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Essays in Product Diversity and Urban Transportation

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Contents

Introduction

This dissertation is about the understanding of markets, defined as a large number of individuals taking decentralized decisions. The tool I use to model markets is game theory: when individuals take decisions that do not depend only on exogenous parameters, but also on the decisions the others take. More specifically, in the games I study here, decisions depend on beliefs: what people decide depends on what they expect other people to do. And the same population, with different beliefs, can take different decisions. Game theorists denote this by "multiple equilibria". Hence, a first common feature of the three chapters of this dissertation is that they involve problems of coordination among a large number (in fact, a continuum) of players. A second common feature comes from the concept of horizontal differentiation. I am interested in studying worlds were what distinguishes individuals and products are not their intrinsic quality, but a matter of taste

The first chapter, "On goods and premises" relates to one of the most long lasting questions in economics: how efficient is a decentralized market to match buyers and sellers - to exhaust the potential gains from trade - and how the surplus of this trade is shared among them. In a world of perfect information, the typical answer is known as the "Bertrand Paradox": firms compete in prices, and always have an incentive to set a lower price than their competitors, until the price is so low that firms make zero profit. Hence, no one searches, all the gains from trade are exhausted, and the entire surplus goes to consumers. In a world of positive search costs, the picture is entirely reversed, and known as the "Diamond Paradox": as long as buyers have a positive cost to search for another seller, it is a best response for every individual seller to slightly increase the price above the market one. As all sellers have the same best response, the market price increases until it is exactly the monopoly price. Hence, no one searches and the sellers receive the same surplus as a monopolist. Those two polar cases do not correspond to what is observed in reality: there is price dispersion – homogeneous goods are sold at different prices, even on the Internet where search costs are almost zero – and search on actual markets. To explain this, several models have introduced heterogeneity of information among consumers, vertical differentiation of products, and more complex game structure.

What I do is coming back to the very simple specification of the two aforementioned paradoxes. I model homogeneous goods and services sold on premises, e.g. in a shop or on a website. Since premises differ across sellers, even homogeneous goods become somehow differentiated through the purchasing experience. Differentiation affects all the goods sold on given premises, and it is costly for buyers to get used to a different shop. I study the impact of such differentiation on the equilibrium level of horizontal diversity (between premises) and of prices in an otherwise competitive market. Sellers choose between two categories of premises. There are two types of buyers with different tastes on premises. I show that the market outcome never exhausts all the gains from trade. The market failure is independent of search costs, even when they become arbitrarily small. When buyers care sufficiently about the premises, the market may appear efficient: there is search, product diversity, price dispersion and all buyers accept an offer. However it is not efficient, since only the buyers of the minority type search until they find a good match, while majority types accept any offer. It is important to note that, if the market inefficiency I characterize comes from a problem of coordination, it is not a coordination failure. Indeed, when I consider only Coalition-Proof Nash-Equilibrium – to avoid inefficiencies that could be overcome by allowing communication among coalitions of sellers – the market still fails to exhaust the gains from trade. The sellers coordinate towards the equilibrium that yields the highest profit for them, but this profit is still lower than what a monopoly would get. Forcing the market to produce the category desired by the majority always increases aggregate consumer welfare. It also increases total welfare when the market does not supply enough of the category desired by the majority. In any case, a monopolist owning all the premises would increase total welfare by extracting the entire surplus. Hence, competition increases buyer surplus at the cost of creating inefficiencies in the matching process.

The second chapter, "On the economic impact of smoking bans in bars and restaurants" adds another layer to this model to allow for what is a common feature of many of the leisure consumption goods: people do not buy alone. Hence, I assume that individuals buy in groups, potentially with mixed preferences, and that utility within the group is non-transferable: it is not possible to transfer utility from a member of the group to another. This specification allows explaining what has been, and still is in some countries, a largely debated policy issues: smoking bans. Preference for a smoking or nonsmoking environment is a matter of taste, that can yield to externalities within given groups. The policy debate is about deciding whether it is a good idea to impose a norm corresponding to the taste of the majority to limit those externalities. Four stylized facts stand out in the empirical literature on smoking bans in bars and restaurants: (i) bans do not hurt business for restaurants; (ii) before a law is voted, the market equilibrium typically fails to offer non-smoking environments; (iii) the impact on bars is more contrasted, most likely a decrease in employment after the ban; (iv) the support for smoking bans typically increases after a smoking ban is enforced. The literature does not explain those results. Assuming an economy with a majority of nonsmokers, I model a market with arbitrarily small search costs where buyers differ in taste and consume in groups. I show that in a decentralized equilibrium, unless all restaurants and bars ban smoking, nonsmoking premises are patronized only by nonsmokers while smokers, mixed groups and even some groups of nonsmokers attend smoking premises. In the presence of smoking premises, there are two sources of inefficiency that go on the opposite directions. Too many smoking premises imply a large share of the nonsmokers being mismatched. Too many nonsmoking premises imply a large share of mixed groups leaving the market without buying. I derive a necessary and sufficient condition for smoking bans to be welfare improving. If this condition is not met, the welfare-maximizing policy is to sell a very small share of licenses to allow smoking. A general smoking ban (weakly) decreases the profit of owners. However, substitution effects can lead to measurement errors: if the ban applies to restaurants only, the impact on the profit of restaurants is ambiguous, while the profit of bar decreases and the number of smoking bars increases.

The third chapter, co-authored with Quentin David, is about coordination problems in the discrete choice of modal transportation within a city: "Private car or public transportation". Urban transportation is characterized by two types of externalities. First, in the short run, cars generate congestion: this is also a cross-modal externality, as congestion also affects public transportation. Second, in the medium and long run, public transportation exhibits positive network externalities: the highest the share of public transportation users, the highest the frequency of public transportation for a given cost – or, similarly, the lowest the per capita price of a given investment. Transportation economics mostly focuses on the first effect, and the idea that the taxation of congestion can be welfare improving is consensual among economists. However, considering commuters with heterogeneous preferences for a car can allow for multiple equilibria when one considers the second externality. If a large share of commuters does not have strong preferences in favor of the car or public transportation, the following can happen: if they believe most commuters take the car, then it is a best response to take the car as public transportation is inefficient. But, if they believe most commuters take public transportation, it may be a better option to choose for public transportation. Hence, a social planner focusing on the marginal impact of policies may miss the largest source of inefficiency. We discuss two policy tools: taxation and traffic separation (e.g. exclusive lanes for public transportation). Taxation helps reaching a local maximum by internalizing marginal externalities. Traffic separation decreases cross-modal externalities and therefore marginally increases the relative efficiency of public transportation. If this policy allows commuters to coordinate towards a better equilibrium, it can help reaching a global maximum in total welfare.

Part I

Essays in product diversity

Chapter 1

On Goods and Premises

Price dispersion, homogenous goods sold at different prices in different shops, is a well documented fact, both online and offline (Baye, Morgan and Scholten (2004), Sorensen (2000)). Typical explanations rely on hidden quality of shops (Clemons, Hann and Hitt (2002)), sequential search with either noise (Burdett and Judd (1983)), heterogeneous information (Salop and Stiglitz (1977), Varian (1980)), or bounded rationality (Baye and Morgan (2004)). In this paper, I study how price dispersion can be an equilibrium, even with homogenous production costs (among sellers) and information about the prices (among buyers). I show that when a decentralized market displays product diversity in equilibrium, it is inefficient to match sellers with the tastes of buyers.

Most goods, even if perfectly homogeneous, are sold in what I here denote under the general term of 'premises'. When you buy a particular brand of shoes, you buy it in a shop. The utility you derive from your purchase also depends on the shop itself. Search costs within a shop are negligible. Information about the distribution of prices and the existing categories of premises is common knowledge. However, changing for another shop with different rules, procedures, or organization has a cost.[1] One needs to get used to the shop, its prices and check whether it sells the

[1]Even when you buy on the Internet, the website - its graphical organization and presentation -

brand of shoes you want. While trying to remain as close as possible to the pure competition case, I consider search costs that are arbitrarily small[2] and model product diversity as follows. Sellers choose to sell a good or service on premises of either category *A* or *B*, at no cost. There are buyers of type *a* and type *b* in the market, in a proportion that is common knowledge, with a strict majority of type *a*. A buyer awards a higher valuation to a good match (*A*&*a* or *B*&*b*) than to a mismatch (*A*&*b* or *B*&*a*). Buyers are price takers but have the option of not buying, and of searching for other premises selling a different category or at a lower price.

In the absence of search costs, all buyers would be correctly matched and firms would make zero profit. However, any strictly positive search cost makes the market inefficient: some buyers are not correctly matched. I find that when search costs are arbitrarily small and buyers care sufficiently about the premises, the typical[3] equilibrium involves price dispersion, diversity of the premises and full extraction of the surplus of the majority type *a* by the sellers. The buyers of the minority type *b* keep searching until they find a good match. As long as search costs are not exactly zero, the inefficiency remains and is independent of the size of these costs. The mechanism behind this equilibrium is the following: (i) sellers of either category set a price that corresponds to the participation constraint of type *a* buyers. Therefore category *A* sellers do not meet the participation constraint of type *b* buyers, (ii) sellers of category *B* leave some surplus to buyers of type *b*. As a consequence (iii) buyers of type *a* accept any offer as they get exactly their reservation utility from both categories of firms, (iv) buyers of type *b* keep searching until they find a seller of category *B*, where they receive some surplus, (v) the profit of firms of category

affects your experience. If you are already signed in, you can buy more easily - and feel more secure - than if you have to register to a new one.

[2]The idea of the existence of a potential tradeoff between the taste for diversity of buyers and the minimization of transaction costs has been recently studied by Woodruff (2002).

[3]The equilibrium concept used here is Coalition-Proof Nash Equilibrium, as defined by Bernheim, Peleg and Whinston (1987). Others Nash Equilibria exist, but are not robust to self-enforcing deviations of a mass of sellers. Using this concept allows characterizing a market failure that is not caused by a coordination failure.

B is decreasing in their number (since they share the buyers of type b that search) while the profit of category A firms is independent of their number (since there is no search component in their demand). Hence in equilibrium (vi) the share of category B firms is such that their profit is exactly equal to that of category A firms.

As search costs are almost inexistent, this seems to be efficient: there is product diversity, search and price dispersion. However it is not: a share of buyers of type a is not correctly matched. Forcing the market to produce the category desired by the majority always increases aggregate consumer welfare by decreasing average price and increasing the share of good match. It also increases total welfare when the market does not supply enough of category A. In any case, a monopolist owning all the premises would increase total welfare by providing both categories at a price that gives incentive to the buyers to search. With search costs going to zero, this allows extracting almost the entire surplus of buyers. Hence, competition increases buyer surplus at the cost of creating inefficiencies in the matching process. This model applies to a variety of monopolistically competitive markets: clothing, theaters, bars, restaurants, bookstores, retail stores, shoe stores... Even to service providers as solicitors, physicians, etc. as long as there is a choice to be made by sellers and that it is not possible for a seller to satisfy all types of buyers. Here are two very simple examples.

Example 1 ***The market for hair salons***. *Each hairdresser can provide a large choice of haircuts, colors, at various prices. But when she must decide whether to opt for a 'first come, first served' policy or accept appointments, the two decisions are mutually exclusive (at least for a given time slot). By choosing a system of appointments, the hairdresser satisfies a share of the customers (those who prefer to plan ahead), but disappoints the other share that prefers to come in unexpectedly. The production costs are roughly the same but it is impossible to please both types of customers at the same time of the day.*

Example 2 ***A TV screen in a bar.*** *Bars can serve a variety of drinks, but when they have to decide whether or not to install a TV, it affects all consumers in the room. The barman can offer you any drink you want, your drinking experience is always influenced - positively or negatively, depending on your type - by the television. Consider that a majority of buyers prefer to go to bars that have a television, and that this preference is sufficiently important. The model predicts that bars with television will be more expensive, and that most buyers will therefore be indifferent between bars that have a television and those that do not, while the buyers who do not like having a TV in a bar will only go to places that cater specifically to their taste.*

Product diversity is a well-documented topic. The idea that the characteristics of goods can be valued differently by different types of individuals has been formalized by Lancaster (1966). The novelty of my results comes from a combination of 3 elements. First, the level of horizontal diversity is endogenously determined by the sellers, as opposed to random utility models (Perloff and Salop (1985), Deneckere and Rotschild (1992), Anderson and Renault (1999)). This implies that a continuum of firms may produce homogenous goods in equilibrium if it is a best response for them. Second, search costs are independent of the localization of the sellers, as opposed to models à la Hotelling (Salop (1979), Stahl (1982), Dudey (1990),Gabszewicz and Thisse (1986)). That is, in my model, sellers do not become more accessible to some buyers by changing their good category (position in a Hotelling/Salop model). Third, I model a market, as opposed to the many models using a representative agent (as in Dixit and Stiglitz (1977)). Hence, the inefficiency is not related to the cost structure, but to the failure of the market mechanism itself.

Price dispersion with homogenous goods has been studied in several papers, mostly in the context of the Internet. See Baye, Morgan and Scholten (2005) for a survey. The price structure in horizontal matching models has been studied by Besley and Ghatak (2005), Clark (2007) and Klumpp (2009), but all assume diversity to be exogenously given. Endogenous diversity in oligopoly is studied among others by Chen and Riordan (2006), Kuksov (2004) and Bar-Isaac, Caruana and Cunat (2008). The market failure in my model comes from the presence of nontransferabilities in the matching process (buyers are price takers and there is no bargaining). Legros and Newman (2007) study how non-transferable utility affects matching when differentiation is vertical. As in Nocke, Peitz and Stahl (2007), I find that a monopoly can extract more gains from trade then a competitive ownership.

The result that firms can extract consumer surplus in my model is close to Diamond (1971). In the classical formulation of price competition, firms set a price equal to the marginal cost in equilibrium. Introducing search costs in the specification yields the so-called 'Diamond Paradox': a model of search with a large number of buyers and a large number of sellers does not converge to a competitive equilibrium à la Bertrand. In finite time, the price becomes the one that maximizes joint profit. The logic behind Diamond (1971) is the following. Consider that a time period is the time it takes for a buyer to visit a store. At the beginning of each period, the seller sets the price for the whole period. The only way for a consumer to learn the price set by a specific store is to enter it. The commodity is bought once. Consumers know the distribution of prices today, and are aware that the price might change tomorrow. There is no product differentiation. Search costs take a very general form. The utility of a buyer is given by $U(p,z)$ with p the price and z the number of periods needed to buy. The condition is that U is decreasing in both arguments. For a given price level in the market, sellers always have an incentive to slightly increase their price until they reach the monopoly level. Indeed, by charging a little more than their competitors, sellers make sure that a buyer who enters their shop will not keep searching for a lower price elsewhere.

In this paper, since there is no capacity constraint for the sellers, and as buyers buy either zero or exactly one unit, a Diamond Paradox would be an efficient outcome (with the entire surplus extracted by the sellers). However, product differentiation changes the picture and, in equilibrium, profit maximizing sellers are not maximizing joint profit. Hence, while the price mechanism is closely related to the one of Diamond (1971), my results do not imply a similar paradox. Once horizontal differentiation is feasible, there is search and price dispersion in equilibrium, but another inefficiency arises, due to the mismatching of a share of buyers.

I present the setup of the model in the next section. Section 2 characterizes the equilibrium result: I first show that none of the Nash Equilibria are efficient. Even if search costs are arbitrarily small, not all buyers are properly matched. Two cases may arise:[4] when buyers do not care sufficiently about the premises, only one category is produced and when buyers do care sufficiently about the premises, there is product diversity and search in the only Coalition Proof Nash Equilibrium. In Section 3, I discuss the policy implications of these results. I extend the results to a continuum of types and larger search costs in Section 4. Section 5 concludes.

1.1 Setup

The economy is composed of two groups, each of them being a continuum of mass 1. The first group is the buyers, with an exogenous fraction α of type a and $1-\alpha$ of type b. In this presentation of the model, I consider $\alpha \in (\frac{1}{2}, 1)$.[5] The second group is composed of sellers, who endogenously choose a category A or B. The choice of category is costless: price dispersion does not come from switching costs. A 'good' match (a&A or b&B) generates surplus V and a 'bad' match (a&B or b&A)

[4] I characterize necessary and sufficient conditions for both cases.

[5] The results for $\alpha \in (0, \frac{1}{2})$ are symmetric. I exclude the non-generic possibility of having exactly $\alpha = \frac{1}{2}$. Heterogeneous buyers implies $\alpha < 1$.

generates surplus $v < V$.[6] The surplus is received by the buyer if she accepts the price set by the seller. The outside option is set to $r \in (0, v)$.[7] Both categories are produced with no fixed cost and marginal cost normalized to 0. A buyer can buy either 0 or exactly 1 unit of either good. Parameters α, V, v, r and discount factor δ are common knowledge The stages of the game are as follows:

1. Sellers simultaneously choose a category of premises (either A or B) and price offer ;

2. Buyers learn the share γ of sellers of category A, and the distribution of prices ;

3. Each buyer is randomly matched with a seller.[8] She observes the price and the chosen category of the seller she is matched with. Each buyer decides whether to **Accept** the offer, **Leave** the market and receive the outside option r or to **Search** for another seller. If a buyer searches, she is randomly matched with another seller, but her payoffs are discounted with a parameter $\delta < 1$. There is no limit for search, but the cumulated discount factor decreases to δ^s after s searches.[9]

1.2 Equilibria

In this Section, I assume arbitrarily small search costs ($\delta \to 1$).[10] I first show that there are only two potential prices in equilibrium and that the market outcome

[6]The fact that only two values exist for the surplus is not crucial for my results. I show in section 1.4.2 that the necessary condition is to have a sufficiently high density of buyers sharing close preferences.

[7]The outside option cannot be normalized to zero, as it would imply that any positive outcome, even if discounted a large number of times, is always higher than r.

[8]This first match is assumed to be costless. Making it costly would generate extra equilibria, not robust to the strongest concept of equilibrium used below.

[9]The search cost is supported only by the buyer (wasting time a given day) and not as postponed sales. Therefore, only the surplus of buyers is discounted. When search costs are arbitrarily small, none of the results is affected by this assumption.

[10]This assumption is relaxed in section 1.4.1. The main equilibrium presented in this section is robust to an increase in search costs.

is never efficient. I concentrate on the case where buyers care sufficiently about premises, show there are multiple Nash Equilibria and that only one is Coalition Proof. Then, I briefly summariz the other case. In conclude the section by proposing a theorem on price dispersion and market efficiency.

1.2.1 Main characteristics of the Nash Equilibria

The equilibrium price only takes two values that I denote by 'high price' $p = V - r$ and 'low price' $p = v - r$. The results of this subsection are mainly driven by a mechanism that can be related to the one used by Diamond (1971) while introducing search costs in a homogeneous market.

The difference here comes from the heterogeneous tastes of the buyers. The low price corresponds to the participation constraint of mismatched buyers. A seller that has positive demand from those buyers when the price is exactly $v - r$ certainly loses the demand from the whole group by slightly increasing the price. Therefore, there can be an incentive for sellers not to increase the price above this threshold. Similarly, the high price corresponds to the participation constraint of buyers with a good match, and any price above this value implies zero demand for the seller.

Lemma 1 *There are only two possible prices in a Nash Equilibrium:* $p = V - r$ *and* $p = v - r$

Sketch of the Proof. The formal proof is given in Appendix 1.6.2. Sellers are free to choose their category at no cost. Therefore, if the expected profit of a seller of category i is higher than the expected profit of a seller of category j, this is not an equilibrium. While deciding whether to accept an offer or to search for another, a buyer considers the distribution of prices in the market $\hat{p}$. As there is a continuum of sellers, a single seller has no influence on $\hat{p}$. However, any seller knows $\hat{p}$, and can set her price in order to make buyers of a given type accept her offer. It is always a best response for a seller to slightly increase her price as long as she does not lose consumers by doing so. This can only happen for two levels of price: $v - r$ and $V - r$. At those levels, any increase in the price implies that one of the participation constraints is no longer satisfied. ■

The impossibility of having an equilibrium price different from those two values is the key factor that drives the inefficiency of any Nash Equilibrium of this model. Indeed, either sellers sell to both types of buyers - this implies that some buyers are not correctly matched - or they specialize in only one type and set a high price, such that search never occurs - this implies either a share of mismatches or some buyers leaving the market.

Proposition 1 *The market outcome never exhausts all gains from trade.*

Proof. *(by contradiction)* Assume both types of buyers search until they find a good match. This implies that each seller is specialized in one type. Hence, as shown in Lemma 1, it is a Best Response for every seller to set a price slightly above the market level, even when it is exactly $p = v - r$. The only price in a Nash Equilibrium is therefore $p' = V - r$. At this level of price, the expected surplus on the market is at most r and buyers never search. This is a contradiction. ■

In any equilibrium, a share of buyers is not correctly matched. This implies that the total gains from trade are strictly lower than $V - r$. If there is a mass of sellers of each type and if the price is strictly lower than $V - r$, all buyers search and the total gains from trade tend to $V - r$ (as δ goes to 1). This Proposition shows the existence of a market failure. Indeed, consider instead that a monopoly owns all the sellers. It is easy to show that, by setting a price slightly below $V - r$ and producing both categories, all buyers search and all the potential surplus of the economy is extracted. However, this monopoly would eventually let the buyers with zero surplus. As will be made clear below, competition leaves some buyers with surplus, at the cost of an inefficiency in the matching process. Whether regulation can help increase welfare is discussed in Section 1.3.

