Money or Projects: 
How Should Altruistic Donors Give Aid?

Patrick Legros
SBS-EM, ECARES, Université libre de Bruxelles and CEPR

Mohamed Sraieb
SBS-EM, ECARES, Université libre de Bruxelles

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Patrick Legros∗  Mohamed Sraieb

ECARES & CEPR  ECARES

Abstract

While evidence suggests that donors provide aid both under the form of budget and project assistance, most of the literature suggests that it is optimal to provide either budget or infrastructure assistance, but not both. We revisit this question by developing a model in which the donor cares about helping the recipient accumulate income but cares more about redistributing income to the poor than the recipient country. We show that it is often optimal for the donor to offer both budget and infrastructure assistance. Under perfect information, the aid package involves a fixed project and positive transfer to recipient countries which exhibit a low willingness or ability to redistribute to the poor. For countries with a larger willingness to redistribute, larger infrastructure projects are offered but those countries co-finance the cost of the project. When the recipient’s ability to distribute is private information, the donor optimally chooses to offer pooling contracts. Separating contracts are possible, but create additional distortions; for instance grant matching programs exclude more recipient countries from aid than the pooling contract.

Keywords: aid modalities, pooling contract, targeted infrastructure.

JEL classification: D86, F35, O12, O19.

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1 Introduction

The literature on aid effectiveness has focused on several imperfections linked to the characteristics of donors and recipients of development aid. These imperfections include the lack of institutional capacities and corruption in the recipient country as well as the lack of coordination among donors and the inherent enforcement problems (Doucouliagos and Paldam, 2009; Bourguignon and Sundberg, 2007; Tarp, 2006). More recently, the focus has shifted from the recipient country’s internal limitations to the more controversial issue of the content of aid. The argument is that the ineffectiveness of aid in promoting growth and combating poverty also depends on the form under which aid is delivered. Despite the existence of a large consensus on the importance of the instruments to provide aid, the question of the best choice of aid modalities is still open.

OECD data show that for both bilateral and multilateral donors, there is indeed a mix of budget and infrastructure support to LDCs and LMICs, even if there seems to be a regular decrease in the share of budget assistance of total aid. A striking example is ODA provided by the U.K to Tanzania for which budget support was 794 times larger than the amount of project assistance provided in 2006. This ratio fell to 25 times in 2009 and to 1.5 times for 2012. If we consider multilateral aid to LDCs, the same pattern prevailed. Budget assistance amounted 14.25% of total aid in 2013 while the share was 37.12% in 2004. This tendency is reversed for some particular donors. This is the case for International Development Association (one of World Bank’s lending arms) with a budget support accounting for 13% of its total aid in 2001. By 2004, that share had risen to 43% and by 2005 to 50%. Other examples include the European Commission and the UK’s Department for International Development (DfID) that have provided almost 50% of their total aid under the form of budget support. As we discuss below, existing research does not seem to help in explaining the observed mix of aid modalities since results suggest that donors should use only one instrument.

In this paper, we visit the question of aid modality in a static framework, both under perfect and imperfect information and focus on two modalities of aid: budget or infrastructure project. We follow the literature in assuming that

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1These two categories are discussed in OECD (2006) where budget support is defined as aid channeled to the partner government using the country’s own allocation, procurement and accounting system, and this support is not linked to specific project activities. Budget support is transferred to the recipient government’s treasury, and is managed in accordance
the donor would like to transfer income to the recipient country. Because the donor and the recipient have different preferences in terms of income redistribution between the rich and the poor in the recipient country, any dollar given as a budget support will not be distributed in a way that is optimal from the donor’s perspective. We also make the common assumption that projects are complement to income and facilitate development: road networks facilitate distribution and therefore may enhance production activities, electrical and telecommunication networks may allow entrepreneurial activities outside the main urban areas, schools increase local human capital. We depart from the literature by assuming that infrastructure projects can benefit differently the rich and the poor: building a school in a rural area benefits mainly the poor, while building a freeway through the same area facilitates the distribution of manufactured goods, and may benefit mainly the rich.\footnote{The argument implicitly relies on a static framework. In a dynamic setting, more educated individuals may enable firms to shift to more complex technologies, freeways may facilitate migration to urban areas.}

Because of the complementarity between budget and project assistance, and the fact that some projects may create a bigger multiplier effect to the income of one group, a donor can therefore use a mix of these instruments to increase the efficiency of aid, that is achieve the desired level of redistribution and development at least cost.

Under perfect information about the preferences of the recipient country and its ability to transfer income to the poor, we show that monetary transfers are decreasing in the initial ability of a country to transfer wealth to the poor. Optimal projects, are constant for poor countries but larger projects are required for countries more willing to transfer wealth, and the richer the recipient country is, the more the project should be targeted toward the poor. Hence for poor countries, budget and projects are complement while they are substitute for richer countries.

Obviously, the assumption of perfect information is an idealization and an often voiced argument for using only infrastructure projects rather than giving a non-targeted budget to the recipient country is that it will be inconsistent with incentive compatibility.\footnote{The argument has parallels with those made in the literature on welfare benefits and the role, e.g., of food stamps versus more cash subsidies.} For this reason, we introduce asymmetry of information with the partner country’s budgetary procedures. With project assistance, the donor directly participates in the design and the implementation of a developmental project, decides the inputs to be provided, and usually uses its own disbursement and accounting procedures, this is off-budget (Foster and Fozzard, 2000).
tion along two dimensions: the ability of the recipient to transfer income to the poor, an adverse selection aspect, as well as the non-contractibility of consumption flows in the recipient country, a moral hazard aspect. These assumptions reflect the idea that even if the donor may have a good knowledge of the total GDP in the country, the effective share of this GDP that could be transferred from the rich to the poor is private information to the recipient country, e.g., because it will depend on the degree of corruption within the country and the shadow cost of transfers which are less observable.

Our main result is that while it is possible to separate types via contracting, the donor prefers to use a pooling contract. There are two reasons for this result. First, contrary to usual screening models where the principal benefits from reducing transfer payments while the agent suffers from this reduction, here both the donor and the recipient value transfers. Second, this is a model with type-dependent outside options. In the first best, the recipient’s payoff net of its autarky payoff is a decreasing function of its type. Therefore, providing rents to higher types is not as valuable as in standard screening contexts with uniform outside options, and the donor finds it too costly to extract information about types since this information would require paying additional rents. The pooling contract will involve both a positive monetary transfer to the recipient and a project when the expected ability to engage in income redistribution is small. In this case, the project coincides with the optimal project under perfect information. However, if the expected ability to transfer to the poor is high, the aid contract involves a positive co-payment from the recipient country and therefore precludes some low income countries from receiving aid. As the expected income of the recipient country increases, the donor will offer projects that are more targeted toward the rich.

