\widetilde{\pi}_{hA}$. Since $\widetilde{\pi}_{hA} = \pi_{hA}$ when $n_{lB} = 0$, $\Omega(\pi_{lB} = 0) < \Phi(\pi_{lB} = 0)$. When $\pi_{lB} + \pi_{lA} = 1$, $\Omega = \pi_{hB}$. Hence, $\Omega(\pi_{lB} + \pi_{lA} = 1) > \Phi(\pi_{lB} + \pi_{lA} = 1)$.

Step 2. Fix a $\delta^* \in (0,1)$. Suppose $\Omega(\pi_{lA}, \pi_{lB}) - \delta^* = 0$ and rewrite with n_{lA} as a function of n_{lB} (with n, n_A, n_B given): $n_{lA} = \omega(n_{lB})$. This function gives all of the pairs (i.e. initial conditions) such that the social benefit of moving to full segregation is exactly equal to the cost. In the same way, suppose $\Phi(\pi_{lA}, \pi_{lB}) - \delta^* = 0$ and rewrite with n_{lA} as a function of n_{lB} (with n, n_A, n_B given): $n_{lA} = \phi(n_{lB})$. This function yields all of the pairs (i.e. initial conditions) such that the social benefit of moving to half-segregation is exactly equal to the cost. ω and ϕ are both twice continuously differentiable (on their domains) and strictly convex functions. By steps 1 and 2, ω and ϕ cross each other an odd number of times.

Step 3. To show that these two functions cross only once, consider $\Lambda(\pi_{lA}, \pi_{lB})$, suppose

$\Lambda(\pi_{lA}, \pi_{lB}) - \delta^* = 0$ and rewrite with n_{lA} as a function of n_{lB} (with n, n_A, n_B given): $n_{lA} = \lambda(n_{lB})$. λ is a twice continuously differentiable, linear and strictly decreasing function. Now, let us prove that that ω and ϕ cross exactly once.

Step 4. Suppose by contradiction that ω and ϕ cross more than once, say three times (remember that they must cross an odd number of times), at $(n_{lA}^1, n_{lB}^1), (n_{lA}^2, n_{lB}^2), (n_{lA}^3, n_{lB}^3)$. Consider the four areas Z_i $(i = 1, 2, 3, 4)$ between the two bounds ($[\pi_{lB} = 0, \pi_{lB} + \pi_{lA} = 1]$, the two functions ($\omega$ and ϕ) and the three crossing points:

1) in $Z_1 = [n_{lB} = 0, n_{lB}^1) \times \{n_{lA} : \omega(n_{lB}) > n_{lA} > \phi(n_{lB})\}$, we have a set of initial conditions such that it is optimal to have half-segregation but not full segregation

2) in $Z_2 = (n_{lB}^1, n_{lB}^2) \times \{n_{lA} : \phi(n_{lB}) > n_{lA} > \omega(n_{lB})\}$, we have a set of initial conditions such that it is optimal to have full segregation but not half-segregation

3) in $Z_3 = (n_{lB}^2, n_{lB}^3) \times \{n_{lA} : \omega(n_{lB}) > n_{lA} > \phi(n_{lB})\}$, we have a set of initial conditions such that it is optimal to have half-segregation but not full segregation

4) in $Z_4 = (n_{lB}^3, n_{lB} : \pi_{lB} + \pi_{lA} = 1) \times \{n_{lA} : \phi(n_{lB}) > n_{lA} > \omega(n_{lB})\}$, we have a set of initial conditions such that it is optimal to have full segregation but not half-segregation.

Remember that λ is a linear and strictly decreasing function, above which it is better to have full segregation than half-segregation, while the converse is true below. Note also that in $n_{lB} = 0$, $n_A > \lambda = \delta n_A = \omega > \phi > 0$. Since inside a Z_i it is optimal to have either half or full segregation, but not the two simultaneously, λ cannot cross an area, and hence can only cross ω and ϕ at a crossing point. Moreover, since ω and ϕ are strictly convex and λ is linear and strictly decreasing, λ can cross ω and ϕ only once .

a) Suppose this occurs at (n_{lA}^3, n_{lB}^3). Then, at any point below the function, it is better to have half-segregation rather than full segregation. But this contradicts the interpretation of Z_2. Hence, it cannot occur at (n_{lA}^3, n_{lB}^3).

b) Suppose this occurs at (n_{lA}^2, n_{lB}^2). Then, at any point below the function, it is better to have full segregation than half-segregation. But this contradicts the interpretation of Z_3. Hence, it cannot occur at (n_{lA}^2, n_{lB}^2).

c) Suppose this occurs at (n_{lA}^1, n_{lB}^1). Then, at any point below the function, it is better to have half-segregation than full segregation. This is coherent with the interpretation of Z_1. Above the function, it is better to have full segregation than half-segregation. This means that we can only have areas of type Z_2 above the function, and the only way of having this is that ω and ϕ do not cross again.

We have thus proved that ω and ϕ can cross only once, at $(n_{lA}^1, n_{lB}^1) = (\overline{\pi}_{lA}, \overline{\pi}_{lB})$. ■

This proposition tells us there are distribution of types such that full segregation is optimal but not half-segregation, and others for which the converse is true. Since when $n_{lB} = 0$, $n_A > \lambda = \delta n_A = \omega > \phi > 0$, there are also cases in which both conditions are met, and others in which none of them are fulfilled. This result is depicted in Figure III

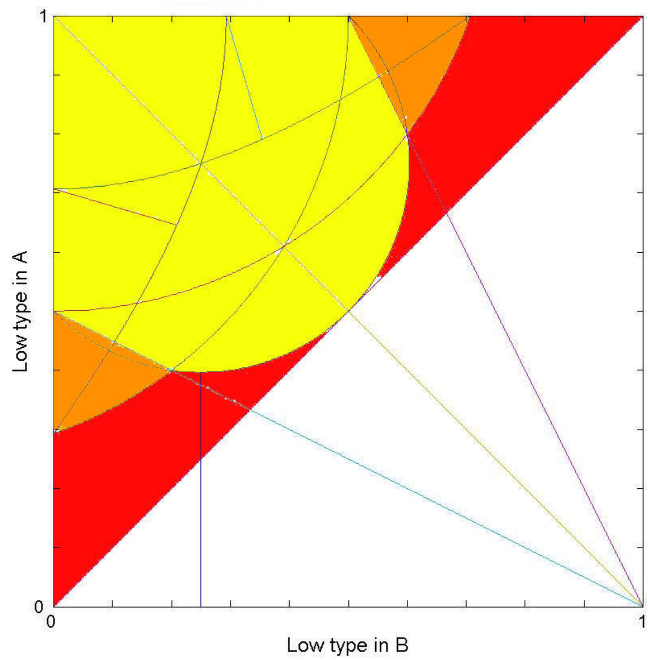

Figure III: Distribution of types and Social optimality

Axes and diagonal lines have the same interpretations as in Figure I. The yellow area represents the pairs for which it is socially optimal to have full segregation. The orange area is the one where it is socially optimal to have half-segregagtion. Finally, the red area corresponds to intial conditions such that it is not optimal to segregate groups.

Part 2

Once we know what is socially optimal, and what is not, we must study whether consumers individually take the optimal decision or not. Since we are investigating the case $\pi_{lB}^0 + \pi_{lA}^0 < 1$

it is obvious that

$$\left(\pi_{lA}-\pi_{lB}\right)\left(\pi_{lA}+\pi_{lB}\right)<\left(\pi_{lA}-\pi_{lB}\right)\left[2-\left(\pi_{lA}+\pi_{lB}\right)\right]$$

Lemma 10 *The individual benefit of moving is always strictly smaller than the social benefit of having "half-segregation", that is*

$$\left(\pi_{lA}-\pi_{lB}\right)\left(\pi_{lA}+\pi_{lB}\right)<\left(\pi_{lA}-\pi_{lB}\right)\left[2-\left(\pi_{lA}+\pi_{lB}\right)\right]<\pi_{hB}-\pi_{hA}\widetilde{\pi}_{hA}$$

Proof. $\Leftrightarrow\left(\pi_{lA}-\pi_{lB}\right)\left[2-\left(\pi_{lA}+\pi_{lB}\right)\right]\gtrless\pi_{hB}-\pi_{hA}\widetilde{\pi}_{hA}$

$\Leftrightarrow 2\left(\pi_{lA}-\pi_{lB}\right)-\left(\pi_{lA}-\pi_{lB}\right)\left(\pi_{lA}+\pi_{lB}\right)\gtrless\pi_{hB}-\pi_{hA}\widetilde{\pi}_{hA}$

$\Leftrightarrow 2\left[\left(1-\pi_{hA}\right)-\left(1-\pi_{hB}\right)\right]-\left[\left(1-\pi_{hA}\right)-\left(1-\pi_{hB}\right)\right]\left[\left(1-\pi_{hA}\right)+\left(1-\pi_{hB}\right)\right]$

$\gtrless\pi_{hB}-\pi_{hA}\widetilde{\pi}_{hA}$

$\Leftrightarrow 2(\pi_{hB}-\pi_{hA})-\left(\pi_{hB}-\pi_{hA}\right)\left(2-\pi_{hA}-\pi_{hB}\right)\gtrless\pi_{hB}-\pi_{hA}\widetilde{\pi}_{hA}$

$\Leftrightarrow 2(\pi_{hB}-\pi_{hA})-2(\pi_{hB}-\pi_{hA})-(\pi_{hB}-\pi_{hA})(-\pi_{hA}-\pi_{hB})\gtrless\pi_{hB}-\pi_{hA}\widetilde{\pi}_{hA}$

$\Leftrightarrow\pi_{hB}^{2}-\pi_{hA}^{2}\gtrless\pi_{hB}-\pi_{hA}\widetilde{\pi}_{hA}$

$\Leftrightarrow\pi_{hB}(\pi_{hB}-1)<\pi_{hA}(\pi_{hA}-\widetilde{\pi}_{hA})$ ■

This lemma means that as soon as one type decides to move, it is at least social welfare improving to have half-segregation. Moreover, even if nobody moves, it could be SWI to have half-segregation.

Part 3: proof of Proposition 6

Point 1 is obvious by definition of the equilibria and the social welfare function.

For point 2, by Proposition 7, neither Ω nor Φ is always the largest. Hence, if the moving cost is higher than the larger of the two, then there is no advantage to move, and the decision taken individually by agents is optimal. If, on the other hand, at least one of the two functions is higher than the cost, then it would have been optimal for agents to move to half or full segregation, depending on the values of Ω, Φ, and Λ. In that case, individual decisions are suboptimal.

Concerning point 3, we know by lemma 10 that as soon as agents of one type decide to move, it is social welfare improving to have at least half-segregation. Depending on the size of the moving cost with respect to the social benefit of having full segregation, the individual decision will be optimal (if it is relatively too expensive to fully segregate) or suboptimal (if both lemmas 8 and 9 are fulfilled).

B Computation

B.1 Utility in ECG when nobody moves

Proof. $\Leftrightarrow -n_{lA}\left|y_l-\overline{y}_A\right|-n_{hA}\left|y_h-\overline{y}_A\right|-n_{lB}\left|y_l-\overline{y}_B\right|-n_{hB}\left|y_h-\overline{y}_B\right|$

$$\Leftrightarrow -n_{lA}(\overline{y}_A-y_l)-n_{hA}(y_h-\overline{y}_A)-n_{lB}(\overline{y}_B-y_l)-n_{hB}(y_h-\overline{y}_B)$$

$$\Leftrightarrow (n_{lA}y_l-n_{hA}y_h)+\overline{y}_A(n_{hA}-n_{lA})+(n_{lB}y_l-n_{hB}y_h)+\overline{y}_B(n_{hB}-n_{lB})$$

$$\Leftrightarrow (n_l y_l-n_h y_h)+(\pi_{lA}y_l+\pi_{hA}y_h)(n_{hA}-n_{lA})+(\pi_{lB}y_l+\pi_{hB}y_h)(n_{hB}-n_{lB})$$

$$\Leftrightarrow n_l y_l-n_h y_h+(\tfrac{n_{lA}}{n_A}y_l+\tfrac{n_{hA}}{n_A}y_h)(n_{hA}-n_{lA})+(\tfrac{n_{lB}}{n_B}y_l+\tfrac{n_{hB}}{n_B}y_h)(n_{hB}-n_{lB})$$

$$\Leftrightarrow \frac{1}{n_A n_B}\left\{\begin{array}{c} n_A n_B n_l y_l-n_A n_B n_h y_h+(n_B n_{lA}y_l+n_B n_{hA}y_h)(n_{hA}-n_{lA}) \\ +(n_A n_{lB}y_l+n_A n_{hB}y_h)(n_{hB}-n_{lB}) \end{array}\right\}$$

$$\Leftrightarrow \frac{1}{n_A n_B}\left\{\begin{array}{c} n_A n_B n_l y_l-n_A n_B(n-n_l)y_h+(n_B n_{lA}y_l+n_B(n_A-n_{lA})y_h)(n_A-2n_{lA}) \\ +(n_A n_{lB}y_l+n_A(n_B-n_{lB})y_h)(n_B-2n_{lB}) \end{array}\right\}$$

$$\Leftrightarrow \frac{1}{n_A n_B}\left\{\begin{array}{c} n_A n_B n_l y_l-n_A n_B n y_h+n_A n_B n_l y_h+(n_B n_{lA}y_l+n_B n_A y_h-n_B n_{lA}y_h)(n_A-2n_{lA}) \\ +(n_A n_{lB}y_l+n_A n_B y_h-n_A n_{lB}y_h)(n_B-2n_{lB}) \end{array}\right\}$$

$$\Leftrightarrow \frac{1}{n_A n_B}\left\{\begin{array}{c} n_A n_B n_l y_l-n_A n_B n y_h+n_A n_B n_l y_h+n_A n_B n_{lA}y_l+n_B n_A^2 y_h-n_A n_B n_{lA}y_h \\ -2n_B n_{lA}^2 y_l-2n_A n_B n_{lA}y_h+2n_B n_{lA}^2 y_h+n_A n_B n_{lB}y_l+n_A n_B^2 y_h-n_A n_B n_{lB}y_h \\ -2n_A n_{lB}^2 y_l-2n_A n_B n_{lB}y_h+2n_A n_{lB}^2 y_h \end{array}\right\}$$

$$\Leftrightarrow \frac{1}{n_A n_B}\left\{\begin{array}{c} -3n_A n_B n_{lA}y_h-3n_A n_B n_{lB}y_h+2n_B n_{lA}^2 y_h-2n_A n_{lB}^2 y_l+2n_A n_B n_l y_l+2n_A n_{lB}^2 y_h \\ -2n_B n_{lA}^2 y_l+n_A n_B n_l y_h+n_B n_A^2 y_h+n_A n_B^2 y_h-n_A n_B n y_h \end{array}\right\}$$

$$\Leftrightarrow \frac{1}{n_A n_B}\left\{\begin{array}{c} -3n_A n_B y_h(n_{lA}+n_{lB})+2n_B n_{lA}^2(y_h-y_l)+2n_A n_B n_l y_l+2n_A n_{lB}^2(y_h-y_l) \\ +n_A n_B n_l y_h+n_B n_A y_h(n_A+n_B-n) \end{array}\right\}$$

$$\Leftrightarrow \frac{1}{n_A n_B}\left\{-2n_A n_B n_l y_h+2n_A n_B n_l y_l+2(y_h-y_l)\left[n_B n_{lA}^2+n_A n_{lB}^2\right]\right\}$$

$$\Leftrightarrow \frac{1}{n_A n_B}2(y_h-y_l)\left\{-n_A n_B n_l+n_B n_{lA}^2+n_A n_{lB}^2\right\}$$

$\Leftrightarrow 2(y_h - y_l)\left[-n_l + \frac{n_{lB}^2}{n_B} + \frac{n_{lA}^2}{n_A}\right]$

$\Leftrightarrow -2(y_h - y_l)\left(n_l - n_{lA}\pi_{lA} - n_{lB}\pi_{lB}\right)$ ■

B.2 Utility at the optimum in the PPG

Proof.

$$-(1-\alpha)\left[y_i - (1-\alpha)y_i - \alpha\overline{y}_k\right]^2 - \alpha\left[\overline{y}_k - (1-\alpha)y_i - \alpha\overline{y}_k\right]^2$$

$$-(1-\alpha)\left[\alpha(y_i - \overline{y}_k)\right]^2 - \alpha\left[(1-\alpha)(\overline{y}_k - y_i)\right]^2$$

$$-\alpha^2(1-\alpha)\left[y_i - \overline{y}_k\right]^2 - \alpha(1-\alpha)^2\left[\overline{y}_k - y_i\right]^2$$

$$-\alpha(1-\alpha)\left[y_i - \overline{y}_k\right]^2\left\{\alpha + (1-\alpha)\right\}$$

$$-\alpha(1-\alpha)\left[y_i - \overline{y}_k\right]^2$$

■

B.3 Useful Computation

Proof.