1.2.2 When buyers care sufficiently about the premises

When buyers do not care sufficiently about the premises, the equilibrium corresponds to a classical result of standard setting. The market provides only one of the two categories, at a low price, there is no search and no one is excluded from the market (I develop this result in Section 1.2.3).

When the importance of the premises increases sufficiently, or when the majority is sufficiently large, the incentive for sellers to extract the surplus of a good match also increases and product diversity starts to become a Nash Equilibrium. Condition 1 is necessary and sufficient to be in this case.

Condition 1 $\alpha > \frac{v-r}{V-r}$

In this Section, I consider that condition 1 is true. The interpretation is twofold. If the ratio on the right hand side of the equation is sufficiently low, it means that sellers can make large surplus by setting the high price and selling only to the majority type. If the left hand side is sufficiently high, it means that the demand from the majority is large enough to compensate the loss from not attracting minority buyers.

In this subsection, I show that there are potentially four Nash Equilibria in the economy. Three of them coexist, depending on the values of the parameters. I list them below. Then, I explain why I consider a more restrictive concept of equilibrium: Coalition-Proof Nash Equilibrium (CPNE). I show that only one equilibrium, $AS_{\min}$, is a CPNE.

Definition 1 ***Tyranny of the majority (high price):*** *TM_H. All sellers choose the category desired by the majority and sell it at the high price.*

In this equilibrium, the buyers of the minority are excluded from the market. There is no search.

Definition 2 ***Tyranny of the majority (low price):*** *TM_L. All sellers chose the category desired by the majority and sell it at the low price.*

Therefore, all buyers accept the offer. There is no search.

Definition 3 ***Asymmetric supply - Some surplus left to the majority*** *AS_{maj}. There are sellers of both categories in the market. The sellers of category A (corresponding to the majority type a) sell at the low price and the sellers of category B sell at the high price.*

Here, the buyers of the minority type accept any offer, while the buyers of the majority type search until they find a good match.

Definition 4 ***Asymmetric supply - Some surplus left to the minority*** *$AS_{\min}$. There are sellers of both types in the market. The sellers of category B (corresponding to the minority type b) sell at the low price and the sellers of category A sell at the high price.*

This equilibrium is the only CPNE. The buyers of the majority type accept any offer, while the buyers of the minority type search until they find a good match.

Lemma 2 *If condition 1 is true, there are exactly four potential Nash Equilibria in this game: TM_H, TM_L, AS_{maj} and $AS_{\min}$. For all parameters, either TM_L or AS_{maj} exist but never both.*

Sketch of the Proof. The formal proof is given in Appendix 1.6.3. Here is the intuition for each of the equilibria.

- TM_H: it is not a best response for sellers to lower the price unless it is at most $p\prime = v - r$. If the price is exactly p', the expected profit is $\pi' = v - r$. This is lower than the equilibrium profit $\pi = \alpha(V - r)$ by condition 1. It is not a best response for a seller to sell the other category, as the profit of the deviating seller is at most $\pi'' = (1 - \alpha)(V - r)$. Which is lower than π, since $\alpha > \frac{1}{2}$.

- TM_L: a seller that slightly increases the price never increases her profit, as she automatically loses a large share of the buyers. The profit of each firm is $\pi' = v - r$. The only possibility to increase profit is to sell the category desired by the minority at price $p = V - r$. Therefore, TM_L is a Nash Equilibrium if and only if $\pi'' < \pi'$.

- AS_{maj}: the share γ^* of firms of category A is such that their profit is exactly the same as the firms of category B, $\pi'' = (1-\alpha)(V-r)$. A firm can increase its profit by deviating and selling category A at the low price if and only if $\pi'' < \pi'$. Therefore, if TM_L is a Nash Equilibrium, AS_{maj} is not.

- $AS_{\min}$: the share $(1-\gamma^*)$ of firms of category B is such that their profit is exactly the same as the firms of type A, $\pi = \alpha(V-r)$. Therefore, a deviating firm can at most receive profit π' or π'' which are lower by definition.

■

Considering the general definition of Nash Equilibrium, this economy displays a multiplicity of equilibria, and there is no way to predict which one is expected to be realized in practice. In the next Proposition, I use an alternative concept of equilibrium: Coalition-Proof Nash Equilibrium, as defined by Bernheim, Peleg and Whinston (1987). This more restrictive definition implies that there is no *self-enforcing* deviation by a coalition of sellers that can gain from deviating. Take for instance the first equilibrium, TM_H, which is a Nash Equilibrium because no single seller can make buyers search for it. However, there is a coalition of mass $(1-\gamma')$ that would benefit from selling the category desired by the minority at a low price. If $(1-\gamma')$ is not too high, those sellers can attract a sufficiently large number of buyers of type b to increase their profit. This deviation is *self-enforcing*, as none of those deviating sellers has any incentive to change her price or category. And there is no *self-enforcing* deviation by a sub-coalition that can increase her profit by doing so.

By definition, in every CPNE, the sellers get the highest possible equilibrium profit. Thus, the market failure identified here does not come from a coordination failure among sellers. Moreover, this concept is much more realistic in the context of this paper. Indeed, firms can communicate and exchange ideas, even if they do not explicitly coordinate. Some sellers can for reasons unrelated to profit maximization try to sell the other category. This can even be created from the demand side: a coalition of buyers of the minority type could start its own business, or simply give a certification or a label to firms who accept to sell their preferred category. All of those effects would make the equilibrium TM_H disappear.

Proposition 2 *When condition 1 is fulfilled, the only Coalition-Proof Nash Equilibrium of this game is* $AS_{\min}$*: the sellers of category B (corresponding to the minority type b) sell at a low price and the sellers of category A sell at a high price. The buyers of the majority type accept any offer, while the buyers of the minority type search until they find a good match.*

Sketch of the Proof. The formal proof is provided in Appendix 1.6.4. In TM_H there exists a *self-enforcing* coalition that can increase its profit by selling the type desired by the minority at the low price. In TM_L and AS_{maj}, the profit of sellers is strictly lower than in $AS_{\min}$. Therefore, a *self-enforcing* deviation of a coalition of mass 1 increases its profit by playing $AS_{\min}$. ■

A key factor to understand the equilibrium outcome is that, as sellers are free to choose their category at no cost, the expected profit of all sellers is equivalent. The profit of sellers of category A is independent of their number and is $\pi = \alpha(V - r)$. Hence, the number of firms of category B is determined by the difference between high and low price (how much extra surplus a seller can extract by specializing in the majority type) and the share of buyers of the minority (how many buyers will search to reach a seller of category B). This equilibrium value is given by

$$\gamma^* = 1 - \frac{1-\alpha}{\alpha}\frac{v-r}{V-v}. \tag{1.1}$$

It must be noted that the category desired by the majority of buyers can be produced by a minority of sellers in equilibrium.

Coming back to the TV screen example provided in the introduction, this result implies that if a majority of bar goers prefer TV screen, they are indifferent between high price bars with a TV screen and low price bars without. Buyers who dislike TV screens however, would never consume in a high-price bar with a TV screen. They keep searching until they find a low-price bar corresponding to their preferences, i.e. without a TV. If high profit can be made by sellers specializing in those customers that like TV screen, the model predicts the market to reflect the preferences of buyers. But this is not always true, and the market outcome can perfectly display a large majority of bars without TV screen, patronized by all types of buyers, and only a few bars with a TV screen, only patronized by TV lovers. Bars without a TV have no incentive to increase their price, as they would lose TV lovers. Bars with a TV have no incentive to decrease their price, as it would not be sufficient to attract consumers with other preferences.

1.2.3 When buyers care less about the premises

Assume now that condition 1 is not satisfied, i.e.

$$\alpha < \frac{v-r}{V-r}$$

This corresponds to assuming that the relative surplus generated by a good match with respect to a mismatch is quite small, given the size of the majority. The intuition is that of a standard setting. There is only one category sold in equilibrium, at the low price.

Proposition 3 *When condition 1 is not fulfilled, there is only one category provided in equilibrium. It is sold at low price, there is no search and no buyers are excluded from the market.*

Sketch of the Proof. The formal proof is given in Appendix 1.6.5. The profit of each firm is $\pi' = v - r$. There is no incentive for any firm to increase the price, as the profit would be at most $\pi = \alpha(V - r)$. This is lower than π' as condition 1 is not fulfilled. As the potential surplus from specializing in one category is not high enough, all sellers attract both types. Product diversity is not a Nash Equilibrium. If sellers set the low price but attract only one type of buyers, each seller has an incentive to slightly increase the price. And if one category is sold at the high price and the other at the low price, the profit of the latter sellers is at least $\pi' = v - r$, higher than what she can get by specialization. ■

As in the previous case, the market alone fails to provide an efficient level of product diversity. Note that a market where only category B, preferred by the minority, is produced is also a Nash Equilibrium. Back to the TV example again, this depicts a world where most buyers like TV screen and no bar provides this service. This is not a sufficiently important issue for buyers, and a bar would not increase its profit by installing the screen and specialize in TV lovers.

1.2.4 Price dispersion and market efficiency: a Theorem

Theorem 1 *When a decentralized market displays product diversity in Equilibrium, there is always price dispersion and a share of buyers searching until they find a good match. Still, the market is inefficient to match the sellers with the tastes of the buyers.*

Proof. By propositions 2 and 3, when there is product diversity in a Coalition Proof Nash Equilibrium, there is also price dispersion, and a strictly positive share of buyers of type a are matched with sellers of category B. By lemma 2, in the only non CPNE with product diversity, there is price dispersion and a positive share of buyers of type b are matched with sellers of category A ■

When buyers care sufficiently about the premises, a decentralized market where rational buyers share the same information and where sellers have the same production costs can display price dispersion. But this price dispersion is a feature of an equilibrium where a share of the buyers buy to sellers that do not match their preferences. This result holds regardless of the equilibrium concept, as it holds for any Nash Equilibrium.

1.3 Welfare and sandardization

The objective of this Section is to discuss the welfare implications of the results presented above. I mainly focus on the case where condition 1 is true. First, I discuss the welfare impact of the simplest possible regulation: standardization, to force the sellers to choose the majority type. I show that this always increases aggregate consumers' surplus. This regulation also increases total welfare when the market does not provide enough of the type desired by the majority. However, this policy is never Pareto Improving, as it decreases the surplus of the buyers of the majority. Then, I explain how setting a market for licenses can increase total surplus and be Pareto Improving: there is an extra revenue from licenses that does not decrease the surplus of buyers or the profit of sellers. I close this Section by presenting the result for the case when condition 1 is not fulfilled.

Lemma 3 *If firms are only allowed to sell category A, the only Coalition-Proof Nash Equilibrium is TM_L: all firms sell category A, at the low price. All buyers accept the offer. There is no search.*

Proof. The two potential Nash Equilibria are TM_L, with $p = v - r$ and TM_H, with $p'' = V - r$.

(i) TM_H is not a CPNE.[11] There exists a coalition ξ of sellers that can increase its

[11] Even if there is no product diversity in this game, this result differs from the paradox raised by Diamond (1971). TM_H is the equilibrium that maximizes joint profit.

profit by setting $p = v - r$, as buyers will search until they find a seller at price $v - r$. Slightly increasing the price decreases profit, so the deviation is *self-enforcing*.
(ii) It follows that TM_L is a CPNE. A mass 1 of sellers can increase its profit by setting p'', but it is not *self-enforcing* since a subcoalition ξ of sellers can increase its profit by setting $p = v - r$. ■

The question is to find out whether standardization can be welfare improving. A social planner can broadly have two main objectives: (i) maximize the total surplus (ii) maximize the surplus of consumers.[12]

While considering the aggregate surplus of consumers, it is easy to show that this regulation always leads to an improvement. Indeed, in the unregulated equilibrium $AS_{\min}$, the minority buyers receive surplus $S = V - (v - r)$, corresponding to the difference between their valuation for the good match and the low price, while the majority buyers receive a surplus equivalent to the outside option. In the regulated equilibrium TM_L, the majority buyers receive S while the minority buyers receive a surplus equivalent to the outside option.

While considering total surplus, there is a trade-off between the profit of sellers, which is lower in the regulated equilibrium, and the surplus of buyers. The key factor is to know if the loss in profit due to specialization is compensated by the gain in consumer surplus. Still, this is not Pareto Improving, as the expected utility of the buyers of type b is reduced.

Proposition 4 *If the gains from specialization are sufficiently high, the unregulated market supplies a large share of the category desired by the majority, and regulation decreases total welfare. Otherwise, regulation increases total welfare.*

Sketch of the Proof. The formal proof is provided in Appendix 1.6.6. Regulation increases welfare if the total surplus is higher in TM_L than in $AS_{\min}$. This can

[12]For instance, the latter is the official statement of the European Commission for market regulation policies.

be written as

$$\begin{aligned} \alpha V + (1-\alpha)v &> V - (1-\alpha)(v-r) \\ &\Leftrightarrow V - v < v - r \end{aligned} \tag{1.2}$$

■

This relates to the value of γ^* presented in equation 1.1. If the gain from specialization is high, the share of sellers of the majority type is also high, and regulation is not welfare improving. But when the gain is lower, the market does not supply enough of the majority type and regulation increases total welfare.

While price regulation seems to be mostly a theoretical object,[13] a more realistic alternative policy is to implement a market for licenses. Assume that a license is the only legal way for sellers to choose the category desired by the minority. The social planner sets a number of licenses sufficiently high for the minority type to search when sellers set the low price. In our case with search cost arbitrarily small, the number of licenses is also arbitrarily small. In this case, one can expect almost every buyer to benefit from a good match, and therefore the equilibrium to be close to the first best. More generally, for a share $1-\gamma^*$ of licenses, the market price of a licence, L, is the solution to

$$\alpha(V-r) = \alpha(v-r) + \frac{1-\alpha}{1-\gamma^*}(v-r) - L$$

Depending on the level of search costs δ, the optimal share of licenses $1-\gamma^*$ is the smallest that ensures search from buyers of type b. On can show (see appendix 1.6.3) that this is the solution to

$$1-\gamma^* = \frac{r}{V-v}\frac{1-\delta}{\delta}$$

[13] Among other because $v-r$ is not constant through time and the market outcome is the only way to measure it. Also because this model only reflects the specific profits of the platform that manages the premises but not the price of the good itself.

The sellers of the minority type still set the low price.[14] Those sellers set the lowest price but are also the ones that pay for the license. The profit of sellers and the surplus of buyers is exactly the same as in $AS_{\min}$. The difference in total welfare comes from the revenue of the licenses.

When condition 1 is not fulfilled, it is straightforward that if firms are only allowed to sell category A, this weakly increases total welfare. If a system of licenses is introduced as described before, the equilibrium changes to $AS_{\min}$ (with a constrained share of sellers of category A), therefore increasing the price of the majority category for which no license is paid. Thus, licensing decreases aggregate consumers' welfare - but increases total welfare as more consumers obtain a good match.

1.4 Extensions

1.4.1 Increasing search costs

The objective of this extension is to present the additional conditions on discount factor δ for the existence of the various equilibria when search costs are not arbitrarily small. The computations are provided in the formal proof of each of the equilibria in Appendix 1.6.3.

a. When buyers care sufficiently about the premises

When condition 1 is true:

- TM_H is always an equilibrium.
- TM_L is an equilibrium if $(1-\alpha) < \frac{(v-r)}{(V-r)}$ and $\delta > \frac{\alpha V-(v-r)}{\alpha(V-(v-r))}$. The first condition excludes specialization in the minority type, the second condition ex-

[14] Slightly increasing the price is not a best response since it implies losing the demand from buyers of the majority type. And there is no self-enforcing deviation of a coalition towards a higher price.

cludes the possibility for a seller of category A to increase the price, still have demand from majority buyers and increase its profit.

- $AS_{\min}$ is an equilibrium if $\delta > \frac{\alpha r}{(1-\alpha)(v-r)+\alpha r}$. This condition ensures that there is a sufficiently large number of sellers of category B for buyers of type b to actually search.

- AS_{maj} is an equilibrium if $1-\alpha > \frac{v-r}{V-r}$ and $\delta > \frac{(1-\alpha)r}{\alpha(v-r)+(1-\alpha)r}$. The first condition ensures there are enough buyers of the minority type and that enough surplus can be extracted from them. The second condition ensures that there is a sufficiently large number of sellers of category A for buyers of type a to actually search.

The main result is that for values of α not too close to 1, the equilibria are robust to increases in search costs. When search costs increase too much, the number of equilibria decreases. One can show that if TM_L is a Nash Equilibrium, $AS_{\min}$ is also a Nash Equilibrium (the reverse is not true).

Example 3 *Consider values of r, v, V such that TM_L exists, and therefore AS_{maj} does not: $r = 1, v = 2, V = 3$. TM_H is always an equilibrium, TM_L is an equilibrium for any $\delta > \frac{3\alpha-1}{2\alpha}$ (the right-hand side is always lower than 1 and increasing in α), $AS_{\min}$ is an equilibrium for any $\delta > \alpha$.*

Example 4 *Consider values of r, v, V such that AS_{maj} exists, and therefore TM_L does not: $r = 1, v = 2, V = 4$. TM_H is always an equilibrium, AS_{maj} is an equilibrium (iff $\alpha < \frac{2}{3}$) for any $\delta > 1-\alpha$ and $AS_{\min}$ is an equilibrium for any $\delta > \alpha$.*

b. When buyers care less about the premises

When condition 1 is not fulfilled, both Nash Equilibria presented in Section 1.2.3 hold for any value of δ. Equilibria with product diversity can arise when δ decreases. One can show that, as long as $\delta < \frac{2r}{2r+V-v}$, there exists values of $\gamma^- < \gamma^+$

such that for any $\gamma \in (\gamma^-, \gamma^+)$ buyers buy any type without search. If buyers accept the offer regardless of the premises, sellers are indifferent between both categories. The higher the search cost, the broader the interval in which those equilibria exist.

1.4.2 A continuum of types

The specification of my model relies on assuming two discrete types, and two different values of consumer surplus. The objective of this extension is to give a continuous interpretation of the main equilibrium of my model, $AS_{\min}$.

As in the basic specification, there is a continuum of buyers and sellers of mass 1 and a fraction $\alpha > \frac{1}{2}$ of buyers of type a. A good match yields surplus V and buyers have an outside option of value r. Assume now that the valuation of a mismatch is drawn, for each buyer, from a continuous distribution f with support $[r,V]$. We try to characterize an equilibrium where sellers of categories A and B extract all the surplus of buyers of type a, and where buyers of type b search for sellers of type B that leave them some surplus.

First, I assume this equilibrium exists. I define the total profit for a firm of category A, and the total profit (and the maximization problem) for a firm of category B. Then, I derive the new value of γ. Finally, I show under which conditions it is actually an equilibrium.

The total profit for a firm of category A is

$$\pi_A = \alpha(V - r)$$

as in the discrete case.

To compute **the total profit of a firm of category** B**,** with price $p_B \in (r, V - r)$, one has to distinguish:

- *The demand from buyers of type a* who pick up the seller first. In this equilibrium, firms of type A give no more surplus than the outside option. This demand can be rewritten as $D_a(p_B)$, with $D_a(V-r)=0$ and $D_a(r)=\alpha$

- *The demand from buyers of type b who pick up the seller first, plus the demand from buyers of type b who actually search* (as $p_B^* < V-r$): $\frac{1-\alpha}{1-\gamma}$,

$$\pi_B = (D_a(p_B) + \frac{1-\alpha}{1-\gamma})p_B.$$

Hence, the maximization problem yields

$$\frac{D'_a(p_B^*)p_B^*}{D_a(p_B^*)} = \frac{1-\alpha}{1-\gamma}.$$

A first comment is that, for such an equilibrium to exist, one needs a point with sufficiently high density (so the elasticity of the demand is high enough), and a share of buyers of type a sufficiently high (so the relative importance of this part of the demand is high enough).

Moreover, one needs a sufficiently small value of γ, such that the 'search' part of the demand, which is constant as long as $p_B < V-r$, is not too high. Other things being equal, the smaller γ, the highest the elasticity of total demand for a seller of category B.

However, even if γ is taken as exogenous in the maximization problem of an individual seller, it is still determined by the expected profit of a seller of category B. The expected profit of both categories of sellers being equal, γ must satisfy.

$$\gamma^* = 1 - \frac{(1-\alpha)p_B}{\alpha(V-r) - D_a(p_B)p_B}$$

which, for any $p < p_B^*$ is strictly decreasing in p_B. This is quite intuitive, as for any $p < p_B^*$ increasing the price increases the profit of a firm of category B, it also increases the number of firms of category B, and decreases the 'search' component of the demand.

Those prices correspond to a Nash Equilibrium if there exists a solution $\gamma^* \in (0,1)$. This is true if there is a sufficiently high concentration of types, at a point that yields a sufficiently high surplus of a mismatch, and with a sufficiently large majority of buyers of type a. This corresponds to the same intuition as the conditions of existence of this equilibrium in the discrete case.

If those conditions are satisfied, setting another price is not a best response for a firm of category A (a price higher then $V - r$ yields zero demand, a price lower does not increase demand but decreases profit). The price of firms of category B is an equilibrium by definition, as it is the result of individual profit maximization.