Hence, whether under perfect or imperfect information, the co-existence of the two forms of aid, monetary transfers and infrastructure projects, is a characteristic of the optimal contract. We later contrast the second-best contract with matching grant policies, another mechanism used for aid allocation. While matching grant policies allow separation of types, they are not second-best optimal and also tend to prevent more countries from getting aid as compared to the second-best.
Literature Review

The costs and benefits of different aid instruments have been debated in the literature. Arguments in favor of budget support include the absence of crowding effect, the low level of transaction and administrative costs and the freedom given to the recipient for allocating the money to the sectors that are most in need (Hefeker, 2006; Jelovac and Vandeninden, 2008; Leiderer, 2012). Budget support, however, suffers from being more vulnerable to fungibility and corruption (Leiderer, 2012; Dollar and Pritchett, 2013). It can be diverted by the recipient to be used for purposes other than the ones for which it was initially intended.

On the other side, project assistance gives more control to the donor to monitor the use of aid but suffers from high transaction costs especially when aid is fragmented (many donors giving small amounts in the same recipient country), a feature characterizing aid giving in practice. It may also be undermined by the lack of coordination between donors yielding over investment or even duplicated investments.

Surprisingly, the debate on the “right choice” of aid instruments stayed long at the descriptive level. No analytical attempt has been made to investigate the question until the seminal work of Cordella and Dell’Ariccia (2007). The authors consider separately two modalities of aid. First, a budget aid that is conditional on (observable) capital investments made by the recipient country. Second, a project aid that directly imposes a capital expenditure. They find that budget support always dominates project assistance when the preferences of donors and recipients are aligned, and when the assistance is small relative to the recipients’ own resources. They do not consider however the possibility of having both budget and project aid in the same package.

By contrast, Jelovac and Vandeninden (2008) use a unified framework for the comparison between aid modalities. They show that all aid should be given via budget support, independently of whether conditionality can be used or not. Moreover, they show that conditionality is optimal to use only if the recipient’s developmental preferences, the productivity of inputs and the level of aid compared to the recipient’s budget are all high enough. Otherwise, unconditional budget support is optimal.

Another paper allowing for the simultaneous use of both of the aid modalities is Hefeker (2006) who asks which instrument is better suited to increase the economic situation of underprivileged groups in recipient countries. The finding is
that budget support is a better instrument than project aid, if aid programs are small with respect to the government’s resources and if the difference between the agents’ preferences is large.

Both Jelovac and Vandeninden (2008) and Hefeker (2006) consider that the donors and the recipients are perfectly informed about their environment and their mutual characteristics. As such, these models overlook the issue of asymmetric information and its impact on aid provision and composition. In addition, all the models cited above fail to explain why evidence suggests that donors provide aid under both modalities.

Our model borrows from the work by Azam and Laffont (2003) who analyze aid as a contract where the North gives a transfer to the South in return for poverty reduction. The issue of conditionality is discussed under asymmetric information about the degree of altruism of the government of the South; that is the preferences of the recipient government for transfers. By allowing for an endogenous aid budget and assuming observability of the transfers to the poor, they make the issue of conditionality central, that is the size of the budget that should be given to the recipient country as a function of observables. By contrast, we assume that the transfers to the poor are not observable and focus on the interplay between two aid modalities.

Finally, notice that the result on the optimality of pooling contracts, from the donor perspectives, is somewhat unusual in the mechanism design literature. As we already discussed, the explanation lies in the (relative) lack of conflict of interests between the donor and the recipient concerning the direction of transfers: they both benefit when the income of either group, rich or poor, is increased. They only differ in the way income should be distributed. Hence, if the preferences for redistribution to the poor are more aligned, or if the recipient has more ability to transfer income to the poor, it becomes less costly to transfer income to the recipient country.

In a principal-agent model (Bourguignon et al., 2014) also note that if the principal has a paternalistic utility function, whenever he is able to put the agent at his reservation utility level, he will respond to the rise in altruism or domestic governance by offering to the agent a less attractive contract, or by tightening external discipline (through monitoring and punishment). They do not address, however, the question of aid modalities.

The remainder of the paper is organized as follows: the next section introduces the model. Section 3 presents the benchmark case. The optimal aid under full information is characterized. Section 4 extends the analysis to asymmet-
ric information concerning the consumption of the poor and the willingness of the recipient to transfer wealth to the poor. It investigates the combination of lump-sum transfers and project assistance in order to enhance aid effectiveness. Matching grant contracts are introduced and contrasted with the second-best contract. Section 5 concludes.

2 A Model of Structural and Monetary Aid

A developed country, hereafter the donor, provides development assistance to a developing country, the recipient. In the recipient country, there is a measure of rich and a measure of poor. Rich have total income $y$ and the consumption levels of the rich and the poor are denoted by $r, p$ respectively. The rich can engage in income redistribution subject to the budget constraint $r + np = y$, in which case their welfare is $\log r + na \log p$. The donor country has the same objective function as above with $a = 1$. Hence $a \leq 1$ indexes the degree of preference alignment between the recipient and the donor.\(^4\)

There may be a lump-sum transfer $T$ from the donor to the recipient, in which case total income in the country is $y + T$. $T$ can be positive or negative, in the later case we interpret it as a co-financing of the project by the recipient country.\(^5\) The lump-sum transfer benefits the low income agents only if the decision makers in the recipient country decide to channel this additional income to them. When $a < 1$, the recipient country will channel funds to the poor at a rate that is lower than the one the donor would like.

In addition, or instead of lump sum transfers, the donor can help the recipient country build an infrastructure project like a road, a school, a bridge, a telephone network. Projects increase the benefits of consumption for different classes of individuals. For instance, electricity networks in rural areas may benefit industries but also low income agents, the same for road development. However, digging of water wells or building a school in a rural area tend to benefit mainly low income agents. We model this by assuming that projects create a multiplier effect of consumption for the poor and the rich. Projects are parametrized by $(\rho, \pi)$, with $\rho \geq 1$ is the multiplier effect of the project for the rich and $\pi \geq 1$ is

\(^4\)Preference alignment refers to the degree of coincidence in terms of the developmental priorities between the donor and the recipient. Priorities include which policy to adopt, which sector must be handled first, what steps to undertake, etc.

\(^5\)The idea is that aid comes as a combination of budget support and infrastructure projects. As such, when the transfer is negative, it is interpreted as a contribution of the recipient to the project aid.
the multiplier effect for the poor. Since consumption levels are \( r, p \), total welfare becomes\(^6\)

\[
\log(pr) + na \log(\pi p).
\]

There is an opportunity cost of \( \phi \geq 1 \) of transferring income from the donor to the recipient country and a project \((\rho, \pi)\) costs \( c(\rho + \pi - 2) \), linear in the total multiplier effect, with a zero cost when \( \rho = \pi = 1 \).\(^7\) We assume that the total marginal cost is small enough:

\[ c\phi \leq 1. \]

### 3 Full information

**Autarky**

To simplify, we will assume that the recipient country does not have the capabilities to self-finance projects.