$$\begin{aligned}
(\overline{y}_B - \overline{y}_A) &= \pi_{lB}y_l + \pi_{hB}y_h - \pi_{lA}y_l - \pi_{hA}y_h \\
&= (\pi_{lB} - \pi_{lA})y_l + (\pi_{hB} - \pi_{hA})y_h \\
&= (\pi_{lB} - \pi_{lA})y_l + (1 - \pi_{lB} - 1 + \pi_{lA})y_h \\
&= (\pi_{lB} - \pi_{lA})y_l - (\pi_{lB} - \pi_{lA})y_h \\
&= (\pi_{lB} - \pi_{lA})(y_l - y_h)
\end{aligned}$$

$$
\begin{aligned}
(\overline{y}_B + \overline{y}_A) &= \pi_{lB}y_l + \pi_{hB}y_h + \pi_{lA}y_l + \pi_{hA}y_h \\
&= (\pi_{lB} + \pi_{lA})y_l + (\pi_{hB} + \pi_{hA})y_h \\
&= (\pi_{lB} + \pi_{lA})y_l + (1 - \pi_{lB} + 1 - \pi_{lA})y_h \\
&= (\pi_{lB} + \pi_{lA})y_l - (\pi_{lB} + \pi_{lA})y_h + 2y_h \\
&= (\pi_{lB} + \pi_{lA})(y_l - y_h) + 2y_h
\end{aligned}
$$

$$
\begin{aligned}
(\overline{y}_A - \overline{y}_B) &= \pi_{lA}y_l + \pi_{hA}y_h - \pi_{lB}y_l - \pi_{hB}y_h \\
&= (\pi_{lA} - \pi_{lB})y_l + (\pi_{hA} - \pi_{hB})y_h \\
&= (\pi_{lA} - \pi_{lB})y_l + (1 - \pi_{lA} - 1 + \pi_{lB})y_h \\
&= (\pi_{lA} - \pi_{lB})y_l - (\pi_{lA} - \pi_{lB})y_h \\
&= (\pi_{lA} - \pi_{lB})(y_l - y_h)
\end{aligned}
$$

■

B.4 Utility when nobody moves in the PPG

Proof. $\Leftrightarrow -n_{lA}\alpha(1-\alpha)(y_l - \overline{y}_A)^2 - n_{hA}\alpha(1-\alpha)(y_h - \overline{y}_A)^2 - n_{lB}\alpha(1-\alpha)(y_l - \overline{y}_B)^2$
$-n_{hB}\alpha(1-\alpha)(y_h - \overline{y}_B)^2$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{\begin{array}{c} n_{lA}(y_l^2 - 2y_l\overline{y}_A + \overline{y}_A^2) + n_{hA}(y_h^2 - 2y_h\overline{y}_A + \overline{y}_A^2) + n_{lB}(y_l^2 - 2y_l\overline{y}_B + \overline{y}_B^2) \\ +n_{hB}(y_h^2 - 2y_h\overline{y}_B + \overline{y}_B^2) \end{array}\right\}$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{(n_l y_l^2 + n_h y_h^2) + n_A\overline{y}_A^2 + n_B\overline{y}_B^2 - 2y_l(n_{lA}\overline{y}_A + n_{lB}\overline{y}_B) - 2y_h(n_{hA}\overline{y}_A + n_{hB}\overline{y}_B)\right\}$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{\begin{array}{c} (n_l y_l^2 + n_h y_h^2) + n_A(\dfrac{n_{lA}^2 y_l^2 + n_{hA}^2 y_h^2 + 2n_{lA}n_{hA}y_l y_h}{n_A^2}) \\ +n_B(\dfrac{n_{lB}^2 y_l^2 + n_{hB}^2 y_h^2 + 2n_{lB}n_{hB}y_l y_h}{n_B^2}) \\ -2y_l\left(\dfrac{n_{lA}^2 y_l + n_{lA}n_{hA}y_h}{n_A} + \dfrac{n_{lB}^2 y_l + n_{lB}n_{hB}y_h}{n_B}\right) \\ -2y_h\left(\dfrac{n_{lA}n_{hA}y_l + n_{hA}^2 y_h}{n_A} + \dfrac{n_{lB}n_{hB}y_l + n_{hB}^2 y_h}{n_B}\right) \end{array}\right\}$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{\begin{array}{c}(n_l y_l^2+n_h y_h^2)\\ +\dfrac{1}{n_A n_B}\left(\begin{array}{c}n_B n_{lA}^2 y_l^2+n_B n_{hA}^2 y_h^2+2n_B n_{lA} n_{hA} y_l y_h\\ +n_A n_{lB}^2 y_l^2+n_A n_{hB}^2 y_h^2+2n_A n_{lB} n_{hB} y_l y_h\end{array}\right)\\ -2n_B n_{lA}^2 y_l^2-2n_B n_{lA} n_{hA} y_l y_h-2n_A n_{lB}^2 y_l^2-2n_A n_{lB} n_{hB} y_l y_h\\ -2n_B n_{lA} n_{hA} y_l y_h-2n_B n_{hA}^2 y_h^2-2n_A n_{lB} n_{hB} y_l y_h-2n_A n_{hB}^2 y_h^2\end{array}\right\}$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{\begin{array}{c}(n_l y_l^2+n_h y_h^2)\\ +\dfrac{1}{n_A n_B}\left[\begin{array}{c}\left(-n_B n_{lA}^2-n_A n_{lB}^2\right)y_l^2+\left(-n_B n_{hA}^2-n_A n_{hB}^2\right)y_h^2\\ +\left(-2n_B n_{lA} n_{hA}-2n_A n_{lB} n_{hB}\right)y_l y_h\end{array}\right]\end{array}\right\}$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{(n_l y_l^2+n_h y_h^2)-\frac{1}{n_A n_B}\left[\begin{array}{c}n_B\left(n_{lA}^2 y_l^2+n_{hA}^2 y_h^2+n_{lA} n_{hA} y_l y_h\right)\\ +n_A\left(n_{lB}^2 y_l^2+n_{hB}^2 y_h^2+2n_{lB} n_{hB} y_l y_h\right)\end{array}\right]\right\}$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{(n_l y_l^2+n_h y_h^2)-\frac{n_{lA}^2 y_l^2+n_{hA}^2 y_h^2+n_{lA} n_{hA} y_l y_h}{n_A}-\frac{n_{lB}^2 y_l^2+n_{hB}^2 y_h^2+2n_{lB} n_{hB} y_l y_h}{n_B}\right\}$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{(n_l y_l^2+n_h y_h^2)-\frac{(n_{lA} y_l+n_{hA} y_h)^2}{n_A}-\frac{(n_{lB} y_l+n_{hB} y_h)^2}{n_B}\right\} \blacksquare$$

B.5 Utility when one side moves: $\pi_{lB}+\pi_{lA}>1$

Proof. $\Leftrightarrow -n_{lB}\alpha(1-\alpha)(y_l-\widetilde{y}_B)^2-(n_{hB}+n_{hA})\alpha(1-\alpha)(y_h-\widetilde{y}_B)^2-n_{hA}C$

$$\Leftrightarrow -\alpha(1-\alpha)\left[n_{lB}(y_l-\widetilde{y}_B)^2+(n_{hB}+n_{hA})(y_h-\widetilde{y}_B)^2\right]-n_{hA}C$$

$$\Leftrightarrow -\alpha(1-\alpha)\left[n_{lB}\left(y_l^2+\widetilde{y}_B^2-2y_l\widetilde{y}_B\right)+n_h\left(y_h^2+\widetilde{y}_B^2-2y_h\widetilde{y}_B\right)\right]-n_{hA}C$$

$$\Leftrightarrow -\alpha(1-\alpha)\left[(n_{lB} y_l^2+n_h y_h^2)+(n_{lB}+n_h)\,\widetilde{y}_B^2-2\widetilde{y}_B\left(n_{lB} y_l+n_h y_h\right)\right]-n_{hA}C$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{\begin{array}{c}(n_{lB} y_l^2+n_h y_h^2)+(n_{lB}+n_h)\left(\dfrac{n_{lB}^2 y_l^2+n_h^2 y_h^2+2n_{lB} n_h y_l y_h}{(n_{lB}+n_h)^2}\right)\\ -2\left(\dfrac{n_{lB} y_l+n_h y_h}{(n_{lB}+n_h)}\right)(n_{lB} y_l+n_h y_h)\end{array}\right\}$$
$$-n_{hA}C$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{\begin{array}{c}(n_{lB} y_l^2+n_h y_h^2)\\ +\dfrac{n_{lB}^2 y_l^2+n_h^2 y_h^2+2n_{lB} n_h y_l y_h-2n_{lB}^2 y_l^2-2n_{lB} n_h y_l y_h-2n_h n_{lB} y_l y_h-2n_h^2 y_h^2}{n_{lB}+n_h}\end{array}\right\}$$
$$-n_{hA}C$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{(n_{lB} y_l^2+n_h y_h^2)-\frac{n_{lB}^2 y_l^2+n_h^2 y_h^2+2n_{lB} n_h y_l y_h}{n_{lB}+n_h}\right\}-n_{hA}C$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{(n_{lB} y_l^2+n_h y_h^2)-\frac{(n_{lB} y_l+n_h y_h)^2}{n_{lB}+n_h}\right\}-n_{hA}C \blacksquare$$

B.6 Utility when one side moves: $\pi_{lB} + \pi_{lA} < 1$

Proof. $\Leftrightarrow -(n_{lA} + n_{lB})\alpha(1-\alpha)(y_l - \widetilde{y}_A)^2 - n_{hA}\alpha(1-\alpha)(y_h - \widetilde{y}_A)^2 - n_{lB}C$

$$\Leftrightarrow -\alpha(1-\alpha)\left[(n_{lA} + n_{lB})(y_l - \widetilde{y}_A)^2 + n_{hA}(y_h - \widetilde{y}_A)^2\right] - n_{lB}C$$

$$\Leftrightarrow -\alpha(1-\alpha)\left[n_l\left(y_l^2 + \widetilde{y}_A^2 - 2y_l\widetilde{y}_A\right) + n_{hA}\left(y_h^2 + \widetilde{y}_A^2 - 2y_h\widetilde{y}_A\right)\right] - n_{lB}C$$

$$\Leftrightarrow -\alpha(1-\alpha)\left[(n_l y_l^2 + n_{hA}y_h^2) + (n_l + n_{hA})\,\widetilde{y}_A^2 - 2\widetilde{y}_A\left(n_l y_l + n_{hA}y_h\right)\right] - n_{lB}C$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{\begin{array}{c} (n_l y_l^2 + n_{hA}y_h^2) + (n_l + n_{hA})\left(\dfrac{n_l^2 y_l^2 + n_{hA}^2 y_h^2 + 2n_l n_{hA} y_l y_h}{(n_l + n_{hA})^2}\right) \\ -2\left(\dfrac{n_l y_l + n_{hA}y_h}{(n_l + n_{hA})}\right)(n_l y_l + n_{hA}y_h) \end{array}\right\}$$
$$-n_{lB}C$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{\begin{array}{c} (n_l y_l^2 + n_{hA}y_h^2) + \\ \dfrac{n_l^2 y_l^2 + n_{hA}^2 y_h^2 + 2n_l n_{hA} y_l y_h - 2n_l^2 y_l^2 - 2n_l n_{hA} y_l y_h - 2n_{hA} n_l y_l y_h - 2n_{hA}^2 y_h^2}{n_l + n_{hA}} \end{array}\right\}$$
$$-n_{lB}C$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{(n_l y_l^2 + n_{hA}y_h^2) - \frac{n_l^2 y_l^2 + n_{hA}^2 y_h^2 + 2n_l n_{hA} y_l y_h}{n_l + n_{hA}}\right\} - n_{lB}C$$

$$\Leftrightarrow -\alpha(1-\alpha)\left\{(n_l y_l^2 + n_{hA}y_h^2) - \frac{(n_l y_l + n_{hA}y_h)^2}{n_l + n_{hA}}\right\} - n_{lB}C \quad \blacksquare$$

B.7 Full Segregation function

Proof. $\Leftrightarrow \dfrac{n_{lA}}{n_A}\dfrac{n_{hA}}{n_{hA} + n_{lB}} + \dfrac{n_{hB}}{n_B}\dfrac{n_{lB}}{n_{hA} + n_{lB}} = \delta$

$$\Leftrightarrow n_B n_{lA} n_{hA} + n_A n_{hB} n_{lB} = n_A n_B \left(n_{hA} + n_{lB}\right)\delta$$

$$\Leftrightarrow n_B n_{lA}\left(n_A - n_{lA}\right) + n_A\left(n_B - n_{lB}\right)n_{lB} = n_A n_B\left(n_A - n_{lA} + n_{lB}\right)\delta$$

$$\Leftrightarrow n_A n_B n_{lA} - n_B n_{lA}^2 + n_A n_B n_{lB} - n_A n_{lB}^2 - n_A^2 n_B\delta + n_A n_B n_{lA}\delta - n_A n_B n_{lB}\delta = 0$$

$$\Longleftrightarrow -n_B n_{lA}^2 + \left[n_A n_B + n_A n_B\delta\right]n_{lA} + \left[n_A n_B n_{lB} - n_A n_{lB}^2 - n_A^2 n_B\delta - n_A n_B n_{lB}\delta\right] = 0$$

$$\Leftrightarrow n_{lA} = \tfrac{1}{-2n_B}\left(\begin{array}{c} -\left[n_A n_B + n_A n_B\delta\right] + \\ \sqrt{\left[n_A n_B + n_A n_B\delta\right]^2 + 4 * n_B * \left[n_A n_B n_{lB} - n_A n_{lB}^2 - n_A^2 n_B\delta - n_A n_B n_{lB}\delta\right]} \end{array}\right)$$

$$\frac{\partial n_{lA}}{\partial n_{lB}} = \frac{1}{-2n_B}\frac{1}{2}\frac{1}{\sqrt{\left[n_A n_B + n_A n_B\delta\right]^2 + 4 * n_B * \left[n_A n_B n_{lB} - n_A n_{lB}^2 - n_A^2 n_B\delta - n_A n_B n_{lB}\delta\right]}}$$
$$(4n_A n_B^2 - 8n_A n_B n_{lB} - 4n_A n_B^2\delta)$$

$$\Rightarrow \frac{\partial n_{lA}}{\partial n_{lB}} = 0 \Leftrightarrow n_{lB} = \frac{4n_A n_B^2(1-\delta)}{8n_A n_B}$$

$$\frac{\partial^2 n_{lA}}{\partial n_{lB}^2} > 0 \quad \blacksquare$$

B.8 Half Segregation function (<1)

Proof. $\Leftrightarrow \frac{n_{hB}}{n_B} - \frac{n_{hA}}{n_A}\frac{n_{hA}}{n_A + n_{lB}} = \delta$

$$\Leftrightarrow n_A\left(n_A + n_{lB}\right)\left(n_B - n_{lB}\right) - n_B n_{hA}^2 = n_A n_B\left(n_A + n_{lB}\right)\delta$$

$$\Leftrightarrow n_A^2 n_B - n_A^2 n_{lB} + n_A n_B n_{lB} - n_A n_{lB}^2 - n_B(n_A^2 + n_{lA}^2 - 2n_A n_{lA}) - n_A^2 n_B\delta - n_A n_B n_{lB}\delta = 0$$

$$\Leftrightarrow n_A^2 n_B - n_A^2 n_{lB} + n_A n_B n_{lB} - n_A n_{lB}^2 - n_A^2 n_B - n_B n_{lA}^2 + 2n_A n_B n_{lA} - n_A^2 n_B\delta - n_A n_B n_{lB}\delta = 0$$

$$\Leftrightarrow -n_B n_{lA}^2 + 2n_A n_B n_{lA} - [n_A^2 n_{lB} - n_A n_B n_{lB} + n_A n_{lB}^2 + n_A^2 n_B\delta + n_A n_B n_{lB}\delta] = 0$$

$$\Leftrightarrow n_{lA} = \frac{-2n_A n_B + \sqrt{4n_A^2 n_B^2 - 4 * n_B * [n_A^2 n_{lB} - n_A n_B n_{lB} + n_A n_{lB}^2 + n_A^2 n_B\delta + n_A n_B n_{lB}\delta]}}{-2n_B}$$

$$\frac{\partial n_{lA}}{\partial n_{lB}} = \frac{1}{-2n_B}\frac{1}{2}\frac{1}{\sqrt{4n_A^2 n_B^2 - 4 * n_B * [n_A^2 n_{lB} - n_A n_B n_{lB} + n_A n_{lB}^2 + n_A^2 n_B\delta + n_A n_B n_{lB}\delta]}}$$
$$(-4n_A^2 n_B + 4n_A n_B^2 - 8n_A n_B n_{lB} - 4n_A n_B^2\delta)$$

$$\Rightarrow \frac{\partial n_{lA}}{\partial n_{lB}} = 0 \Leftrightarrow n_{lB} = \frac{(1-\delta)n_B - n_A}{2}$$

$$\frac{\partial^2 n_{lA}}{\partial n_{lB}^2} > 0 \quad \blacksquare$$

B.9 Full rather than Half function (<1)

Proof. $\Leftrightarrow \frac{n_{lA} + n_{lB}}{n_A + n_{lB}} = \delta$

$$\Leftrightarrow n_{lA} + n_{lB} = \delta n_A + \delta n_{lB}$$

$$\Leftrightarrow n_{lA} = \left(\delta - 1\right) n_{lB} + \delta n_A \quad \blacksquare$$

Part III

Social Identity, Advertising and Market Competition

1 Introduction

In everyday life, one observes groups of people who act in a similar way: they may listen to the same type of music, buy the same good, etc. The fashion market fits this observation particularly well. For example, fancy young men from rich neighborhoods all dress in the same way, but completely differently from people in poor neighborhoods or ghettos. In school playgrounds, children naturally form groups according to the type of shoes or shirts they wear, or the way their hair is cut.