1.5 Conclusion

This model relies heavily on two assumptions: the sellers have to make a choice - they cannot serve both types - and no seller has sufficient market power to attract buyers by changing her price. The first assumption is the reason why this model applies to premises - which affect all the goods and services - and not to the goods and services themselves. The second assumption is the key reason for the differences between the results of this model - with competitive markets - and the recent oligopoly models cited in the review of the literature.

I have shown that when buyers care sufficiently about the premises, the market does not exhaust all the gains from trade: a share of the buyers is not correctly matched. As long as search costs are not too high, the efficiency is independent of search costs, and so is the average price on the market. Therefore, the perfectly competitive price is a knife-edge case, since any arbitrarily small search costs make it disappear.

When the market outcome does not reflect the preferences of buyers, a social planner can increase aggregate welfare by forcing firms to sell only the type desired by the majority. This can be a rationale to consider standardization laws, assuming they reflect the tastes of the majority, as something potentially more effective than a simple transfer of utility from the minority to the majority type. However, this not Pareto Improving among consumers, and this is also expected to lower the profit of firms, which may therefore oppose such a regulation. Another policy enabling to increase total welfare without hurting sellers is to create a market for licenses, with as few licenses of the minority type as needed to make the buyers of this type actually search. Under this policy, all sellers and buyers get exactly the same surplus as in the unregulated market, and the extra surplus from a higher matching rate is extracted by the social planner, or by whoever owns the property rights on the licenses.

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1.6 Technical Appendixes

1.6.1 Two preliminary lemmas

The two following lemmas are used in several of the following proofs:

Lemma 4 *Both categories of sellers have the same expected profit in equilibrium.*

Proof. Sellers are free to choose their type at no cost. Therefore, if the expected profit of a seller selling a service of category i is higher than the expected profit of a seller of category j, this is not an equilibrium. It is a best response for a seller of category j to sell category i. ■

Lemma 5 *For a given level of price, a seller sets her price in a way to decide what kind of buyers accept her offer, but has no influence on who searches for her.*

Proof. This Lemma is close to Diamond (1971). While deciding whether to accept an offer or to search for another, a buyer considers the distribution of prices in the market $\hat{p}$. As there is a continuum of sellers, a single seller has no influence on $\hat{p}$. However, a seller knows $\hat{p}$, and can set her price in order to make buyers of a given type accept her offer. ■

1.6.2 Proof of Lemma 1

Consider first the following definitions:

Definition 5 ***The participation constraint for a seller of category** j **is satisfied for a buyer of type** i if the seller offers a price leaving the buyer of type i utility higher than the reservation utility r. I denote this by* PC_i^j*, with* $i \in \{a,b\}$, $j \in \{A,B\}$.

Definition 6 ***The incentive compatibility constraint for a seller of type** j **is satisfied for a buyer of type** i if the seller offers a utility higher than her (discounted) expected utility if she stays in the market. I denote this by* IC_i^j*, with* $i \in \{a,b\}$, $j \in \{A,B\}$.

Definition 7 *A buyer of type i accepts the offer of a seller of category j if and only if PC_i^j and IC_i^j are satisfied.*

This is the proof of Lemma 1:

Proof. PART 1: $p > V - r$ is never a NE.

(i) Consider a pair i, j with $i \neq j$.For a firm i setting price p, PC_i^i is given by $V - p \geq r, \Leftrightarrow p \leq V - r$ and PC_j^i is given by $v - p \geq r \Leftrightarrow p \leq v - r$.

(ii) A seller always makes a positive profit. Assume all firms sell at price zero and make zero profit. Slightly increasing the price is a profitable deviation for a firm of type i, as there exist $p' > 0$ such that IC_i^i is satisfied, i.e. $V - p' \geq \delta V$.

(iii) As a corollary of (ii), it is never a best response for a firm to have zero demand. Hence, $p > V - r$ is never a best response, because no buyer ever accepts the offer.

PART 2: $v - r < p < V - r$ is never a NE.

Consider a firm of category i. Denote the expected surplus proposed by a firm of category i to a buyer of type j. S_j^i. As $p > v - r$, PC_j^i is not satisfied. The firm only considers buyers of type i, hence fulfilling IC_i^i and PC_i^i. As $p < V - r$, PC_i^i is already satisfied.

IC_i^i is always satisfied when the firm sets a price p' such that

$$S_i^i(p') \geq \max(\delta S_i^i(\hat{p}_i), \delta S_i^j(\hat{p}_j)).$$

(i) If $\max(\delta\ S_i^i(\hat{p}_i), \delta\ S_i^j(\hat{p}_j)) = \delta\ S_i^i(\hat{p}_i)$, there exists a price $p' > \hat{p}_i$ such that IC_i^i is satisfied. Indeed, $S_i^i(p') \geq \delta\ S_i^i(\hat{p}_i)$ for some $p' > \hat{p}_i$. Hence, it is always a best response for a firm to increase the price as long as PC_i^i is satisfied.

(ii) If $\max(\delta\ S_i^i(\hat{p}_i), \delta\ S_i^j(\hat{p}_j)) = \delta\ S_i^j(\hat{p}_j)$, then it is a BR for the firm to change and sell category j at price $p' > p_j$. Indeed:

a) By Lemma 4, in equilibrium, $\pi_i = \pi_j$.

b) As $\max(\delta\ S_i^i(\hat{p}_i), \delta\ S_i^j(\hat{p}_j)) = \delta\ S_i^j(\hat{p}_j)$, IC_j^j is satisfied and from PART 1, $p < V - r$.

c) For the same reason, if there exist some firms selling at price $\hat{p}_i$ in equilibrium, they must have non negative demand. Hence, IC_i^i is satisfied. Then, as $S_i^i(\hat{p}_i) < S_i^j(\hat{p}_j)$, IC_i^j is also satisfied, with $S_i^j(\hat{p}_j) > \max(\delta\ S_i^i(\hat{p}_i), \delta\ S_i^j(\hat{p}_j))$.

d) Hence, there exists some $p' > p_j$ such that IC_i^j is still satisfied.

PART 3: $p < v - r$ is never a NE.

Here, for both categories of firms and both types of buyers, PC is satisfied.

(i) For both categories of firms selling to both types of buyers to be a Nash Equilibrium, profit must be the same. As demand is 1 for any firm, it is only possible if p is the same for any firm. Also, IC_B^a and IC_a^B have to be binding. Indeed, if it is not satisfied, a type of buyer searches. And if it is not exactly binding, slightly increasing the price is a best response, i.e. for IC_B^a

$$\begin{aligned} v - p &= \delta\gamma(V - p)\sum_{i=0}^{\infty}\delta^i(1-\gamma)^i \\ v - p &= \frac{\delta(1-\gamma)(V-p)}{1-\delta(1-\gamma)}. \end{aligned}$$

And similarly, IC_a^B

$$v - p = \frac{\delta(1-\gamma)(V-p)}{1-\delta\gamma}.$$

This is only possible if $\gamma = \frac{1}{2}$. But then, both equalities yield

$$\begin{aligned}(1-\delta\gamma)(V-p) &= \delta\gamma(V-p)\\(1-\delta\gamma) &= \delta\gamma\\1-\frac{\delta}{2} &= \frac{\delta}{2}\\\delta &= 1\end{aligned}$$

this is, no search cost at all.

(ii) If a firm is interested in only one type of seller, the reasoning becomes the same as in PART 2, there is always an incentive to increase the price. ■

1.6.3 Proof of Lemma 2

To prove the Lemma, I first prove that each of the four potential Nash equilibrium exist under some conditions, and then that no other equilibrium can exist.

a. TM$_H$ is a Nash Equilibrium:

Proof. (i) It is not a best response for a seller to sell category B and set price $v-r < p' \leq V-r$, because the profit will be at most $(1-\alpha)(V-r) < \alpha(V-r)$.

(ii) It is not a best response for a seller to sell category B and set price $p' \leq v-r$. By Lemma 5 a seller cannot make people search for it. So, the profit will be at most $v-r < \alpha(V-r)$ (by condition 1).

(iii) It is not a best response for a seller to sell category A at price $p' \leq v-r$. On her own, a seller can't make buyer search for her. So, the profit will be at most $v-r < \alpha(V-r)$ (by condition 1).

(iv) It is not a best response for a seller to sell category A at price $v-r < p' < V-r$. By condition 5 this yields demand α and therefore profit strictly lower than $\alpha(V-r)$. ■

b. TM$_L$ is a Nash Equilibrium iff $(1-\alpha)(V-r) < v-r$**:**

Proof. Profit in equilibrium is $\pi_A = v-r$.

(i) It is not a best response for a firm to sell category B and set price $p = v-r$ as it will lose all buyers of type a.

(ii) It is not a best response for a firm to sell category B and set price $p' = V-r$, because the profit will be $(1-\alpha)(V-r) < v-r$.

(iii) It is not a best response for a firm of category A to increase the price. Consider $\tilde{p}$, the threshold price such that for any $p'' > \tilde{p}$ buyers of type a start searching. There is no incentive to set $p > \tilde{p}$ as it leads to zero profit. Neither is it an incentive to set $V-r < p < \tilde{p}$. If $\tilde{p} < V-r$, it is not a BR to set $p''' < \tilde{p}$ as this yields profit $\alpha p''' < \alpha\tilde{p}$. Therefore, we only consider an increase of price to exactly $\tilde{p}$. Define $\tilde{p} = v-r+\varepsilon$. Buyers of type a do not search as long as

$$\begin{aligned} V-(v-r+\varepsilon) &\geq \delta(V-(v-r) \\ \Leftrightarrow \varepsilon &= (1-\delta)(V-(v-r)) \end{aligned}$$

since the condition is binding. Therefore, it is a best response for a firm to increase the price iff

$$\begin{aligned} \alpha(v-r+\varepsilon) &\geq v-r \\ \Leftrightarrow \delta &> \frac{\alpha V-(v-r)}{\alpha(V-(v-r))} \end{aligned} \quad .$$

■

c. AS$_{\min}$ is a Nash Equilibrium:

Proof. A buyer waits if her expected surplus is higher than the reservation utility, i.e.

$$\begin{aligned}
&\Leftrightarrow \quad r < \delta(1-\gamma)\sum_{i=0}^{\infty}\gamma^i\delta^i(V-v+r) \\
&\Leftrightarrow \quad r < \frac{\delta(1-\gamma)}{1-\gamma\delta}(V-v+r).
\end{aligned}$$

this simplifies to

$$V - v > \frac{r}{1-\gamma}\frac{1-\delta}{\delta}.$$

The profit in equilibrium is given by:

For a firm of category A: $\pi_A = \alpha(V-r)$.

For a firm of category B: $\pi_B = \alpha(v-r) + \frac{1-\alpha}{1-\gamma}(v-r)$.

I want to find γ such that $\pi_A = \pi_B$. Write:

$$\begin{aligned}
&\alpha(V-r) = \alpha(v-r) + \frac{1-\alpha}{1-\gamma}(v-r) \\
\Leftrightarrow \quad &(1-\gamma)\alpha(V-v) = (1-\alpha)(v-r) \\
\Leftrightarrow \quad &\gamma^* = 1 - \frac{1-\alpha}{\alpha}\frac{v-r}{V-v}.
\end{aligned}$$

It is actually an equilibrium: (i) At isoprofit, consumers of type b actually search. We know $\gamma^* = 1 - \frac{1-\alpha}{\alpha}\frac{v-r}{V-v}$. I want $V - v > \frac{r}{1-\gamma}\frac{1-\delta}{\delta}$ for buyers of type b to wait. Replacing γ by γ^* yields $\frac{1-\alpha}{\alpha} > \frac{r}{v-r}\frac{1-\delta}{\delta}$. This is always true when $\delta \to 1$. The condition on δ can be conveniently rewritten as $\delta > \frac{\alpha r}{(1-\alpha)(v-r)+\alpha r}$.

(ii) In equilibrium, it is not a best response to sell category A at price $p'_A \leq v-r$. This yields at most profit $\pi_A = v - r$ which is lowering the profit by condition 1.

(iii) In equilibrium, no one wants to produce category B at price $v - r < p'_B \leq V - r$. Increasing the price make consumers of type a lose, and yields at most profit $\pi'_B = (1-\alpha)(V-r)$ which is lower than π_B as I have assumed $\alpha \geq \frac{1}{2}$.

(iv) In equilibrium, it is not a best response to sell category A at price $v - r < p_A <$

$V - r$. Demand is at most α and the firm therefore makes lower profits.

(v) It is not a best response to change of category. There is isoprofit at equilibrium and, for any $\gamma > \gamma^*$ the best response of any firm is to supply category B (as π_B is an increasing function of γ). ■

d. AS$_{maj}$is a Nash Equilibrium iff $(1-\alpha)(V-r) > v-r$**:**

Proof. (i) Selling category A at price $p' > V - r$ yields lower profit as, by definition buyers of type a reject the offer and search.

(ii) Selling category A at price $p' = V - r$ is not a best response as long as buyers of type a reject the offer and search.

(iii) Selling category A at price $v - r < p' < V - r$ is not a best response, as by Lemma 5 it does not increase the demand, but, by lowering the price it lowers the profit.

(iv) Selling category A at price $p' < v - r$ is not a best response as by Lemma 5 it does not increase the demand, but, by lowering the price it lowers the profit.

(v) Selling category B at price $p' < v - r$ is not a best response as by Lemma 5 no one specifically search for the firm and therefore profit is at most $v - r$, which is lower than $(1-\alpha)(V-r)$.

(vi) Selling category B at price $v - r \leq p' < V - r$ is not a best response as by Lemma 5.

it does not increase the demand from buyers of type b and as long as buyers of type a reject the offer and search.

(vii) Selling category B at price $p' > V - r$ yields lower profit as, by definition buyers of type a reject the offer and search.

(viii) Using the same reasoning as for $AS_{\min}$, (ii) and (iv) are true when $\delta > \frac{(1-\alpha)r}{\alpha(v-r)+(1-\alpha)r}$. ■

Now, to complete the proof, it only remains to show that no other equilibrium may exist.

e. No other equilibria exist under condition 1

Proof. I want to show that the previous equilibria are the only existing ones when condition 1 is true. Therefore, I still have to get rid of the following alternatives.

(1) All sellers sell category B at price $p_B = V - r$. Selling category A at price $p_A = V - r$ is a profitable deviation as it yields profit $\alpha(V - r) > (1 - \alpha)(V - r)$, by $\alpha > \frac{1}{2}$.

(2) All sellers sell category B at price $p_B = v - r$. Selling category A at price $p_A = V - r$ is a profitable deviation as it yields profit $\alpha(V - r) > v - r$, by condition 1.

(4) A fraction γ of sellers sell category A at price $p_A = v - r$ while a fraction $(1 - \gamma)$ sells category B at price $p_B = v - r$. (i) If buyers don't search for the seller of their category then, by setting $p_A = V - r$ a seller of category A does not lose buyers of type a and therefore increases profit (ii) If buyers do search then, a seller only serves buyers of its category, and there must exist a price $p' > v - r$ such that buyers still accept the offer. Thus setting p' is a profitable deviation.

(5) A fraction γ of sellers sell category A at price $p_A = V - r$ while a fraction $(1 - \gamma)$ sells category B at price $p_B = V - r$. As $\alpha \geq \frac{1}{2}$ one can never have the same profit when $\gamma \neq 1$.

(6) Given Lemma 1, I have exhausted all the potential Nash Equilibria. ■

1.6.4 Proof of Proposition 2

Proof. AS$_{\text{min}}$ is coalition-proof. By Lemma 1, any Nash-Equilibrium implies either $p = v - r$ or $p = V - r$. Selling category B at $p = V - r$ is not a profitable deviation, as demand would be zero. Selling category A at $p = v - r$ can increase joint profit of a coalition of sellers, but is not *self-enforcing*. Indeed, as the demand for such firms only comes from buyers of type A, each seller has an incentive to slightly increase the price - the participation constraint of buyers of type b is non-binding.

TM$_H$ is not coalition-proof. There exist a coalition of size $(1 - \gamma') < (1 - \gamma^*)$ that would increase her profit by selling a good of category B at price $p = v - r$. Such a deviation is *self-enforcing*, as this price is the best response of any member of the coalition given that all the other members play the same strategy. Hence, Tyranny of the majority at high price is not a coalition-proof Nash Equilibrium.

TM_L and AS_{maj} are not coalition-proof. As the profit of each firm is higher in $\mathrm{AS}_{\min}$, and as $\mathrm{AS}_{\min}$ is coalition-proof, a coalition of mass 1 always has an incentive to choose the equilibrium $\mathrm{AS}_{\min}$, and this strategy is *self-enforcing*. ■

1.6.5 Proof of Proposition 3

There is a Nash Equilibrium where all sellers sell the good desired by the majority at the high price:

Proof. (i) Buyers of type a buy without search (surplus $V - v + r \geqslant r$).

(ii) Buyers of type b buy without search (surplus r).

(iii) It is not a BR to sell category A at price $v - r < p'_A \leq V - r$. This decreases the profit to at most $\pi'_A = \alpha(V - r) < v - r$

(iv) It is not a BR for a firm to sell category B at price $p_B \leq v - r$. This yields at most profit $\pi_B = (1 - \alpha)(v - r)$. Why? Because buyers of type a wait[15] to match a buyer of their type, while buyers of type b do not search for the deviating firm.

(v) It is not a BR for a firm to sell category B at price $v - r < p'_B \leq V - r$. This yields at most profit $\pi'_B = (1 - \alpha)(V - r)$. This is smaller because condition 1 is not fulfilled and $\alpha \geq \frac{1}{2}$.

■

There is a Nash Equilibrium where all sellers sell the good desired by the minority at high price:

Proof. (i) Buyers of type b buy without search (surplus $V - v + r \geqslant r$).

(ii) Buyers of type a buy without search (surplus r).

(iii) It is not a BR for a firm to sell category A at price $v - r < p'_A < V - r$. This

[15] If condition 2 is false, sellers are indifferent.

decreases the profit to at most $\pi_A = \alpha(V-r)$ [profit loss by the fact that condition 1 is not fulfilled].

(iv) It is not a BR for a firm to sell category A at price $p'_A \leq v-r$. This yields profit at most $\pi'_A = \alpha(v-r)$. Why ? Because consumers of type 2 wait[16] to match a buyer of their type, while buyers of type a do not search for the deviating firm.

(v) It is not a BR for a firm to sell category B at price $v-r < p'_B \leq V-r$. This yields profit $\pi'_B = (1-\alpha)(V-r)$. This is smaller because condition 1 is not fulfilled and $\alpha \geq \frac{1}{2}$.

■

Consider the following condition:

Condition 2 $\delta < \frac{2r}{V-v+2r}$

If condition 2 is true, there exist threshold values (γ^-, γ^+) **such that, for any** $\gamma^- < \gamma < \gamma^+$**, buyers accept any offer without search and sellers are indifferent between both types. Any** $\gamma \in (\gamma^-, \gamma^+)$**, with** $\gamma^- = \frac{V-v+r}{V-v} - \frac{r}{\delta(V-v)}$ **and** $\gamma^+ = 1-\gamma^-$**, with price** $p_B = p_A = v-r$ **is a Nash Equilibrium.**

Proof. (i) Matched buyers buy without search (surplus $V-v+r \geqslant r$).

(ii) Mismatched buyers buy without search as long as $\gamma \in (\gamma^-, \gamma^+)$.

(iii) It is not a BR for a firm to sell category A at price $v-r < p'{}_A \leq V-r$. This decreases the profit to at most $\pi'_A = \alpha(V-r)$ [profit loss by the fact that condition 1 is not fulfilled].

(iv) It is not a BR for a firm to sell category B at price $v-r < p'_B \leq V-r$. This yields at most profit $\pi'_B = (1-\alpha)(V-r)$. This is smaller because condition 1 is not fulfilled and $\alpha \geq \frac{1}{2}$.

(iv) Firms are indifferent between producing category A or category B at price $p_A = p_B = v-r$, as, for any value of γ,we have $\pi_A = \pi_B = v-r$, by (i) and (ii), which satisfy isoprofit.

■

[16]If condition 2 is false, sellers are indifferent.

Proof that no other equilibria exist when condition 1 is not fulfilled

Proof. I want to show that the previous equilibria are the only existing ones when condition 1 is not fulfilled. Therefore, I still have to get rid of the following alternatives:

(1) All sellers sell category B at price $p_B = V - r$. Selling category A at price $p_A = V - r$ always yields higher profit since $\alpha \geq \frac{1}{2}$.

(2) All sellers sell category A at price $p_A = V - r$. As condition 1 is not fulfilled, reducing price to $p'_A = v - r$ increases profit.

(4) A fraction γ of sellers sell category A at price $p_A = V - r$ while a fraction $(1-\gamma)$ sells category B at price $p_B = v - r$. Sellers selling category A do not make buyers of type a search (yields surplus 'r'). Then, profit is at most $\alpha(V - r) < v - r$ since condition 1 is not fulfilled.

(5) A fraction γ of sellers sell category A at price $p_A = V - r$ while a fraction $(1-\gamma)$ sells category B at price $p_B = V - r$. Such a price is too high to attract. Hence, as $\alpha \geq \frac{1}{2}$ one can never have the same expected profit for both categories.

(6) A fraction γ of sellers produces category A at price $p_A = v - r$ and a fraction of sellers $1-\gamma$ produce category B at price $p_B = V - r$. This means sellers of category B make profit $\pi_B = (1-\alpha)(V - r)$. Then, if isoprofit is satisfied, it is a BR for any seller to produce category A at price $p_A = V - r$ and get profit $\pi_A = \alpha(V - r) > \pi_B$.