In autarky, the recipient country chooses consumption levels, in order to solve the following problem

\[
\max_{r,p} \log r + na \log p \\
\text{s.t. } r + np \leq y
\]

which solution is:

\[ r = \frac{y}{1 + na}; \quad p = \frac{ay}{1 + na}. \]

The welfare under autarky is therefore

\[ U(a, y) = (1 + na) \log y + A(a) \]

where

\[ A(a) \equiv na \log a - (1 + na) \log(1 + na). \]

\(^6\)By using a log-utility, we can separate the effects of monetary transfers and project aid, which is a common specification in screening models. When there is not separability, the analysis becomes a lot more involved, see for instance Guesnerie and Laffont (1984); Ruiz del Portal (2012).

\(^7\)The linearity of the cost of projects is assumed for convenience but does not affect the qualitative results of the model. In general, a convex cost function \( c(\cdot) \) with support \([0, \infty]\) and satisfying \( c(0) = 0 \) and \( c'(0) = c \) can be used; remember that \( \rho = \pi = 1 \) means that the donor does not engage in project development, hence \( c(0) = 0 \) means that not providing infrastructure projects is costless to the donor.
Aid

If the aid is \((T, \rho, \pi)\), the recipient country solves

\[
\max_{r,p} \log r + na \log p + \log \rho + na \log \pi \\
\text{s.t. } r + np \leq y + T
\]

yielding optimal consumption levels

\[
r = \frac{y + T}{1 + na}; p = \frac{a(y + T)}{1 + na}
\] (1)

and a total welfare

\[
U(T, \rho, \pi; a, y) = (1 + na) \log(y + T) + \log \rho + na \log \pi + A(a) 
\] (2)

Regardless of the level of optimal transfer, the poor gets a share \(\frac{a}{1+na}\) while the rich gets a share of \(\frac{1}{1+na}\). Although the aid is initially intended for the poor, a share of \(\frac{a}{1+na}\) is captured by the rich.

Optimal Aid

Anticipating the choices (1) made by the recipient country, the donor will choose \((T, \rho, \pi)\) to solve:

\[
\max_{T,\rho,\pi} (1 + n) \log(y + T) + \log \rho + n \log \pi + n \log a \\
- (1 + n) \log(1 + na) - \phi(T + c(\rho + \pi - 2)) \\
\text{s.t. } (1 + na)[\log(y + T) - \log y] + \log \rho + na \log \pi \geq 0 
\] (3)

where (3) is the individual rational constraint of the recipient country.

If the constraint does not bind, the optimal lump-sum transfer is

\[
T^*(y) = \frac{1 + n}{\phi} - y 
\] (4)

and it is optimal to give a positive monetary transfer only if \(y\) is smaller than \((1 + n)/\phi\). For richer countries, the optimal transfer is negative, suggesting that the recipient country has to co-finance the infrastructure project. This implicitly assumes that the recipient country has a certain financing capacity allowing him to contribute to the project. We also examine the case of recipients
that cannot co-finance projects, $T \geq 0$, (see the appendix) and we show that the results are qualitatively the same.

The optimal project solves

$$\pi^*(y) = n\rho^*$$

$$\rho^*(y) = \frac{1}{\phi c}$$

with $\pi^*(y) > \rho^*(y) > 1$, since $n > 1$ and since we have assumed that $\phi c < 1$.

This yields a welfare for the recipient country of

$$U^*(a, y) = (1 + na) \left[ \log(1 + n) - \log(\phi c) \right] - (1 + na) \log \phi + na \log n + A(a)$$

The recipient accepts the aid when its participation constraint (3) holds, that is if $U^*(a, y) \geq U(a, y)$, or when

$$(1 + na) \left[ \log(1 + n) - 2 \log \phi - \log c - \log y \right] + na \log n \geq 0$$

$$\Leftrightarrow y \leq y^*(a) = \frac{1 + n}{\phi^2 c} \cdot \frac{1 + na}{n + \phi}$$

Since $\phi c < 1$ and $n > 1$, $y^*(a)$ is larger than $\frac{1 + n}{\phi}$. The aid for a poor country then takes the form of a project that is independent of the income of the recipient country, a positive monetary transfer to recipient countries with $y \leq \frac{1 + n}{\phi}$, decreasing with the income of the recipient country, but a negative transfer for richer countries, those with income larger than $(1 + n)/\phi$ but smaller than $y^*(a)$. Richer countries should co-finance the project.

If $y$ is larger than $y^*(a)$, the donor’s best aid will bind the individual rationality constraint of the recipient’s country. In this case, there is a shadow price associated to the participation constraint, and if $\lambda(y)$ denotes this shadow price, the solution is $T^*(y), \rho^*(y), \pi^*(y), \lambda^*(y)$ such that:

$$y + T^*(y) = \frac{1 + n + \lambda^*(y)(1 + na)}{\phi}$$

$$\rho^*(y) = \frac{1 + \lambda^*(y)}{\phi c}$$

$$\pi^*(y) = \frac{n(1 + \lambda^*(y)a)}{\phi c}$$
A missing condition is the binding participation constraint. Substituting the values \( T^*(y), \rho^*(y), \pi^*(y) \) into the binding constraint (3), we find the solution \( \lambda^*(y) \). This shadow price is decreasing in \( a \) (when the recipient country cares more about the welfare of the poor, the IR constraint is easier to satisfy) and increasing in \( y \) (when the recipient country is richer, its autarky payoff is larger).

The constraint binds if

\[
(1 + na) \log(y + T^*(y)) + \log \rho^*(y) + na \log \pi^*(y) = (1 + na) \log y
\]

Since \( \log \rho^*(y) + na \log \pi^*(y) \) is positive, it must be the case that \( T^*(y) \) is negative: rich countries necessarily co-finance the project in the first best.

Notice that \( \frac{\pi^*(y)}{\rho^*(y)} = \frac{n(1 + \lambda^*(y)a)}{1 + \lambda^*(y)} \); hence as \( y \) increases, this ratio decreases since it is decreasing in \( \lambda^*(y) \) and \( \lambda^*(y) \) is increasing in \( y \). Richer recipient countries should co-finance larger infrastructure projects that are increasingly targeted towards the rich. Since \( \lambda^*(y) \) is an increasing function of \( y \), the transfer \( T^*(y) \) in (7) has slope \(-1 + \lambda^*(y)\frac{1+na}{\phi} > -1 \). Hence, while the recipient country will co-finance the project, this will be done at a rate lower than the one chosen by relatively less poor recipient countries (those with \( y \in [(1 + n)/\phi, y^*(a)] \)).

The total cost of the project is \( c(\rho^*(y) + \pi^*(y) - 2) \), which is equal, using (8) and (9), to \( T^*(y) + y - 2c \), and therefore, the marginal cost when \( y \) varies is increasing in \( y \) since \( \lambda^*(y) \) is increasing in \( y \).

For \( y < y^*(a) \), the variation of the transfer is inversely related to the variation of income, that is \( T^*(y) = -1 \). For \( y \geq y^*(a) \), the variation of the transfer is \( T^*(y) = -1 + \lambda^*(y)\frac{1+na}{\phi} \), that is the recipient country is reimbursed for the marginal cost of the project but co-finances the project at the same rate as recipient countries with \( y < y^*(a) \). Because of this reimbursement of the marginal cost, the final income available for allocation is increasing in \( y \) when \( y \geq y^*(a) \).

