Interferometric profiling of transparent objects in zero-fringe mode using 2D Fan Wavelets

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Abstract: Complex closed fringe images are analysed using a novel 2D Wavelet Transform based algorithm outperforming in speed the current state-of-the-art by 10 times and 'Windowed Fourier' techniques by up to 200 times.

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1. Problem Description

In the present contribution we are using interferometry (i.e. Quantitative Phase Imaging) for profiling transparent objects. The objects in question are liquid droplets imaged in a classic Mach-Zehnder interferometer. The innovation of the contribution lies in the image analysis algorithm used. With the use of 2D Wavelet Transform techniques, we are able to obtain excellent noise-resiliency in a time which is up to 10 times faster than existing Wavelet techniques and up to 200 times faster than 'Windowed Fourier' algorithms. Currently analysing a single 1024x1024 image requires approximately 1.5 seconds on a recent desktop computer (Intel Core i7-5820 with 2133Mhz DDR4 memory) and using 90 different scales. The methodology described here forms the subjet of a paper under review in Applied Optics [1].
The typical image the algorithm needs to analyse is shown in Figure 1(a). From this single image, it needs to extract the local phase and the local phase gradient which are proportional to the local height and slope respectively. The main issue encountered is the inherent sign ambiguity present in the phase retrieval; i.e. from a single image it is impossible to determine in which direction the phase is increasing or decreasing. While this problem has been solved many times in literature already, we show here how it can be done with the use of two 2D Fan wavelet transforms. In this way we minimize the amount of directional transforms to only 2, whereas up to 20 were needed before [2].

2. Algorithm Description

At the basis of this algorithm are the Fan wavelet and Wavelet Ridge phase extraction algorithms. The mathematical formulation of the Fan wavelet is described in [3] and a good description of the basic use of the Wavelet Ridge extraction technique for fringe analysis is given in [4]. Basically, Fan wavelets are a simple summation of several well-chosen Morlet wavelets in order to gain a certain amount of direction-insensitivity. The two Fan wavelets used here are constructed so as to accept mainly horizontal or vertical directions. For instance, the result of applying the 'vertical' Fan wavelet to the typical image shown in Figure 1(a) is shown in Figure 1(b) and (d). Discussing first the local amplitude image (d), we notice that a high and uniform contrast is picked up when the local phase gradient direction remains somewhat vertical. Similarly for the 'horizontal' Fan wavelet, Figure 1(c) and (e) show the local phase and contrast image. Now combining images (d) and (e) to (i) is quite straightforward and only a matter of pixel by pixel comparison between two matrices to use the highest found value. Another output of the Wavelet Ridge technique, which is not shown here, is the scale which is best-fitting the local phase evolution. This scale can be transformed into the local phase gradient (see for instance [5]). This image can also be composed directly based on the same comparison between the contrast images, so as to retain the local scale corresponding to that one for which the maximal cross-correlation was found.
However, before the phase images can be merged, the phase sign ambiguity problem needs to be solved first. This will be done by the construction of the so-called 'phase flipping' masks shown in Figure 1(f) and (g). With these masks we can flip the sign of the phase in the bottom half of Figure 1(b) and the left half of (c). With these corrections, the final phase image can be simply composed based on the local maximum contrast. The result is shown in Figure 1(h). The only difficult part is therefore the construction of these phase-flipping masks. In our paper which is under review [1], we describe three different approaches to this problem. For lack of space, we will only briefly go into two of these approaches here.