(7) Given Lemma 1, I have exhausted all the potential Nash Equilibria. ■

1.6.6 Proof of Proposition 4

First, I compute the total surplus in the two potential equilibria, second I compare the two surpluses to find the condition for TM_L to be preferred to $AS_{\min}$:

	Total surplus in TM_L:
Sellers:	$v-r$
Buyers of type a :	$V-v+r$
Buyers of type b :	r
Total:	$(v-r)+\alpha(V-v+r)+(1-\alpha)r=\alpha V+(1-\alpha)v$

	Total surplus in $AS_{\min}$
Sellers:	$\alpha(V-r)$
Buyers of type a :	r
Buyers of type b :	$V-v+r$ (*)
Total:	$\alpha(V-r)+\alpha r+(1-\alpha)(V-v+r)=V-(1-\alpha)(v-r)$

(*) (as $(1-\gamma)(V-v+r)\sum_{i=0}^{\infty}\gamma^i\delta^i=\frac{(1-\gamma)(V-v+r)}{1-\gamma\delta}$, and $\delta\to 1$)

Hence, TM_L is preferred to $AS_{\min}$ iff

$$\begin{aligned}\alpha V+(1-\alpha)v &> V-(1-\alpha)(v-r)\\ &\Leftrightarrow V-v<v-r.\end{aligned}$$

Chapter 2

On the economic impact of smoking bans in bars and restaurants

The introduction of smoking ban laws in Europe and the in US over the last decades has generated an important debate on public health and economic concerns. Four stylized facts which I discuss thoroughly below stand out from the empirical literature: first, smoking bans do not reduce the profitability of restaurants, quite the contrary. Second, in most cases, the market alone fails to supply non smoking premises before a ban is enforced. Third, the support for smoking bans typically increases after a smoking ban is enforced. Fourth, the impact on bars is more contrasted, most likely a decrease in employment after the ban. Why does the market fail to adjust before the ban ? Why should the effect be different in restaurants than in bars ? Do people actually change of preferences due to the existence of a ban? The latter questions are difficult to answer rationally based on the classical economic theory.[1]

The decision to allow smoking or not can be related to the type of horizontal differentiation I study in the first chapter. Bars and restaurants have to take a decision (allow smoking or not) that affects all consumers in the room. However, it

[1]For instance, Adda et al. (2009) show that, in a Hotelling environment, the market provides only smoking premises. But it does not explain how this can be the case with a majority of nonsmokers. And why the support for bans would be influenced by the presence of the bans.

differs in the sense that (i) people typically do not go alone to a bar or a restaurant. There is an externality within groups when they do not share the same preferences regarding the smoking environment. I assume utility to be non transferable within groups ; (ii) bars and restaurants are imperfect substitutes. Substitution effects can lead to a misinterpretation of the changes in the profit of firms.

In this paper, I extend the model presented in the first chapter to account for the existence of groups of consumers, with heterogeneous preferences and Non Transferable Utility (NTU) within the groups. There are three types of groups: groups of smokers, nonsmokers, and mixed groups (groups with both smokers and nonsmokers). Due to NTU, a seller wanting to attract mixed groups must meet both the participation constraints of smokers and nonsmokers. These groups are therefore not a "third" type of buyers, but a combination of the two first types, with a participation constraint more complex to meet. I show that, in the presence of product diversity, only homogenous groups of nonsmokers attend nonsmoking premises while all other groups attend smoking premises. A mixed group initially matched with a nonsmoking restaurant leaves the market without buying: the nonsmokers of the group do not want to pay the search cost to find a smoking place. And the nonsmoking premises do not meet the participation constraint of smokers. In this case, a policy of licenses (defining a small share of smoking restaurants) decreases total demand, as it increases the number of those ‘conflictual’ cases. A policy of smoking ban increases total demand, but decreases the profit of firms (as it decreases the prices). Without mixed groups, a policy of licenses is always maximizing total welfare. In the presence of mixed group, it may be dominated by a policy of complete smoking bans. One of the two policies is always maximizing the total welfare. None of the policies is Pareto Improving.

To understand better the empirical puzzles I also add the possibility of substitution between bars and restaurants. Following the four stylized facts mentioned above, I show that: (i) substitution effect explains that, if the bans are not enforced simultaneously in restaurants and bars, restaurants may increase their profit due to the ban ; (ii) in presence of a majority of nonsmokers, the market can provide a minority of nonsmoking premises ; (iii) before a ban, nonsmoking premises are a niche market, only attended by nonsmokers. After a ban, those premises have an incentive to accommodate all types of consumers, by lowering prices. This change in the price structure explains the change in the support for nonsmoking environments by consumers ; (iv) bars may be seriously hurt by a ban affecting them if it has been previously enforced in restaurants. The effect of a ban in restaurants alone is to increase the share of smoking bars, and therefore increases their specialization in the minority type.

In the presence of premises diversity in the market, there are two sources of inefficiency that go in opposite directions. The first one, already documented in the first chapter, is that the market may provide *too many premises corresponding to the taste of the minority*, implying a large share of the majority buyers being mismatched. The second one, related to the group effect, is that the market may not provide *enough premises corresponding to the taste of the minority*, implying a large share of mixed groups leaving the market without buying.

In general, smoking ban decisions are enforced for reasons of health policy.[2] The objective of this paper is not to discuss this issue, but to understand the market mechanism. The smoking ban example is convenient to study as it is a largely debated policy issue that has inspired a vast empirical literature. Moreover, the question of the efficiency of a decentralized market to provide product diversity when consumers buy in groups is much more general. Examples include most social or leisure activity when decisions have to be taken cooperatively, such as (movie) theaters, holidays, ...

[2]For instance Allwright (2004, p.811) argues that: ‘Given the seriousness of the health consequences of exposure to passive smoke, the economic argument is hardly relevant. For example, would anyone seriously propose that because removing asbestos from buildings costs money and may put marginal businesses out of business, workers should continue to work in dangerously contaminated buildings?’

In the next section, I present the existing analysis of smoking ban policies, and show how empirical results contradict those predictions. Section 2 presents the setup of the model. The decentralized equilibrium is characterized in section 3. Different scenarios of smoking bans are studied in section 4. I conclude in section 5.

2.1 Three theories and why they fail to match the facts

A vast literature has focused on the questions of smoking bans. Since the earlier stages, it was mostly aimed at estimating the potential impact on the profit of firms, and used by advocates and opponents to the laws. Besides, some authors have tried to add a theoretical structure to these results. Before setting up the model, I briefly summarize the three main explanations to the empirical puzzles to be found in the literature, and why I consider they may be insufficient to explain those results.

2.1.1 The market reflects the preferences of buyers

A simple economic statement on the subject of smoking bans may be the following: if individuals and restaurant owners are rational utility maximizers, there is no rationale for introducing a law to decides of what the best choice is for individuals. Second-hand smoking is a well-known phenomenon, and consumers who go to a restaurant that allows smoking are informed of the risk they are taking. Even employees that work in a smoking environment are in fact making an optimal choice: they know the risk they are taking and they are rewarded for this. This point is raised by Boyes and Marlow (1996), in their paper on the public demand for smoking bans. They conclude that Coase's theorem can be applied to property rights on the air quality in restaurants. The Coase Theorem predicts that private markets internalize negative externalities when there are zero transaction costs and property rights are clearly assigned to all resources. The authors argue that, as the air space within privately-owned premises is also private, owners of these premises are owners of the air space and are free to allocate it between two distinct demanders: smokers and nonsmokers. Property rights are thus clearly assigned. The authors assume that negotiation between smokers and nonsmokers occurs at no cost via the owners of the private businesses. Hence, smoking bans shift ownership of the airspace away from owners of firms to nonsmokers. By voting a law, the government allocates air space at zero price to nonsmokers. Smokers transfer income to nonsmokers without being compensated. Therefore, a smoking ban law can be seen as a way for the majority to get themselves an income transfer from the minority. As the *laissez faire* is expected to maximize the profit, the ban should of course decrease it. Yet, this is not what can be observed.

Among others, positive or non significant impact has been found by: Huang and McCusker (2002) (in El Paso, Texas), Bartosh and Pope (2002) (in Massachusetts), Glantz and Smith (1994) (15 communities in California and Colorado), Sciacca and Ratliff (1998) (in Flagstaff, Arizona), Huang et al. (1995) (in West Lake Hills, Texas). Dunham and Marlow (2000) and Dunham and Marlow (2003) put those results into perspective, mostly because they do not take into account distributional effects.[3] This is mostly because it mixes smoking bans and other regulations. Those results are consistent with the model I develop, as they allow emphasizing the dif-

[3] Moreover, the authors analyze a survey carried out on 600 owners of restaurants randomly chosen in the US. They conclude that an important percentage of owners (39%) predicted a decrease in their revenues if a ”smoking law” was voted. They argue that the predictions of the owners of restaurants not affected by a smoking regulation did not differ from those of the owners of restaurants affected by the law. A first problem is that they consider as *Law States* not only those that voted a 100% smoking ban, but also those who voted laws of regulation (for instance, laws asking the restaurants to have at least some percentage of their seating in nonsmoking zones) or partial smoking bans. Indeed, while looking more carefully at the 32 States defined as *Law States*, only 3 of them had a 100% smoking ban on bars and restaurants at the time the study was published. In fact, the only necessary condition to be considered as a *Law State* is *to have voted laws allowing or requiring nonsmoking sections in restaurants*. Moreover, the question that has been asked refers to expected variation of profit if a smoking law is voted, without defining precisely what kind of ban or restriction is considered, allowing the owners to implicitly interpret this regarding the kind of law they face or fear to face.

ferences between total and partial smoking bans. Another approach has been used by Alamar and Glantz (2004). They tested the impact of a 100% smoking law in two US States (Utah and California). They conclude that there was a slightly positive effect on the value of restaurants. This does not necessarily means that sales increased, because costs are lower when restaurants are nonsmoking. Adams and Cotti (2007), in a difference in differences analysis for the entire USA, found no significant impact on employment in restaurants.

Looking again at the survey presented by Dunham and Marlow (2000), we learn that only a very small number of restaurants ban smoking in the US (100% of their seating dedicated to nonsmoking consumers) in States where no smoking ban law has been voted. In the UK, Adda et al. (2009) find that more than 95% of the pubs allowed smoking before the ban. In France, the website of the city of Paris (paris.fr) counted slightly more than 100 restaurants or bars offering a hermetic nonsmoking environment (the city counted 12 699 restaurants and bars in 2005) the day before the ban. The existing websites trying to reference nonsmoking restaurants in Brussels (rookvrij.be, thinkabout.be) counted slightly more than 20 nonsmoking restaurants or bars in the city (among the more than 3000 referenced by the Belgian institute of statistics) before the smoking ban. The supply of nonsmoking restaurant seats was thus scarce, even if those countries all appear to have a majority of nonsmokers in the adult population.[4]

2.1.2 Restaurants and bars have different cultures

Two papers undertake separate analysis for bars and restaurants: Phelps (2006) and Adams and Cotti (2007). Both find no significant effect of a ban in restaurants, and a negative effect of a ban in bars. The variable they study is employment in

[4]Among recent surveys for those countries: 20,9% of US adults are cigarette smokers (NHIS, 2005), 22% in the UK (NHS, 2007), 27% in Belgium (CRIOC, 2004) and 27% in France (INSEE, 2001)

the considered sector. (Adams and Cotti , 2007, p.5) justify to undertake a separate analysis of bars and restaurants by the idea that 'Smoking seems to be part of the bar culture and not necessarily part of the restaurant culture'. Dunham and Marlow (2000) argue that it is more difficult to separate smokers and nonsmokers in bars, and that nonsmokers have a lower preference for nonsmoking environment in bars.

This difference cannot be ignored, but it does not allow to understand why so few restaurants ban smoking without a ban and why the support increases after the ban. Moreover the 'cultural' argument may be the consequence of the bans and not its cause, as bans have been voted for trains, planes, and many other public places where smoking was part of the 'culture'. Indeed, as emphasized by Adams and Cotti (2007), bars and restaurants are differently affected by bans, but the number of bans is also different. Many bans in bars are implemented when a ban is already enforced in restaurants. And many jurisdictions still ban smoking only in restaurants.

2.1.3 Consumers do not know their preferences

Finally, there is potentially a 'behavioral' explanation. The main result of the empirical literature on the perception of bans is that the support for smoking bans in the population, and particularly among smokers, increases after the smoking ban has been voted. This result is validated both in cross section and time series analysis. For instance, Fong et al. (2006) carry out a survey in Ireland showing that 'support for total bans among Irish smokers increased in all venues', Gilpin et al. (2004) discussing the smoking bans in California, conclude that 'a strong, comprehensive tobacco programme such as California's can influence population norms, including those of smokers'. Comparing the answers of smokers in four countries (USA, Canada, UK and Australia), Borland et al. (2006) show that 'support for bans is related to the presence of bans' and that 'smokers adjust and both accept and comply with smoke-free laws'. Similarly, the survey of Gallus et al. (2006) in Italy stated

‘Once smoke-free policy were introduced support for them in the public opinion tended to increase’. This can go together with the general idea that the preferences for ‘sin goods’ can be time inconsistent Gruber and Kószegi (2004)

This behavioral explanation seems unable to explain the whole phenomenon. In the US, many bans have been enacted at the community level. And it does not appear that the surrounding communities experienced a decentralized change towards nonsmoking restaurants. If the experience of nonsmoking places changes the utility functions, one should expect a contagion effect at the borders of countries with a smoking ban. This did not happen. Moreover, Phelps (2006) shows that a ban is more likely to increase the profit of a restaurant if the community is the first one to enact it than if it is surrounded by other nonsmoking communities. This goes exactly the opposite way as *learning* our own preferences.

2.2 The model

2.2.1 Setup

The economy is composed of two groups: customers and sellers.

The sellers are a continuum of restaurants of mass 1 and a continuum of bars of mass 1. The sellers endogenously choose whether to allow smoking or not. Sellers also individually set a price. Production costs are equal to zero, and the smoking policy can be chosen at no cost.[5]

Among the customers, there is a mass 1 of *drinkers*, a mass 1 of *eaters* and a mass 2λ of *good timers*. *Drinkers* look for a bar, *eaters* look for a restaurant, and *good timers* have only decided to go out, either to a bar or to a restaurant. Hence, λ represents the level of substitutability between bars and restaurants. Consumers go out in groups: there is a share α of nonsmokers, μ of mixed groups and $1-\mu-\alpha$

[5] In fact, a restaurant does not set a ‘single price’, but one can interpret this price as a profit margin.

of smokers in the population, with $\alpha > 1-\mu-\alpha$ (more nonsmokers than smokers). I assume those proportions are the same for all types of customers.[6]

When the smoking policy of the premises coincides with the preferences of the customer, the surplus of the matching is V, when it does not, the surplus is $v < V$. The surplus is received by the buyer if she accepts the price set by the seller. The outside option is set to $r \in (0, v)$. A group of customer can buy either 0 or exactly 1 unit of either good. Parameters α, μ, V, v and r are common knowledge. The search costs are modeled by a discount factor δ tending towards 1.[7] The stages of the game are the following:

1. Sellers simultaneously choose their smoking policy and price ;
2. Customers learn the share γ_R and γ_B of nonsmoking restaurants and bars, and the distribution of prices in each category ;
3. Each group of customers is randomly matched with a seller. He observes the price and the chosen category of the seller he is matched with. Each group decides whether to **Accept** the offer, **Leave** the market and receive the outside option r or to **Search** for another seller. The decision rules within the groups are presented in the next subsection. If a group searches, he is randomly matched with another seller, but her payoffs are discounted with a parameter δ tending towards 1. There is no limit for search, but the cumulated discount factor decreases to δ^s after s searches. A group of *drinkers* is matched only with bars, a group of *eaters* is matched only with restaurants, a group of *good timers* first chooses whether to go to a bar or a restaurant, and then stays in the chosen category.

[6]The objective is to isolate the market effect that is not explained by an intrinsic difference of 'culture'. However, it is easy to change this and it does not affect the reasoning.

[7]In the first chapter, I show how increasing the discount factor does not affect the main equilibria.

2.2.2 Decision rules

For a group to accept an offer, there must be an agreement on the chosen bar or restaurant. An homogenous group (smokers or nonsmokers) accepts an offer if and only if the participation constraint (utility of accepting is higher than the outside option) and the incentive compatibility (the expected utility of search is lower than the utility of accepting) of all its members are met. As they share the same utility function, the reasoning is the same as for an individual.

Within mixed groups, a decision rule must be defined. One can consider two polar cases:

An easygoing group accepts an offer if all the participation constraints of its members are met and if searching is not - in expectation and given the decision rule - Pareto Improving.

A confrontational group accepts an offer if both the participation constraints and the incentive compatibility of all its members are met.

As will be made clear below, these decision rules lead, in practice, to equivalent outcomes.

Regarding mixed groups of *good timers*, I assume that the groups choose the category between restaurants and bars that maximizes the probability of accepting an offer. If both probabilities are equal, the tie-breaking rule is to choose each category with probability $\frac{1}{2}$.

2.3 The decentralized equilibrium

I first derive two lemmas, similar in their intuition to Lemma 1 and 2 of the first chapter. In this decentralized market, the potential equilibria for bars and restaurants are similar, as the problem is symmetric. This does not imply that the realized equilibria are equivalent. I assume there is a majority of nonsmokers. The opposite can also be true and leads to symmetric conclusions. To understand the first lemma, one has to remember that, as the utility is nonstranferable within group, it is necessary to meet all participation constraints of the members of the groups for the participation constraint of the group to be met.

Lemma 6 *There are only two possible prices in a Nash Equilibrium: $p = V - r$ and $p = v - r$*

Sketch of the Proof. The formal proof is similar to the proof of lemma 1 of chapter 1. Due to the existence of a continuum of sellers and the presence of search costs, the equilibrium prices always converge towards the participation constraints of either type of consumers. There are only two participation constraints: for a good match ($p = V - r$) or for a mismatch ($p = v - r$). As utility is nontransferable, the participation constraint of a mixed group is also the participation constraint of a mismatch, regardless of the decision rule. Hence, it is a best response for any seller to slightly increase the price unless it is exactly $v - r$ or $V - r$. ■

The incentive for a seller to set the low price is higher in the presence of mixed groups. Specializing in the majority type - nonsmokers - can imply attracting a minority of groups, as mixed groups only accept offers when the price meets both participation constraints.

Lemma 7 *If $\alpha(V - r) < v - r$, restaurants and bars play a pure strategy. Hence, all restaurants either allow or ban smoking, and all bars either allow or ban smoking*

Proof. The formal proof is similar to the proof of lemma 2 of chapter 1, except for the impact of mixed groups. If smoking is allowed on all premises, the first seller to ban smoking can set a high price, but specializes only in the nonsmoking groups. Indeed, there exists a continuum of sellers and positive search costs, hence a single seller cannot make a customer search for him. Assume all restaurants allow smoking. The first one to ban smoking can either keep the same price, and reduce its demand by losing all smoking groups, or increase the price until it is exactly $p = V - r$, but loose both the smoking and the mixed groups. Again, this is independent of the decision rule, as the high price never allows meeting the participation constraint of both types of consumers. Hence, $\gamma_R \in \{0,1\}$ and $\gamma_B \in \{0,1\}$ ■

This implies that the social norm is not necessarily the same in bars and restaurants. In the two decision rules proposed for mixed groups, the probability to go either to a bar or a restaurant would be the same ($P = 1$), and all the buyers accept an offer. Hence, we can observe different "cultures", not based on different preferences, but on different beliefs.[8] This case is also close to the idea that the failure of the market to provide a nonsmoking environment can be the result of a coordination failure, as put forward by (Adams and Cotti , 2007, p.6). However, in the equilibrium I present here, bars and restaurants never take a wrong decision: they are indifferent between both pure strategy equilibria. The difference in welfare only comes from the share of mismatched groups. Hence, if the beliefs in the population are such that bars and restaurants are smoking premises, it is a best response for all owners to keep allowing smoking unless they can increase their profit by increasing their price and losing any demand from smoking and mixed groups.

The expected profit π for all sellers is identical and is:

$$\pi = (1+\lambda)(v-r)$$

Lemma 8 *If $\alpha(V-r) > v-r$, the only Coalition Proof Nash Equilibrium, Asymmetric Supply (some surplus left to the minority) - $AS_{\min}$ - is such that a share $\gamma^* = \gamma_R^* = \gamma_B^* \in (0,1)$ of bars and restaurants ban smoking while the reminder allow it. The nonsmoking premises set high price $p = V - r$, while the smoking premises set low price $p = v - r$.*

[8] However, an equilibrium with different 'cultures' is not Coalition Proof unless the mass of "good timers" is exactly zero.