In economics, different explanations have been proposed for similarity of consumption inside given groups. Bikhchandani et al. (1992) outline four possible mechanisms for uniform behavior: first, the possibility of sanctions on deviants leads to sustainability of norms (Kandori, 1992), and hence similarity in behavior. For example, if someone lives in a very religious neighborhood, not going to church may be sanctioned by unbearable social shame, leading to uniform behavior in the group. The existence of positive payoff externalities is a second way to explain uniform behavior. This is the case when using the same good makes exchange possible (VHS versus BETAMAX systems, PC versus MAC, etc.). A third possibility is linked to a matter of information: the idea here is that when choosing one's consumption, not only is personal information important, but also the information held by others (revealed by their actions). This effect is explained in Banerjee (1992), where in a sequential game, the action of a player is influenced by those of his predecessors. Finally, conformity preference may also lead to uniform behavior. This means that the choice of a consumer is guided by the fact that he wants to consume the same good as other consumers (see for example Karni and Schmeidler, 1990, who study, in a dynamic game, the demand for a good available in three colors when the choices of others matter).

The study of consumption choice behavior has of course not been exclusively restricted to economics. Sociology for example also investigates consumption behavior. In this field, consumption is seen as serving several purposes. Campbell (1995) outlines that besides the role of satisfying needs or indulging wants and desires, consumption can also serve to compensate the individual for feelings of inferiority, insecurity or loss, to symbolize achievement, success or power, to communicate social distinctions, etc. Among all these interpretations, an interesting one is representing consumption as an activity which conveys information about the consumer's identity to those who witness it. For example, Zerubavel (1997) considers that clothing may be a tool people use to be identified when meeting a stranger.

The notion of identity has been widely studied in sociology. Following Jenkins (2004), identity is a matter of knowing who's who, that is, our understanding of who we are and who other people are, and, reciprocally, other people's understanding of themselves and of others (which includes us). He also outlines that one of the first things we do when meeting others is to try to guess who they are, what their identity is, and that we work at presenting ourselves so that others will work out who we are along the lines we wish them to. Similarly, Goffman (1959) argues that people seek to be and to be seen to be. Identity is then not only about who we are and who others are, but also how others see us.

Coming back to Zerubavel (1997), the possibility of signalling means that not only is the agent's action of consumption important, but what this action means for agents interacting with him also matters. Davis (1992) refers to this phenomenon as being captured by the concept of *social identity*, which basically *"concerns the configuration of attributes and attitudes persons seek to and actually do communicate about themselves"* (Davis, 1992, p.16). For example, an agent of type β may choose to consume good x instead of good y if consuming x allows others to identify him as being a $\beta - type$. Besides this individual approach, there is the *social identity theory*, which mainly focuses on groups: "*[Social Identity Theory] starts from the assumption that social identity is derived primarily from group memberships. It further proposes that people strive to achieve or maintain a positive social identity (...), and that this positive identity derives largely from favorable comparisons that can be made between the in-group and relevant out-groups*" (Brown, 2000, p.746-747). Social identity theory was first developed by Tajfel and Turner (1979) and is composed of three elements: categorization, identification, and comparison. The idea is that people classify others into categories, identify themselves as belonging to one or many categories, and then compare their situation to that of others (and derive positive utility from that comparison).

Examples of categories are gender, race, ethnicity, etc. Of course, categories are not always clear at first sight, and signals may be needed. In that case, consumption choice can act as a signal of type. Consumption then signals type through group membership, with the whole group consuming the same good.

Based on social identity and social identity theory, an agent should choose a consumption similar to agents from the same group/the same type, but distinct from out-groups/different type, ensuring that his identity (characteristics) is well understood by others. This is obviously the case in the fancy young men versus people in ghettos example. Another example is the dress code of someone who likes hard rock versus the clothes of a rap fan. In all of these cases, clothes are indeed not just about keeping you warm or making you look beautiful, but also, and above all, about signalling one's type in order to be matched with similar agents. Note that in this paper we do not take into account any effect such as in Pesendorfer (1995) where agents tend to buy an expensive good to be matched with a high type agent, rather than a low type. There is no vertical differentiation between consumers, only horizontal differentiation. Furthermore, we do not consider any other group behavior explanations such as herd behavior or peer pressure.

For the remainder of the paper, we refer to *social identity* as the fact that people value being well understood by others and that they use consumption to signal who they are to others.

The first goal of this paper is to study the effect of social identity, a specific form of interpersonal relationships, on the market outcome. More precisely, we investigate how social identity modifies the classical result of the Bertrand price competition model. We then ask ourselves how identity is linked to a given product. A possible answer studied in this paper is the use of advertising by firms. Again, we look at how this feature, coupled with social identity, modifies the classical result that prices equal marginal costs. The idea behind the advertising effect is that firms can create an image for their product, focussing on specific consumers, which in turn ensures a rigid demand. For example, Coke Light is targeted at women, Jupiler beer at men, and Seat cars are targeted at young people. Note that we do not consider any fashion effect, which would mean that someone consumes a good not because its characteristics are in line with his type, but because others are doing it, but some kind of "anti-fashion" effect, i.e. consumers dislike others consuming the same good. Of course, we allow firms to not advertise at all. This can be interpreted as a "no brand" product.

The literature presents three different views on the role played by advertising: *informative*, *persuasive* and *complementary* (see Bagwell, 2007). The *informative* view states that advertising provides information about the existence of the good, its price and/or quality, etc. This view depicts advertising as a way to increase the quality of matching between agents and goods. It should increase competition, and hence lower prices on the market and increase the elasticity of demand as well as entry. The *persuasive* view, on the other hand, considers advertising as having an anti-competitive effect, increasing the price and decreasing elasticity, by altering consumers' tastes. The *complementary* view regards advertising and the good which is advertised as complements in the utility function. According to this view, consuming a good which is advertised yields utility through the consumption of the good, but also through the image of the good created by the advertising. Empirically, however, the effects of advertising on various dimensions such as price, quantity, quality, entry and brand loyalty are uncertain, as they depend on the sector and the good which is considered.

Our model is based on both social identity and advertising. The notion of identity has not really been studied by economists, except by Akerlof and Kranton (2000). In their paper, they consider a utility function that depends on both agents' actions and identity. Identity is modeled as the fact that agents categorize others and are categorized, and that agents expect specific behavior from each category. They give various examples on how identity affects the utility of the agents through their own actions as well as those of others. As far as advertising is concerned, we rely on some existing studies that modelize situations similar to those we consider. In an extension of the Dorfman-Steiner Model, Bagwell (2007) uses a model in which the valuation of the product by agents depends on the type of the agent (vertical differentiation of type) and on advertising. A monopoly firm can then influence valuation. The main results are in line with the persuasive view since an increase in advertisement leads to a decrease in elasticity and an increase in price. In Grossman and Shapiro (1984), products are horizontally differentiated by the distance between an agent and firms. Firms can provide costly and random information about the existence of the product and its price. In line with the informative view, they find that increasing the level of advertising leads to an increase in price elasticity and a decrease in price. In Meurer and Stahl (1994) two firms produce two differentiated goods. Each agent is a good match with one of the two products, and has the same reservation value as everyone. Firms can provide information through advertising. The authors outline that advertising has a non monotonic effect on social surplus. The intuition behind this is that advertising increases information, hence the number of good matches, but on the other hand, it increases product differentiation and market power, which makes firms increase their price, and thus reduces expected sales.

Our model considers a duopoly in which each firm produces one good. Both goods have the same intrinsic value for agents and are perfect substitutes as long as neither signals a consumer's type. Indeed, an agent's valuation of a good depends on both social interactions and advertising. Consumption is used to signal types, in such a way that agents get utility if their type is well understood by others. Since only two goods are available, agents have to form groups, and the more heterogenous a group, the less utility agents obtain since their type cannot be deduced with precision. Hence, an agent will choose the good which is consumed by the group that is the closest to him in terms of type. Firms choose to produce a costly exogenously fixed level of advertising or not (we assume that they cannot choose its level), and determine the targeted type (the type on the basis of which the advertising is designed). For agents, consuming an advertised good is valuable. By choosing or not to advertise and its target, firms can then influence agents' valuations. Our model thus includes aspects from both the complementary and persuasive views. The complementary view is taken into account by the fact that we introduce a parameter which translates agents' taste for advertising. The persuasive view will be introduced by way of the targeted market theory, in which advertising is centered on some specific types which will buy the good since it seems to be perfect for them. Although goods are undifferentiated, social identity and advertising create differentiation.

The main results are the following: taking social identity into account leads to higher prices and profits for firms compared to the classical Bertrand price competition model, as well as to the formation of groups. This process leads to multiple equilibria, each of which corresponds to a particular partition of types (section 3). By adding advertising to the problem, agents become able to coordinate on some particular equilibria. On the other hand, this increases market power for firms, implying higher prices and profits. The type and the number of equilibria depend on the cost of advertising as well as the value of advertising to agents (section 4). Obviously, there always exist equilibria in which both firms advertise (when advertising costs are low enough) and in which no firm advertises (for high advertisement costs). On top of this, for low values of advertising, only symmetric equilibria can arise, with the possibility of multiple equilibria for medium advertising costs. If the value of advertising is high enough, then asymmetric equilibria can arise. We may then end up with one firm targeting its advertising toward the average type, and the other being considered as a "no-brand" firm. The various equilibria are a result of a combination of three effects: the competition effect, which leads firms to decrease their prices, the market stealing effect, which corresponds to the use of advertising by firms in order to take some of the market share of others, and the decrease in price elasticity due to the presence of social identity as well as advertising. We also investigate welfare, and we show that, depending on the value of advertising, it is optimal from the aggregate consumer welfare point of view to have zero, one or two firms advertising. At the individual level, we show that some consumers may lose, while others may gain from advertising (section 5).

The paper is organized as follows. Section 2 presents the setup of the model. In section 3, we develop a model of social identity, analyzing the formation of groups and price competition between firms. Advertising is introduced in section 4. Welfare implications of the presence of both social identity and advertising are investigated in section 5. Finally, section 6 concludes.

2 Setup of the model

We consider an economy with a continuum of agents of mass one. Each agent i has a type θ_i, which is privately known and uniformly distributed on $[0,1]$[1]. Agents are sorted according to their types. Two goods are produced in the economy: a and b. Each consumer buys one unit of one of the goods. There are two firms. Firm A produces good a, and firm B produces good b. Both firms use the same production technology and have the same cost structure. The marginal cost is equal to c, which is, for simplicity, normalized to 0. There are no fixed costs. a and b are perfect substitutes and perfectly undifferentiated, as long as there is no social identity or advertising. a (b) is sold at a unit price p_a (p_b). Each firm may also decide to advertise at cost γ and with a target θ_k. Firms choose prices and advertising targets in order to maximize profit, which is given by

$$\pi_j = \begin{cases} p_j q_j - \gamma & \text{if firm } j \text{ advertises} \\ p_j q_j & \text{otherwise} \end{cases}$$

The timing of the game is the following: first, firms choose simultaneously to advertise or not, as well as their advertising targets. Then, firms compete in prices. Finally, agents choose one of the goods, on the basis of the price, social identity theory mentioned earlier, as well as advertising.

As already explained, the idea behind social identity is that agents use consumption behavior to signal who they are to others, getting more utility if the type is well understood (remember the example of a hard rock fan wearing specific clothes which are completely different from those of a rap fan). Since the number of goods is limited, and given the absence of any outside option, groups will be formed. The more heterogenous a group, the less utility agents get, since their type cannot be inferred with precision. Although goods are initially undifferentiated, social identity then acts as a differentiating tool on goods, given that it modifies the value of the good for consumers. Hence, when choosing a consumption

[1] Another possibility would be to put consumers on a circle. However, computations then become much more complicated.

good, agents consider not only the price of each good, but also who else consumes that good in order to choose the good consumed by the group in which members' types are the closest to his own. To take into account this aspect, we need a utility function which reflects the composition of groups consuming a given good. Define $\Psi(k)$ as the set of agents consuming good $k \in \{a, b\}$. The social identity part of the utility of agent i consuming good k is then given by

$$-\int_{\{\theta_j : j \in \Psi(k)\}} |\theta_i - \theta| \, d\theta$$

The latter expression reaches it's maximum value in two situations: first if agent i is the only one who is consuming good k; second, if all agents consuming good k have exactly the same type. In both cases, this function is in line with the social identity theory since reaching the maximum value means that the type of agent i is well identified by others. On the contrary, the more $\Psi(k)$ is heterogenous, the lower the utility.

Advertising is introduced in the following way. If a firm decides to advertise (at a fixed level and a fixed cost γ), it must choose a type (θ_k) toward which the advertising is targeted. The idea is that the good is intrinsically the same for everyone, but the image created by advertising influences valuations in function of types. The further a type is from the advertising target, the less utility he gets from consuming that good. This comes from the literature on advertising, more precisely from the target market theory (see for example Aaker et al., 2000). This theory is based on the user positioning approach, in which a brand is closely associated with a particular user or customer. In line with the persuasive view, persuasion is enhanced by a match between the characteristics of the advertisement and those of the consumer, relative to when there is no such match. On the contrary, people in the nontarget market fail to buy the good since their type is not targeted by advertising, leading those agents to consider that their tastes are such that the good "is not for them". On top of this target market effect, we also assume that advertising generates a positive gain for agents consuming the advertised good. This is coherent with the complementary view of advertising and implies that not only does the targeted type has an incentive to buy the good, but so do a range of types around him. The advertising part of the utility of agent i consuming good

k is then given by

$$I_k(\alpha - |\theta_i - \theta_k|)$$

where α represents the intrinsic utility of advertising (taste for advertising). I_k is an indicator function which takes the value 1 if good k is advertised, and 0 otherwise. We further assume that the set of possible advertising targets is limited to $\left\{\theta_0, \theta_{\frac{1}{2}}, \theta_1\right\}$. One can interpret this as the fact that firms cannot discriminate exactly between all types. As a consequence, potential targets are limited. Moreover, it seems natural that firms can target the two extremes as well as the center (average) more easily. However, modifying the potential targets (for example $\left\{\theta_{\frac{1}{4}}, \theta_{\frac{1}{2}}, \theta_{\frac{3}{4}}\right\}$) does not change the results significantly.

Putting the two parts together, and subtracting the price of the good (p_k), the utility function of agent i choosing good k ($k \in \{a, b\}$) is then

$$U_i(k) = I_k(\alpha - |\theta_i - \theta_k|) - \int_{\{\theta_j : j \in \Psi(k)\}} |\theta_i - \theta| \, d\theta - p_k$$

3 A model of Social Identity

In a first step, we investigate how social identity affects the equilibrium in terms of prices, quantities, and profits, and how it drives the formation of groups.

As mentioned earlier, due to the limited number of goods and the absence of any outside option, groups will be created by the consumption choice of agents. Formation of groups means here that the continuum of types is partitioned into subsets, inside each of which agents consume the same good, in such a way that no one has an incentive to change his consumption. In other words, looking for groups means looking for an equilibrium partition of the continuum. Each group is a convex set of strictly positive measure of buyers consuming the same good. For a given number of subsets n, we must then determine the $n-1$ cutoffs such that $U_i(k) = U_i(-k), \forall i \in \{1, 2, ..., n-1\}, k \in \{a, b\}$. We denote by $\theta_1^*, ..., \theta_{n-1}^*$ the $n-1$ indifferent consumers. However, solving the problem for any possible n leads to many technical difficulties. For the sake of simplicity, we restrict the analysis to the possibility of having two or three groups (the possibility of having one group will appear in the next section).[2]

For each number of group, we first do not take into account price competition by firms (we consider prices as exogenously given) and advertising, but focus only on the formation of groups. As a result, the utility function is limited to $U_i(k) = -\int_{\{\theta_j : j\in\Psi(k)\}} |\theta_i - \theta|\, d\theta - p_k$. In a second step, we introduce price competition, based on the groups formed through social identity.

3.1 Two groups

If there are two groups ($n = 2$), we assume without loss of generality that groups of consumption divide $[0,1]$ following a sequence $a - b$. The indifferent consumer θ_1 is such that

$$-\int_0^{\theta_1} (\theta_1 - \theta) d\theta - p_a = -\int_{\theta_1}^{1} (\theta - \theta_1) d\theta - p_b$$

implying

$$\theta_1^* = \frac{1}{2} - (p_a - p_b)$$

Knowing the consumer behavior, a firm maximizes its profit by setting its price, taking into account the other firm's behavior. Hence, profit functions of firm A and firm B are respectively

$$\pi_A(p_a \mid p_b) = p_a \left(\frac{1}{2} - (p_a - p_b)\right) = p_a \left(\frac{1}{2} - \Delta p\right)$$

and

$$\pi_B(p_b \mid p_a) = p_b \left(1 - \left(\frac{1}{2} - \Delta p\right)\right)$$

The best response functions are then given by

$$\begin{cases} p_a = \frac{1+2p_b}{4} \\ p_a = \frac{-1+4p_b}{2} \end{cases}$$

[2] Results for larger values of n are available upon request.

meaning that equilibrium prices, quantities and profits are

$$\begin{aligned} p_a^* &= p_b^* = \frac{1}{2} = q_a^* = q_b^* \\ \pi_A^* &= \pi_B^* = \frac{1}{4} \end{aligned}$$

3.2 Three groups

If there are three groups ($n = 3$), we assume without loss of generality that groups of consumption divide $[0,1]$ following a sequence $a - b - a$[3]. The indifferent consumers θ_1 and θ_2 are such that

$$\begin{cases} -\int_0^{\theta_1}(\theta_1-\theta)\,d\theta - \int_{\theta_2}^1(\theta-\theta_1)\,d\theta - p_a = -\int_{\theta_1}^{\theta_2}(\theta-\theta_1)d\theta - p_b \\ -\int_0^{\theta_1}(\theta_2-\theta)\,d\theta - \int_{\theta_2}^1(\theta-\theta_2)\,d\theta - p_a = -\int_{\theta_1}^{\theta_2}(\theta_2-\theta)d\theta - p_b \end{cases}$$

$\Leftrightarrow$

$$\begin{cases} -\int_0^{\theta_1}(\theta_1-\theta)\,d\theta - \int_{\theta_2}^1(\theta-\theta_1)\,d\theta - \left[-\int_{\theta_1}^{\theta_2}(\theta-\theta_1)d\theta\right] = p_a - p_b \\ -\int_0^{\theta_1}(\theta_2-\theta)\,d\theta - \int_{\theta_2}^1(\theta-\theta_2)\,d\theta - \left[-\int_{\theta_1}^{\theta_2}(\theta_2-\theta)d\theta\right] = p_a - p_b \end{cases}$$

$\Leftrightarrow$

$$\begin{aligned} & -\int_0^{\theta_1}(\theta_1-\theta)\,d\theta - \int_{\theta_2}^1(\theta-\theta_1)\,d\theta - \left[-\int_{\theta_1}^{\theta_2}(\theta-\theta_1)d\theta\right] \\ = & -\int_0^{\theta_1}(\theta_2-\theta)\,d\theta - \int_{\theta_2}^1(\theta-\theta_2)\,d\theta - \left[-\int_{\theta_1}^{\theta_2}(\theta_2-\theta)d\theta\right] \end{aligned}$$

$\Leftrightarrow$

$$-1 + \theta_1 + \theta_2 = 0$$

$\Leftrightarrow$

$$\theta_2 = 1 - \theta_1$$

[3]Reversing this assumption just reverses the results below.