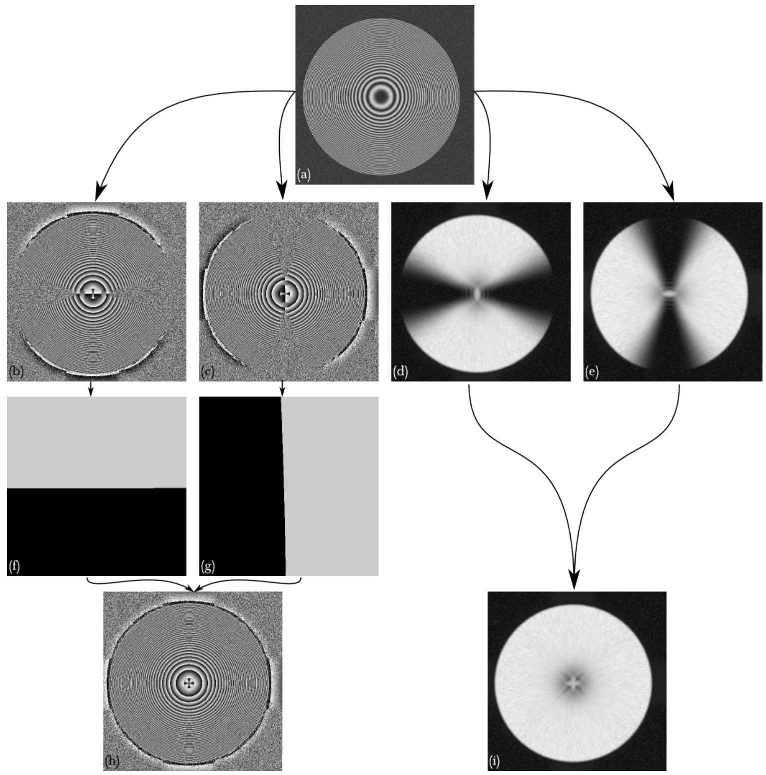

Figure 1. Overview of the algorithm steps. (a) is the input image. (b) and (c) are 'partial' phase images. With the use of the phase-flipping masks (f) and (g), these will be combined into (h). (d) and (e) are the partial contrast images which can be recombined immediately into (i).

The simplest approach only works for images with a single stagnation point such as Figure 1(a). This is the same problem that was tackled in the DH conference of 2013 using 1D Wavelet transforms [6]. However, the current approach will prove to be many times faster because of the absence of image reinterpolation. The algorithm starts by composing the absolute phase image (i.e. $abs(\varphi)$), which can be determined without knowledge of the true sign. From this image, one can now extract and label some closed fringes. Now, as it turns out, the dividing lines we want to construct go through the 'turning' points of these objects, i.e. the minimal and maximal column and minimal and maximal row extents. Through these points, a parabola is then fitted from which the phase flipping masks shown in Figure 1(f) and (g) are easily constructed. More details are given in our paper under review [1]. To demonstrate that this technique is also capable of analysing slightly more complicated images, its application on a sliding droplet is shown in Figure 2(b), where the two parabolas are clearly shown.

When the image becomes more complex, a more traditional approach is used, where first the fringe orientation (0-π) is determined and later transformed into direction (0-2π). This classic approach was used for instance in [2], but also in many articles before that. First step here is obtaining orientational information on the fringes and as mentioned previously, we have sacrificed this ability by the use of the Fan wavelet to gain time. However, what we have access to is a 'noise-free' absolute phase image, which can be used as input for the fast accumulated differences algorithm from [7]. In this way, we get a good orientation map in a fraction of the time with the powerfull noise resiliency lended by the Wavelet transform filtering. The transformation to direction can then be obtained in many ways, here we chose to adapt our phase unwrapping algorithm of choice ([8]) to perform this task. A typical application of this technique to a complicated droplet image is shown in Figure 2(e).

Finally, following a segmentation step based on local amplitude thresholds, we are able to compute not only local height (e.g. Figure 2(f)) and local slope (e.g. Figure 2(c)) but also droplet volume and local contact angle values for each image and track these quantities over time. This is something which can be used in many applications (e.g. [9]). Uncertainty figures for these measurements as a function of SNR are described in the paper under review [1].