Sketch of the Proof. A formal proof of the uniqueness of this equilibrium is given in appendix 2.6.1. The concept of Coalition Proof Nash Equilibrium ensures that a *self-enforcing* deviation of a share of sellers is not enough to make the equilibrium disappear. As the switching cost is zero, sellers must be indifferent in expectation between both policies. A seller allowing smoking attracts: (i) nonsmoking groups matched with him (ii) mixed groups matched with him (iii) smoking groups matched with him (iv) smoking groups initially matched with nonsmoking premises and searching for a good match. Hence, the total profit of a smoking bar or restaurant is:

$$\pi_s = (1+\lambda)[\alpha+\mu+\frac{1-\alpha-\mu}{1-\gamma}](v-r) \tag{2.1}$$

Sellers that ban smoking only attract nonsmoking groups, as they do not meet any other participation constraint and as no group has an incentive to search for them. Hence, their profit is:

$$\pi_{ns} = (1+\lambda)\alpha(V-r) \tag{2.2}$$

The share of nonsmoking premises is obtained by equating those two profit levels, since the profit on nonsmoking premises is independent of γ and the profit of smoking premises is decreasing in γ. Solving $\pi_s = \pi_{ns}$ yields:

$$\begin{aligned} \gamma^* &= 1-\frac{(1-\alpha-\mu)(v-r)}{\alpha(V-r)-(\alpha+\mu)(v-r)} \\ \gamma^* &= \frac{\alpha(V-r)-(v-r)}{\alpha(V-v)-\mu(v-r)} \end{aligned} \tag{2.3}$$

■

Nonsmoking premises are attended only by nonsmokers, while smoking premises attract all groups of consumers. Groups of smokers search until they find smoking premises. Mixed groups matched with nonsmoking premises leave the market without buying. Mixed groups matched with smoking premises accept the offer, while nonsmoking groups accept any offer without search. The share of nonsmoking premises is increasing in the share of nonsmoking groups, and decreasing in the share of all other groups. Specifically, close to the limit of inequality $\alpha(V-r) < v-r$, the share of nonsmoking premises γ is expected to be very low. Mixed groups initially matched with nonsmoking premises never accept the offer, but do not search either: they leave the market without buying. This decision is, again, independent of the decision rule within the group. As both smoking and nonsmoking premises offer the same utility for nonsmokers, the decision is never confrontational.

Define the surplus of a mixed group by $\tilde{v}$, with $v < \tilde{v} < V$.[9] The total surplus generated by each matching is the following:

	Average surplus of the matching	Average surplus for the customers
Smokers	V	$V-(v-r)$
Nonsmokers	$\gamma V + (1-\gamma)v$	r
Mixed	$\gamma r + (1-\gamma)\tilde{v}$	$\gamma r + (1-\gamma)(\tilde{v}-(v-r))$

Hence, the total gains of trade (as compared to everyone taking the outside option) are

$$W = \alpha(\gamma V + (1-\gamma)v) + \mu(\gamma r + (1-\gamma)\tilde{v}) + (1-\alpha-\mu)V \tag{2.4}$$

Lemma 9 *Even when it is an equilibrium, providing diversity is always dominated in terms of total welfare. Depending on the share of mixed groups, the social optimum in the presence of a positive mass of smoking premises is either only smoking premises, or an arbitrarily small share of nonsmoking premises.*

Proof. The share of nonsmoking premises has an ambiguous effect in terms of total welfare. Indeed, on the one hand it increases the share of nonsmoking groups with

[9] The size of the group does not influence our results. If one consider groups of two people, mixed groups are composed of one smoker and one nonsmoker, and therefore $\tilde{v} = \frac{V+v}{2}$.

a good match, but it also increases the share of mixed groups leaving the market without buying anything. Hence, the first best share of nonsmoking premises, when $\gamma < 1$,[10] can be either γ tending towards 1 or $\gamma = 0$, depending on whether the gain for nonsmokers is higher than the loss for mixed groups, i.e. the solution to the maximization of equation (2.4) is to choose γ tending towards 1 iff

$$\alpha(V - v) > \mu(\tilde{v} - r) \tag{2.5}$$

■

This implies that if a social planner, instead of a smoking ban, can define an optimal share of licenses for premises allowing smoking, this would be either exactly 1[11] or a number arbitrarily small but not exactly zero. However, as discussed in the next section, the outcome where all the premises allow smoking is always dominated by a smoking ban in total welfare. Hence, when inequality (2.5) does not hold, the 'best' outcome allowing smoking bans is to allow smoking everywhere, and the total gains from trade are lower than in the presence of a smoking ban.

Proposition 5 *In a decentralized equilibrium, unless all restaurants and bars ban smoking, nonsmoking premises are attended only by nonsmokers, while smokers and mixed groups attend smoking premises.*

Proof. When all premises ban smoking, from lemma 2, the price is low enough, and all groups of consumers accept the offer of nonsmoking bars and restaurants. When all premises allow smoking, all groups go to smoking restaurants. In the mixed equilibrium defined in lemma 3, nonsmokers go to nonsmoking premises, mixed groups either go to smoking premises or leave the market, and smokers only go to smoking premises. ■

[10] This condition ensures that smokers continue to search.

[11] Implying a negative price for the licenses, since the share of smoking premises is higher than the equilibrium one.

Hence, a majority of nonsmokers is absolutely compatible with a minority of nonsmoking premises. First, because if the beliefs are such that smoking is allowed everywhere, it can be a best response for all owners to allow smoking. Second, because even if it is not the case, the nonsmoking premises will be a niche market, while a diversified population including smokers and nonsmokers, and even some groups composed only by nonsmokers, go to smoking premises.

To repeat, this is totally unrelated to matters of quality, ‘culture’ or willingness to go out. Nonsmoking premises appear to be preferred only by very few people, not because they provide lower utility, but because of the price structure in equilibrium.

2.4 On the economic impact of smoking bans in bars and restaurant: three scenarii

2.4.1 A smoking ban everywhere

A smoking ban everywhere gives an incentive to nonsmoking premises to attract all types of consumers. It is easy to show[12] that the only Coalition Proof Nash Equilibrium is $\gamma = 1$ and $p = v - r.$ Hence, there is a positive effect of the ban on the number of good matches for nonsmokers and a negative effect on the number of good matches for smokers.

On top of that there is the mixed group effect. If the decentralized market outcome is to provide both smoking and nonsmoking premises, the mixed groups matched with nonsmoking premises leave the market without buying. In presence of the smoking ban, they accept any offer since it meets all the participation constraints.

[12] The intuition is similar to the proof of lemma 3 in the first chapter.

Lemma 10 *A smoking ban on both bars and restaurants decreases aggregate welfare iff (i) the market provides both smoking and nonsmoking premises in equilibrium (ii) the welfare loss from mismatched smokers outweighs the gains from good matches of nonsmokers and mixed groups not leaving the market.*

Proof. If the market only provides nonsmoking premises, it is obvious that the ban has no impact. If the market only provides smoking premises, the ban is welfare improving as all consumers are still matched, but a largest share benefits from a good match. If the market provides diversity, a ban is welfare improving iff the gains derived from the matches of nonsmokers and the mixed groups is higher then the losses from smokers

$$\alpha(\gamma V+(1-\gamma)v)+\mu(\gamma r+(1-\gamma)\tilde{v})+(1-\alpha-\mu)V<\alpha V+\mu\tilde{v}+(1-\alpha-\mu)v$$

This is equivalent to:

$$\alpha(1-\gamma)(V-v)+\mu\gamma(\tilde{v}-r)>(1-\alpha-\mu)(V-v) \tag{2.6}$$

A sufficient condition for inequality (2.6) to hold is:

$$\mu(\tilde{v}-r)>(1-\alpha-\mu)(V-v) \tag{2.7}$$

Indeed, we already know that when inequality (2.5) does not hold, the socially optimal share of smoking premises is 1 when it is not exactly zero. The total welfare in presence of a smoking ban is always higher than when all the premises allow smoking (since the share of nonsmokers is higher than the share of smokers). If inequality (2.5) holds, the smoking ban is beneficial if the total welfare with the ban is higher than with almost zero smoking premises. This simplifies to inequality (2.7): the smoking ban allows mixed groups to get a good match at the cost of the smokers having a mismatch. ■

Inequality (2.7) is a sufficient but not necessary condition for a smoking ban to be welfare improving. Indeed, in equation (2.6), the sufficient and necessary condition, γ is endogenously determined by α, V, v, r and μ. On the one hand, a high equilibrium value of γ implies that many nonsmokers have a good match initially, and that the smoking ban may be detrimental. However, taking into account the mixed groups has exactly the opposite effect. Those groups benefit more from the smoking ban when the market was already providing nonsmoking environment. Indeed, the nonsmoking premises in an equilibrium with product diversity exclude mixed groups from the market. As no one leaves the market without buying in the presence of a smoking ban, this policy is good for mixed groups in the cases where the market was already providing a large share of nonsmoking environments.

Lemma 11 *A smoking ban on both bars and restaurants (weakly) decreases the profit of firms. In presence of product diversity, the negative impact on profit is negatively correlated with the share of nonsmoking premises in the decentralized equilibrium.*

Proof. The profit of the sellers either decreases or remains constant. In an equilibrium with $\gamma \in (0,1)$, it decreases from $\pi = (1+\lambda)\alpha(V-r)$ to $\pi' = (1+\lambda)(v-r)$. Therefore, the smoking ban hurts businesses, but the higher the initial share of nonsmoking premises, the higher the profit losses. Indeed, as $\gamma^* = \frac{\alpha(V-r)-(v-r)}{\alpha(V-v)-\mu(v-r)}$, the numerator decreases when π is close to π', and is exactly 0 when $\pi = \pi'$. If the equilibrium share of nonsmoking premises is close to zero, the profit loss due to the smoking ban is also close to zero. In an equilibrium where $\gamma \in \{0,1\}$, the ban does not affect profit. ■

This result can seem counter intuitive: if the market outcome provides a large share of nonsmoking premises, then a ban has a high negative impact on profits. But this is explained by the fact that the share of nonsmoking premises in equilibrium reflects the extra profit sellers make by specializing only in this type of consumers, and extracting their entire surplus. In presence of a smoking ban, this extra surplus goes back to the customers, thereby decreasing the profit of sellers.

Proposition 6 *The two policies that maximize total welfare are either an arbitrarily small share of licenses for smoking premises, or a total smoking ban. One of the two policies is always dominating the market equilibrium in terms of total surplus, but none is Pareto improving.*

Proof. By lemma 9, 10 and 11. The welfare maximizing policy is either almost no smoking premises, or a full smoking ban. However, a smoking ban decreases both the welfare of smokers and the surplus of sellers, while a policy of licenses decreases the surplus of mixed groups. ■

The existence of mixed groups makes the world more complex than when decision are taken individually. It is impossible to define Pareto improving policies. Therefore, even if it is beneficial in terms of total welfare, any regulation is a matter of political choices, not of market efficiency.

2.4.2 A smoking ban in restaurants only

a. Baseline

Assume the ban does not apply to bars. Restaurants are all nonsmoking and set price $p = v - r$.

Assume first that bars do not change their policy, i.e. stay in $AS_{\min}$. I conclude this subsection by checking under which conditions $AS_{\min}$ is still an equilibrium.

Lemma 12 *If a smoking ban applies to restaurants only, and if some bars still allow smoking then a smoking ban has an ambiguous effect on the profit of restaurants*

Proof. In this case, substitution effects have to be taken into account. Among the *good timers*, consumers who have the choice, all nonsmoking groups choose restaurants. Under any of the decision rules, mixed groups do the same, they buy and receive expected surplus above the reservation utility. All the smokers go to bars. This means that the new profit for restaurants is given by

$$\pi = (1 + (\alpha + \mu)2\lambda)(v - r) > (1 + \lambda)(v - r)$$

■

The economic impact of the smoking ban on the profit of a restaurant can be either positive or negative. On the one hand, restaurants benefit from an increase in the number of consumers. On the other hand, they suffer from a decrease in price and there is no longer any search. Hence, if α, μ and λ are sufficiently large, a smoking ban in restaurants can increase the profit of restaurants, due to substitutability. This can be put in parallel with the 'border' effect found by Phelps (2006). If there is some substitutability between communities, then the first community to ban smoking can enjoy increased profits.

Lemma 13 *If a smoking ban applies to restaurants only, and if some bars still allow smoking then a smoking ban decreases the profit of bars and increases the share of bars that allow smoking*

Proof. The profit of nonsmoking bars, and of all the bars as expected profit is the same, decreases due to the desertion of the *good timers*, of both nonsmoking and mixed groups,

$$\pi = \alpha(V - r) < \alpha(1 + \lambda)(V - r)$$

The demand for smoking bars is now

$$D = \alpha + \mu + (1 + 2\lambda)\left(\frac{1 - \alpha - \mu}{1 - \gamma}\right)$$

Hence, the share of smoking bars is γ^* is higher than before the ban, as it is the solution to:

$$\begin{aligned}\alpha(V-r) &= [\alpha+\mu+(1+2\lambda)(\frac{1-\alpha-\mu}{1-\gamma})](v-r) \\ &\Leftrightarrow \gamma = 1-\frac{(1+2\lambda)(1-\alpha-\mu)(v-r)}{\alpha(V-v)-\mu(v-r)}\end{aligned}$$

Hence, if $AS_{\min}$ is still a Coalition Proof Nash Equilibrium, the smoking ban in restaurants implies that the profit of bars decreases, and that the share of smoking bars increases. ■

However, it is not obvious that $AS_{\min}$ is still a Coalition Proof Nash Equilibrium. The condition to be in $AS_{\min}$ has changed because (i) The profit in equilibrium has decreased (ii) the share of smokers in the demand has increased. A bar that decides individually to set price $p = v - r$ has no influence on the decision of *good timers* from nonsmoking or mixed groups, who decide to go to restaurant anyway.

Condition 3 *Therefore, the condition to have product diversity is:*

$$\alpha(V-r) > (1+2\lambda(1-\alpha-\mu))(v-r)$$

Proposition 7 *If condition 1 is true i.e. if some but not all bars allow smoking after a smoking ban in restaurants, then this smoking ban (i) has an ambiguous impact on the profit of restaurants (ii) has a negative impact on the profit of bars (iii) increases the share of bars that allow smoking*

Proof. Using lemma 12 and 13 and condition 1. ■

Hence, a smoking ban has two possible outcomes:

1. The share of smoking bars increases and their profit decreases

2. There is a norm (either all bars ban smoking or allow smoking) and profit decreases

In the latter case, if bars all allow smoking, the profit is given by

$$\pi = (1+2(1-\alpha-\mu)\lambda)(v-r)$$

which is even smaller than before. If all bars ban smoking, then the profit of restaurants also changes, and both firms get the same profit as in the case where the ban affected both bars and restaurants.

b. A smoking ban in restaurants only, and then a smoking ban in bars

Consider now the short run impact of a smoking ban on bars, when a smoking ban already applies to restaurants. If all consumers are flexible, this brings us back to the equilibrium where a ban applies to both types of firms. But if one assumes that in the short run consumption behaviors presents some rigidity, i.e. that *good timers* cannot change of subsets, the picture is different.

Indeed, one can then expect the following profits respectively for bars and restaurants.

$$\begin{aligned}\pi^R &= (1+2(\alpha+\mu)\lambda)(v-r)\\ \pi^B &= (1+2(1-\alpha-\mu)\lambda)(v-r)\end{aligned}$$

This means that the short run effect of a smoking ban for bars, when they are specialized in smoking consumers, can be quite negative, while it does not affect restaurants. This implies that a policy that aims to protect the bar industry by temporarily exempting them from the smoking ban for some time can have the exact opposite effect. A legislator who decides to take such a decision must be convinced that smoking will be allowed in bars forever, or accept to see a large decrease in profit for bars, at least in the short term.[13]

2.5 Conclusion

This model aims to explain how group and substitution effects can affect the effective and the desirable level of horizontal diversity of premises. Using the example of smoking bans in bars and restaurants I show that, even when product diversity exists in practice, it is not always desirable. Regardless of the market equilibrium, the premises that specialize in what the majority of people prefer are always a niche market. Mixed groups always choose to go to the places preferred by the minority of people. This decision is independent of the potential existence of conflicts within the group. The only necessary condition for consumption of a group to occur is that the participation constraints of all the members of the group must be satisfied.

The impact of smoking bans on aggregate welfare is however ambiguous. Trivially, it always decreases the surplus derived from the matching of smokers. When a large share of premises ban smoking in the decentralized equilibrium, many mixed groups never buy, and a smoking ban can be beneficial for them. However, it decreases the surplus from smokers without increasing the share of nonsmokers with a good match very much. When few premises ban smoking, then a smoking ban increases the surplus from the matching of nonsmokers, but does not increase the share of mixed groups that stay in the market very much.

A complete smoking ban, affecting both bars and restaurants, always (weakly) decreases the profit of sellers. However, this profit loss can be very low when the decentralized market provides only few nonsmoking premises. Moreover, if a smoking ban is enacted in restaurants and not in bars, the profit of the former may increase while the profit of the latter always decreases. As a consequence, the share of smok-

[13] A political economy implication of this is that bars may lobby in favor of such an exemption, because it will be more difficult for a legislator to ban smoking afterwards.

ers in bars increases, which may convince bar owners that a smoking ban in a second period may hurt them even more, specifically if consumers have habits and ‘stick’ to the same establishments for more than one period.

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2.6 Technical Appendix

2.6.1 Proof of Lemma 8

If $\alpha(V-r) > v-r$, the only Coalition Proof Nash Equilibrium is such that a share $\gamma^* = \gamma_R^* = \gamma_B^*$ of bars and restaurants ban smoking while the reminder allow it. The nonsmoking premises set high price $p' = V-r$, while the smoking premises set low price $p = v-r$. Nonsmoking premises are attended only by nonsmokers, while smoking premises attract all groups of consumers. Groups of smokers search until they find smoking premises. Mixed groups matched with nonsmoking premises leave the market without buying. Mixed groups matched with smoking premises accept the offer, while nonsmoking groups accept any offer without search.

- $\gamma_R^* = 0$ or $\gamma_B^* = 0$ and price $p = v - r$ is not a Nash Equilibrium. A seller deciding to ban smoking and setting price $V - r$ increases its profit, as $\alpha(V - r) > v - r$

- $\gamma_R^* = 1$ or $\gamma_B^* = 1$ and price $p = v - r$ can be a Nash Equilibrium. A seller deciding to allow smoking and setting price $V - r$ increases its profit only if $(1 - \alpha - \mu)(V - r) > (v - r)$. This is not necessarily the case, as $\alpha > 1 - \alpha - \mu$. However, it is not Coalition-Proof. Indeed, if a sub-coalition of sellers of strictly positive mass decides to allow smoking and set a price $v - r$, the best response of owners of nonsmoking premises is to increase the price until it is exactly $V - r$. Hence, the deviation is self-enforcing, as there exists coalitions sufficiently large, such that it is a best response for its member not to increase the price (not to lose nonsmokers and mixed groups).

- $\gamma_R^* = 0$ or $\gamma_B^* = 0$ and price $p' = V - r$ is not an equilibrium, as a seller that bans smoking and set price $p' = V - r$ increases its profit $\pi = (1 + \lambda)\alpha(V - r) > (1 + \lambda)(1 - \alpha - \mu)(V - r)$

- $\gamma_R^* = 1$ or $\gamma_B^* = 1$ and price $p' = V - r$ can be a Nash Equilibrium. However, it is not Coalition-Proof. Indeed, if a sub-coalition of sellers of strictly positive mass decides to allow smoking and set a price $v - r$, there exist coalitions sufficiently large, such that it is a best response for its member not to increase the price (not to lose nonsmokers and mixed groups).

$\gamma_R^* \in (0, 1)$ and $\gamma_B^* \in (0, 1)$ are the only remaining candidates. Now, we need to show that any other possibility than $\gamma^* = \gamma_R^* = \gamma_B^* = \frac{\alpha(V-r)-(v-r)}{\alpha(V-v)-\mu(v-r)}$ and $p_{ns} = V - r$, $p_s = v - r$ is not a coalition proof Nash Equilibrium.

- $p_{ns} = p_s = v - r$ is not Coalition-Proof, as it yields a profit lower than $(1 + \lambda)\alpha(V - r)$

- $p_s = V - r$ is not Coalition-Proof, as it yields a profit lower than $(1+\lambda)\alpha(V-r)$
- For any $\gamma^* \neq \frac{\alpha(V-r)-(v-r)}{\alpha(V-v)-\mu(v-r)}$, one of the two category of sellers (smoking or nonsmoking) has a higher profit than the other. Hence, it is not an equilibrium.

Part II

An essay in Urban Transportation

Chapter 3

Modal Choice and Optimal Congestion

(joint with Quentin David)

The cost of congestion is an increasingly important issue in urban areas. For instance, Duranton and Turner (2011) estimate that a typical American household spends 161 person-minutes in a car every day. Goodwin (2004) expected the annual cost of congestion in the UK to reach 30 billion £ in 2010. De Palma and Lindsey (2011) report congestion costs between 0.5 and 1.5 % of GDP in urban areas. Most of the congestion is due to the use of private cars. On the one hand, cars generate both congestion - on other cars and on public transportation - and pollution. On the other hand, cars are necessary for the economy. Unfortunately, screening commuters to reach the optimal share of cars is a complex policy challenge. This is why policies must identify tools that affect peoples' behavior and improve efficiency. In practice, the most widely-used policies addressing traffic issues are taxation,[1] subsidies and traffic separation.[2]

[1] See de Palma and Lindsey (2011) for a survey of the different methods and impacts of congestion tolls.

[2] For instance through the use of exclusive lanes for public transportation. See Cain et al. (2006) and Echeverry et al. (2005) for larger discussion of the case of Bogota's Transmilenio and its application to other countries.