Hence, at equilibrium, θ_1^* is such that

$$-\int_0^{\theta_1} (\theta_1 - \theta)\, d\theta - \int_{1-\theta_1}^{1} (\theta - \theta_1)\, d\theta - p_a = -\int_{\theta_1}^{1-\theta_1} (\theta - \theta_1) d\theta - p_b$$

that is

$$\theta_1^* = \frac{1}{6}\left(3 - \sqrt{3}\sqrt{1 + 4\left(p_a - p_b\right)}\right)$$

Profit functions of firm A and B are respectively

$$\pi_A(p_a \mid p_b) = p_a\left(2\theta_1^*\right)$$
$$\pi_B(p_b \mid p_a) = p_b\left(1 - 2\theta_1^*\right)$$

The best response functions are then given by

$$\begin{cases} p_a = \frac{1}{6}\sqrt{2}\sqrt{1 - 2p_b} + 4p_b \\ p_a = \frac{1}{4}(-1 + 6p_b) \end{cases}$$

Solving this system leads to equilibrium prices, hence to equilibrium quantities and profits.

$$\begin{aligned} p_a^* &= \frac{2 + 3\sqrt{6}}{25} < \frac{1}{2} \\ p_b^* &= \frac{11 + 4\sqrt{6}}{50} < \frac{1}{2} \end{aligned}$$

$$\begin{aligned} q_a^* &= \frac{2}{15}\left(6 - \sqrt{6}\right) < \frac{1}{2} \\ q_b^* &= \frac{1}{15}\left(3 + 2\sqrt{6}\right) > \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \pi_A^* &= \frac{4}{375}\left(-3 + 8\sqrt{6}\right) < \frac{1}{4} \\ \pi_B^* &= \frac{1}{750}\left(81 + 34\sqrt{6}\right) < \frac{1}{4} \end{aligned}$$

3.3 Equilibria

Let us compare the different results. First, with respect to the classical result of the Bertrand price competition model, prices and profits are higher in both cases. This comes from the fact that social identity creates rigidities on the market, hence, market power for firms. This is one of the main results of the paper. The explanation for this is the following: in Bertrand, goods are undifferentiated, meaning that consumers only care about relative prices. Here, social identity gives some value to the good depending on who consumes it. Hence, initially undifferentiated goods are in fact differentiated by consumption: social identity acts as a differentiating tool on goods.

Second, we have multiple equilibria, in the sense that for each n, there exists an equilibrium partition of the continuum as well as equilibrium prices and profits. However, these are different in terms of prices and profits. When $n = 2$, equilibrium prices and quantities are equal to $\frac{1}{2}$ and profits are equal to $\frac{1}{4}$. When $n = 3$, prices and profits are smaller than when $n = 2$, while the quantity is larger for one firm and smaller for the other one. The explanation for this is the following. When $n = 2$, each firm "owns" exactly half of the market, each firm owing one extreme of the continuum, while when $n = 3$, one firm "owns" the two "extreme" subsets. This means that firms in the odd case are more exposed to competition. Hence, it is the structure of partitioning that increases competition in the odd case, leading to lower prices: when there are three subsets, good b is only consumed in the central subset, while good a is consumed in the two extreme subsets. This means that the group of people consuming b is more homogenous than the group consuming a. Hence, firm B has more market power than A, which leads to higher price and quantity for firm B.

Proposition 1 *A model including social identity and price competition leads to the formation of groups as well as to higher prices and profits compared to the classical Bertrand price competition model.*

Concerning consumers' total surplus, the total surplus (TS) for $n = 2$ is $-\frac{7}{12}$, while for $n = 3$ it is $\frac{-777-178\sqrt{6}}{2250}$, so it is better in terms of consumers' total surplus to have three subsets than only two. This result is clearly linked to the difference in competition outlined above.

In order to ensure that the reader understands clearly what we are talking about, let us consider the following example. Adidas and Bellerose are two firms that produce sweatshirts among other things. Basically, the sweatshirts are the same: they are available in almost the same colors and the same design, they both are equally warm, etc. However, people wearing Adidas sweatshirts are usually not the same as those who wear Bellerose sweatshirts. The idea is that the brand you wear will signal who you are to others. Hence, groups of consumption have been created. One way this signal can be created is by advertising, which focuses the image of the good on a particular type of agents, each firm targeting a different type of consumer. This possibility is investigated in the next two sections: section 4 presents a model including social identity as well as advertising, with the objective of describing firms' equilibrium behavior in terms of advertising. Section 5 studies the welfare implications of having both social identity and advertising, at the aggregate level as well as at the individual level.

4 Social Identity, Advertising and Market Competition

As we just showed, in the absence of coordination, social identity leads to multiple equilibria. We now allow firms to advertise in order to create an image for the good. For the simplicity of the analysis, we restrict ourselves, for the benchmark case, to considering only the case with two groups if no advertising takes place. This assumption does not generate any result which would not exist if we were also considering three groups. Every result would hold qualitatively. Moreover, given that firms' profits are higher with two groups than with three groups, taking the two group case as a benchmark means considering the lower bound for firms to advertise.

Remember that with two groups, the presence of social identity in the utility function increases prices and profits, in such a way that

$$\pi_A = \pi_B = \frac{1}{4}$$

We now add the possibility for firms to advertise. We then do not only have to take into account the effect of social identity, but also how advertising modifies the formation of groups and prices on the market. On top of the possibility of "no advertising", each firm can choose between three advertising targets: $\left\{\theta_0, \theta_{\frac{1}{2}}, \theta_1\right\}$. Since there are two firms, 16 possible situations may arise, which are represented in Table I below. In order to keep things simple, we lower the number of cases to study by making two assumptions: first, if only one firm advertises, we call it firm B[4]. Second, if both firms advertise, then, again without loss of generality, we set $\theta_b \leqslant \theta_a$. Note that by symmetry, $\{\theta_a = \emptyset, \theta_b = 0\}$ is the same as $\{\theta_a = \emptyset, \theta_b = 1\}$, so we do not need to investigate the latter.

The remainder of section 4 is organized as follows: subsections 4.1, 4.2 and 4.3 analyze the different firms' strategies. Subsection 4.4 studies firms' equilibrium behavior.

For each case, the table indicates the section in which the situation is analyzed.

		Firm B			
		$\theta_b = \emptyset$	$\theta_b = 0$	$\theta_b = \frac{1}{2}$	$\theta_b = 1$
	$\theta_a = \emptyset$	Section 3	4.1.1	4.1.2	.
Firm A	$\theta_a = 0$	.	4.3	.	.
	$\theta_a = \frac{1}{2}$	.	4.2.1	4.3	.
	$\theta_a = 1$	.	4.2.2.	4.2.3	4.3

Table I: Strategy of each firm

We start by analyzing cases in which only one firm advertises, that is $\{\theta_a = \emptyset, \theta_b = 0\}$ and $\left\{\theta_a = \emptyset, \theta_b = \frac{1}{2}\right\}$. We then investigate cases where both firms advertise, but use different targets: $\left\{\theta_a = \frac{1}{2}, \theta_b = 0\right\}$, $\{\theta_a = 1, \theta_b = 0\}$, $\left\{\theta_a = 1, \theta_b = \frac{1}{2}\right\}$. Finally, we look at cases in which both firms choose the same target. Once this is done, we look for Nash equilibria in all cases.

[4] This assumption implies that when we will study Nash equilibria, we will only look at firm B potentially deviating from the "no advertising" equilibrium, and if B advertises, we will look at how A behaves. This is done without loss of generality, other cases yield results symmetric to the ones we obtain.

4.1 One firm advertises

4.1.1 If $\theta_a = \emptyset$ and $\theta_b = 0$

Lemma 1 *If only one firm advertises, with an advertising targeted at one endpoint of the continuum* $[0,1]$*, the partition is composed of exactly two subsets.*

Proof. See appendix ■

This first case is asymmetric, since one firm advertises but the other does not. Moreover, the target chosen by the firm is the extreme left of the continuum. Adding advertising modifies the problem considerably, since agents now have a coordination tool.

The indifferent consumer's type θ_i is given by

$$\alpha - \theta_i - \int_0^{\theta_i} (\theta_i - \theta) d\theta - p_b = - \int_{\theta_i}^{1} (\theta - \theta_i) d\theta - p_a$$

which allows to compute the market share of each firm

$$\begin{aligned} q_a &= \frac{\frac{3}{2} - (\alpha - p_a + p_b)}{2} \\ q_b &= \frac{\frac{1}{2} + \alpha - p_a + p_b}{2} \end{aligned}$$

Since each firm maximizes its profit by choosing its price, given the price of the other firm and the value of α, the two best response functions are

$$\begin{aligned} BR_A &: p_b^{opt} = \frac{-3}{2} + \alpha + 2p_a^{opt} \\ BR_B &: p_b^{opt} = \frac{1}{4}\left(1 + 2\alpha + 2p_a^{opt}\right) \end{aligned}$$

Solving the price system leads to (with $p_i^{\theta_a,\theta_b}$ the price of firm i when firm A chooses θ_a and firm B chooses θ_b)

$$\begin{aligned} p_a^{\emptyset,0} &= \frac{7 - 2\alpha}{6} \\ p_b^{\emptyset,0} &= \frac{5 + 2\alpha}{6} \end{aligned}$$

corresponding quantities are

$$\begin{aligned} q_a^{\emptyset,0} &= \frac{7-2\alpha}{12} \\ q_b^{\emptyset,0} &= \frac{5+2\alpha}{12} \end{aligned}$$

and the firms' profits are given by

$$\begin{aligned} \pi_A^{\emptyset,0} &= \frac{(7-2\alpha)^2}{72} \\ \pi_B^{\emptyset,0} &= \frac{(5+2\alpha)^2}{72} - \gamma \end{aligned}$$

Analyzing these results, we first observe that even if $\alpha = 1$, i.e. that no one suffers from a negative effect of advertising, the market share of B is not 1. This is due to the arbitrage agents face between the taste for advertising and the taste for similarity when interacting. Second, advertising adds rigidities in prices to those created by the presence of social identity. Indeed, for each α, $p_b^{\emptyset,0} > p_b^{\emptyset,\emptyset}$ and for each $\alpha < 2, p_a^{\emptyset,0} > p_a^{\emptyset,\emptyset}$. Third, if $\alpha > \frac{1}{2}$, group B is larger than half of the population (i.e. $q_b^{\emptyset,0} > q_a^{\emptyset,0}$), and $p_b^{\emptyset,0} > p_a^{\emptyset,0}$. This means that the majority of the population buys the more expensive good because of the combination of advertising and social identity. An explanation for this result is that if the taste for advertising is high enough, the firm using advertising steals some of the market share of the other firm (this market share stealing effect is defined as the fact that, because of the taste for advertising, a firm which advertises its good can attract new consumers). Fourth, the use of advertising by a firm generates a profit externality for the other firm, whose sign depends on the size of α. Indeed, if $\alpha < \frac{7-3\sqrt{2}}{2}$, then the externality is positive, the underlying idea being that the taste for advertising is not high enough so the effect of stealing some of the market share is smaller than the increase in market power for both firms. Fifth, note that the minority is better off in terms of social interactions than in the no advertising case, but the price it has to pay is higher, as soon as $\alpha < 2$. Finally, if $\alpha \geqslant \frac{7}{2}$, firm B covers the whole market.

To sum up these effects, the use of advertising by a firm creates a decrease in price elasticity for this firm, leading to a higher profit. This effect will also affect the other firm, thanks to an externality effect, but providing that the taste for advertising is not too high. Indeed, if the taste for advertising is high, then the market stealing effect becomes more important than the externality effect, and the non-advertising firm is forced to decrease its price in order to retain a positive market share.

4.1.2 If $\theta_a = \emptyset$ and $\theta_b = \frac{1}{2}$

Lemma 2 *If only one firm advertises, with an advertising targeted at the center of the continuum* $[0, 1]$*, the partition is composed of three subsets, with two symmetric indifferent consumers.*

Proof. See appendix ■

This case is similar to the previous one, in the sense that only one firm advertises. However, the target is located at the center of the continuum. Three groups will now be formed, consumers in the central one consume the advertised good.

Computing the first cutoff, using the demand side equilibrium condition leads to

$$\theta' = \frac{1}{3}(2 - \sqrt{1 + 3p_a - 3p_b + 3\alpha})$$

Since each firm maximizes its profit by choosing its price, given the price of the other firm and the value of α, the two best response functions are obtained, i.e.

$$\begin{aligned} BR_A &: p_a^{opt} = \frac{2}{27}\left(1 + 9p_b^{opt} - 9\alpha + 2\sqrt{7 - 9p_b^{opt} + 9\alpha}\right) \\ BR_B &: p_a^{opt} = \frac{1}{24}\left(-7 + 36p_b^{opt} + \sqrt{1 + 24p_b^{opt}} - 24\alpha\right) \end{aligned}$$

Solving the price system, prices are

$$\begin{aligned} p_a^{\emptyset,\frac{1}{2}} &= \frac{1}{135}\left(37 - 54\alpha + 3\sqrt{45 + 60\alpha} + 2\sqrt{10}\sqrt{43 + 54\alpha - 3\sqrt{45 + 60\alpha}}\right) \\ p_b^{\emptyset,\frac{1}{2}} &= \frac{1}{30}\left(9 + 12\alpha + \sqrt{15}\sqrt{3 + 4\alpha}\right) \end{aligned}$$

and quantities

$$
\begin{aligned}
q_a^{\emptyset,\frac{1}{2}} &= \frac{4}{3}-\frac{1}{9}\sqrt{\frac{2}{5}}\sqrt{83+54\alpha-3\sqrt{45+60\alpha}+4\sqrt{10}\sqrt{43+54\alpha-3\sqrt{45+60\alpha}}} \\
q_b^{\emptyset,\frac{1}{2}} &= \frac{1}{45}\left(-15+\sqrt{10}\sqrt{83+54\alpha-3\sqrt{45+60\alpha}+4\sqrt{10}\sqrt{43+54\alpha-3\sqrt{45+60\alpha}}}\right)
\end{aligned}
$$

The profit of each firm is given by

$$
\begin{aligned}
\pi_A^{\emptyset,\frac{1}{2}} &= -\frac{1}{6075}\left(\begin{array}{c}\left(37-54\alpha+3\sqrt{45+60\alpha}+2\sqrt{10}\sqrt{43+54\alpha-3\sqrt{45+60\alpha}}\right)\times \\ \left(-60+\sqrt{10}\sqrt{83+54\alpha-3\sqrt{45+60\alpha}+4\sqrt{10}\sqrt{43+54\alpha-3\sqrt{45+60\alpha}}}\right)\end{array}\right) \\
\pi_B^{\emptyset,\frac{1}{2}} &= \frac{1}{1350}\left(\begin{array}{c}\left(9+12\alpha+\sqrt{45+60\alpha}\right)\times \\ \left(-15+\sqrt{10}\sqrt{83+54\alpha-3\sqrt{45+60\alpha}+4\sqrt{10}\sqrt{43+54\alpha-3\sqrt{45+60\alpha}}}\right)\end{array}\right)-\gamma
\end{aligned}
$$

Most observations made above remain valid. In particular, the market stealing effect still appears, but for lower values of α. Indeed, as soon as $\alpha > \frac{3}{16}$, firm B sells more than firm A, and at a higher price. Similarly, now firm B owns the whole market as soon as $\alpha = 3$. These two results are different from the previous ones since the advertising target is now located in the middle of the continuum. Hence, the market power of the advertising firm is stronger.

4.1.3 Firm B's optimal strategy if A does not advertise

To investigate this, we have to compare the profit made by firm B when the target is, respectively, $\theta_b = 0$ and $\theta_b = \frac{1}{2}$ (remember that $\theta_b = 1$ is symmetric to $\theta_b = 0$). First, note that $\frac{\partial \pi_B^{\emptyset,0}}{\partial \alpha} > 0$ and $\frac{\partial \pi_B^{\emptyset,\frac{1}{2}}}{\partial \alpha} > 0$. Second, we also have that $\pi_B^{\emptyset,0}(\alpha = 0) > \pi_B^{\emptyset,\frac{1}{2}}(\alpha = 0)$ and $\pi_B^{\emptyset,0}(\alpha = 3) < \pi_B^{\emptyset,\frac{1}{2}}(\alpha = 3)$. Third, $\pi_B^{\emptyset,0}$ and $\pi_B^{\emptyset,\frac{1}{2}}$ cross only once on the interval $\alpha \in [0, 3]$.

