



Mergers and the Dynamics of Innovation

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MERGERS AND THE DYNAMICS OF INNOVATION ^{*}

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Abstract

This paper analyzes the impact of mergers on innovation in the context of a dynamic oligopoly. We show that the distribution of technological states has a fundamental impact on innovation. If some states are very innovative, but extremely unlikely, they do not contribute to the average level of innovation. Mergers can decrease innovation. This effect is particularly severe for mergers to duopoly. More efficient R&D by dominant players harms consumers. Mergers that increase rivalry and leave enough firms in the market are more likely to foster innovation. However, these dynamic benefits to consumers are not sufficient to outweigh the corresponding price increases.

Keywords: mergers, innovation, dynamic models, consumer welfare

JEL: C61, G34, O31

Introduction

On 19 April 2011, Seagate notified to the European Commission the acquisition of the hard disk drive ("HDD") business of Samsung Electronics. The day after, on 20 April 2011, Western Digital ("WD") notified the acquisition of Hitachi's ("HGST") HDD business. Taken together, these two mergers are tantamount to a move from five to three major players in the market for Mobile (2.5") HDD and from four to two in the market for Desktop (3.5") HDD. While the first acquisition was accepted unconditionally, the second was accepted provided that Western Digital would divest assets for Desktop (3.5") HDD production. This business was subsequently acquired by Toshiba, already active on the Mobile (2.5") HDD market.¹ The main rationale for these decisions was to safeguard the existence of a third viable actor on every market. The European Commission's analysis showed that the Original Equipment Manufacturers ("OEM"), who are the main customers of HDD manufacturers, multi-source their HDD in order to limit their operational risks of the shortage of supply. The Commission's remedy explicitly mentions

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¹Non-confidential final decisions are available on the European Commission's website both for Seagate's acquisition (M6214) and Toshiba's acquisition of parts of WD's Desktop HDD business (M6531), while only the press release is available so far for WD's notified acquisition of HGST's HDD business (M6203).

the necessity to have a third player with research ability such that the third player is not rapidly outdistanced and so that the remedy is persistent.

However, the Commission's remedy did not explicitly intend to address direct consequences of the acquisitions on the pace of innovation itself. Naturally, innovation should not replace the more classical issue of unilateral effects and incentives to increase prices in any technological state when a market concentrates. Nevertheless, "quality", and hence innovation, are also key drivers of consumer welfare in this type of industries. Therefore, if the merger adversely affects the pace of innovation, this leads to significant consumer harm. In fast-growing, innovative industries, it is therefore necessary to also address the impact of mergers on firms' incentives to innovate. Decreased incentives to innovate make market power more persistent and thereby adversely impact the industry's aggregate rate of innovation. Yet, despite these policy stakes, economic literature so far has not addressed this issue.

This paper intends to fill this gap. It shows that some mergers can indeed increase the dynamic rivalry and hence increase the pace of innovation in the market. However, this always comes at a cost of higher prices and, overall, the welfare effect of these mergers still tends to be detrimental to consumers. Moreover, mergers to duopoly always very dramatically decrease rivalry between firms and, overall, the dynamic effect adds to the classical detrimental effects of mergers. To sum up, it is not unlikely that an unconditional clearance of the two mergers in the HDD market would have significantly reduced the pace of innovation in the market.

Mainstream theoretical literature on endogenous growth (Romer 1990, Aghion and Howitt 1992, Grossman and Helpman 1991) argues that competition decreases innovation and hence growth. These results rely on the classical Schumpeterian intuition that incentives to innovate are highest when innovation is drastic and the innovator becomes a monopolist. Thus, in a nutshell, reducing ex-post rents decreases the ex-ante incentives to innovate, and for this reason, in the long run, antitrust is counterproductive.

Even though this general argument is still influential, it finds little empirical support. Kamien and Schwartz (1972), as well as Cohen and Levin (1989), find that the impact of "competition" on innovation might be positive or negative. This depends on the market characteristics, such as its structure or technologies, for instance, and the features of the potential innovations. Moreover, Nickell (1996), Blundell, Griffith, and Van Reenen (1995) and Blundell, Griffith, and Van Reenen (1999) find indications of positive correlation between their measures of product market competition and measures of innovation.