We have therefore established the following proposition.

**Proposition 1.** The recipient’s country is made strictly better off by aid only if its income is less than \( y^*(a) \).

(i) As \( y \) increases in \([0, y^*(a)]\), the transfer to the recipient decreases but the project is independent of \( y \). For \( y < \frac{1+n}{\phi} \), the recipient receives a positive transfer from the donor, but for \( y \in \left[ \frac{1+n}{\phi}, y^*(a) \right] \), the recipient country receives a negative transfer, hence co-finances the project. Any additional income above \( \frac{1+n}{\phi} \), is used to co-finance the project at a marginal rate
equal to the increase in income.

(ii) As \( y > y^*(a) \), the donor invests in larger projects, that are increasingly targeted towards the poor. The recipient country is reimbursed for the marginal cost of the project, and pays the marginal increase in income.

(iii) Project assistance is more targeted toward the rich as \( y \) increases.

Clearly, monetary aid and projects are substitute in the first best. Countries that are not very willing or able to transfer wealth to the poor (low \( y \)) get high transfers and small (constant) projects, the reverse being true for countries with high \( y \). Consider now the aggregate aid \( T^*(y) + c(\rho^*(y) + \pi^*(y) - 2) \). This aggregate aid is decreasing in \( y \) for \( y < y^*(a) \), since the monetary transfer is decreasing in \( y \) while projects are constant.

**Corollary 1.** In the first best, monetary transfers are decreasing in \( y \) for all recipients whether they are poor or rich. Optimal projects, however, are constant for poor countries but increasing and more poor-oriented for countries more willing to transfer wealth to the poor.
Preference alignment

Let us examine the impact of the degree of preferences alignment between the donor and the recipient. Notice first that $y^*(a)$ is increasing in $a$. This means that for a given income schedule, the more preferences are aligned, the larger is the interval $\left[\frac{1}{y^*}, y^*(a)\right]$ over which IR holds; and therefore, the lower the region where it binds. Hence, an increase in $a$ increases the cutoff income above which recipient countries are required to co-finance projects. Put it differently, an increase in $a$ results in a situation where a co-financing is required from increasingly richer countries, and where it is optimal for the donor to use constant projects.

Moreover, using (7), (8) and (9) and recalling that the shadow price $\lambda$ is decreasing in $a$, recipient countries with preferences different from those of the donor should be allocated less monetary transfers for more projects tailored to the poor.

Finally, Note that the share of project that goes to the poor $\frac{\pi^*(y)}{\rho^*(y)}$ is increasing in $a$; Indeed, $\frac{\partial}{\partial a} \left( \frac{\pi^*(y)}{\rho^*(y)} \right) = \frac{\partial \lambda^*(y)}{\partial a} n(a - 1) + n \lambda^*(y)(1 + \lambda^*(y)) \geq 0$, since $a \leq 1$ and $\lambda$ is decreasing in $a$. Hence, for a given $y$, the more preferences are aligned (i.e. increasing $a$), the more recipients are required to co-finance projects targeting the poor.

4 Imperfect Information on $y$

We focus here on imperfect information on $y$, that is on the ability of the recipient to transfer income to the poor. Even if there is good information on the total GDP in the country, the effective share of this GDP that could be transferred from rich to poor (that is $y$ in the model) is private information, because it depends on knowledge about the internal functioning of institutions that is difficult to acquire by third parties.\footnote{An alternative would have been to follow the literature and consider imperfect information on $a$. The two approaches would lead to similar trade-offs but it is technically simpler to consider imperfect information on $y$.}

4.1 Pooling is the Second-Best Optimum

Let us define $P = \log(\rho) + na \log(\pi)$ the aggregate value of the project to the recipient. Then, the donor has at its disposal two instruments to separate types: transfers $T$ and overall value of projects $P$, leading to an indirect payoff
for the recipient country of \( u(T, P, y) \equiv (1 + na) \log(y + T) + P \). We note that 
\[
\frac{\partial^2 u}{\partial T \partial y} = -(1 + na) \left( \frac{1}{y+T} \right)^2 < 0 \text{ while } \frac{\partial^2 u}{\partial P \partial y} = 0.
\]
Hence, the ratio \( \frac{\partial u}{\partial T} / \frac{\partial u}{\partial P} \) is a strictly decreasing function of \( y \), implying that the Spence-Mirrlees condition holds as long as \( T'(y) \leq 0 \).

Incentive compatibility, indeed, requires that \( T(y) \) be non-increasing in \( y \), and, in order to separate the types, that \( P(y) \) be non-decreasing in \( y \). While the first best satisfies these two properties, it is nevertheless not incentive compatible, as we show first (all proofs not appearing in the text are in the Appendix.).

**Lemma 1.**  
(i) Incentive compatibility requires that \( T(y) \) be non increasing in \( y \) and \( P(y) \equiv \log \rho(y) + na \log \pi(y) \) to be non-decreasing in \( y \).

(ii) For any value of \( a, y, \hat{y} \), the first-best allocation does not satisfy the incentive compatibility conditions.

While separation is possible, we show that it is not optimal in this environment. It is standard to show that incentive compatibility implies that the recipient has indirect utility \( U(y) = U(y) + \int_{y}^{\hat{y}} \frac{1+na}{x+T(x)} \, dx \).

We claim that for any non-increasing incentive compatible schedule \( T \), individual rationality is satisfied. Clearly if \( T(y) \) is positive, recipient countries cannot be worse-off since \( P(y) \) is non-negative. If \( T(y) \leq 0 \), then for any \( y \),
\[
U(y) = U(y) + \int_{y}^{\hat{y}} \frac{1+na}{x+T(x)} \, dx \\
\geq U(y) + \int_{y}^{\hat{y}} \frac{1+na}{x} \, dx \\
\geq U(y) + \int_{y}^{\hat{y}} U'(x) \, dx.
\]
Therefore, if \( U(y) \geq U(y) \), individual rationality is satisfied for all \( y \). The last case is when \( T(y) > 0 \) and there exists \( y^* \) such that \( T(y^*) = 0 \). In this case,
$T(y) \leq 0$ for all $y > y^*$, and we have:

\[
U(y) = U(y^*) + \int_{y^*}^{y} U'(y) dy \\
\geq U(y^*) + \int_{y^*}^{y} \frac{1 + na}{x + T(x)} dx \\
\geq U(y^*) + \int_{y^*}^{y} \frac{1 + na}{x} dx \\
\geq U(y).
\]

Lemma 2. Consider an incentive compatible aid $T(y), P(y)$. If there exists $y^* \in (y, \overline{y})$ such that $T(y^*) = 0$, individual rationality holds for all $y$. If $T(y) < 0$, individual rationality holds for all $y \geq y$ whenever it holds at $y$.