Lemma 3 $\exists \alpha' \in [0,3]$ *such that*

if $\alpha \leqslant \alpha' : \pi_B^{\emptyset,0} \geqslant \pi_B^{\emptyset,\frac{1}{2}}$

if $\alpha > \alpha' : \pi_B^{\emptyset,0} < \pi_B^{\emptyset,\frac{1}{2}}$

The interpretation of the result is the following: if α is not too high, the market stealing effect is too weak with respect to the price competition effect. In this case, targeting the center of the continuum is not sufficiently profitable for firm B (in terms of market power), while, by choosing to locate at the border, firm B is protected against part of the price competition effect. If, on the other hand, α is high enough, then being fully exposed to the price competition effect is not too bad since the market stealing effect is high.

4.2 Both firms advertise using different targets

Lemma 4 *When both firms advertise using different targets there are exactly two subsets.*

Proof. See appendix ■

When both firms advertise, they both have to choose an advertising target on the continuum. Remember that we only study cases in which $\theta_b \leqslant \theta_a$, meaning, in this section, that $\theta_b < \theta_a$.

We are looking for the indifferent consumer, i.e.

$$\alpha - |\theta_i - \theta_b| - \int_0^{\theta_i} (\theta_i - \theta) d\theta - p_b = \alpha - |\theta_i - \theta_a| - \int_{\theta_i}^{1} (\theta - \theta_i) d\theta - p_a \tag{1}$$

Market shares for both firms are then given by

$$\begin{aligned} q_a &= \frac{\frac{5}{2} - (\theta_a + \theta_b) - p_a + p_b}{3} \\ q_b &= \frac{\frac{1}{2} + (\theta_a + \theta_b) + p_a - p_b}{3} \end{aligned}$$

Since each firm maximizes its profit by choosing its price, given the price of the other firm, the value of α and both advertising targets, the two best response functions are

$$\begin{aligned} BR_A &: p_a^{opt} = \frac{5 + 2p_b - 2(\theta_a + \theta_b)}{4} \\ BR_B &: p_a^{opt} = -\frac{1}{2} + 2p_b - 2(\theta_a + \theta_b) \end{aligned}$$

Solving the price system, we obtain

$$
\begin{aligned}
p_a^{\theta_a,\theta_b} &= \frac{11-2(\theta_a+\theta_b)}{6} \\
p_b^{\theta_a,\theta_b} &= \frac{7+2(\theta_a+\theta_b)}{6}
\end{aligned}
$$

corresponding quantities are

$$
\begin{aligned}
q_a^{\theta_a,\theta_b} &= \frac{11-2(\theta_a+\theta_b)}{18} \\
q_b^{\theta_a,\theta_b} &= \frac{7+2(\theta_a+\theta_b)}{18}
\end{aligned}
$$

The profit of each firm is then given by

$$
\begin{aligned}
\pi_A^{\theta_a,\theta_b} &= \frac{(11-2(\theta_a+\theta_b))^2}{108} - \gamma \\
\pi_B^{\theta_a,\theta_b} &= \frac{(7+2(\theta_a+\theta_b))^2}{108} - \gamma
\end{aligned}
$$

4.2.1 If $\theta_a = \frac{1}{2}$ and $\theta_b = 0$

With these targets, we have

$$
\begin{aligned}
\theta_a^* &= \frac{1}{2}; \theta_b^* = 0 \\
p_a^{\frac{1}{2},0} &= \frac{10}{6}; p_b^{\frac{1}{2},0} = \frac{8}{6} \\
q_a^{\frac{1}{2},0} &= \frac{10}{18}; q_b^{\frac{1}{2},0} = \frac{8}{18} \\
\pi_A^{\frac{1}{2},0} &= \frac{100}{108} - \gamma; \pi_B^{\frac{1}{2},0} = \frac{64}{108} - \gamma
\end{aligned}
$$

4.2.2 If $\theta_a = 1$ and $\theta_b = 0$

With these targets, we have

$$\begin{aligned}
\theta_a^* &= 1; \theta_b^* = 0 \\
p_a^{1,0} &= \frac{9}{6}; p_b^{1,0} = \frac{9}{6} \\
q_a^{1,0} &= \frac{1}{2}; q_b^{1,0} = \frac{1}{2} \\
\pi_A^{1,0} &= \frac{81}{108} - \gamma; \pi_B^{1,0} = \frac{81}{108} - \gamma
\end{aligned}$$

4.2.3 If $\theta_a = 1$ and $\theta_b = \frac{1}{2}$

With these targets, we have

$$\begin{aligned}
\theta_a^* &= 1; \theta_b^* = \frac{1}{2} \\
p_a^{1,\frac{1}{2}} &= \frac{8}{6}; p_b^{1,\frac{1}{2}} = \frac{10}{6} \\
q_a^{1,\frac{1}{2}} &= \frac{8}{18}; q_b^{1,\frac{1}{2}} = \frac{10}{18} \\
\pi_A^{1,\frac{1}{2}} &= \frac{64}{108} - \gamma; \pi_B^{1,\frac{1}{2}} = \frac{100}{108} - \gamma
\end{aligned}$$

Obviously $\{\theta_a = 1, \theta_b = 0\}$ will never be an equilibrium since each firm has an incentive to deviate. Hence, only $\left\{\theta_a = \frac{1}{2}, \theta_b = 0\right\}$ and $\left\{\theta_a = 1, \theta_b = \frac{1}{2}\right\}$ can potentially be equilibria. In each case, one firm targets the border of the continuum, and the other one the center. The "border firm" sells a smaller quantity but at a higher price than when there is only social identity. The "central firm" exhibits both a higher price and a higher quantity than in the benchmark case. Hence, advertising increases prices, by creating market power for firms, but the effect on quantities depends on the position of the advertising (note here that the market stealing effect disappears since both firms advertise, meaning that the taste for advertising no longer plays any role). Moreover, the "central firm" obtains a higher profit than the "border firm". Of course, the final decision to advertise or not depends crucially on the size of the advertising cost.

4.3 Both firms advertise using the same target

In this case, advertising no longer signals anything. Indeed, when choosing the same target, firms modify the problem in equation (1) in the sense that advertising disappears from the problem, meaning that we are back to the case where only the social identity effect takes place, thereby leading to the same prices and quantities, but to smaller profits since firms have to pay the advertising cost. Since, in the best possible case, firms have a profit of $\frac{1}{4}$, it is obvious that targeting the same type is never an equilibrium.

4.4 Equilibrium behavior

So far, we have shown that $\{\theta_a = \theta_b \neq \emptyset\}$ and $\{\theta_a = 1, \theta_b = 0\}$ are not equilibria. Let us take a look at the other cases. Remember that in case of social identity when there is no advertising and an even number of groups, profits are

$$\pi_A^{\emptyset,\emptyset} = \pi_B^{\emptyset,\emptyset} = \frac{1}{4}$$

This is an equilibrium if no firm has an incentive to deviate, i.e. to advertise. Firm B will decide to advertise its good, that is, to deviate from no advertising if the payoff it gets from advertising is higher than by not, given than A does not advertise[5]. Moreover, firm B also has to decide which advertising target to choose. Hence, to see if B deviates or not, we need to look at two different expressions, each one corresponding to one advertising target.

$$\begin{aligned} \Delta_1 &= \pi_B^{\emptyset,0} + \gamma - \frac{1}{4} \\ \Delta_2 &= \pi_B^{\emptyset,\frac{1}{2}} + \gamma - \frac{1}{4} \end{aligned}$$

Note that $\Delta_1$and Δ_2 respectively share the same specificities as $\pi_B^{\emptyset,0}$ and $\pi_B^{\emptyset,\frac{1}{2}}$. Firm B then advertises if one of the two Δ_i is larger than γ. If both Δ_i are larger than γ, B chooses the target corresponding to the Δ_i with the highest value. If, on the other hand, both

[5]This comes from the assumption that if only one firm advertises, it is firm B. This is done without loss of generality, symmetric results can be found by assuming that firm A unilaterally deviates

expressions are smaller than γ, the equilibrium is a situation in which no firm advertises. If B chooses to advertise, we still have to check if A also chooses to advertise to characterize the equilibrium. Depending on the target chosen by B, A advertises if the corresponding expression is larger than γ.

$$\begin{aligned}
\Delta_3 &= \pi_A^{1,\frac{1}{2}} + \gamma - \pi_A^{\emptyset,\frac{1}{2}} \\
\Delta_4 &= \pi_A^{\frac{1}{2},0} + \gamma - \pi_A^{\emptyset,0}
\end{aligned}$$

If one of these last two Δ_i is larger than γ, we end up in an equilibrium in which both firms advertise. Otherwise, we have an asymmetric equilibrium in which one firm advertises, and the other one does not. $\Delta_1$and Δ_2 are two increasing and convex functions, while Δ_3 and Δ_4 are increasing but concave functions. Note also that

$$\begin{aligned}
\Delta_1(\alpha = 0) &< \Delta_2(\alpha = 0) < \Delta_3(\alpha = 0) < \Delta_4(\alpha = 0) \\
\Delta_3(\alpha = 3) &< \Delta_4(\alpha = 3) < \Delta_2(\alpha = 3) < \Delta_1(\alpha = 3)
\end{aligned} \tag{2}$$

In fact, $\Delta_4 > \Delta_3 \ \forall \alpha \geqslant 0$, which can be interpreted as follows: when both firms advertise, it is always better for firm A to choose $\theta_a = \frac{1}{2}$ if possible. By Proposition 3, we know that there exists α' such that $\Delta_2(\alpha') = \Delta_1(\alpha')$. Moreover, we have

$$\Delta_1(\alpha') = \Delta_2(\alpha') < \Delta_3(\alpha') < \Delta_4(\alpha') \tag{3}$$

By the properties of the $\Delta's$ and (2) and (3), there exists $\alpha'' > \alpha'$ such that $\Delta_2(\alpha'') = \Delta_3(\alpha'')$, which leads to Proposition 4.

Proposition 2 $\exists 0 < \alpha' < \alpha'' < 3$ *such that*

1) For $\alpha : 0 \leqslant \alpha < \alpha'$

if $\gamma > \Delta_4$: *no firm advertises*

if $\gamma < \Delta_2$: *both firms advertise and* $\{\theta_a, \theta_b\} = \left\{\frac{1}{2}, 0\right\}$

if $\Delta_2 \leqslant \gamma \leqslant \Delta_4$: *there are two equilibria. In one equilibrium, no firm advertises. In the* *other,* $\{\theta_a, \theta_b\} = \left\{\frac{1}{2}, 0\right\}$

2) For $\alpha : \alpha' \leqslant \alpha < \alpha''$

if $\gamma > \Delta_3$ *: no firm advertises*

if $\gamma < \Delta_1$ *: both firms advertise and* $\{\theta_a, \theta_b\} = \left\{1, \frac{1}{2}\right\}$

if $\Delta_1 \leqslant \gamma \leqslant \Delta_3$ *: there are two equilibria. In one equilibrium, no firm advertises. In the other,* $\{\theta_a, \theta_b\} = \left\{1, \frac{1}{2}\right\}$

3) For $\alpha : \alpha'' \leqslant \alpha \leqslant 3$

if $\gamma > \Delta_1$ *: no firm advertises*

if $\gamma < \Delta_3$ *: both firms advertise and* $\{\theta_a, \theta_b\} = \left\{1, \frac{1}{2}\right\}$

if $\Delta_3 \leqslant \gamma \leqslant \Delta_1$ *: there is an asymmetric equilibrium in which only one firm advertises:* $\{\theta_a, \theta_b\} = \left\{\emptyset, \frac{1}{2}\right\}$

These equilibria are represented in Figure I

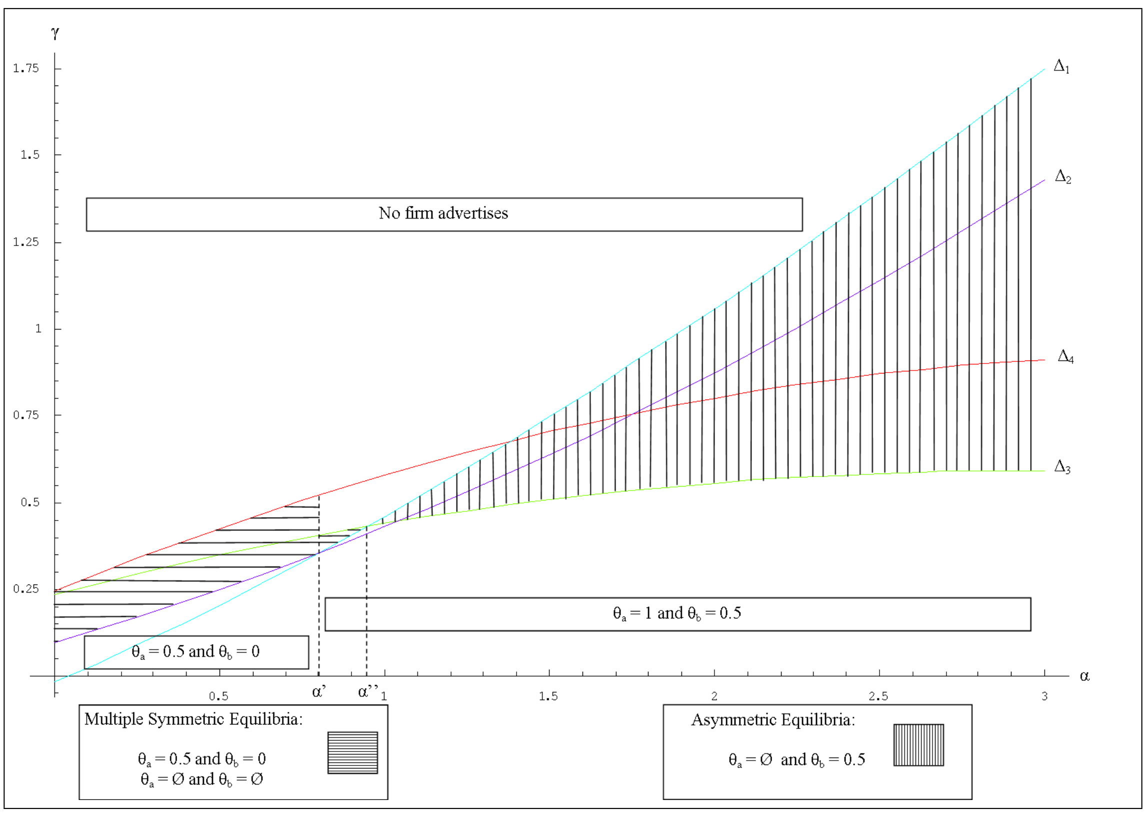

Figure I: Equilibria

Let us now turn to the interpretation of Proposition 2. For low values of α, as already explained, if firm B advertises, it chooses $\theta_b = 0$. Indeed, choosing $\theta_b = \frac{1}{2}$ is worse since the decrease in price elasticity and the market stealing effect created by advertising are not high enough to overcome the higher price competition effect when the advertising target is located at the middle of the continuum. Hence, if firm A decides to advertise, it chooses $\theta_a = \frac{1}{2}$. Results for high and low values of γ are quite intuitive. If the advertising cost is too high, no one has an incentive to invest in advertising. On the other hand, a low advertising cost implies that both firms advertise. Multiple equilibria arise for medium values of γ since, given the low value of α, firm B may not have an incentive to move and start an advertising project, meaning that no one deviates from the no-advertising equilibrium. However, suppose both firms advertise, then the profit of A may be high enough to prevent this firm from ceasing to advertise. A similar reasoning explains results for average values of α, with the difference that now α is high enough for firm B to choose $\theta_b = \frac{1}{2}$, meaning that firm A has to choose $\theta_a = 1$. For high values of α, the incentive for firm B to invest in advertising is strong, and becomes stronger with α when A does not advertise. On the contrary, the incentive for firm A to advertise when B advertises is bounded below the level of incentive for B[6]. Hence, there exists a range of values of γ such that B advertises, but not A, given the advertising cost and the limit on the advertising incentive[7]. Of course, results for really high and low levels of γ still exist. Once again, we want to underline the fact that in this "Equilibrium Behavior" section we only look at the possibility for B to deviate (or not) unilaterally from the "no-advertising" equilibrium. This assumption is simply meant to ensure that the reasoning is not too complicated, and it generates no loss of generality, since one can find all the other results (i.e. when A deviates from the "no-advertising" equilibrium) by symmetry.

Our model, more precisely this section about advertising, yields several interesting results: first, we showed that there exist equilibria in which firms use advertising in order to increase their profits. This is partly due to the parameter measuring the taste for advertising, but not only. Indeed, advertising also acts as a coordination tool for consumers, which in turn creates more market power for firms. Beyond the trivial cases in which both firms advertise (for low advertising costs) or do not (for high advertising costs), our model also generates

[6] This comes from the fact that when both firms advertise, α no longer plays any role.

[7] Again, the same result exists for A advertising but not B.

multiple equilibria, as well as an asymmetric equilibrium. The latter arises for high values of taste for advertising, and average advertising costs. This equilibrium may explain why there are cases in which one firm advertises, focussing on the average type, while the other behaves as a "no brand" product.

5 Consumer welfare implications

After computing the different equilibria, we now investigate how consumer welfare is affected by advertising. To do this, we use the utilitarian definition of welfare, and split the problem into two parts. First, what is the effect of advertising on aggregate consumer welfare? Second, what is the individual effect on agents?