In this paper, we build a theoretical model in which heterogeneous commuters decide simultaneously whether to use a private car or public transportation. Car users generate congestion on all the commuters and users of public transportation enjoy a positive network externality.[3] We do not specifically model pollution costs, as this externality affects all commuters regardless of their modal choice, and therefore does not affect this decision. In practice, considering the impact of pollution would lower the socially optimal share of car users obtained with our model.

First, we show that the market outcome never maximizes aggregate welfare. This is a classic result, in presence of externalities. Second, we explain how ex-ante similar cities might end up with very different modal shifts. This is a problem of coordination when a large share of commuters have similar preferences. In the presence of such multiple equilibria, the one involving the highest share of public transportation always Pareto dominates all the others. Therefore, the market presents two types of inefficiencies. The first one is at the margin: the market provides a too large share of car users in any decentralized equilibrium. The second is more substantial: coordination failures may lead to the presence of inefficient equilibria.

We study two policies: taxation (which, in our discrete choice setup is equivalent to a fare subsidy) and traffic separation. Both can be used to enforce coordination. We show that the main problem of taxation is when the number of car users is small. Shrinking the tax base can be very detrimental for the remaining car users. The main problem of traffic separation is when the increase in congestion costs outweighs the benefits resulting from the decrease of the share of car users. This happens when the share of car users is high in equilibrium. In practice, a social planner considering marginal changes in commuting patterns should focus on taxation, while a social planner interested in more substantial changes should focus on traffic separation.

The question of optimal congestion has been addressed by many scholars from different fields. Among economists, it is rather consensual that pigouvian taxation

[3] We show in Extension 5.1 how introducing discomfort externalities increases the likelihood of ending up with multiple equilibria.

should be the preferred way to deal with congestion problems (Beesley and Kemp, 1987, Calfee and Winston, 1998). The idea is that, given both the structure of cities and the intrinsic preferences that many consumers have for the car, one should focus on the best way to accommodate traffic flows and make car user pay for the marginal external cost they produce (Anas and Small, 1998). The 'games of congestion' have been largely studied in economic theory (Rosenthal, 1973) and many applied papers deal with congestion costs and car taxation. One of the most famous results is due to Vickrey (1963). He argues that pricing should vary at different times of the day as to make commuters pay for the marginal cost of congestion. The question of public transportation has often been of minor interest though some authors (e.g. Mirabel, 1999, Dobruzkes and Fourneau, 2007) addressed the so-called 'crossed modal externalities' (the impact of the congestion generated by one mode of transportation on the other one). The congestion costs have been shown to be convex both in terms of pollution (De Vlieger et al., 2000) and in terms of perceived cost (Wardman, 2001).[4]

Another group of papers focuses on urban planning. It emphasizes the fact that the structure of the city is the main driver of commuting patterns. The main idea to improve the performance of urban transportation is to have a shift towards 'transit-oriented development'. Belzer and Aultier (2002) define such a development as follows: 'mixed-use, walkable, location-efficient development that balances the need for sufficient density to support convenient transit service with the scale of the adjacent community'.[5] Some economists indirectly address this dimension by considering a form of traffic separation (see Berglas et al., 1984, Arnott et al., 1992, de Palma and Lindsey, 2002, de Palma et al. 2008). These papers propose various

[4]Time is valued 50% higher when spent in congestion. Hence, the cost of congestion is convex, as congestion (i) increases travel time, and (ii) increases the marginal cost of travel time. This principle is applied by Santos and Bhakar (2005) to assess the benefits of the congestion toll in London.

[5]Cervero et al. (2002, p.2), emphasize that it does 'involve some combination of intensifying commercial development around stations, inter-mixing land uses, layering in public amenities (e.g., civic spaces, landscaping), and improving the quality of walking and bicycling'. One should also consider the book by Dittmar and Ohland (2003) that summarizes the literature and 'good practices' in transit oriented development.

approaches for road pricing and tolls in the presence of alternative roads, modes of transportation and consumer preferences. Their settings differ from our in various dimensions, but all have in common to find a unique equilibrium and an optimal policy, corresponding to the idea of pricing the marginal externality. In this paper, we show that in the presence of multiple equilibria, internalizing marginal externalities may not be efficient. Then, some physical planning (traffic separation, in our model) must be used as a coordination device.

The multiple equilibria come from the conjunction of congestion, positive externalities from public transportation, and commuters' heterogeneity. A relatively large literature exists on the network effect of the number of transit users on the efficiency of public transportation. In a seminal contribution, Mohring (1972, p.591) explains that 'Transportation differs from the typical commodity price theory texts in that travelers and shippers play a producing, not just a consuming role'. The underlying idea is the existence of a so-called 'dynamic network externality'. If the demand for bus service doubles, a company is expected to double the number or buses serving the route, at the same per capita price. Thus, the waiting time for an individual commuting by bus decreases, which improves the efficiency of public transportation. The combination of network externalities in public transportation and congestion by cars is a feature of several economic models (Tabuchi, 1993, Parry and Small, 2009). To repeat, those models focus on a unique equilibrium. Commuters differ in their preference for the use of a private car (Beirão and Cabral, 2007, Handy et al., 2005, Jensen, 1999, Steg, 2005, Hiscock et al., 2002 and Van Vught et al., 1996). Berhoef and Small (2004) encompass this dimension by considering heterogeneous agents in a model of pricing for car use only. Batarce and Ivaldi (2011) test this feature in a model of modal choice applied to Santiago, Chile.

The existence of similar cities characterized by different modal shifts has already been documented in the late eighties by Pucher (1988). He observed that 'Urban transportation and traveler behavior vary widely, even among countries with similar per capita income, technology and urbanization'. Kenworthy and Laube

(1999) show that the fraction of workers using transit is 6 times higher in wealthy Asian cities compared to the US.[6] They also find that the commuting time is lower (and cheaper) where the use of public transport is higher and that the cost recovery of transit increases with the share of passengers using it. Cities where transit is intensively used appear to need a smaller share of subsidies for operating it.

The paper is organized as follows. In the next section, we provide an illustrative example that contains the basic intuitions behind our main results. Section 3 presents the model, shows that a Nash Equilibrium always exists, gives conditions for the existence of multiple equilibria and discusses their relative efficiency. In Section 4, we derive the optimum of the social planner and study taxation and traffic separation. We extend the model in section 5, addressing the possible existence of capacity constraints and congestion within public transportation, and considering the possibility of underground transit. We conclude in Section 6.

3.1 An illustrative example

Suppose a continuum of commuters simultaneously choose between using a car or public transportation in order to minimize the cost associated to their modal choice. These costs (T^c and T^{pt}) are respectively given by:

$$T^c = t + f$$

and

$$T^{pt} = t + W + \varepsilon,$$

where t denotes the time spent in congestion. In the benchmark case, it is identical for both modal choices and we assume that $t = 0$ if less than 50% of the population uses public transportation and $t = 1$ otherwise. f is the fixed cost associated with

[6] Similarly, Pucher and Renne (2003) computed that, in the US, public transport accounted for less than 2% of urban travel in 2001.

the use of a car, and is set at 1. W is a cost associated with the use of public transportation that is characterized by a network externality such that if less than 50% of the population uses public transportation, $W = 2$, otherwise, $W = 1/2$. Finally, ε represents the value of an intrinsic preference for the use of a car, compared to public transportation.

Consider three groups (A, B and C) of commuters of equal size (each group represents $1/3$ of the population), characterized by different levels of ε. Group A displays strong preferences for the use of a car ($\varepsilon = 2$), group B is indifferent ($\varepsilon = 0$) and group C prefers public transportation ($\varepsilon = -2$).

We define z as the equilibrium share of the population using a car and $\hat{z}$ as the beliefs over the outcome z of the game.

Depending on the expectations, the outcome of the game for an individual belonging to one of the groups is

	if $\hat{z} = \frac{2}{3}$		if $\hat{z} = \frac{1}{3}$	
	T^C	T^{pt}	T^C	T^{pt}
Group A	**2**	5	**1**	2.5
Group B	**2**	3	1	**0.5**
Group C	2	**1**	1	**−1.5**

There exist two Nash Equilibria in Pure Strategy. Group A always uses a car, group C always uses public transportation and group B uses a car if its members believe that the other commuters in the group will do so and public transportation otherwise. With the same exogeneous set of parameters, one may end up either in a world where a majority of people commute either by car or by public transportation. However, the latter equilibrium Pareto dominates the former. We refer to these two equilibria as the "good" and the "bad" equilibrium when $z = 1/3$ and $z = 2/3$, respectively.

We now study the ability of two policies (taxation and traffic separation) to avoid the "bad" equilibrium.

Policy 1: taxation

Consider the simplest taxation scheme: a tax is levied on car users and is thrown away. To ensure that group B uses public transportation in the presence of the tax, its level must be set in such a way that $2+T>3$. As a consequence, we remain with the "good" equilibrium only but, this equilibrium is no longer Pareto improving as the group A is worse off with this tax (the cost of taxation outweighs the decrease in congestion).

Policy 2: physical planning

Consider now a policy of traffic separation. This policy consists in separating the traffic lanes for cars and for public transportation. Under the assumption of a fixed number of traffic lanes, this implies an increase of the congestion for car users and a decrease for users of public transportation. Assume that, in the presence of congestion, the cost of congestion for cars is doubled ($t^c = 2$ if $z > 50\%$, $t^c = 0$ otherwise) while it is divided by two for public transportation ($t^{pt} = 1/2$ if $z > 50\%$, $t^c = 0$ otherwise). The 'bad' equilibrium disappears again and the game becomes

	if $\hat{z} = \frac{2}{3}$		if $\hat{z} = \frac{1}{3}$	
	T^C	T^{pt}	T^C	T^{pt}
Group A	**3**	4.5	**1**	2.5
Group B	3	**2.5**	1	**0.5**
Group C	3	**0.5**	1	$-$**1.5**

This policy is Pareto Improving, since every commuter is better off in the equilibrium with policy than in the "bad" equilibrium without policy.

3.2 The model

3.2.1 Basic assumptions

We consider a closed city with a unit mass of commuters who have to make a discrete choice between using a private car or public transportation. The use of a car generates congestion on the other commuters. The space is finite and it is possible to increase neither the number of roads nor the number of the traffic lanes. The degree of separation of public transportation from the rest of the traffic is given by $\alpha \in [0,1]$.[7] Commuters are heterogeneous as they have different intrinsic preferences for the use of a car (relative to public transportation).[8] The outcome of the game is a share z of car users, and $(1-z)$ of public transportation users.

The utility[9] of a commuter i, traveling in a private car or with public transportation, is respectively given by

$$U_i^c(\alpha,z) = -f_c - t^c(\alpha,z) + \frac{\varepsilon_i}{2} \tag{3.1}$$

and

$$U_i^{pt}(\alpha,z) = -W(z) - t^{pt}(\alpha,z) - \frac{\varepsilon_i}{2}. \tag{3.2}$$

The fixed cost associated with the use of the car is denoted by $f_c > 0$.[10]
The functions $t^c(\alpha,z)$ and $t^{pt}(\alpha,z)$ ($\in IR^+$) represent the congestion faced respectively by cars and public transportation. They are assumed to be equal if there is no traffic separation between cars and public transportation, and equal to zero if there are no users of cars (i.e. $t^c(0,z) = t^{pt}(0,z)$ and $t^c(\alpha,0) = t^{pt}(\alpha,0) = 0$ respectively). Both functions are increasing and convex in z, and a higher degree of traffic separation (higher α) generates more congestion for cars (because there is less space for them) and less congestion for public transportation. This last effect

[7] α is exogenous in this section, but we allow the social planner to choose its level in the next section. Note also that we use a very general definition of α, one that encompasses many possibilities to protect public transportation. The condition being that increasing α decreases congestion for public transportation and increases congestion for car users. This excludes the possibility of building an underground (which is briefly discussed in section 5).

[8] This preference can be negative. One can imagine various alternative ways of modelling heterogeneity: different valuation for time and money, different location within the city, ease of access to the public transportation network, etc. We use the simplest formulation for the tractability of the model.

[9] Utility functions are expressed in monetary terms. All components (fixed costs, congestion, individuals' heterogeneity and waiting time) are expressed in monetary terms.

[10] This is the additional cost compared to the use of public transportation, which is normalized to 0.

is assumed to be amplified by z (that is, separation has an impact only if there is actually a problem of congestion). Hence, we have

$$\begin{array}{rcl} \frac{\partial t^c}{\partial z}(\alpha,z) & > & 0,\ \frac{\partial^2 t^c}{\partial z^2}(\alpha,z)\geqslant 0,\ \frac{\partial t^c}{\partial \alpha}(\alpha,z)>0,\ \frac{\partial^2 t^c}{\partial z\partial \alpha}(\alpha,z)>0 \\ \frac{\partial t^{pt}}{\partial z}(\alpha,z) & > & 0,\ \frac{\partial^2 t^{pt}}{\partial z^2}(\alpha,z)\geqslant 0,\ \frac{\partial t^{pt}}{\partial \alpha}(\alpha,z)<0,\ \frac{\partial^2 t^{pt}}{\partial z\partial \alpha}(\alpha,z)<0. \end{array}$$

The individual parameter, ε_i, is the preference for the use of a car, compared to public transportation. It comes from a cumulative distribution function $\varepsilon_i \sim F(\varepsilon)$. F is assumed to be continuous and differentiable over its support $(-\infty,+\infty)$. This support implies that some individuals love public transportation so much that they would never accept not to use it ($\varepsilon_i \to -\infty$), while others will never use public transportation ($\varepsilon_i \to +\infty$). Without loss of generality, we split ε_i equally between the two utility functions.

The waiting time for public transportation is $W(z) \in \mathbb{R}_0^+$. It displays a positive network externality for public transportation users. The idea is that, if there are more users, the frequency of public transportation increases and the waiting time decreases.[11] For simplicity, we assume this network externality to be linear. If there are $(1-z)$ users of public transportation, the waiting time of each of them is given by $W(z)$, with

$$W'(z) > 0 \text{ and } W''(z) = 0.$$

Further on in the paper, it will be useful to define $G(x) = F^{-1}(\varepsilon)\ \forall \varepsilon \in IR$. Given the assumptions over $F(\varepsilon)$, the support of G is $x \in [0,1]$.[12]

Definition 8 $\Delta(\alpha,z)$ *is the additional congestion faced by car users in comparison*

[11] An alternative interpretation: a lower price for a given quality of service.

[12] With $G(0) = +\infty$, $G(1) = -\infty$ and $G(x) = \varepsilon_i$ such that there is a mass x of commuters with $\varepsilon > \varepsilon_i$.

to the congestion faced by public transportation, i.e.

$$\Delta(\alpha,z) = t^c(\alpha,z) - t^{pt}(\alpha,z).$$

Using the properties of $t^c(\alpha,z)$ and $t^{pt}(\alpha,z)$, we have

Lemma 14 *Properties of* $\Delta(\alpha,z)$.
(i) $\frac{\partial\Delta(\alpha,z)}{\partial\alpha} > 0$;
(ii) $\frac{\partial\Delta(\alpha,z)}{\partial z} > 0$, $\forall\alpha > 0$;
(iii) ***Supermodularity of*** $\Delta(\alpha,z)$***.:*** *the effect of separation on the differential of commuting time increases with congestion (with the number of car users), i.e.* $\frac{\partial^2\Delta(\alpha,z)}{\partial z\partial\alpha} > 0$.

Proof. (i) and (ii) are straightforward from the properties of $t^c(\alpha,z)$ and $t^{pt}(\alpha,z)$. Property (iii), the supermodularity of $\Delta(\alpha,z)$ is obtained using $\frac{\partial^2 t^c(\alpha,z)}{\partial z\partial\alpha} > 0$ and $\frac{\partial^2 t^{pt}(\alpha,z)}{\partial z\partial\alpha} < 0$. The definition of $\Delta(\alpha,z) = t^c(\alpha,z) - t^{pt}(\alpha,z)$ leads to:

$$\frac{\partial^2\Delta(\alpha,z)}{\partial z\partial\alpha} = \frac{\partial^2 t^c(\alpha,z)}{\partial z\partial\alpha} - \frac{\partial^2 t^{pt}(\alpha,z)}{\partial z\partial\alpha} > 0.$$

■

3.2.2 The game

The modal choice is a simultaneous game among a unit mass of commuters. It consists in each commuter choosing the mode of transportation (either car of public transportation) that maximizes her utility given her expectation on z. Hence, commuter i commutes by car if $U_i^c(\alpha,z) > U_i^{pt}(\alpha,z)$, i.e.

$$\varepsilon_i > f_c - W(z) + [t^c(\alpha,z) - t^{pt}(\alpha,z)].$$

If it is a best response *ex post* for a commuter j with $\varepsilon_j > \varepsilon_i$ to commute using public transportation, it is also a best response for commuter i to do so.

Using Definition (8), the condition for commuter i to use a car becomes

$$\varepsilon_i > f_c - W(z) + \Delta(\alpha, z). \tag{3.3}$$

3.2.3 Decentralized Equilibria

In this section, we first show that a Nash equilibrium always exists. Second, we derive the conditions for the presence of multiple equilibria. Third, we characterize the most efficient one.

a. Existence

The existence of at least one Nash equilibrium is relatively easy to show. Stability *ex post* comes from the fact that there always exists an equilibrium where a share of commuters strictly prefers public transportation while the other prefers to use a car.

Proposition 8 *There exists at least one pure strategy Nash equilibrium.*

Proof. Remember $F(\varepsilon)$ is assumed to be continuous and differentiable over its support $(-\infty, +\infty)$. This implies that there exists at least one commuter k with taste parameter ε_k such that, if all commuters with parameter $\varepsilon_j < \varepsilon_k$ use public transportation, and all commuters with $\varepsilon_k < \varepsilon_i$ take the car,

$$G(z_k) = f_c - W(z_k) + \Delta(\alpha, z_k). \tag{3.4}$$

Commuter k is indifferent between the private car and public transportation. Sharing the same beliefs, commuters with $\varepsilon_j < \varepsilon_k$ strictly prefer public transportation and $\varepsilon_k < \varepsilon_i$ strictly prefer their car. Thus, it is a Nash Equilibrium. ■

b. Multiplicity

The intuition behind the existence of multiple equilibria is the following. Assume that there is a large share of commuters with similar preferences (ε) for the use of a car. When they believe that most of them use public transportation, it is a best response for them to do so. This is a Nash equilibrium with a low z. If, on the contrary, most of them believe that they will use a car, they expect public transportation not to be efficient and, indeed, it will not be. This is also a Nash equilibrium, involving a high z.

Proposition 9 *There exist multiple equilibria if and only if there exists a solution z_k such that*

$$\frac{\partial G(z_k)}{\partial z} > \frac{\partial [f_c - W(z_k) + \Delta(\alpha, z)]}{\partial z}. \tag{3.5}$$

Proof. The proof is presented in Appendix 3.6.1. ■

For this condition to be fulfilled, the difference in the costs between the two modes of transportation must be sufficiently low and a sufficiently high mass of commuters must have similar preferences. Consider the particular case of unimodal preferences: few people with polarized preferences, and a large fraction of people with similar preferences. This is likely to lead to the presence of three equilibria, as plotted in Figure 1 (with $\alpha = 0$). There are two stable[13] equilibria, one with few users of public transportation (a share z_1 of car users) and one with a large fraction (a share $z_3 < z_1$ of car users). There is also one unstable equilibrium, z_2.

c. Efficiency

Proposition 10 *If there are multiple equilibria, the equilibrium involving the higher use of public transportation Pareto dominates all the other equilibria. The Pareto dominant equilibrium is denoted $\widehat{z}$.*

Proof. The formal proof is provided in Appendix 3.6.2. ■

Figure 1 illustrates this proposition. Define three groups of people A, B and C. Group A uses a car in both equilibria, group C uses public transportation in both

[13] Those equilibria are locally stable in the sense that agents' best-response to any small perturbation to the equilibrium z would bring this share back to equilibrium.

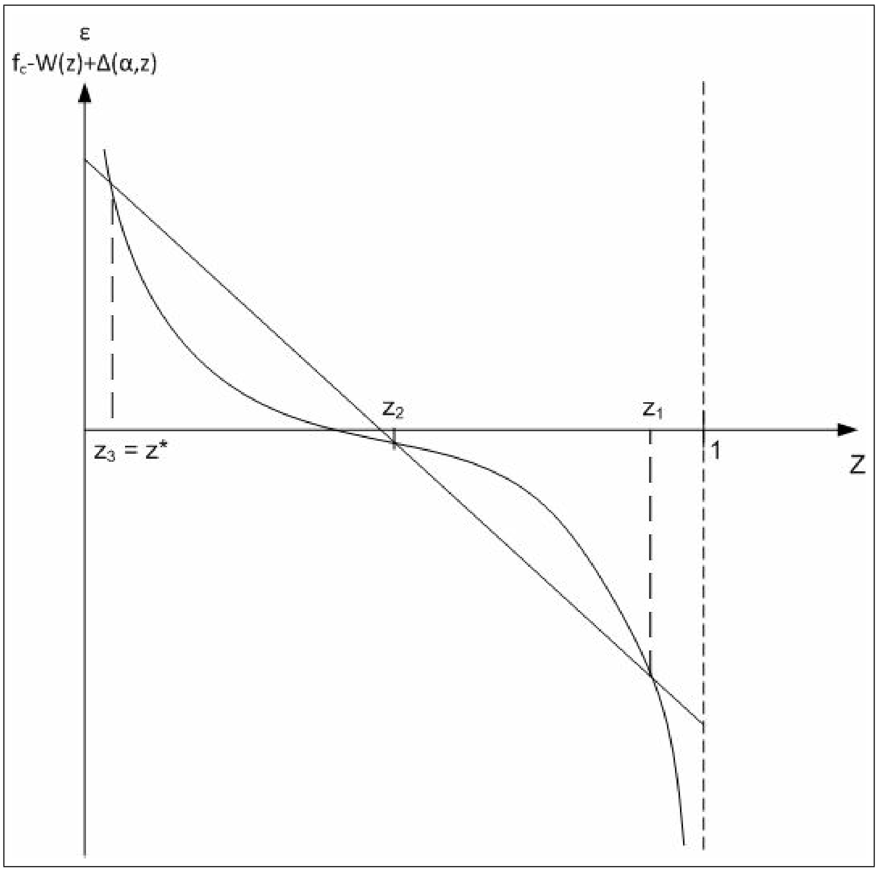

Figure 3.1: Illustration with multiple equilibria

equilibria, and group B uses public transportation when $z = z_3$ and a car otherwise. As the cost of both public transportation and car use are lower in z_3, groups A and C are strictly better off. By revealed preferences, group B is also better off in that equilibrium. They are better off by using a car in z_3 than in z_1, but they use public transportation instead.