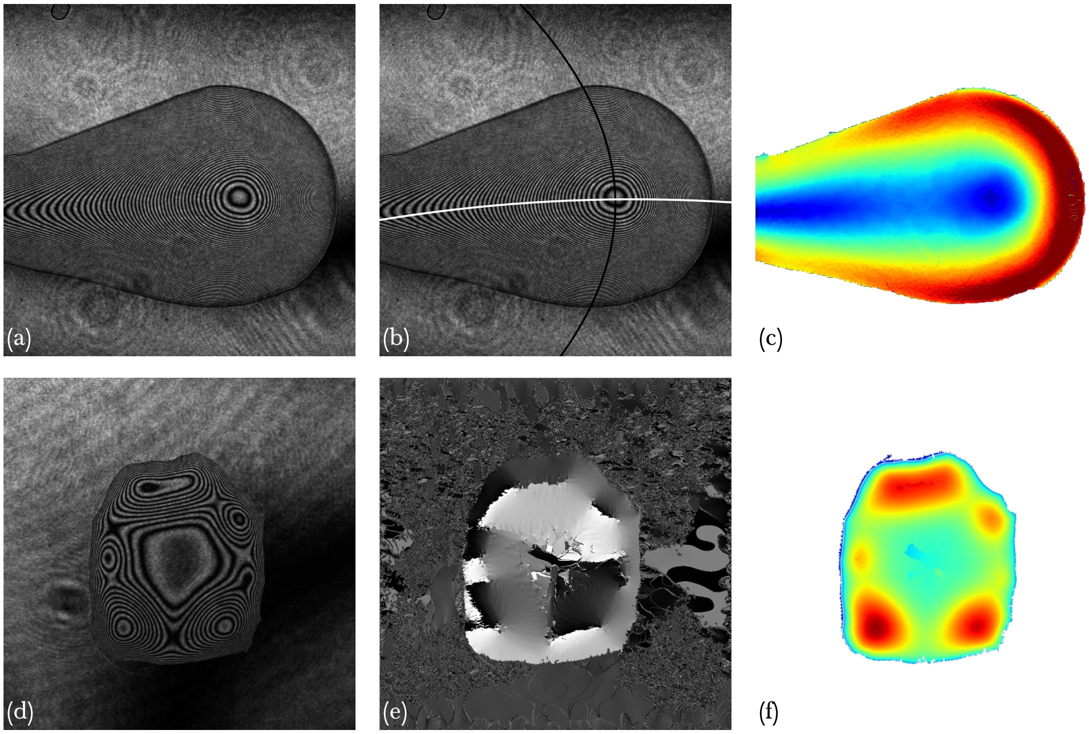

Figure 2. (a) Sliding droplet image. (b) Parabolic dividing lines used for phase-flipping mask construction. (c) Resulting local slope map. (d) Image of droplet in dewetting transition (e) Obtained direction map. (f) Resulting height map.

3. Conclusions

In this contribution, we have demonstrated how Fan wavelets can be used to analyse complex closed fringe images. The resulting algorithm has the noise robustness of the Wavelet Ridge analysis techniques achieving a precision of 1/30th of a fringe for noise levels going up to 1/5th of the input contrast. Our current algorithm only requires 1.5 seconds per image, which improves on the state-of-the-art by an order of magnitude and up to 200 times on the Windowed Fourier techniques.

References

1. S. Dehaeck, Y. Tsoumpas, and P. Colinet, "Analysing closed-fringe images using 2d fan wavelets." Applied Optics **(under review)** (2015).
2. J. Ma, Z. Wang, B. Pan, T. Hoang, M. Vo, and L. Luu, "Two-dimensional continuous wavelet transform for phase determination of complex interferograms," Applied Optics **50**, 2425–2430 (2011).
3. F. Kirby, "Which wavelet best reproduces the fourier power spectrum?" Comput. Geosci. **31**, 846–864 (2005).
4. M. A. Gdeisat, D. R. Burton, and M. J. Lalor, "Spatial carrier fringe pattern demodulation by use of a two-dimensional continuous wavelet transform," Applied Optics **45**, 8722–8732 (2006).
5. Q. Kemao, S. Soon, and A. Asundi, "Instantaneous frequency and its application to strain extraction in moire interferometry," Applied Optics **42**, 6504–6513 (2003).
6. S. Dehaeck, Y. Tsoumpas, and P. Colinet, "Analyzing droplets through digital holography and a 1d wavelet transform technique," in "Digital Holography and Three-Dimensional Imaging, OSA Technical Digest," (2013), p. DW3A.5.
7. X. Yang, Q. Yu, and S. Fu, "An algorithm for estimating both fringe orientation and fringe density," Optics Communications **274**, 286–292 (2007).
8. M. A. Herraez, D. R. Burton, M. J. Lalor, and M. A. Gdeisat, "A fast two-dimensional phase unwrapping algorithm based on sorting by reliability following a non-continuous path,," Applied Optics **41**, 7437–7444 (2002).
9. Y. Tsoumpas, S. Dehaeck, M. Galvagno, A. Rednikov, H. Ottevaere, U. Thiele, and P. Colinet, "Nonequilibrium gibbs criterion for completely wetting volatile liquids," Langmuir **30**, 11847–11852 (2014).