5.1 Aggregate consumer welfare

We start by evaluating total consumer welfare when there is no advertising. This does not lead to a unique solution since, as outlined above, when there is no coordination device, multiple equilibria exist, each one corresponding to a particular partition of the continuum. Letting W be the total consumer welfare, we have in the case of no advertising

$$\begin{aligned} W^0(2 \text{ groups}) &= -\frac{7}{12} \\ W^0(3 \text{ groups}) &= \frac{-777 - 178\sqrt{6}}{2250} \end{aligned}$$

If both firms advertise, we have a unique solution (for reasons of symmetry), and total consumer welfare is given by

$$W^2 = \alpha - \frac{593}{324}$$

Finally, if only one firm advertises, we have

$$W^1 = \frac{1}{72900}\left(\begin{array}{c} -49005 - 1620\sqrt{45+60\alpha} - 1080\sqrt{10}\sqrt{43+54\alpha-3\sqrt{45+60\alpha}} \\ +436\sqrt{10}\sqrt{83+54\alpha-3\sqrt{45+60\alpha}+4\sqrt{10}\sqrt{43+54\alpha-3\sqrt{45+60\alpha}}} \\ +10\left(-3\sqrt{6}\sqrt{3+4\alpha}+8\sqrt{43+54\alpha-3\sqrt{45+60\alpha}}\right) \\ \times\sqrt{83+54\alpha-3\sqrt{45+60\alpha}+4\sqrt{10}\sqrt{43+54\alpha-3\sqrt{45+60\alpha}}} \\ +108\alpha\left(270+\sqrt{10}\sqrt{83+54\alpha-3\sqrt{45+60\alpha}+4\sqrt{10}\sqrt{43+54\alpha-3\sqrt{45+60\alpha}}}\right) \end{array}\right)$$

Note that W^1 and W^2 are increasing in α. Moreover, $W^0(\alpha = 0) > W^1(\alpha = 0) > W^2(\alpha = 0)$ and $W^2(\alpha = 3) > W^1(\alpha = 3) > W^0(\alpha = 3)$. Hence, there are three zones, each defining a range of α in which it is optimal, from the point of view of aggregate consumer welfare, that respectively no firm advertises, one firm advertises or both firms advertise.

Proposition 3 $\exists\ \alpha^*, \alpha^{**} : 0 < \alpha^* < \alpha^{**} < 3$ *such that*

1) $\forall\alpha \leqslant \alpha^*$, *it is optimal for aggregate consumer welfare that no firm advertises*

2) $\forall\alpha^* < \alpha \leqslant \alpha^{**}$, *it is optimal for aggregate consumer welfare that only one firm advertises*

3) $\forall\alpha > \alpha^{**}$, *it is optimal for aggregate consumer welfare that both firms advertise*

This result is illustrated by Figure II.

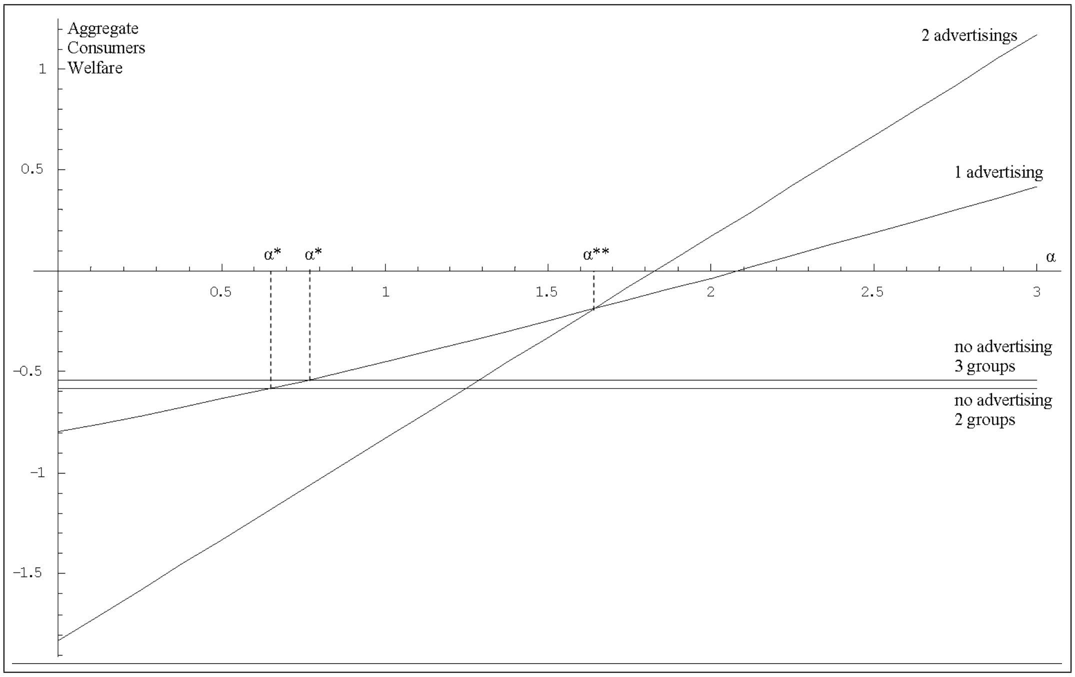

Figure II: Welfare

For low values of α, advertising generates benefits for very few people, those located around the advertising target, while most are hurt by it. Moreover, advertising creates market rigidities, thus higher prices. This means that the cost of advertising for consumers is really higher than its benefit. For average levels of α, the benefit of advertising is positive for more people. However, it is still too small to overcome the rigidities created when both firms advertise. Having only one firm advertise is then optimal. Finally, for high values of α, the benefit of advertising is high enough so it becomes optimal for consumers if both firms, although advertising increases prices. The reason for that is that, if only one firm advertises, the price would be much more high since it would depend on α, which is not the case when both firms advertise.

Putting the results we have here together with those obtained when studying firms' optimal behavior, it is clear that whatever the value of α, given that firms take their decisions based on both the value of the taste for advertising by consumers as well as on the cost of advertising, these decisions may be optimal or not for consumers, depending on the cost of advertising. For example, if the cost of advertising is low, both firms will advertise their goods. If α is low, this decision will not be optimal from the point of view of consumers.

If, on the other hand, α is high, then the decision to advertise is optimal for both firms and consumers. The same reasoning applies for the various values of γ and α.

5.2 Individual consumer welfare

This part of the analysis is a bit more complicated. The purpose of this section is to see, beyond the aggregate effect of advertising, if in some cases, some agents lose while others gain.

Let us start by investigating the case $\alpha \leqslant \alpha''$. In this case, no asymmetric equilibrium can arise, and cases in which only one firm advertises musn't be taken into account.

Example 1 *Let $\alpha = \frac{3}{4}$. If both firms advertise, advertising targets are $\{\theta_b, \theta_a\} = \left\{0, \frac{1}{2}\right\}$. U_0^0(2 groups) $= U_1^0$(2 groups) $= -0.625$, U_0^0(3 groups) $= U_1^0$(3 groups) $= \frac{1}{50}\left(-93 - 2\sqrt{6}\right)$. With advertising, $U_0^2 = \alpha - \frac{116}{81}$, meaning that the consumer located at the extreme left of the continuum gains from advertising whatever the number of groups. On the other hand, $U_1^2 = \alpha - \frac{188}{81}$, meaning that the comsumer located at the extreme right of the continuum loses, whatever the number of groups.*

Example 2 *Let $\alpha = \frac{9}{10}$. In this case, advertising targets, if any, are $\{\theta_b, \theta_a\} = \left\{\frac{1}{2}, 1\right\}$. Given the modification in advertising targets, $U_0^2 = \alpha - \frac{188}{81}$ and $U_1^2 = \alpha - \frac{116}{81}$, which means that, for any number of groups, θ_0 loses and θ_1 gains from advertising.*

Let us now consider what happens when $\alpha > \alpha''$. This means that no firm advertises, one firm advertises $\{\theta_b, \theta_a\} = \left\{\frac{1}{2}, \emptyset\right\}$ or both firms advertise $\{\theta_b, \theta_a\} = \left\{\frac{1}{2}, 1\right\}$.

Example 3 *Let $\alpha = 1$. At equilibrium, it may be that no firm advertises, one firm advertises ($\{\theta_b, \theta_a\} = \left\{\frac{1}{2}, \emptyset\right\}$) or both firms advertise ($\{\theta_b, \theta_a\} = \left\{\frac{1}{2}, 1\right\}$). In case of no advertising, depending on the initial partition of the continuum, we have: U_1^0(2 groups) $= U_{\frac{1}{2}}^0$(2 groups) $= -0.625$, U_1^0(3 groups) $= \frac{1}{150}\left(-93 - 2\sqrt{6}\right)$, $U_{\frac{1}{2}}^0$(3 groups) $= \frac{1}{60}\left(-7 - 8\sqrt{6}\right)$. If advertising targets are $\{\theta_b, \theta_a\} = \left\{\frac{1}{2}, \emptyset\right\}$, then $U_1^1 = \frac{1}{30}\left(-9 - \sqrt{105}\right)$ and $U_{\frac{1}{2}}^1 = \frac{1}{60}\left(11 - 2\sqrt{105}\right)$. If advertising targets are $\{\theta_b, \theta_a\} = \left\{\frac{1}{2}, 1\right\}$, then $U_1^2 = -\frac{35}{81}$ and $U_{\frac{1}{2}}^2 = -\frac{257}{324}$. In the asymmetric* *equilibrium, $\theta_{\frac{1}{2}}$ gains from advertising, while θ_1 may gain or lose, depending on the initial partition. On the other hand, in a symmetric equilibrium in which both firms advertise, $\theta_{\frac{1}{2}}$ loses but θ_1 gains.*

This leads us to Proposition 4:

Proposition 4 *Depending on the value of α, the initial partitioning of the continuum $[0, 1]$ and the equilibrium chosen by firms, there are cases in which some consumers gain and others lose.*

The interpretation is that the result in terms of individual welfare depends on the distance between the advertising and the agent, and the value of α which gives the taste for advertising, as well as as the equilibrium chosen by firms, since this affects the market price. The idea is that for a given value of α, some people will be hurt by advertising, because of the distance between them and advertising, as well as by higher prices. Others, located close to the advertising target, may gain from advertising, the positive effect of advertising being higher than the negative effect on prices. Of course, with high values of α, more people gain from advertising, the converse being true for low values of α.

6 Conclusion

In this paper, we investigate the effect of social identity, a particular form of interpersonal relationships, on the classical result of the Bertrand price competition model. We find that social identity has two major effects: first, the fact that people get utility if their type is correctly understood by others, and under the constraint of a limited number of goods, groups are formed, i.e. the continuum of agents is partitioned. Second, these interactions create market power for firms, meaning higher prices and profits. Without any coordination device, or in case of coordination failure, multiple equilibria will arise, each corresponding to a particular partition. That is why we also consider the possibility for firms to use advertising. This also generates two effects: first, advertising acts as a coordination device, making agents of similar type consume the same good. Second, it may also increase prices and profits.

We show that the equilibrium depends on the cost of advertising, as well as the taste for advertising, that is, how much agents value it. In fact, which equilibrium arises depends on the result of the combination of three effects. Since we have two firms competing for the market share, the first effect is what we call the competition effect, which basically leads firms to decrease their prices to win market shares. The way the continuum is divided into subsets has an impact on this effect, since it exposes firms more or less to competition. The second effect is created by advertising, and the fact that consumers value it. It is the market stealing effect, which allows a firm that advertises to steal market shares from the other firm, thanks to the taste of consumers for advertising. Finally, the presence of social identity and advertising decreases the price elasticity, leading to higher prices. This is the third effect that plays a role in determining which equilibrium arise.

For any value of taste for advertising, depending on its cost, there are situations in which both firms advertise or no firm advertises. In addition, for given advertising costs, multiple equilibria (for low values of the taste for advertising) and asymmetric equilibria (for high values of the taste for advertising) may arise.

Finally, we investigate welfare implications. In terms of aggregate consumer welfare, it is optimal that no firm advertises, one firm advertises or both firms advertise, depending on the value of the taste for advertising. At the individual level, advertising is welfare increasing or welfare decreasing depending on the initial partition of the continuum, the position on this continuum, the cost of advertising and the taste for advertising.

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Appendices

A Proofs

A.1 Lemma 1

Proof. Let $\theta_b = 0$ and $\theta_a = \phi$, and suppose there can be three groups. This means that there are two indifferent consumers. Let us look at the indifference condition for these consumers: θ_1 and θ_2.

$$\theta_i : \alpha - \theta_i - \int_{\theta_j \in \Psi(b)} |\theta - \theta_i| \, d\theta - p_b = -\int_{\theta_j \in \Psi(a)} |\theta - \theta_i| \, d\theta - p_a, \ i \in \{1, 2\}$$

Taking the indifferent condition between the two goods for each consumer, and equaling these two conditions (using the price vector) leads to:

$$(\theta_2 - \theta_1)(\theta_1 + \theta_2) = 0$$

which is not possible. ■

A.2 Lemma 2

Proof. Let $\theta_b = \frac{1}{2}$ and $\theta_a = \phi$.

Suppose we only have two groups, i.e. only one indifferent consumer, and that good b consumers are located on the left of the continuum. Solving this case leads to

$$\begin{aligned} p_b &= \frac{6 + 2\alpha}{6} \\ q_b &= \frac{6 + 2\alpha}{12} \\ p_a &= \frac{6 - 2\alpha}{6} \\ q_a &= \frac{7 - 2\alpha}{12} \end{aligned}$$

However, in this situation, we have that

$$\begin{aligned} U_{\theta_0}(b) &= -\frac{1}{72}(\alpha-39)(\alpha-3) \\ U_{\theta_0}(a) &= \frac{1}{72}(\alpha-3)(\alpha+33) \end{aligned}$$

meaning that

$$U_{\theta_0}(a) > U_{\theta_0}(b)\ \forall\alpha \in [0,3)$$

meaning that the consumer on the extreme left has an incentive to deviate, soonly one indifferent consumer cannot be an equilibrium.

Suppose that there are three groups. If both cutoffs are located on the same side of the advertising target, the equilibrium condition requires that $\theta' + \theta'' = 2$, which is not possible. If, on the other hand, these cutoffs are on different sides, then theequilibrium condition requires that $\theta' + \theta'' = 1$, meaning that they have to be symmetric. ■

A.3 Lemma 4

Proof.

- Consider the case $\theta_b = 0$ and $\theta_a = 1$, and suppose there can be three groups. This means that we have two indifferent consumers. Let us look at the indifference condition for these consumers: θ_1 and θ_2.

$$\theta_i : \alpha - \theta_i - \int_{\theta_j \in \Psi(b)} |\theta - \theta_i|\, d\theta - p_b = \alpha - (1-\theta_i) - \int_{\theta_j \in \Psi(a)} |\theta - \theta_i|\, d\theta - p_a,\ i \in \{1,2\}$$

Taking the indifference condition between the two goods for each consumer, and equaling these two conditions (using the price vector) leads to:

$$(\theta_2 - \theta_1)(1 + \theta_1 + \theta_2) = 0$$

which is not possible.

- Consider now one firm targeting an endpoint of the continuum, and the other firm targeting the center. The existence of an equilibrium with only one indifferent consumer (located between the two advertising targets) is shown in the text (see section 4.2). Solving the problem with one indifferent consumer located after $\frac{1}{2}$ leads to $\theta_1 < \frac{1}{2}$, a contradiction. With two indifferent consumers, there are three possible cases. If both cutoffs are located between the two targets, the equilibrium condition requires that $\theta_1 < 0$ or $\theta_1 = \theta_2$, a contradiction. If both cutoffs are located on the right of $\frac{1}{2}$, the equilibrium condition requires that $\theta_1 = 1 - \theta_2$ or $\theta_1 = \theta_2$, a contradiction. If cutoffs are located on each side of $\frac{1}{2}$, the equilibrium condition requires that $\theta_1 > \frac{1}{2}$ and $\theta_2 > \frac{1}{2}$.

■

Part IV

Social Capital in Belgium

1 Introduction

Tensions between the two major communities of Belgium have always existed. Recently, the country has reached a sort of breakup point, where no one listens to anyone, and it seems impossible to find a way out. It is quite common to hear that the North and South of the country are totally different, that there is no reason at all to keep them together. Although some differences do exist between the two communities, in particular in terms of culture (for example, television shows, singers, actors, etc. are often different) and in lifestyle, it can not be denied that they also share similarities. This paper does not intend to answer the long and complex debate between separatists and non separatists. Its objective is to find out if there is a difference in the levels of *social capital* between the regions of Belgium.

The concept of social capital, a very popular concept in economics, has been widely used for about thirty years to explain various economic outcomes such as growth, development, education, etc. Most of the related studies try to explain the differences in outcomes between different regions or countries by their difference in social capital. In this paper, we limit ourselves to describing what social capital is about in the particular case of Belgium.

The main results of the paper are the following: after building an index of social capital based on various variables from the European Social Survey, we show that social capital can be decomposed into three different aspects, namely, *trust*, *social networks* and *social activities*. Using control variables, we show that there are not only differences in the levels of social capital of regions, but also in terms of composition of social capital between them. Moreover, we put forward that education is important in the formation of social capital, as well as in its different aspects. Finally, we extend the analysis to European countries. We find that European countries can be divided into three groups with respect to the level of social capital: Western European countries, Eastern European countries and Northern European countries. We also highlight that regional differences exist in most countries, except in Ireland, and that Switzerland has the highest regional differences. Moreover, the level of regional differences in Belgium is higher than in Austria, The Netherlands and France, but smaller than in Switzerland.

The paper is organized as follows: in section 2, we propose a short review of the literature on the definition of social captal, as well as on the use of the concept. Section 3 presents the data and the methodology used to measure and study social capital in Belgium. Section 4 contains the results of the analysis for Belgium, while section 5 investigates European countries. Finally, section 6 concludes.