3.3 Social planner

In this section, we stress the possible presence of two different sources of inefficiencies. The *coordination failure* implies a move from one type of equilibrium to another (e.g. z_1 to z_3 in the previous section). The *sub-utilization of public transportation* implies that, at the margin, the first best equilibrium requires more users of public transportation at any initial Nash equilibrium. This latter result is standard in the presence of externalities.

First, we derive the first best equilibrium conditions for a social planner maximizing the aggregate utilities. We show that this social planner may miss the coordination failure by focusing on local maximization. Second, we study the effect of two policies, taxation and traffic separation at different initial Nash equilibria. Third, we study the conditions for these policies to be Pareto improving and compare their relative efficiency.

3.3.1 The social planner's optimum

Assume that the aim of the social planner is to maximize the sum of all commuters' utilities, i.e.

$$\begin{aligned} & \underset{z,\alpha}{Max} \int_0^1 \left[\phi U_i^c(\alpha,z) + (1-\phi) U_i^{pt}(\alpha,z)\right] dx \\ st.\ \phi &= 1 \text{ if } \varepsilon_i \geq G(z) \\ &= 0 \text{ otherwise} \\ \alpha, z &\in [0,1] \end{aligned}$$

This is equivalent to

$$\min_{z,\alpha} \int_0^z [f_c - t^c(\alpha,z) + \frac{G(x)}{2}]dx + \int_z^1 [W(z) - t^{pt}(\alpha,z) - \frac{G(x)}{2}]dx$$

and the first order conditions are

$$G(z^*) = f_c - W(z^*) + \Delta(\alpha,z^*) + z t_z^{c\prime}(\alpha,z^*) + (1-z^*)[W'(z^*) + t_z^{PT\prime}(\alpha,z^*)] \tag{3.6}$$

and

$$z t_\alpha^{c\prime}(\alpha,z) + (1-z) t_\alpha^{PT\prime}(\alpha,z) = 0. \tag{3.7}$$

Rearranging the terms to compare the private costs and the public benefits for commuter $i : G(z^*) = \varepsilon_i$, equation (3.6) leads to the following condition:

$$W(z^*) - \Delta(\alpha,z^*) - f_c + G(z^*) = z^* t_z^c(\alpha,z^*) + (1-z^*)\left[W'(z^*) + t_z^{PT}(\alpha,z^*)\right]. \tag{3.8}$$

Thus, we have:

Lemma 15 *In any Nash Equilibrium, the share of car users is too high. For the share of car users to be socially optimal, there must exist public transportation* *users that strictly prefer the car.*

Proof. The right-hand side of equation (3.8) corresponds to the social cost of increasing the share of car users. This is clearly positive, as all negative externalities of a car are increasing with z. On the left hand side is the individual preference for the car of the swing commuter z^*, such that all commuters with $\varepsilon < G(z^*)$ take public transportation, and the other use a car. For the equality to hold, this must be positive. This implies that the socially optimal swing commuter strictly prefers to use a car rather than public transportation. ■

If the second FOC leads to an interior solution, $\alpha^* \in (0,1)$, it becomes

$$\frac{z}{(1-z)} = -\frac{t_\alpha^{PT}(\alpha^*,z)}{t_\alpha^c(\alpha^*,z)}. \tag{3.9}$$

The right-hand side of equation (3.9) is positive and represents a measure of the relative efficiency of a traffic separation policy, i.e. the marginal effect of α on the relative commuting time ratio. Defining $\beta(\alpha,z) = -\frac{t_\alpha^{PT}(\alpha^*,z)}{t_\alpha^c(\alpha^*,z)}$, it is reasonable to believe that $\beta_\alpha(\alpha,z) \geq 0 \ \forall \ z \in [0,1]$. Indeed, on the one hand, by increasing the share of roads dedicated to public transportation, the incidence of congestion on public transportation is reduced proportionally. On the other hand, the effect on cars is likely to be different. The creation of dissociated traffic lanes for public transportation generates bottlenecks for cars. The creation of these bottlenecks is likely to increase congestion but at a marginally decreasing rate (by increasing the number of bottlenecks, the impact of each one is reduced).

If the second FOC does not yield an interior solution (i.e. if $\beta_\alpha(\alpha,z) = 0$ or $\frac{z}{(1-z)} \neq \beta(\alpha,z) \ \forall \alpha \in [0,1]$), the social planner will choose $\alpha = 0$ if z is sufficiently high, and $\alpha = 1$ if z is sufficiently small. If there is no interior solution and there is a large share of car users, it is socially beneficial - at the margin - to allow more space for cars. If there is a large share of public transportation users, it is socially beneficial to fully protect public transportation from congestion.

It is important to underline that equations (3.8) and (3.9) give the conditions

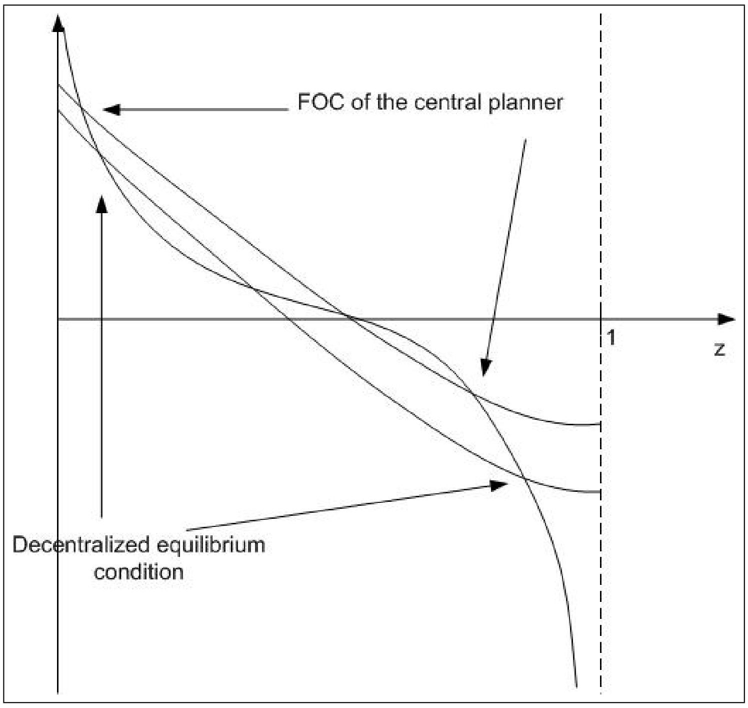

Figure 3.2: Decentralized equilibrium and social planner's first order condition for z^* ($\alpha > 0$)

to reach a local maximum, not necessarily a global one. As for the decentralized equilibria, there is no reason for these optima to be unique.

Therefore, these conditions give an insight into what can be socially optimal to solve the *sub-utilization of public transportation* at the margin: in any decentralized state of the world, there are not enough users of public transportation. This is illustrated in Figure (3.2) where we plot the functions on the left and the right hand sides of equation (3.6) together with equation (3.4), the decentralized equilibrium condition. Compared to the decentralized equilibrium, the right hand side of the equation is associated to a higher intercept and slope, for any value of α, i.e. the difference between the two curves is increasing in z.

A social planner could be misled when he tries to reach a maximum using equations (3.8) and (3.9). The planner may be missing a larger inefficiency: coordination failure. Indeed, consider a Nash equilibrium that implies a high share of cars, z_k, and assume a taxation scheme able to internalize the marginal externality. If the social planner maximizes the aggregate utility by setting $\alpha = 0$ for this value of z_k, there is a possibility that there exists another (lower) value, $\widehat{z} \neq z_k$, another Nash equilibrium, with another optimal value of taxes and $\alpha > 0$. In other words, a social planner must ensure not to target a local maximum when a better, global maximum is reachable.

3.3.2 Policy tools

In the previous sections, we showed that two different kinds of inefficiencies have to be distinguished: coordination failure and sub-utilization of public transportation. The first one comes from poor coordination between individuals in the presence of multiple equilibria, when the prevailing equilibrium does not involve the best use of public transportation. The second is due to the two considered externalities (congestion and network externalities) leading to the sub-optimal use of public transportation, whatever the prevailing equilibrium. To address the first inefficiency properly, it is required to significantly change individuals' behavior. To address the second one, a central planner should affect the behavior of some marginal individuals only: those that are the most likely to use public transportation after the implementation of new policies. In the presence of a unique equilibrium, the only relevant conditions are given by equations (3.8) and (3.9). Otherwise, policies may also have a role by ensuring coordination towards the most efficient equilibrium.

It is worth noting that, in our setting, a government could set a very high tax and then remove it almost instantaneously in order to force commuters to coordinate on the efficient equilibrium. Nevertheless, we believe that this is not realistic. The dynamics of switching from one equilibrium to another is a long, progressive process. To solve this issue, we assume that setting a policy implies that the government will keep it in place forever. This assumption may be considered ad hoc, but it is realistic for an intervention to take effect and be credible.

We assume that the government has two policy tools at its disposal: the **taxation** of car users (T), and the possibility to change the traffic **separation** between cars and public transportation (α). Due to the discrete choice nature of the model, tax is equivalent to a fare subsidy.[14] We do not consider variations of taxation schemes that can have differential effects among car users or time of the day.[15]

We assume that **a taxation policy** implies to levy T on every car user and that this tax is redistributed lump-sum among all commuters.[16] Therefore, every commuter receives a transfer zT and car users pay T. The new utility functions become

$$\begin{aligned} U_i^c(\alpha, T, z) &= -f_c - t^c(\alpha, z) - (1-z)T + \frac{\varepsilon_i}{2}, \\ U_i^{pt}(\alpha, T, z) &= -W(z) - t^{pt}(\alpha, z) + zT - \frac{\varepsilon_i}{2}. \end{aligned}$$

After the introduction of a taxation policy, a commuter i uses public transportation if and only if

$$\varepsilon_i < \Delta(\alpha, z) - W(z) + T + f_c$$

and, by proposition 8, there is always at least one Nash equilibrium. As we do not limit the size of the tax, there always exist a T such that the only Nash equilibrium with taxation involves a lower share of car users than in $\hat{z}$.

The **traffic separation** is the other available policy tool. Assume that the government sets a new traffic separation, $\alpha' : \alpha' > \alpha$. The new utility functions become

$$\begin{aligned} U_i^c(\alpha', T, z) &= -f_c - t^c(\alpha', z) + \frac{\varepsilon_i}{2}, \\ U_i^{pt}(\alpha', T, z) &= -W(z) - t^{pt}(\alpha', z) - \frac{\varepsilon_i}{2}. \end{aligned}$$

[14]An alternative policy would be to allow the social planner to invest in lower W for a given value of z.

[15]One can refer to Parry (2002) for a comparison between a single lane toll, a uniform congestion tax across freeway lanes, a gasoline tax, and a transit fare subsidy for the reduction of congestion.

[16]As will be made clear below, assuming the tax is lost only marginally affects the results.

Now, a commuter i uses public transportation if

$$\varepsilon_i < \Delta(\alpha', z) - W(z) + f_c.$$

Here, one cannot theoretically claim that the impact of traffic separation is sufficient to keep only one equilibrium. It depends on the size of the effect of traffic separation on the difference in commuting times. The effect of these two policies is presented in Figure 3.3. The optimal level of traffic separation has been discussed in the previous section. Comparing equations (3.6) and (3.4), an optimal taxation scheme can easily be obtained. If a social planner wants to reach an optimal modal split, z^*, it must set a level of taxation corresponding to the social marginal effect of the use of a car, computed at the targeted optimum, z^*.

For every locally optimal modal split z^*, there exists an optimal level of taxation T^* corresponding to the social marginal impact of the use of a car in z^*, this level of taxation corresponds to the sum of the social marginal congestion for cars and public transportation and of the social marginal opportunity cost in terms of network externality of car users not using public transportation.

$$T(\alpha, z^*) = z^* t_z^c(\alpha, z^*) + (1 - z^*) \left[W'(z^*) + t_z^{PT}(\alpha, z^*) \right]$$

It is interesting to note that the optimal tax is increasing in z^* ($T_z(\alpha, z^*) \geq 0$). This means that, comparing two similar cities, if the optimal share of car users is higher in one of the two cities, the level of taxation in that city must also be higher. This result is due to the marginal cost of car use (both in terms of congestion and in terms of network externalities) which is increasing in the share of car users.

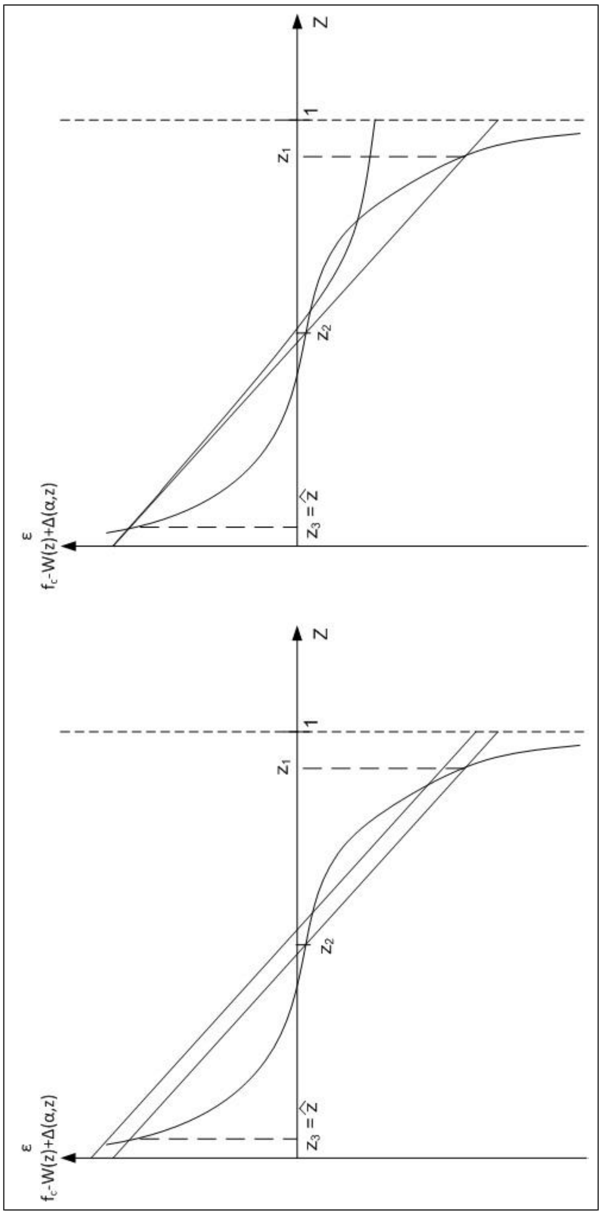

Figure 3.3: Effect of taxation (left) and traffic separation (right).

This optimal taxation is a necessary condition for the decentralized equilibrium to be optimal. It is not a sufficient condition. Indeed, consider a city where the decentralized equilibrium is located in z_1 and the global optimum slightly to the left of $\widehat{z}$. If a social planner sets a taxation compatible with $\widehat{z}$, it is very likely that the equilibrium in the city does not end up at $\widehat{z}$, but rather somewhere between z_1 and $\widehat{z}$, as shown in Figure 3.4, where we duplicate the curves of Figure 3.2 and add the new decentralized equilibrium condition taking the optimal taxation into consideration.

The same approach applies to traffic separation. From the first order condition (3.7), the optimal traffic separation is decreasing in z, the share of car users. In the presence of a large share of car users, the marginal impact of α is to dramatically increase the transportation time for the majority of commuters and to decrease it for a minority. Hence, in the global welfare maximum, the traffic separation is very high, but this is not necessarily the case in the local maximum, and setting the level of traffic separation of the global optimum is not a sufficient condition to reach this equilibrium. This can lead to too much traffic separation in an equilibrium where most of the commuters use their car.

3.3.3 Efficiency of policy tools

Even setting the optimal policy is not a sufficient condition to reach the first best. This is why it is necessary to provide a more general tool to assess policies: efficiency. In this section, we first define the conditions for either taxes or traffic separation to be Pareto-improving. Second, given those conditions, we provide a general result to compare their relative efficiency. If one is not strictly better than the other, then taxation should be preferred for smaller changes of commuting patterns, while traffic separation should be preferred for bigger changes.

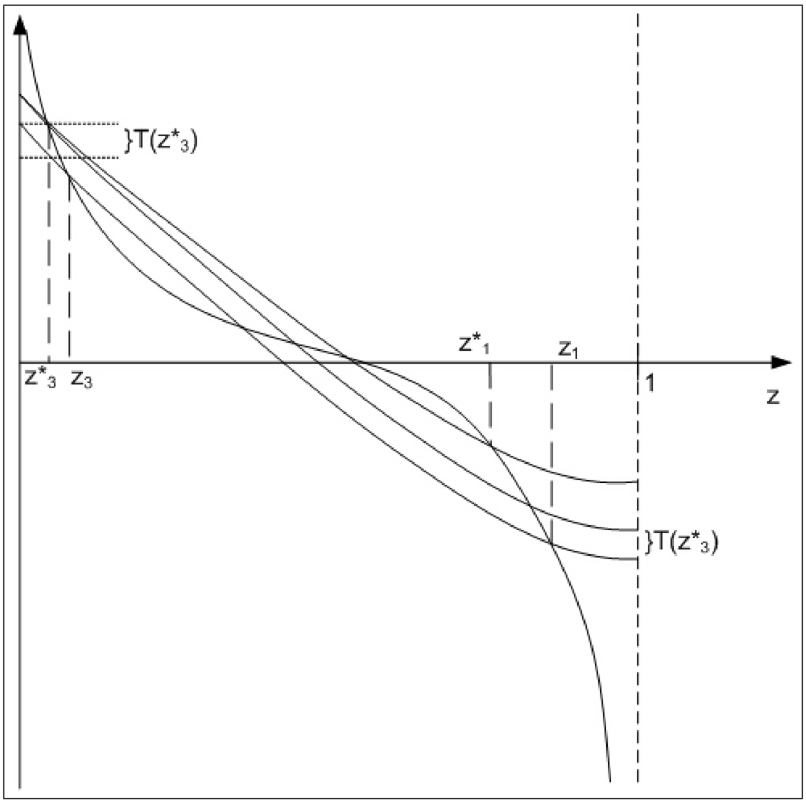

Figure 3.4: Effect of a taxation computed at z_3^* on the different decentralized equilibria.

a. Absolute efficiency of policy tools

Definition 9 *For a given Nash equilibrium, z_k, we define the* ***initial swing commuter*** *as the commuter indifferent between using a car or public transportation before the introduction of a policy, and the* ***final swing commuter*** *as the commuter indifferent between using a car or public transportation after the introduction of a policy.*

Definition 10 z^T *and* z^{α} *are the equilibria after the implementation of, respectively, a policy of taxation and of traffic separation.*

In our setting, the study of policy implementation requires an analysis of its effect on three types of agents. The *initial users of* public transportation (those on the right of the initial swing commuter), *the switching users* (those located between the initial and the final swing commuters), and the *remaining car users* (those on the left of the final swing commuter).

We know, by definition of our externalities, that all the *initial users of public transportation* are better off with the implementation of either policy. Indeed, they enjoy higher network externalities (more users of public transportation), face less congestion (less cars) and, in case of a taxation policy, receive a lump sum transfer. In the case of traffic separation, they enjoy an additional decrease in congestion (since public transportation benefits from a higher share of roads).

Now we study the effect of the policies on *the switching users* and the *remaining car users*. We show that measuring the welfare of the remaining car users is sufficient to assess the Pareto efficiency of these policies. First, we identify the conditions under which the implementation of a policy can be Pareto improving and, second, we compare the effect of those policies on remaining car users.

Lemma 16 *Conditions for the two considered policies to be Pareto improving:*
(i) A policy of ***taxation*** *is Pareto improving if and only if*

$$\left[t^c\left(\alpha, z_k\right) - t^c\left(\alpha, z^T\right)\right] > \left(1 - z^T\right) T. \tag{3.10}$$

(ii) A policy of ***traffic separation*** *is Pareto improving if and only if*

$$t^c(\alpha, z_k) > t^c(\alpha', z^\alpha). \tag{3.11}$$

Proof. See Appendix 3.6.3. ■

A policy of taxation is Pareto improving if the reduction of congestion compensates for the cost of taxation. A policy of traffic separation is Pareto improving if it reduces congestion for the *remaining car users*. This implies a trade-off between fewer car users ($z^\alpha < z_k$), concentrated over fewer traffic lanes ($\alpha' > \alpha$). The combination of these two effects must reduce congestion for the policy of traffic separation to be Pareto improving.