Ignoring the condition $U(y) \geq U(y)$ and the other part of the incentive compatibility condition $T'(y) \leq 0$, the problem can be reduced to:

\[
\max_{\{U(y), T(\cdot), \rho(\cdot), \pi(\cdot)\}} V = U(y) + \int_{y}^{\overline{y}} \left[ \frac{1 + na}{y + T(y)} \frac{1 - F(y)}{f(y)} \\
+ (n - na) \left( \log(y + T(y)) + \log \pi(y) + \log a \right) \\
- \phi T(y) - \phi c(\rho(y) + \pi(y)) \right] f(y) dy
\]

As we show in the Appendix, pointwise maximization would imply that $\rho$ and $\pi$ are independent of $y$. But then, the transfer $T$ must also be independent of $y$.

Proposition 2. It is optimal for the donor to offer a pooling aid-package.

In the second-best, aid will not be conditional on the type of the recipient country and the donor will use a fixed aid contract $(T, \rho, \pi)$.

As long as $T$ is non-negative, recipient countries are willing to get aid, independently of their type $y$. Hence, once an aid contract is offered, the donor cannot exclude any type. Therefore, the donor’s expected utility is

\[
V = \int_{y}^{\overline{y}} \left[ (1 + n) \log(y + T) \right] dF(y) + \log \rho + n \log \pi \\
- \phi(T + c(\rho + \pi - 2)) - (1 + n) \log(1 + na) + n \log a
\]
and therefore projects are first-best (see (5)):

\[ \rho^* = \frac{1}{\phi c}, \quad \pi^* = n\rho^* \]

while the transfer solves

\[ \int_y^\infty \frac{dF(y)}{y + T} = \frac{\phi}{1 + n} \quad (10) \]

By the intermediate value theorem, there exists a unique value \( y_F \in (y, \bar{y}) \) such that

\[ \frac{1}{y_F} = \int_y^{y_F} \frac{dF(y)}{y} \quad (11) \]

For any non-negative \( T \), we have \( \int_y^{\bar{y}} \frac{dF(y)}{y + T} < \frac{1}{y_F} \), and (10) implies that \( y_F < \frac{1+n}{\phi} \).

In order for the aid package described by (5) and (10) to be optimal, it is then necessary that \( y_F < \frac{1+n}{\phi} \); this is trivially true if \( \bar{y} < \frac{1+n}{\phi} \), but not for more general distribution functions.

At \( y_F = \frac{1+n}{\phi} \), the optimal transfer is equal to zero, and all aid takes the form of infrastructure aid. However this is a non-generic case. If \( y_F > \frac{1+n}{\phi} \), it becomes beneficial for the donor to ask for co-financing, and, as in the first-best analysis, it is important to increase the benefit from the project in order to satisfy the participation constraint of recipient countries.

To simplify, assume that \( y = 0 \), and that recipient countries are asked to co-finance projects, that is are required to make a transfer \( t \) to the donor (hence \( T = -t \) in our previous notation). Then, only countries with type \( y > t \) will be able to accept the aid. Among the recipient countries with \( y \geq t \), participation constraint must be satisfied, that is \((1 + na)[\log(y) - \log(y - t)]\) must be smaller than the project benefit \( P = \log \rho + na \log \pi \).

Because all types face the same project, if a total benefit \( P \) has to be given, it is cost-minimizing to offer a project solving

\[
\max_{\rho, \pi} \log \rho + n \log \pi - \phi c(\rho + \pi - 2) \\
\text{s.t. } \log \rho + na \log \pi \geq P,
\]

and if \( \lambda \) is the coefficient of the constraint, the solution is

\[ \rho(\lambda) = \frac{1 + \lambda}{\phi c}; \quad \pi(\lambda) = \frac{n(1 + \lambda a)}{\phi c}. \]
As \( \lambda > 0 \), the constraint binds and there is a one to one relationship between \( P \) and \( \lambda \), and from above, between \( \rho, \pi \) and \( \lambda \).

\[
P(\lambda) = \log(1 + \lambda) + na \log(1 + \lambda a) - (1 + na) \log(\phi c) + na \log n \quad (12)
\]

The donor’s problem can be therefore reduced to the choice of \((t, \lambda)\). For such an aid package, there exists a unique marginal type \( y(t, \lambda) \) who is willing to participate, and this type solves \((1 + na)(\log(y) - \log(y - t)) = P(\lambda)\):

\[
y(t, \lambda) \equiv t \frac{(\exp P(\lambda))^{1/(1+na)}}{(\exp P(\lambda))^{1/(1+na)} - 1}.
\quad (13)
\]

and since \(\log y - \log(y - t)\) is a decreasing function of \( y \), only countries with \( y \geq y(t, \lambda) \) will accept the aid package. Having \( T \) negative involves a loss to the donor since poor countries cannot benefit from the aid package and get their autarky payoff. The optimal aid package solves

\[
\max_{t, \lambda}(1 + n) \left[ \int_{y(t, \lambda)}^{\gamma} \log(y - t) dF(y) + \int_{y}^{y(t, \lambda)} \log(y) dF(y) \right] + (1 - F(y(t, \lambda))) [\log \rho(\lambda) + n \log \pi(\lambda) + \phi t - \phi c(\rho(\lambda) + \pi(\lambda) - 2)]
\]

The optimal aid package gives first best projects and a positive transfer solving (10) when \( y_F \leq \frac{1 + n}{\phi} \). Otherwise, when \( y_F > \frac{1 + n}{\phi} \) recipient countries are asked to co-finance the project.

**Proposition 3.** Letting \( y_F \) be the expected income of recipient countries, the optimal aid package is such that:

(i) If \( y_F \leq \frac{n+1}{\phi} \), all recipient countries get the same aid package that involves a positive monetary transfer.

(ii) If \( y_F > \frac{n+1}{\phi} \), only recipient countries with a high enough income accept the aid package that involves a co-payment from the recipient country towards the financing of the project.

### 4.2 Matching Grant Policies

In a matching grant, the donor commits to match the contribution of the recipient towards an infrastructure project. Hence, if the recipient contributes \( t \), the donor will contribute \( t \) and the project that is chosen must satisfy the budget...
balance condition
\[ 2t \geq c(\rho + \pi - 2) \] (14)

A recipient with income \( y \), will choose to contribute \( t \) in order to solve

\[
\begin{align*}
\max_{r,p,\rho,\pi} \log(r) + na \log(p) + \log(\rho) + na \log(\pi) \\
\text{s.t. } r + np \leq y - t \\
2t = c(\rho + \pi - 2) \\
\rho \geq 1, \pi \geq 1
\end{align*}
\]

Clearly the budget constraint will bind, and the problem reduces to

\[
\begin{align*}
\max_{r,p,\rho,\pi} \log(r) + na \log(p) + \log(\rho) + na \log(\pi) \\
\text{s.t. } r + np + c\frac{\rho + \pi}{2} \leq y + c \\
\rho \geq 1, \pi \geq 1.
\end{align*}
\]

Denoting by \( \lambda \) the shadow price of the budget constraint, the solution is

\[
\begin{align*}
\lambda^*(y) &= \frac{2(1 + na)}{y + c} \\
r^*(y) &= \lambda^{*-1}(y) \\
p^*(y) &= ar^{*}(y) \\
\rho^*(y) &= \max\{1, \frac{2}{c}\lambda^{*-1}(y)\} \\
\pi^*(y) &= \max\{1, na\rho^*(y)\}
\end{align*}
\]

Since \( a \leq 1 \), the feasibility constraints \( \rho \geq 1, \pi \geq 1 \) hold whenever \( \pi^*(y) \geq 1 \), that is when

\[ y \geq y(a) \equiv \frac{c}{na} \]

If \( y \leq y(a) \), the recipient country prefers not to adopt the matching grant program. Note that at \( a = 1 \), \( y(1) = \frac{c}{n} \), and therefore \( y(1) \in \left(0, \frac{1+n}{\phi}\right) \).