2 What is Social Capital ?

The concept of social capital goes back to Loury (1977) and Coleman (1988). During the last twenty years, many authors have used this concept to study various subjects, ranging from economic growth to success in education. It is worth noting that there are many definitions, so it is sometimes hard to know exactly what social capital really represents. Among all the proposed definitions, here are some of the most often used:

Loury (1977) defined social capital as "social connections creating differences in access to opportunities for minority and non minority youth". Around ten years later, Coleman (1988) described social capital as "a variety of entities with two elements in common: they all consist of some aspect of social structures, and they facilitate certain action of actors". Coleman put forward three possible aspects of social capital: obligations, expectations, and trustworthiness of structures (for example trust that exists between sellers on the diamond market), information channels (the netwok to which I belong helps me to get informed about, for example, what is fashionable, and what is not), and norms and effective sanctions (that can be used to obtain good grades in high school, or decrease crime in some neighborhoods). Granovetter (1973) put forward the role played by social networks to find jobs. His theory is based on what he calls the "strength of weak ties". The idea is that an agent may have a number of close friends (strong ties) with whom he forms a dense network, all members interacting with each other. On top of these strong ties, agents may also have less close friends who are not connected to the other members of his dense network. These weak ties may be helpful to the agent, in the sense that they allow him to be in touch with distant networks, being then able to get information on various subjects, in particular job openings.

Knack and Keefer (1997) defined social capital as trust and civic behavior (participation in elections, charity, helping others, etc.). In 2000, Putnam wrote the famous "Bowling Alone", in which he describes social capital as taking part in activities and being members of organizations. He also makes a distinction between "bonding social capital", which means strenghtening the relationships inside a network, and "bridging social capital", meaning creating bridges between different networks thereby expanding the circle of reciprocity. Bowles and Gintis (2002) define social capital as being composed of "trust and a willingness to live by the norms of one's community and to punish those who do not". Durlauf and Fafchamps (2005) described it as referring to the community relations that affect personal interactions. Across all these definitions and many more, Durlauf and Fafchamps isolated a basic underlying mechanism common to most authors which works as follows: *the existence of social networks and associations leads to shared trust, norms and values, which in turn generate positive externalities.* In this paper, we do not restrict social capital to any definition in particular, but we rather create an index of social capital by selecting questions in the database, each question instrumenting one or more elements used to describe social capital (such as trust, norms, network, civic behavior, participation in activities).

Following Durlauf and Fafchamps (2005), empirical studies can be classified into two categories: individual-level studies and aggregate studies. As far as individual-level studies are concerned, these can also be divided into two: those related to development (immigration, reduction of poverty, land development, trader profitability, etc.), and those focussing on OECD countries (mental health, dropping out of high school, criminal activity, etc.). Aggregate studies mainly deal with growth. On top of all these studies, some also investigate the determinants of social capital such as watching TV, neighborhood homogeneity, female labor force participation, neighborhood benefits, etc. The idea is to determine what kind of factors impact the formation of social capital.

Our goal in this paper is to study social capital in Belgium, more precisely, to find out if there exists a difference in the levels of social capital between the regions of the country. Such a regional investigation has already been performed by some authors: Helliwell and

Putnam (1995), for example, explain the more rapid growth of Northern Italy relative to Southern Italy because of differences in social capital measured by group membership and civic participation. Beugelsdijk and van Schaik (2001) show that social capital, measured by generalized trust and associational activity is positively related to growth differentials in European regions. Van Oorschot et al. (2006) describe how social capital, by its various aspects, is distributed geographically among European countries and regions (North, West, South and East). They find some differences between countries and regions, although those differences are not very large, except for Northern European countries.

3 Measuring Social Capital

The data we use in this paper comes from the European Social Survey. There have already been three waves of this survey, carried out respectively in 2001-2002, 2003-2004 and 2005-2006. In the last wave, the database covers more than 20 countries, contains more than 150 questions on 43000 observations. The data concern various subjects such as political life, social life, family, employment, health, etc. Given that new questions are added in each wave, and that three periods is not enough to use panel data techniques, we will focus on the last wave, which contains questions allowing us to create a more precise index of social capital.

To measure social capital, we select various variables, each one instrumenting one or more aspects of social capital as defined earlier. The chosen variables are given here under. Each time, the coding of the variable follows the name of the variable.

- *Most people can be trusted or you can't be too careful* (TRUST), [from 0 to 10; 0: you have to be careful, 10: most people can be trusted]

The reason why we chose this first variable is quite obvious: trust is one of the aspects of social capital which is most put forward. Trust was popularized by Fukuyama (1999) and is the most commonly used empirical measure of social capital (Fidrmuc, 2008); this is probably due to its availability in many databases. The main idea of this aspect is that the more people trust each other, the more efficient economic outcomes will be. For example, two traders who trust each other will spend less time and money writing contracts (remember in particular the example of the diamond traders). Another example of the positive effect of trust is that a community inside which trust is high does not have to spend money on the surveillance of its members' actions. Note that trust can be understood as generalized trust, when this concerns the whole population (of a club or a network), and personalized trust, when it is specific to some specific agents who, for example, interact repeatedly.

The second variable we use is

- *Involved in work for voluntary or charitable organizations, how often during the past 12 months* (CHARITY), [from 1 to 6; 1: never, 2: less than once every six months, 3: at least once every six months, 4: at least once every three months, 5: at least once a month, 6: at least once a week]

The main goal of this variable is to instrument civic behavior. This aspect is seen as also having a positive impact on economic outcomes. Someone involved in charitable activities is expected to behave in the same way on various subjects, such as tax payment, participation in elections, etc. The link to social capital appears in the fact that such behavior can take place either because there is a norm of civic behavior (see for example Knack and Keefer, 1997), because you are expecting others to behave in the same way if you need it one day (idea of reciprocity), or simply because of altruism, which is also sometimes seen as social capital because you care about other members of the community.

The third variable is

- *Most people try to take advantage of you or try to be fair* (FAIR), [from 0 to 10; 0: most people try to take advantage of me, 10: most people try to be fair]

The idea here is close to that of the two first variables. Fairness can be seen as way to improve economic outcomes, by lowering costs (if I believe others will trade with me in a fair way, then contracts do not have to be long, complicated and expensive). In this sense, fairness plays the same role as trust. On the other hand, fairness is also linked to civic behavior. Indeed, one can consider fairness as a behavior arising inside a well-defined community, because of reciprocity for example.

The fourth variable is

- *Help or attend activities organized in local area, how often during the past 12 months* (LOCAL ACTIVITIES), [from 1 to 6; 1: never, 2: less than once every six months, 3: at least once every six months, 4: at least once every three months, 5: at least once a month, 6: at least once a week]

This variable can also be seen as a proxy for civic behavior depending on the type of activities (note here that since these questions come from a large survey, and since questions are not necessarily precise, respondents may have various interpretations). Another possibility is that agents participate in such activities in order to build a network, either with weak or strong ties, in order to be able to achieve a given goal. In that sense, this variable is more related to the network aspect of social capital.

This fifth variable is

- *There are people in my life who care about me* (PEOPLE CARE), [from 1 to 5; 1: Disagree strongly, 5: Agree strongly]

Here, we focus on the strong ties, that is the dense and potentially close network around an individual. As already said, this kind of network is the best place to ensure that norms will be sustained. It is also a way for agents to use their connections to reach goals (although weak ties can also fill this role as explained by Granovetter) as well as the place in which trust between members is the most likely to arise.

The sixth variable is

- *How often do you socially meet with friends, relatives, or colleagues* (SOCIAL MEETING), [from 0 to 7; 0: never, 7: every day]

The idea here is the same as above, except that this question does not necessarily focus on very close agents, but allows for more distant friends and colleagues. Hence, both strong and weak ties are supposed to be instrumented by this question.

Finally, the last one is

- *Take part in social activities compared to others of same age* (SOCIAL ACTIVITIES), [from 0 to 5: 0: much less than most, 5: much more than most]

The idea is that taking part in social activities is clearly a way for agents to get into a network which can be used to achieve their goal, but it is also a way to pursue a common goal with other people.

From these seven variables, we create an index of social capital using principal component analysis[1]. This will not only allow us to have a single variable to measure social capital, but it will also help us to analyze various aspects of social capital.

3.1 Methodology

We use Principal Component Analysis (PCA) to develop our index of social capital. More precisely, we take all variables related to one or more aspects of social capital and use data reduction to compute our index. The idea is to reduce the number of variables by creating new ones containing enough information to characterize social capital. Let X be an N individuals $\times$ P variables matrix and X^* be the matrix of centered and reduced data such that

$$x^*_{ip} = \frac{x_{ip} - \bar{x}_p}{s_p} \; \forall i \in \{1, ..., N\} \, \forall p \in \{1, ..., P\}$$

where $\bar{x}_p$ and s_p are respectively the mean and the standard deviation of variable p. Let $\aleph^*$ be the representation of the N individuals in a P dimensional space ($\aleph^* = \{I^*_1, ..., I^*_N\}$). The first component is obtained in the following way: we determine the straight line Δ_1 such that

$$I(\aleph^*, \Delta_1) = \min_{\Delta \text{ passing through } O} I(\aleph^*, \Delta)$$

[1]Even if none of the variables we use are continuous, they are all ordered and have multiple modalities. Hence, for the sake of simplicity, we use principal component analysis instead of multiple correspondence analysis.

with

$$I(\aleph^*, \Delta_1) = \frac{1}{n}\sum_{i=1}^{N} d^2(I_i^*, P_{\Delta_1}(I_i^*))$$

where $P_{\Delta_1}(I_i^*)$ is the orthogonal projection of I_i^* on Δ_1. Let

$$\underline{u}_1 = (u_{1,1}, ..., u_{1,P})'$$

be the unit vector generating Δ_1. The coordinate of individual i on the first component is then given by

$$\phi_{i1} = \sum_{p=1}^{P} u_{1,p}x_{ip}^*$$

Other components are computed using the same procedure, with the additional constraint that principal components are orthogonal to each other. The number of principal components we use depends on the percentage of the variance explained by the h first components.

In other words, the goal is to reduce the number of dimensions, by creating new variables that are uncorrelated with each other and constructed as a linear combination of the "old" ones. The results are then more easily obtained and interpreted, while being careful to keep enough information in the data to retain some explanatory power.

3.2 Index of social capital

Taking the seven variables (after centering and reducing them) presented above, and performing PCA leads to

Table I: PCA

	ϕ_1	ϕ_2	ϕ_3
TRUST	0.532	0.671	-0.146
FAIR	0.474	0.722	-0.113
SOCIAL ACTIVITIES	0.683	-0.295	0.021
CHARITY	0.579	-0.330	-0.342
LOCAL ACTIVITIES	0.590	-0.392	-0.296
SOCIAL MEETING	0.504	-0.192	0.478
PEOPLE CARE	0.337	0.067	0.737

These three components (whose eigenvalues are above 1[2]) explain together almost 65% of the variance. Although this may not appear to be very good, it is not too bad, since the seven original variables (before performing PCA) are not highly correlated (75% of the correlations are below 20%). The first component (ϕ_1) is our index of social capital. It can be interpreted as the level of social capital of each individual since it is positively correlated with all aspects of social capital. This means that as soon as one variable among the seven increases, the component, that is the level of social capital, increases. Though this new variable will be very helpful to investigate the differences in social capital between regions of the country as well as studying the role of various socioeconomic factors in the formation of social capital, it does not allow us to disentangle the different aspects of social capital that may be present. This is possible with the two other components. ϕ_2 opposes TRUST and FAIR with all others, which means that FAIR is considered to be similar to TRUST by people. Hence, we call these two variables by the same name i.e. *trust* oriented social capital: for a given level of ϕ_1 and ϕ_3, an observation with a higher coordinate on ϕ_2 has a more *trust* oriented social capital. The third component, ϕ_3, opposes SOCIAL MEETING and PEOPLE CARE to the five other variables. Given the different possible explanations given above, one can interpret

[2] This cutoff comes from the fact that the mean of the eigenvalues (of the correlation matrix) is one, each of the eigenvalues giving the inertia of the corresponding component (the share of the total variance explained by each component is given by the eigenvalue corresponding to the component divided by the sum of all eigenvalues). Hence, taking components whose eigenvalues are larger than one guarantees that we retain enough information.

these two variables as being the *social networks* side of social capital. The idea is here that if you often meet friends, colleagues, family and the like, and if you have people who care about you, you have a lot of ties (both strong and weak) that can be used in purposive actions. As a consequence, the last three variables, SOCIAL ACTIVITIES, CHARITY, and LOCAL ACTIVITIES can be put together. We call them the *social activities* side of social capital. The underlying idea is that someone who is involved in these activities (sports clubs, religious organizations, youth groups, political parties, charities, etc.) binds with other people and is then able to pursue common goals with them, with or without exerting externalities on the rest of society. We want to outline that *social activities* and *social networks* are obviously very close to each other. In particular, participating in social activities often means building a network, while being part of a social network is often synonymous of taking part in social activities. However, given the way questions were asked in the survey, our classification takes into account that the *social networks* are more people (and hence links) oriented, while the other one is more activities oriented. The distinction we made between more individual purpose in *social networks* versus common action in *social activities* follows also from the question of the survey, even if again, the distinction may not be perfectly clear. Given the presence of CHARITY and LOCAL ACTIVITIES, we assume that it is more common goal-oriented. It is important to underline that this distinction between aspects of social capital is very close to different analyses found in the literature (see for example Paldam (2000) who decomposes social capital into trust, ease of cooperation and network, or van Oorschot et al. (2006) who also consider three aspects, namely trust, networks and civism).

So far, starting from several variables, we constructed an index of social capital for Belgium, while disantangling three aspects, i.e. *trust*, *social activities* and *social network*, as well as the distinction between common goals versus individual objectives.

4 Belgian's Regional Differences in Social Capital

Let us first look at some descriptive statistics. The following table gives the number of observations, the mean, maximum and minimum values, as well as the standard deviation for the three components, for the whole country as well as for the regions.

Table II: Descriptive statistics

Component	Region	N	Mean	Max	Min	Std.deviation
ϕ_1	All	1788	0.01	6.14	-6.53	2.03
	Flanders	1123	0.2	6.14	-6.02	2.01
	Brussels	98	0.02	5.33	-4.51	2.12
	Wallonia	567	-0.38	5.19	-6.53	2.03
ϕ_2	All	1788	0.01	3.80	-5.19	1.37
	Flanders	1123	0.18	3.80	-4.20	1.31
	Brussels	98	-0.28	2.56	-3.91	1.44
	Wallonia	567	-0.31	2.87	-5.19	1.39
ϕ_3	All	1788	0.01	2.21	-4.75	1.02
	Flanders	1123	0.05	2.21	-4.75	0.96
	Brussels	98	-0.03	1.65	-2.39	1.01
	Wallonia	567	-0.09	2.08	-3.94	1.11

Let us briefly comment these results: concerning the first component (our index of social capital), the Flemish region has the highest mean, and Walloon region the lowest. The maximum value can be found in the Flemish region, while the minimum is in Wallonia. For the second and third components (trust aspect and social networks aspect of social capital), it is still the Flemish region whose mean is the highest and Wallonia's which is the lowest. To test the equality of means between the three regions, we first perform a general ANOVA test of equality of means on all three regions simultaneously (H_0 = equality of means).

Table III: ANOVA

	Mean Square	F	P value
ϕ_1	62.61	15.388	0.000
ϕ_2	50.13	27.641	0.000
ϕ_3	3.9	3.782	0.023

This table shows that we reject the null hypothesis of equality of means[3,4]. In others words, the mean population of each component is not the same accross regions. To be a bit more precise, we now run a t-test on means, taking regions two by two. Results are given in Table IV.

Table IV: t-test on means

	H_0	Mean difference	Std. error difference	P value
ϕ_1	FL=BR	0.18	0.21	0.401
	FL=W	0.58	0.10	0.000
	BR=W	0.4	0.22	0.075
ϕ_2	FL=BR	0.47	0.14	0.001
	FL=W	0.49	0.07	0.000
	BR=W	0.03	0.15	0.855
ϕ_3	FL=BR	0.09	0.10	0.395
	FL=W	0.14	0.05	0.007
	BR=W	0.06	0.12	0.642

Table IV gives some interesting results. Concerning our index of social capital, it appears that the means of Flanders and Brussels are not statistically different, while Wallonia's is lower. For the trust aspect of social capital, it is Brussels and Wallonia which are now not statistically different, while Flanders is higher. Finally, only Flanders and Wallonia are

[3] At a 5% significance level (probability of rejecting the null hypothesis when it is true)

[4] Runing the ANOVA test requires to have both the normality of the data and equality of variances between regions assumptions fulfilled. This is true for the two first components. For the third, however, variances are not equal. Hence, the result cannot really be interpreted for the third component

statistically different with respect to the third component.

Until now, we have only looked at the data in a descriptive way. However, the characteristics of each region could be different, which would of course have an impact on the results of potential regional differences. For example, the regional differences in social capital may come from the fact that one region is more educated than another, and not because of social capital itself. We must control for this.