From equation (3.10), if z is sufficiently large, the condition is not extremely restrictive. Indeed, the tax levied on car users is largely compensated by the payoffs resulting from the lump-sum benefit of the considered tax. However, when z decreases, the tax base shrinks, making this condition more and more restrictive.

One can conveniently rewrite equation (3.11) by separating two effects: a positive effect (decrease in congestion due to the lower number of cars) and a negative effect (increase in congestion due to the increase of traffic separation):

$$t^c(\alpha, z) - t^c(\alpha', z^\alpha) = [t^c(\alpha, z) - t^c(\alpha, z^\alpha)] - [t^c(\alpha', z^\alpha) - t^c(\alpha, z^\alpha)]. \tag{3.12}$$

The likelihood for a traffic separation policy to be Pareto improving depends on the relative importance of these two forces.

b. Relative efficiency of policy tools

It is not possible to compare the absolute efficiency of these two policies without considering specific functional forms for congestion. Nevertheless, a general intuition of the relative efficiency of these two policies can be derived from the following proposition.

Proposition 11 *Assume there exist two distinct policies, α_1 and T_1, that yield the same equilibrium, z_1, and that car users enjoy the same utility under either of these two policies. Then, for any other two distinct policies, α_2 and T_2, yielding another equilibrium, z_2 ($z_2 < z_1$), car users always prefer the policy of traffic separation to the policy of taxation.*

Proof. See Appendix 3.6.4. ■

It follows from this proposition that even though we cannot theoretically exclude the possibility that one of the two policies is always better than the other, if this is not the case, taxation should be preferred for small changes in z, while separation should be preferred for larger changes.

This relates to the two schools of thought we presented in the literature review. If a social planner is convinced that the city is car-dependent, and that any policy can only have a marginal impact on the modal split, then a policy of taxation may be the best policy. But, if one believes that a large shift can take place, traffic separation might be a better choice.

3.4 Extensions

3.4.1 Capacity constraints and discomfort externalities

There are two possible types of congestion in public transportation. First, public transportation is a source of congestion. For instance, there may be so many buses on a bus lane that the travel time on that lane increases with the number of public transportation users. Second, congestion can occur when commuters cannot enter in the first bus and face a queue to access public transportation. This would increase the waiting time for commuters.

Remember that an equilibrium is such that $\varepsilon_i = \Delta(\alpha, z) - W(z) + f_c$. Congestion between public transportation leads $\Delta(\alpha, z)$ to decrease after some threshold, say, $\bar{z}$. Congestion within public transportation implies that instead of enjoying network externalities among the users of public transportation, $W(z)$ decreases below a certain threshold.

Capacity constraints, viewed in a strict way, would be that there is no mean to serve the demand for public transportation if this demand is higher than a given threshold, say $(1-\bar{z})$. In this case, for people exceeding this threshold, waiting time goes to infinity. In Figure 3.5, we illustrate the case of a strict capacity constraint, i.e. no possibility to transport more than a share $(1-\bar{z})$ of the population by public transportation. Note that any line located between the dashed line (no congestion within or between public transportation) and the vertical line could be achieved in the presence of different degrees of capacity constraints and/or congestion in public transportation.

Another representation of congestion within the public transportation system is to explicitly model discomfort externalities. Crowding costs (Kraus, 1991) are imposed by every marginal passenger on other passengers while the Mohring effect could be seen as a discrete process. Consider a bus lane where a bus is added when a given number of passengers per bus is reached. Every time a bus is added, the quality of public transportation "jumps" by discretely reducing travel time. However, within any bus, there is marginal congestion. The discomfort externality locally reduces the incentive to use public transportation when the number of passengers per bus increases. Assuming the network effect outweighs the congestion effect, the cost function is modified as in Figure 3.6. W(z) is locally decreasing in z and stepwise increasing in z. With this modified setup, the likelihood of facing multiple equilibria increases. The incentive to remain locally with an equilibrium share of car users is increased by the fact that, at the margin, increasing the share of public transportation users decreases the average quality of public transportation.

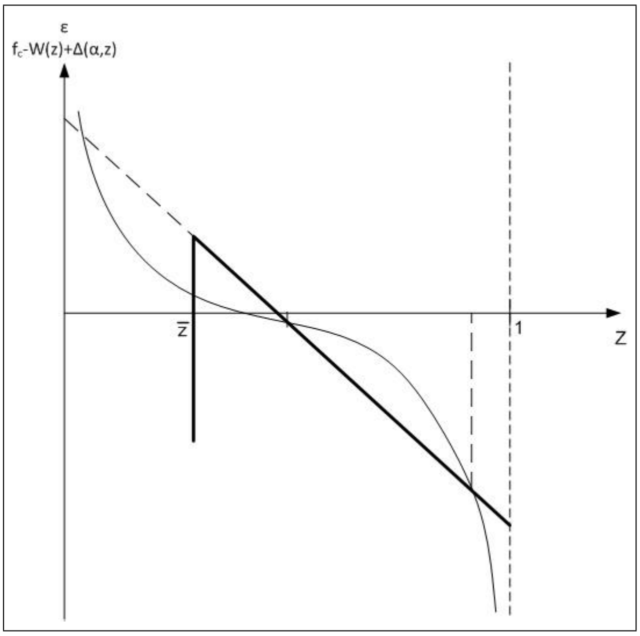

Figure 3.5: Strict capacity constraint in public transportation

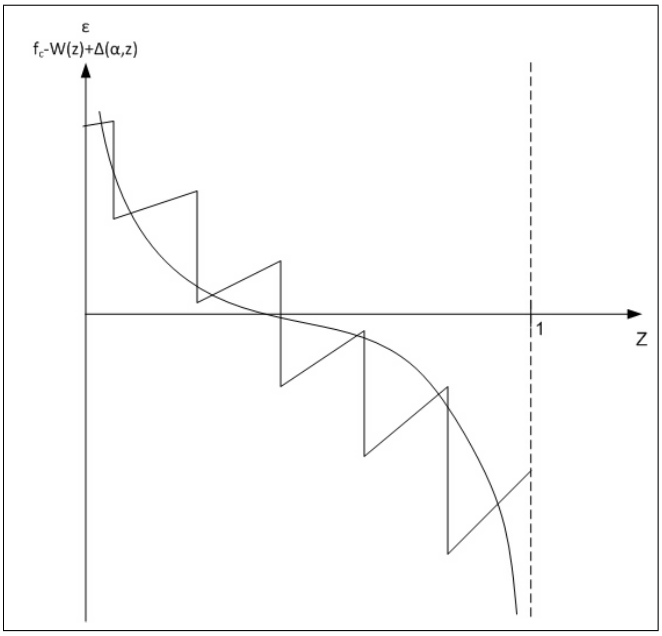

Figure 3.6: Discomfort externalities in public transportation

3.4.2 Building an underground

For the moment, we have considered traffic separation corresponding to bus lanes or light rail: more space for public transportation and less space for cars. Another way to prevent public transportation from congestion is to build an underground. In comparison to delimiting bus lanes (which is almost free), an underground is much more costly to build. Hence, even if one cannot deny that an underground can be an efficient way to provide fast public transportation and decrease congestion within public transport, it is likely to be counter-productive when dealing with coordination problems. Indeed, the underground may actually decrease the travel time both for public transportation and for cars. If, as in Tabuchi (1993), the cost of infrastructure is supported only by public transportation users, building an underground may actually decrease the share of public transportation users in the modal shift.

Assume that the underground is the only available type of public transportation. $\widetilde{\alpha}$ is now the investment in the underground. This investment is associated with $M(\widetilde{\alpha})$, the lump sum cost paid by all commuters, independent of their modal choice.

The effect of $\widetilde{\alpha}$ on time spent commuting by car is now $\frac{\partial t^c(\widetilde{\alpha},z)}{\partial \widetilde{\alpha}} \leq 0$, because the underground does not reduce (and potentially increases) the space for cars in the city. The marginal impact of $\widetilde{\alpha}$ on $t^{pt}(\widetilde{\alpha},z)$ remains negative: more investments in public transportation reduce the commuting time when using public transportation.

Defining $\Delta^u(\widetilde{\alpha},z) = t^c(\widetilde{\alpha},z) - t^{pt}(\widetilde{\alpha},z)$, the utilities of both type of commuters become

$$
\begin{aligned}
U_i^c(\alpha,z) &= -f_c - t^c(\widetilde{\alpha},z) - M(\widetilde{\alpha}) + \frac{\varepsilon_i}{2}, \\
U_i^{pt}(\alpha,z) &= -W(z) - t^{pt}(\widetilde{\alpha},z) - M(\widetilde{\alpha}) - \frac{\varepsilon_i}{2},
\end{aligned}
$$

and the new equilibrium is defined by

$$\varepsilon_i = f_c - W(z) + \Delta^u(\widetilde{\alpha},z).$$

Since $\forall\alpha = \widetilde{\alpha}$, $(\alpha$ and $\widetilde{\alpha} \in [0,1])$, we have $\Delta^u(\widetilde{\alpha},z) \leq \Delta_1(\widetilde{\alpha},z)$, the equilibrium comes with a lower share of public transportation users and, in the case of multiple equilibria, it is even more difficult to avoid the 'bad' equilibrium. This result would be even stronger if the cost of the underground were supported by public transportation users only, as in Tabuchi (1993). However, an argument in favor of the existence of an underground could be linked to congestion in public transportation (as discussed in the previous extension). In that case, an underground can be seen as a means to expand the supply of public transportation in the presence of capcity constraints.

3.5 Conclusion

We show that the combination of externalities of congestion, cross modal externalities and network externalities with heterogeneous commuters can lead to multiple equilibria. This may explain why a priori similar cities end up with very different patterns of car use. We also show that policy tools, namely taxation and traffic separation, are not equivalent in terms of welfare. When one of the two is not strictly more efficient than the other, separation should be preferred for large-scale policies while taxation should be preferred for smaller modifications of commuting patterns. This result partly explains the differences in policy recommendations from the two schools in the literature - physical planning and car-dependant cities - suggesting different reforms to improve the modal choice within a city. On the one hand, a policymaker that believes (as "physical planners") that there must be an important change in the modal split should focus on the allocation of space (α in our model) and increase the share of roads devoted to public transportation only. On the other hand, a social planner only concerned with marginal changes in the cost of cars - or that simply believes that car dependence is the best possible state of the world for a given city - should privilege taxation.

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3.6 Technical Appendixes

3.6.1 Proof of Proposition 9

Proof. We know from proposition (1) that an equilibrium is a solution to

$$G(z_k) = f_c - W(z_k) + \Delta(\alpha, z_k).$$

First, we show that the existence of a Nash equilibrium, z_k, satisfying

$$\frac{\partial G(z_k)}{\partial z} > \frac{\partial f_c - W(z_k) + \Delta(\alpha, z_k)}{\partial z}, \tag{3.13}$$

is a sufficient condition for the existence of multiple equilibria. Second, we show that this is a necessary condition.

(i) If there exist such a z_k, then for any $\eta > 0$ arbitrarily small, we have

$$\begin{aligned} G(z_k) &= f_c - W(z_k) + \Delta(\alpha, z_k) \\ G(z_k + \eta) &> f_c - W(z_k + \eta) + \Delta(\alpha, z_k + \eta) \\ G(z_k - \eta) &< f_c - W(z_k - \eta) + \Delta(\alpha, z_k - \eta). \end{aligned}$$

Since the support of F is $(-\infty, \infty)$ and therefore $G(1) < f_c - W(1) + \Delta(\alpha, 1)$ and $G(0) > f_c - W(0) + \Delta(\alpha, 0)$. It implies that the functions must cross at least three time and there exists at least three equilibria. So, condition (3.13) is a sufficient condition for the existence of multiple equilibria.

(ii) Assume that at any Nash Equilibrium z_k, we have

$$\frac{\partial G(z_k)}{\partial z} < \frac{\partial f_c - W(z_k) + \Delta(\alpha, z_k)}{\partial z}$$

then for any $\eta > 0$, we have

$$\begin{aligned} G(z_k) &= f_c - W(z_k) + \Delta(\alpha, z_k) \\ G(z_k + \eta) &< f_c - W(z_k + \eta) + \Delta(\alpha, z_k + \eta) \\ G(z_k - \eta) &> f_c - W(z_k - \eta) + \Delta(\alpha, z_k - \eta). \end{aligned}$$

Since the support of F is $(-\infty, \infty)$, for any $z', z'' : z' < z_k < z''$, $G(z') > f_c - W(z') + \Delta(\alpha, z')$ and $G(z'') < f_c - W(z'') + \Delta(\alpha, z'')$. This implies that the functions cross only once and condition (3.13) is necessary.

From (i) and (ii), condition (3.13) is, indeed, a necessary and sufficient condition.

■

3.6.2 Proof of Proposition 10

Proof. Assume there exist T equilibria $z_1 < z_2 < ... < z_T$

(1) We want to show that z_1 Pareto dominates any equilibrium z_j, $j = \{2, ..., T\}$

(2) For any pair z_j, z_1 with $z_j > z_1$, there are three categories of commuters:

(a) Commuters with ε_i such that $F(\varepsilon) < 1 - z_j$. Their best response is to use public transportation in both equilibria. Those users are better off in equilibrium z_1 as

$$t^{pt}(\alpha, z_1) < t^{pt}(\alpha, z_j) \text{ and } W(z_1) < W(z_j),$$

then

$$t^{pt}(\alpha, z_1) + W(z_1) + \frac{\varepsilon_i}{2} < t^{pt}(\alpha, z_j) + W(z_j) + \frac{\varepsilon_i}{2}$$

(b) Commuters with ε_i such that $1 - z_j < F(\varepsilon) < 1 - z_1$. Their best response is public transportation in equilibrium z_1 and car in equilibrium z_j. Those users are better off in equilibrium z_1. Indeed, as commuters reveal their preferences by choosing their mode, then for any $F(\varepsilon_j)\varepsilon[1-z_j, 1-z_1]$:

$$W(z_j) + t^{pt}(\alpha, z_j) + \varepsilon_j > f_c + t^c(\alpha, z_j) \tag{3.14}$$

and

$$W(z_1) + t^{pt}(\alpha, z_1) + \varepsilon_j < f_c + t^c(\alpha, z_1). \tag{3.15}$$

As congestion increases in z,

$$f_c + t^c(\alpha, z_j) > f_c + t^c(\alpha, z_1).$$

Hence, it is straightforward that

$$f_c + t^c(\alpha, z_j) > W(z_1) + t^{pt}(\alpha, z_1) + \varepsilon_j$$

(c) Commuters with ε_i such that $1 - z_1 < F(\varepsilon)$. Their best response in both equilibria is to take the car. Those users are better off in equilibrium z_1 as:

$$f_c + t^c(\alpha, z_j) > f_c + t^c(\alpha, z_1).$$

■

3.6.3 Proof of Lemma 16

Proof. Let us divide the population into three families: those who use public transportation before and after the implementation of the new policy (PT-PT), those who use their car before and after the policy (C-C), and the swing commuters, those who used their car before the policy and public transportation afterward (C-PT). We show the effect of both policies on the differents families described above.

In the case of taxation:

1) PT-PT: the variation of their welfare is given by

$$U_T^{pt} - U^{pt} = W(z_k) - W\left(z^T\right) + t^{PT}(\alpha, z_k) - t^{PT}\left(\alpha, z^T\right) + z^T T$$

which can be decomposed into three effects, all welfare enhancing (as long as $z^T < z_k$): $W(z_k) - W\left(z^T\right)$ corresponds to the reduction of waiting time; $t^{PT}(\alpha, z_k) - t^{PT}\left(\alpha, z^T\right)$ comes from the reduction of congestion; and $z^T T$ comes from the lump sum transfer from car users to the user of public transportation.
2) C-C: the policy of taxation increases their welfare if

$$U_T^c - U^c = t^c(\alpha, z_k) - t^c\left(\alpha, z^T\right) - \left(1 - z^T\right)T > 0$$

i.e. it increases their welfare if

$$t^c(\alpha, z_k) - t^c\left(\alpha, z^T\right) > \left(1 - z^T\right)T$$

3) C-PT: It is easy to show that if car users are better off, swing commuters are also better off. Swing commuters prefer public transportation to the car under the policy leading to z^T. Hence, if their utility when using a car is higher then in z_k, so is their utility while using public transportation.
In the case of separation, increasing α to α':
1) PT-PT: the variation of their welfare is given by

$$U_\alpha^{pt} - U^{pt} = W(z_k) - W(z^\alpha) + t^{PT}(\alpha, z_k) - t^{PT}\left(\alpha', z^\alpha\right)$$

which can be decomposed into two effects, both being welfare enhancing (as long as $z^\alpha < z_k$): $W(z_k) - W(z^\alpha)$ corresponds to the reduction of waiting time a,d $t^{PT}(\alpha, z_k) - t^{PT}(\alpha', z^\alpha)$ comes from the reduction of congestion due to two forces: (i) less car and (ii) more traffic lines devoted to PT only.

2) C-C: the policy of separation increases their welfare if

$$U_{\alpha}^{c} - U^{c} = t^{c}\left(\alpha, z_{k}\right) - t^{c}\left(\alpha', z^{\alpha}\right) > 0$$

i.e. it increases their welfare if

$$t^{c}\left(\alpha, z_{k}\right) > t^{c}\left(\alpha', z^{\alpha}\right)$$

3) C-PT: The reasoning is similar as for the taxation policy. If car users are better off, swing commuters are also better off. ■

3.6.4 Proof of Proposition 11

Proof. Starting from $\alpha_0 \geq 0$ and $T_0 = 0$ and given the definition and the properties of $\beta\left(\alpha, z\right)$ (note that $\beta\left(\alpha, z\right) = -\frac{t_{\alpha}^{PT\prime}(\alpha^{*}, z)}{t_{\alpha}^{c\prime}(\alpha^{*}, z)}$ with $\beta_{\alpha}'\left(\alpha, z\right) \geq 0 \; \forall z \in [0,1]$), it is possible to define $\gamma(\alpha)$ such that

$$t^{pt}\left(\alpha_{1}, z_{1}\right) - t^{pt}\left(\alpha_{0}, z_{1}\right) = -\gamma(\alpha_{1})\left[t^{c}\left(\alpha_{1}, z_{1}\right) - t^{c}\left(\alpha_{0}, z_{1}\right)\right], \tag{3.16}$$

with $\gamma'\left(\alpha\right) > 0$.

(i) Consider two policies: α_1 ($\alpha_1 > \alpha_0$ is associated with $T = 0$) and T_1 (associated with α_0) yielding the same equilibrium z_1. By definition, a commuter indifferent between the two policies has an ε_j such that

$$f_{c} + T_{1} - W\left(z_{1}\right) + \Delta\left(\alpha_{0}, z_{1}\right) = \varepsilon_{j} = f_{c} - W\left(z_{1}\right) + \Delta\left(\alpha_{1}, z_{1}\right).$$

This simplifies to

$$T_{1} = \Delta\left(\alpha_{1}, z_{1}\right) - \Delta\left(\alpha_{0}, z_{1}\right),$$

which can be conveniently rewritten as

$$T_1 = \left[t^c(\alpha_1, z_1) - t^c\alpha_0, z_1\right] - \left[t^{pt}(\alpha_1, z_1) - t^{pt}(\alpha_0, z_1)\right].$$

By assumption, this leads to

$$T_1 = (1 + \gamma(\alpha_1))\left[t^c(\alpha_1, z_1) - t^c(\alpha_0, z_1)\right]. \tag{3.17}$$

(ii) Car users are indifferent between these two policies if $\exists\ \alpha_1$, T_1, z_1 such that

$$(1 - z_1)\,T_1 = t^c(\alpha_1, z_1) - t^c(\alpha_0, z_1).$$

These conditions imply

$$\begin{aligned} (1 - z_1)\left[1 + \gamma(\alpha_1)\right]\left[t^c(\alpha_1, z_1) - t^c(\alpha_0, z_1)\right] &= t^c(\alpha_1, z_1) - t^c(\alpha_0, z_1) \\ (1 - z_1) &= \frac{1}{1 + \gamma(\alpha_1)}. \end{aligned}$$

Now consider two alternative policies associated with a higher use of public transportation: α_2, T_2, z_2 with $z_2 < z_1$. Car users are now better off with traffic separation than with taxation iff

$$(1 - z_2)\,T_2 > t^c(\alpha_2, z_2) - t^c(\alpha_0, z_2).$$

From the expression of T in equation (3.17),

$$\begin{aligned} (1 - z_2)\left[1 + \gamma(\alpha_2)\right]\left[t^c(\alpha_2, z_2) - t^c(\alpha_0, z_2)\right] &> t^c(\alpha_2, z_2) - t^c(\alpha_0, z_2) \\ (1 - z_2) &> \frac{1}{1 + \gamma(\alpha_2)}. \end{aligned}$$

This is always true as $z_2 < z_1$, $\gamma(\alpha_2) \geq \gamma(\alpha_1)$ and given that $(1 - z_1) = \frac{1}{1+\gamma(\alpha_1)}$ ■