We know that matching grants are not second-best. Under the condition \( y(a) > y_F \), they also tend to exclude more recipient countries from aid.

**Proposition 4.** Matching grant contracts are second-best inferior to the optimal pooling contract. Furthermore, there exists \( y_F > \frac{1+n}{\phi} \) such that more recipient
countries do not get aid with matching grant contract than the pooling contract.

Proof. Since \( y(a) > y(1) \), all recipient countries with \( y \in [0, y(a)] \) will opt not to get aid if \( y_F \leq \frac{n+1}{\phi} \), we know by proposition 3, that all recipient countries should get aid, while with matching grants only recipient countries with income greater than \( y(a) \) would obtain aid.

5 Concluding Remarks

Our research shows that the choice of the right mix of aid modalities is a determinant factor for the effectiveness of aid, perhaps as important as the aid level itself. In order to limit the extent of fungibility, common wisdom calls for rewarding well-governed countries with monetary transfers and providing low-performing countries with structural aid only. From a first-best perspective, our results suggest that promoting aid effectiveness goes in the opposite direction. Indeed, interpreting \( y \) as the willingness of the recipient to transfer a share of the revenue to the poor, we show that a country with a low \( y \) (generally, characterizing autocracies, corrupt regimes or countries with poor institutions and non-transparent administrative rules) should receive more monetary aid along with small projects. Moreover, the level of aid should be decreasing in the ability of the recipient country to transfer income to the poor. For richer countries it should involve a co-payment by the recipient towards the financing of a (larger) project. Such a schedule is however not incentive compatible and the main result of this paper is that the second-best optimum should take the form of a pooling aid contract.

This somewhat unusual result is due to two features of the screening problem in our model. First, recipients have type-dependent outside options (autarky), and the higher the type of the recipient the higher is his autarky payoff. Second, the donor and the recipient both benefit from higher transfers, and under perfect information this transfer should be decreasing in the type of the recipient country. This implies that the donor has little incentives to provide informational rents to the recipient country, yielding the result on the optimality of the pooling contract.

An interesting feature of the pooling contract is that it is immune to information leakages in a dynamic context. By contrast, other contracts, including matching grant policies, may lead to ratchet like effects. The evidence from
OECD data in the introduction suggests that both the intensity of aid and the relative proportion of budget aid change over time. If it is possible to partially observe how transfers are made between the rich and the poor, our framework suggests that the mix of budget and infrastructure aid may play an important role in a dynamic context. Once information about $y$ is sufficiently precise, as $y$ increases, the aid package should shift from a combination of positive monetary transfer and project to negative transfer and larger projects. However, the full analysis of a dynamic setting is beyond the scope of this paper.\textsuperscript{9}

The analysis in this paper can be extended along several dimensions. First, we have assumed that the projects require know-how that is not available to the recipient country. If the recipient country can develop projects on its own, there will still be a value to aid, and the fact that the donor can control the way the project is targeted towards the poor is useful, but the individual rationality condition will take a more complex form.

Second, we have assumed that the donor is unable to monitor the use that the recipient country will make of monetary transfers, but that the donor can implement the项目 without risk of manipulation by the recipient country. These are simplifying assumptions; what matters is that it is more difficult to verify whether there has been the right monetary transfer to the poor than to verify that a project (like a road infrastructure) has been built. Extending the model along these lines may be useful however since partial monitoring may lead to richer dynamics.

\textsuperscript{9}The literature is somewhat silent on the question of aid dynamic. See however Sraieb (2015) Ph.D. dissertation where he analyzes the empirical and theoretical determinants of the dynamic pattern of aid.
Appendix

Proof of Lemma 1

Consider \( y > \hat{y} \) and two aid packages \((T, \rho, \pi), (\hat{T}, \hat{\rho}, \hat{\pi})\) designed for countries of types \( y, \hat{y} \) respectively. Let \( U(y) \) be the utility of the recipient with the contract \((T, \rho, \pi)\). Then, if \( y \) chooses the contract \( \hat{T}, \hat{\rho}, \hat{\pi} \) that is intended for \( \hat{y} \), his utility is

\[
U(y, \hat{y}) = U(\hat{y}) + (1 + na) [\log(y + \hat{T}) - \log(\hat{y} + \hat{T})]
\]

and the incentive compatibility condition is \( U(y) \geq U(y, \hat{y}) \). There is a similar condition \( U(\hat{y}) \geq U(y, \hat{y}) \) for \( \hat{y} \), and therefore the incentive compatibility conditions for these two types can be written as

\[
(1 + na) [\log(y + T) - \log(\hat{y} + T)] \geq U(y) - U(\hat{y}) \geq (1 + na) [\log(y + \hat{T}) - \log(\hat{y} + \hat{T})] \tag{A.1}
\]

where (A.2) is the condition for \( y \) and (A.1) is the condition for \( \hat{y} \). Concavity of \( \log \) and \( y > \hat{y} \) imply that

\[
\hat{T} \geq T. \tag{A.3}
\]

Therefore, \( U(y) \geq U(\hat{y}) \) and we must have:

\[
\log \rho + na \log \pi \geq \log \hat{\rho} + na \log \hat{\pi}.
\]

Hence, if the transfers are different, it must be the case that they are decreasing in income, while the “aggregate” project must be increasing in income. This establishes the proof of part (i) of Lemma 1.

As for (ii), we consider different cases. If both \( y, \hat{y} \) are smaller than \( y^*(a) \), \( y + T^*(y) = \hat{y} + T^*(\hat{y}) \), and since the aggregate project is the same, \( U(y) = U(\hat{y}) \), but then (A.2) is violated since \( y > \hat{y} \).

If both \( y \) and \( \hat{y} \) are larger than \( y^*(a) \), the participation constraints bind and therefore \( U(y) - U(\hat{y}) = U(y) - U(\hat{y}) \), implying that the incentive conditions reduce to

\[
\log(y + T^*(y)) - \log(\hat{y} + T^*(\hat{y})) \geq \log(y) - \log(\hat{y}) \geq \log(y + T^*(\hat{y})) - \log(\hat{y} + T^*(\hat{y}))
\]

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implying that $T^*(y) < 0 < T^*(\hat{y})$, but $T^*(\hat{y}) > 0$ is not consistent with the first best transfer of recipient countries when $\hat{y} > y^*(a)$ (Proposition 1).