To do this, we now regress ϕ_1 on various variables. The variables are the following: *Born in country* (takes value 1 if yes, 2 if no), *Gender* (1 if male, 2 if female), *Age*, *Level of education*, and dummies for each Region. The three first variables take into account intrinsic characteristics of individuals. The variable *education* is often studied together with social capital. However, this variable may be endogenous. Indeed, if it is easy to imagine that the education received has an impact on the networks one belongs to and on the activities one participates in, the reverse causality is also quite intuitive. Being part of specific networks or organizations may affect educational achievements of individuals. If such endogeneity exists, then estimations will be biased, leading to false conclusions. To avoid this, we use instrumental variables i.e. we search for variables which are correlated with education but uncorrelated with the error term of the social capital equation, which allows us to use them as instruments for education. In the database, we have two variables which can play this role: *education of the father* and *education of the mother*. Literature on education states that education of the parents is usually a good indicator of the education of the children. On top of this, these two variables cannot be endogenous. Indeed, if the education of the parents may have an impact on the social capital of the child, the reverse is clearly not possible. Hence, these two variables seem to be two potential instruments. One must be aware however of the fact that *education of the father* and *education of the mother* may be correlated, meaning that they contain the same information. Moreover, good instruments should not be correlated with social capital. Hence, when needed, we may use a self-computed third variable which is the gap between education of the father and education of the mother (this can be interpreted as an incentive for a child to get educated when observing a difference between the education levels of his parents).

We run a two stage least squares (2SLS) regression to analyze social capital, using instruments for education. Each time, we run several tests to check the validity and the quality of the instruments. The results are given in Table V.

Table V: Determinants of Social Capital in Belgium (2SLS)

Social capital	Coef.	Std. Err.
Born in country	-0.3198*	0.1721
Gender	-0.1101	0.0971
Age	-0.0073**	0.0029
Education	0.3397***	0.1287
Education of the mother	0.1789***	0.0557
Brussels	0.0467	0.2259
Flanders	0.6126***	0.1046
Cons.	-0.9727**	0.4738
R-squared:	0.1351	

instruments: Education of the father, gap
* significant at 10%, ** significant at 5%, *** significant at 1%

In order to avoid endogeneity, since *education of the mother* is correlated with social capital (even when controlling for *education*) and since tests indicate that it is redundant with *education of the father*, *education* is instrumented by *education of the father* and *gap* (remember that *gap* is the absolute value of the difference between the levels of education of the parents). Both are significantly and positively correlated with education in the first stage of the 2SLS, and are good instruments as confirmed by the tests[5]. We now take a closer look at the results. First, with Wallonia as the control group, Brussels is not different from zero, meaning that the level of social capital in that region is the same as the level in Wallonia for people of the same age, gender, nationality and education. This contradicts the previous result obtained by t-tests. The reason is that we use various control variables here, which means that some factors other than regions have affected the previous result. On the

[5] For this regression, as well as for all the next ones, we run three tests: the first two check the weakness of the instruments (one checks if there is at least one good instrument for each endogenous variable, and the other one checks if both instruments are good in the sense that they reduce the bias created by 2SLS with respect to the bias arising from endogeneity problems sufficiently). The third one checks the exogeneity of instruments with respect to the error term of the social capital equation.

contrary, Flanders is significantly different from zero, meaning that there is a difference in the level of social capital between Wallonia and Flanders. More precisely, the results indicate that the level of social capital is higher in the North of the country than in the South. Concerning the other variables, there seems to be a kind of discrimination in terms of social capital: someone who is *born in the country* has a higher level of social capital than someone who is not. This may be linked to fewer social activities, but also to fewer social networks. Gender plays no role in the level of social capital, but *age* does. Indeed, *age* appears to be negatively correlated with the level of social capital i.e. younger people seem to have more time to be involved in activities and creating network than older ones. Finally, *education* and *education of the mother* are significant. Both variables are positively correlated with social capital which means that *education of the mother* has an impact on social capital of the child not only through an indirect effect (*education*), but also through a direct one.

Since there seems to be differences in the levels of social capital between some regions, one can wonder if the effect of the various variables changes depending on the region considered. To investigate this possibility, we look at ϕ_1 by region. However, due to a problem in finding good instruments for Brussels, given that the ones we used up to now are not good for this region alone, we limit our study to the two other regions.

Table VI: Social Capital in Flanders and Wallonia

Social capital	Flanders		Wallonia	
	Coef.	Std. Err.	Coef.	Std. Err.
Born in country	-0.2429	0.2433	-0.3927	0.2841
Gender	-0.097	0.1206	-0.1089	0.1803
Age	-0.0064	0.004	-0.0135**	0.0056
Education	0.2903*	0.1615	0.5778**	0.2501
Education of the mother	0.1701***	0.643	0.0851	0.1227
Cons.	-0.3253	0.6136	-1.1868	0.8374
R-squared:	0.1054		0.0978	

instruments: Education of the father, gap
* significant at 10%, ** significant at 5%, *** significant at 1%

The level of *education* is significant and positively correlated with social capital in both regions, which indicates that this variable is very important in the formation of social capital.

Education of the mother is significant only in Flanders. *Gender* and *country of birth* are not significant, while *age* is significant only in Wallonia, and has the same sign as in the global study just above.

Another interesting question is to find out to what extent the results change when studying each aspect of social capital taken separately. To answer this, we first need to create an index of each aspect of social capital. Using PCA on the orignal variables of each aspect, we obtain the results reported in Table VII.

Table VII: PCA on each aspect

	ϕ_{TRUST}	$\phi_{\text{ACTIVITIES}}$	ϕ_{NETWORKS}
TRUST	0.869		
FAIR	0.869		
SOCIAL ACTIVITIES		0.733	
CHARITY		0.746	
LOCAL ACTIVITIES		0.751	
SOCIAL MEETING			0.760
PEOPLE CARE			0.760
Share of variance explained	75.54%	55.26%	57.685%

Using these results, we construct an index for each aspect of social capital, and then we use 2SLS on each aspect.

There does not seem to be a discrimination concerning the various aspects of social capital, except for *activities*, which means that it is only the participation to various kinds of activities which is affected by the place invididviduals are born, not the *trust* or the belonging to *networks*. Similarly, *gender* is significant only in explaining the *social activities* aspect: being a woman affects the *social activities* aspect of social capital negatively. The variable *age* is significant for all aspects, but its sign changes depending on the aspect. Younger people seem to have more *social activities* and *social networks* than older ones, while the converse is true for the *trust* aspect. *Education* and *education of the mother* are significant

Table VIII: Determinants of social capital by aspects

Social capital	Trust		Activities		Networks	
	Coef.	Std. Err.	Coef.	Std. Err.	Coef.	Std. Err.
Born in country	-0.0509	0.1345	-0.2983**	0.1377	-0.1179	0.1131
Gender	0.0041	0.0737	-0.1876**	0.0809	0.0811	0.0582
Age	0.0046*	0.0022	-0.0062**	0.0025	-0.0066***	0.0018
Education	0.1739*	0.0973	0.2001*	0.1064	0.1278*	0.0749
Education of the mother	0.1243***	0.0425	0.0818*	0.0473	0.0618**	0.0319
Brussels	0.0957	0.1672	0.0322	0.1885	-0.0507	0.1398
Flanders	0.6396***	0.0794	0.0967	0.0867	0.3188***	0.0675
Cons.	-1.3552***	0.3535	0.0197	0.3858	-0.4081	0.2821
R-squarred:	0.0934		0.082		0.0405	

instruments: Education of the father, gap
* significant at 10%, ** significant at 5%, *** significant at 1%

and positive for all aspects, which means that these two variables are really important for all aspects of social capital. Finally, there does not seem to be a regional difference in terms of *social activities*, while Flanders seems to be higher than the two other regions in terms of *trust* and *social networks*[6].

5 Regional differences in Europe?

Up to now we limited our analysis to Belgium. One can wonder if the regional differences that appear in Belgium also exist in other European countries. To check this, we again have to compute an index of social capital, but this time for 23 countries: Austria, Belgium, Bulgaria, Switzerland, Cyprus, Germany, Denmark, Estonia, Spain, Finland, France, the UK, Hungary, Ireland, The Netherlands, Norway, Poland, Portugal, Russia, Slovakia, Slovenia, Sweden and Ukraine. Taking the seven variables used above to proxy social capital, we reduce

[6] We also regressed each of the seven original variables used to compute the indexes on the various socioeconomic variables. The results we found are in line with the ones concerning each aspect of social capital. More precisely, education is always significant, and there exist almost the same regional differences (there is only a difference for the social activities aspect of social capital: charity is more present in Flanders, while local activities are more present in Wallonia).

the dimensions using principal component analysis. The results are given in Table IX below.

Table IX: PCA for European countries

	ϕ_1	ϕ_2	ϕ_3
TRUST	0.631	-0.598	-0.147
FAIR	0.617	-0.620	-0.131
SOCIAL ACTIVITIES	0.554	0.321	0.388
CHARITY	0.583	0.438	-0.408
LOCAL ACTIVITIES	0.565	0.456	-0.421
SOCIAL MEETING	0.487	0.191	0.589
PEOPLE CARE	0.310	-0.075	0.472

The two first components keep the same interpretation as above. The third one is slightly different, since SOCIAL ACTIVITIES are now coupled with SOCIAL MEETING and PEOPLE. In this case, besides the *trust* aspect, we then have a *network* aspect, composed of these three variables, and a *civic behavior* aspect, formed by the last two variables. We concentrate our analysis first on the first two components, namely *social capital* and the *trust* aspect of social capital, and compute the average of both component for each country. The choice of the second component instead of the third is justified by the fact that trust is the most often used measure of social capital. Figure I below presents the results.

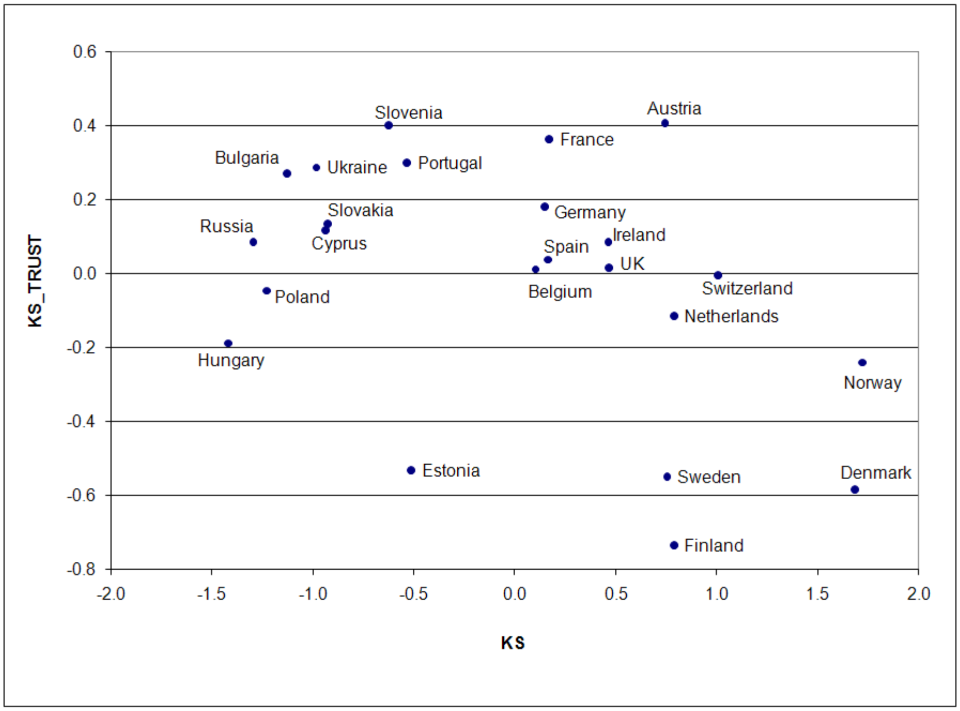

Figure I: Social Capital in Europe

The horizontal axis gives the level of social capital, and the vertical axis presents a measure of trust versus other aspects of social capital (the smaller the value, the higher trust versus other aspects). We immediately see three groups of countries arising: Northern countries, Eastern European countries and Western European countries. Northern countries exhibit levels of social capital among the highest. It is also more trust oriented than in Western European countries. Eastern European countries have lower social capital. Three particular cases: Estonia, which has a more trust-oriented social capital than the other Eastern European countries, and Portugal and Cyprus whose social capital profiles are closer to those of Eastern countries.

In order to study regional differences in Europe, with the objective of comparing them with Belgium, we restrict the sample to similar countries in terms social capital profile, that is, to Western European countries, with the exception of Portugal and Cyprus, because of their profile, as well as the UK and Germany, because of a lack of good instruments. For similar endogeneity problems as those described above, we use two stage least squares (2SLS), with education instrumented by the education of both parents, or education of one of the parents and the gap between their education. In order to be able to compare countries, we need to express these coefficients in terms of elasticity. The results are given in Table X below.

Table X: Determinants of social capital in Europe

Social capital	Austria	Belgium	Switzerland	Spain	France	Ireland	Netherland
Education	0.0085	0.0329***	0.0504***	0.0302***	0.0264***	0.0272*	0.0381***
	[0.0106]	[0.0125]	[0.0086]	[0.0061]	[0.0104]	[0.0157]	[0.0081]
Born in country	-0.0477***	-0.0307*	-0.0359***	-0.0469***	-0.0359**	-0.0577***	-0.0573***
	[0.0174]	[0.0166]	[0.0103]	[1.0162]	[0.0146]	[0.0165]	[0.0138]
Gender	-0.0059	-0.0104	0.0466*	-0.004	-0.0043	-0.0111	0.021**
	[0.0091]	[0.0094]	[0.009]	[0.0075]	[0.0088]	[0.0106]	[0.0084]
Age	-0.0017***	-0.0006**	-0.0009***	-0.0003	0.0011***	0.0019***	0
	[0.0003]	[0.0003]	[0.0003]	[0.0003]	[0.0003]	[0.0004]	[0.0003]
Education of the mother		0.0174***			0.0121**	0.153**	
		[0.0054]			[0.0055]	[0.0068]	
Regions	**Karnten**	**Brussels**	**Espace Mittelland**	**North**	**Est de Paris**	**Border, Midlands and West**	**East**
	0.0344	-0.0575***	0.0251*	0.0574***	0.0053	0.0242	-0.0112
	[0.0288]	[0.0211]	[0.0137]	[0.0159]	[0.02]	[0.0169]	[0.0151]
	Niederösterreich	**Walloon**	**Nordswestchweiz**	**Madrid**	**Ouest de Paris**	**Southern and Eastern**	**West**
	0.0405*	-0.0625***	0.041***	-0.0094	-0.0282	0.0127	-0.0155
	[0.0236]	[0.0099]	[0.0159]	[0.0163]	[0.0207]	[0.0148]	[0.0133]
	Oberösterreich		**Zürich**	**West**	**Nord**		**South**
	0.0469**		0.0051	0.0263*	-0.0181		-0.0303**
	[0.0241]		[0.0135]	[0.0157]	[0.0199]		[0.015]
	Salzburg		**Ostschweiz**	**East**	**Est**		
	0.0279		0.0236	0.0173	-0.0026		
	[0.0272]		[0.0149]	[0.014]	[0.0204]		
	Steiermark		**Zentralschweiz**	**South**	**Ouest**		
	0.0282		0.0498***	0.0264**	0.0308*		
	[0.0253]		[0.0186]	[0.0149]	[0.0183]		
	Tirol		**Ticino**	**Canarias**	**Sud Ouest**		
	0.0233		-0.0394*	0.0311	0.0015		
	[0.0254]		[0.0235]	[0.0232]	[0.0173]		
	Vorarlberg				**Sud Est**		
	0.0315				0.0001		
	[0.0344]				[0.0177]		
	Wien				**Méditerranée**		
	0.0022				-0.0036		
	[0.0252]				[0.0188]		
Control Region	**Burgenland**	**Flemish**	**Lémanique**	**North West**	**Paris**	**Dublin**	**North**
Predicted ks	10.5022	10.2462	10.8306	10.198	10.2911	10.3916	11.1453
R-squared	0.0424	0.1351	0.0668	0.0866	0.096	0.0689	0.0785

Std. Errors between brackets

* significant at 10%, ** significant at 5%, *** significant at 1%

All countries have some regional differences, except Ireland. The differences range from

3% to 9%[7], depending on the country. In the Netherlands, being in the South implies having 3% less social capital. In France and Austria, most regions do not have any significant differences, except for the West of France (3% more social capital than the control region) and the North of Austria (about 4% more than the control region). There seems to be more variance in Switzerland, where one region exhibit almost 4% less social capital while another has 5% more social capital than the control region. In Spain, three regions have a higher level of social capital than the control region. One of these regions has almost 6% more social capital than the control region, which is similar to the differences that exist in Belgium.

6 Conclusion

We first built an index of social capital, which is composed of three aspects. The first measures *trust*, one of the most current measure of social capital. The second aspect is *social activities*, which represents the fact that agents get together in order to achieve a common goal. The last one is *social networks*, which can be used by agents in purposive actions. By running simple t-tests, we showed, among other things, that the level of social capital is the same in Flanders and in Brussels, but different in Wallonia.

In a second step, we study these regional differences more in detail. Using several control variables, we show that education is really important in terms of formation of social capital, and that there do indeed exist regional differences: Brussels and Wallonia do not show a significant difference in the level of social capital, but Flanders has more social capital that the two other regions. We also study social capital in each region, and investigate each aspect of social capital separately. For the latter, we highlight that education is important for all aspects, and that regional differences exist, except for social activities.

Finally, we look at regional differences at the European level. In terms of levels of social capital, three groups of countries appear, namely Northern countries, Eastern European countries and Western European countries. We find that regional differences exist in most

[7]Note that in case of dummy variables, interpreting the coefficients in terms of elasticities requires the following transformation: $e^{\beta} - 1$. However, given the values of the original coefficients, the transformation changes almost nothing, so we do not take this into account.

countries, except in Ireland, and that Switzerland has the highest regional differences. France, Austria and The Netherlands have more or less the same profile, i.e. only a few regions have regional differences in the levels of social capital, and these differences range from 3% to 5% with respet to the control region. Moreover, the level of regional differences in Belgium is higher than in Austria, The Netherlands and France, but smaller than in Switzerland.

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