If $\hat{y} \leq y^*(a) < y$, $U(y) = U(\hat{y})$ and therefore $U(y) - U(\hat{y}) \leq U(y) - U(\hat{y})$, and (A.2) requires that $\log(y) - \log(\hat{y}) \geq \log(y + T^*(\hat{y})) - \log(\hat{y} + T^*(\hat{y}))$, which is possible only if $T^*(\hat{y})$ is positive, that is if $\hat{y} \leq \frac{1 + n}{\phi}$. In this case, $T^*(\hat{y}) = \frac{1 + n}{\phi} - \hat{y} \geq 0$, and we have:

$$U(y) - U(\hat{y}) = (1 + na) \left( \log y - \log \left( \frac{1 + n}{\phi} \right) \right) - (\log \rho^*(\hat{y}) + na \log \pi^*(\hat{y}))$$

violating (A.2).

**Proof of Proposition 2**

Ignoring individual rationality conditions for the moment, let us focus on the incentive problem. Recall that $P(y) \equiv \log \rho(y) + na \log \pi(y)$ is the aggregate value of the project for the recipient. Then, incentive compatibility requires that for $y \geq \hat{y}$,

$$(1 + na)(\log(\hat{y} + T(\hat{y})) - \log(\hat{y} + T(y)))$$

$$\geq P(y) - P(\hat{y}) \geq (1 + na)(\log(y + T(\hat{y})) - \log(y + T(y)))$$

Because we know that $T(y)$ is non-increasing and that $P(y)$ is non-decreasing, the two functions are differentiable almost everywhere. Considering a common point of continuity, we have

$$\lim_{\hat{y} \to y} \frac{T(\hat{y}) - T(y)}{y - \hat{y}} \log(\hat{y} + T(\hat{y})) - \log(\hat{y} + T(y))$$

$$\geq \lim_{\hat{y} \to y} \frac{P(y) - P(\hat{y})}{y - \hat{y}} \geq (1 + na) \lim_{\hat{y} \to y} \frac{T(\hat{y}) - T(y) \log(y + T(\hat{y})) - \log(y + T(y)))}{T(\hat{y}) - T(y)}$$

since the left and right bounds have a common limit, we have:

$$P'(y) = -(1 + na) \frac{T'(y)}{y + T(y)}.$$
which illustrates the negative co-variation of $T(y)$ and $P(y)$. If $y$ is the lower bound of the support of $y$,

$$P(y) = P(y) - (1 + na) \int_{y}^{y} \frac{T'(x)}{x + T(x)} dx;$$

$$U'(y) = \frac{1 + na}{y + T(y)} \text{ and } U(y) = U(y) + \int_{y}^{y} \frac{1 + na}{x + T(x)} dx. \tag{A.4}$$

This local condition, together with $T(y)$ non-increasing, is also sufficient for global incentive compatibility. Indeed, for any $y \geq \hat{y}$, we have

$$U(y) - U(\hat{y}) = \int_{y}^{y} \frac{1 + na}{x + T(x)} dx$$

Now, if $y \geq \hat{y}$, we have $T(\hat{y}) \geq T(y)$, and therefore $\int_{y}^{y} \frac{1 + na}{x + T(x)} dx \leq \int_{y}^{y} \frac{1 + na}{x + T(y)} dx$, implying $U(y) - U(\hat{y}) \geq (1 + na) \log(y + T(\hat{y})) - \log(\hat{y} + T(\hat{y}))$, and $y$ does not want to get the aid package of $\hat{y}$. Similarly, $\int_{y}^{y} \frac{1 + na}{x + T(y)} dx \leq \int_{y}^{y} \frac{1 + na}{x + T(y)} dx$, implying that $U(\hat{y}) \geq U(y) - (1 + na (\log(y + T(y)) - \log(\hat{y} + T(\hat{y}))))$, and $\hat{y}$ does not want to claim the aid package of $y$.

It follows that the objective function of the donor can be written as:

$$V = \int_{y}^{y} \left[ U(y) + (n - na) (\log(y + T(y)) + \log \pi(y)) - \log(1 + na) + \log a \right.$$

$$\left. - \phi T(y) - \phi c(\rho(y) + \pi(y) - 2) \right] f(y) \] dy$$

Using (A.4), and integration by part, we have:

$$\int_{y}^{y} U(y) dF(y) = U(y) + \int_{y}^{y} \frac{1 + na}{y + T(y)} (1 - F(y)) dy.$$

Therefore, the maximization problem of the donor is:

$$\max_{\{U(y), T(y), \rho(y), \pi(y)\}} V = U(y) + \int_{y}^{y} \left[ \frac{1 + na}{y + T(y)} \frac{1 - F(y)}{f(y)} ight.$$

$$\left. + (n - na) (\log(y + T(y)) + \log \pi(y) + \log a) - \phi T(y) - \phi c(\rho(y) + \pi(y)) \right] f(y) dy$$

subject to the participation constraints, the incentive constraint $T'(y) \leq 0$ and the limited liability condition $T(y) \geq -y$. Ignoring these constraints, pointwise
differentiation implies that:

\[
\frac{\partial V}{\partial T} = - \frac{1 + na}{(y+T)^2}(1 - F(y)) + \frac{n - na}{y+T} f(y) - \phi f(y)
\]

\[
\frac{\partial V}{\partial \rho} = -\phi cf(y)
\]

\[
\frac{\partial V}{\partial \pi} = \frac{n - na}{\pi} f(y) - \phi cf(y)
\]

Hence, it is optimal to set \( \rho(y) \equiv 1 \) and \( \pi(y) = \pi_a = \max\{1, \frac{n - na}{\phi c}\} \). Since \( P(y) \) is independent of \( y \), incentive compatibility (A.4) implies that \( T(y) \) is constant. In other words, it is not optimal to screen recipient countries based on \( y \). We solve for the optimal pooling contract in the text.
Preventing co-Financing

If we restrict the analysis to positive transfers, we disregard the possibility of co-financing projects. This section adopts this framework and investigates the effect on the optimal contract.

First-best analysis

If co-financing is not possible, the donor chooses \((T, \rho, \pi)\) to solve:

\[
\begin{align*}
\max_{T, \rho, \pi} & \quad (1 + n) \log (y + T) + \log \rho + n \log \pi + n \log a \\
& \quad - (1 + n) \log (1 + na) - \phi (T + c (\rho + \pi - 2)) \\
\text{s.t.} & \quad (1 + na) [\log (y + T) - \log y] + \log \rho + na \log \pi \geq 0 \\
& \quad T \geq 0 \quad (A.5)
\end{align*}
\]

Let \(\mu\) and \(\lambda\) be the Lagrangian multipliers associated to the IR and the positivity constraint, respectively. Then, when the IR does not bind, the optimal lump-sum transfer is

\[
T^*(y) = \frac{1 + n}{\phi - \lambda(y)} - y, \quad (A.6)
\]

If the positivity constraint does not bind \((\lambda = 0)\), the maximization yields an optimal transfer as in (4) corresponding to first-best without restrictions on \(T\) \((T^*(y) = \frac{1 + n}{\phi} - y)\). When the positivity constraint binds, then \(T = 0\).

Now, when IR binds, the optimal transfer is

\[
T^*(y) = \frac{1 + n + \mu(y)(1 + na)}{\phi - \lambda(y)} - y, \quad (A.7)
\]

which is negative, for all \(y > \frac{1 + n + \mu(y)(1 + na)}{\phi - \lambda(y)}\). Therefore, if the positivity constraint does not bind, \(y > \frac{1 + n}{\phi}\), and \(T = 0\). When the positivity constraint binds, the transfer is null.

To sum up, for poor countries (those having a revenue \(y \leq \frac{1 + n}{\phi}\)), the transfer is as in the first-best without restrictions on \(T\) and the optimal projects are as in (5), which gives
\[ T^\ast(y) = \frac{1 + n}{\phi} - y \]
\[ \pi^\ast(y) = \frac{n}{\phi c} \]
\[ \rho^\ast(y) = \frac{1}{\phi c} \]

The aid will be accepted since for any \( y \leq \frac{1+n}{\phi} \), the welfare level for the recipient is larger than the outside option.

For less poor countries (those having a revenue \( \frac{1+n}{\phi} > y \)), \( T = 0 \). These recipients receive constant projects as long as \( y \leq y^\ast(a) \). The projects are accepted only if

\[
\log \pi \geq \frac{\log \phi c}{na} \\
\log \rho \geq \frac{\log \phi c}{n^2a}
\]

Notice that the second hand member is negative, meaning that any positive project is accepted by the recipient. The optimal allocation is then

\[ T^\ast(y) = 0 \quad (A.8) \]
\[ \pi^\ast(y) = \frac{n}{\phi c} \]
\[ \rho^\ast(y) = \frac{1}{\phi c} \]

However, if \( y > y^\ast(a) \), both the participation constraints and the positivity constraint bind. The transfer is null and the projects are increasing in \( y \), i.e. less poor countries receive larger projects. These are accepted by the recipient. The optimal allocation is then

\[ T^\ast(y) = 0 \quad (A.9) \]
\[ \pi^\ast(y) = \frac{n(1 + \mu^\ast(y)a)}{\phi c} \]
\[ \rho^\ast(y) = \frac{1 + \mu^\ast(y)}{\phi c} \]

**Proposition 5.** When co-financing is not possible, only poor countries end up
receiving a positive transfer. These countries receive constant projects, much as under the first-best with no restrictions on transfers. They are made strictly better off by aid. Furthermore, as \( y \) increases in \([0, \frac{1+n}{\phi}]\), the transfer to the recipient decreases but the project is independent of \( y \). As for less poor countries (those having a revenue larger than \( \frac{1+n}{\phi} \), but \( y \leq y^*(a) \)), they do not receive transfers (\( T = 0 \)). Projects are accepted. For even richer countries (those for which \( y > y^*(a) \)), the transfer is null but the projects are increasing in revenue.

![Figure 2: First-Best without Co-financing](image)

**Second-best**

Here again, (A.1) and (A.3) still hold. For poor recipients, if the transfers are different, it must be the case that they are decreasing in income, while the “aggregate” project must be increasing in income. We find back the same situation as when no restrictions were imposed on \( T \). For richer recipients, the transfers are null (much as in the first-best). As for the “aggregate” project, \( U(y) - U(\hat{y}) = (1 + na)[\log y - \log \hat{y}] \geq 0 \) and the positivity constraint binds. Therefore, the aggregate project must be constant. Indeed, the IC condition translates to

\[
\log \rho(y) + na \log \pi(y) - \log \rho(\hat{y}) + na \log \pi(\hat{y}) = 0.
\]

**(A.10)**

**Lemma 3.** (i) for poor countries, incentive compatibility requires that \( T(y) \) be non-increasing in \( y \) and \( \rho(y) + na\pi(y) \) to be non-decreasing in \( y \). For
richer countries, transfers must be null and the aggregate project must be constant.

(ii) For any value of $a, y, \hat{y}$, the first-best is not incentive compatible.

Proof. (i) has been established in the text. For (ii), we consider different cases.

If both $y, \hat{y}$ are smaller than $\frac{1+n}{\phi}$, $y + T^*(y) = \hat{y} + T^*(\hat{y})$, and since the aggregate project is the same, $U(y) = U(\hat{y})$, but then (A.2) is violated since $y > \hat{y}$.

If both $y$ and $\hat{y}$ are larger than $\frac{1+n}{\phi}$ but are smaller than $y^*(a)$, the participation constraints do not bind but the positivity constraint does. Therefore, $T = 0$, and since the project is the same, $U(y) = U(\hat{y})$, but then (A.2) is violated since $y > \hat{y}$.

If both $y$ and $\hat{y}$ are larger than $y^*(a)$, then both the participation constraints and the positivity constraint bind, $T^*(y) = T^*(\hat{y}) = 0$ and $U(y) - U(\hat{y}) \leq U(y) - U(\hat{y})$. (A.3) and (A.2) then reduce to $(1+na)[\log y - \log \hat{y}] = U(y) - U(\hat{y})$, meaning that we must have constant aggregate projects. A contradiction with the fact that projects are actually increasing in $y$.

If $\hat{y} \leq \frac{1+n}{\phi} < y$, the positivity constraint binds only for type $y$ recipient $(T = 0)$. Recall that $y = \hat{y} + T^*(\hat{y})$ since $T^*(y)$ is null. Therefore, (A.3) and (A.2) yield $(1+na)[\log y - \log \hat{y}] \geq U(y) - U(\hat{y}) \geq (1+na)[\log(y + T(\hat{y}) - \log(\hat{y} + T(\hat{y}))].$ The second inequality implies $\log \rho + na \log \pi + (1+na) \log y \geq \log \hat{\rho} + na \log \hat{\pi} + (1+na) \log \hat{y}$. This yields an aggregate project decreasing in $y$ (since $T(\hat{y}) > 0$), violating the requirement on constant aggregate project.

If $\hat{y} \leq y^*(a) < y$, then the participation constraints for $y$ and the positivity constraint bind for both types, $T^*(y) = T^*(\hat{y}) = 0$ and $U(y) - U(\hat{y}) \geq (1+na)[\log y - \log \hat{y}]$. This yields $(1+na)[\log y - \log \hat{y}] \geq -[\log \hat{\rho} + na \log \hat{\pi}] \geq (1+na)[\log y - \log \hat{y}]$, a contradiction.

Lemma 4. For poor countries ($y \leq \frac{1+n}{\phi}$), incentive compatibility requires that $T(y)$ be non-increasing in $y$ and $\rho(y) + na\pi(y)$ to be non-decreasing in $y$. For these countries, the case runs exactly as under Lemma 1. As for richer countries, because we know that $T = 0$. Incentive compatibility requires that $P(y) - P(\hat{y}) = 0$, and type separation is not possible.

Proof. The proof has been established in the text.
References