The economic intuitions which are consistent with these empirical findings go beyond the pure Schumpeterian incentives to innovate. First, a monopolist has a low incentive to innovate, as by doing so, it mainly destroys his own previous innovation. This is the *replacement effect* (Arrow 1962). Second, if innovation or entry are not drastic, the returns from innovation are weakened by the necessity to share duopoly profits. This is the *efficiency effect* (Gilbert and Newbery 1982).

While all these economic intuitions are perfectly relevant and consistent with each other, they provide competing results as regards to innovation. If the symmetric duopoly profit is very low, then escaping this very intense competition by innovating is profitable and the Schumpeterian effect is maximal. However, then a firm that has a lower technological level has little incentives

to enter the market or to catch up. If the symmetric duopoly profit is very high, the opposite happens. Overall, competition has a different effect on the incentives to innovate for firms of different technological states.

Therefore, it is necessary to build a framework that encompasses all these effects in order to deal with the link between profits and innovation. Aghion, Harris, Howitt, and Vickers (2001) present an elegant dynamic framework to tackle these issues. This modelling exercise is at the basis of the interpretation of Aghion, Bloom, Blundell, Griffith, and Howitt's (2005) empirical results. It is the most complete contribution of the research path also including, among others, Aghion, Harris, and Vickers (1997) and Budd, Harris, and Vickers (1993).²

Aghion, Harris, Howitt, and Vickers (2001) model a duopoly where two firms can differ by a number of technological levels. In the simpler version of this model, firms can only differ by one technological level. The market can then be *levelled*, i.e. the two symmetric (or *neck-and-neck*) firms get symmetric duopoly profits. Alternatively, the market can be *unlevelled*, i.e. one firm gets the leader's profit, and the second one is lagging behind. Then, in each state, in a continuous time framework, firms may invest in costly and uncertain innovation. The comparative statics are straightforward. The leading firm cannot increase her advance and has no incentive to innovate. For given profits for the leader and for the laggard, the laggard's incentive to innovate is increasing in the symmetric duopoly profit. Conversely, for neck-and-neck firms, incentives to innovate are decreasing in the symmetric duopoly profit. This generates interesting dynamics. As a result of these incentives, the probability to be in levelled markets is larger when the symmetric duopoly profit is *ceteris paribus* larger. In particular, this results in an inverted-U pattern between innovation and symmetric duopoly profits when the other profits are fixed.

Aghion, Bloom, Blundell, Griffith, and Howitt (2005) use this simplified version of Aghion, Harris, Howitt, and Vickers (2001) to explain their empirical finding of an inverted-U relationship between competition and innovation. More generally, Aghion, Harris, Howitt, and Vickers (2001) analyze the situation where two firms compete in imperfectly substitutable quantities and innovate to lower their costs. In this setup, not only the symmetric duopoly profits, but also the leader's and laggard's profits change whenever the taste parameters vary. In the version restricted to two levels of innovation with large innovations, Aghion, Harris, Howitt, and Vickers (2001) find the same qualitative pattern between the taste parameter and the average innovation in each market (and hence growth in the economy). However, when considering smaller innovations, proximity of tastes always enhances innovation, even though with decreasing returns (and there is an inverted U in the rate of imitation). Overall, the authors summarize their findings on the effect of product market competition ("PMC") on growth as follows: "*In short, our findings are that the effect of PMC on growth usually is monotonically positive, but sometimes is inverse-U shaped*" (Aghion, Harris, Howitt, and Vickers 2001, p. 470).

Aghion, Bloom, Blundell, Griffith, and Howitt (2005) and its theoretical counterpart Aghion, Harris, Howitt, and Vickers (2001) are milestone contributions on the subject because they show that it is not only post-innovation rents that matter, but rather the difference between the post and the pre-innovation rents. However, Aghion, Harris, Howitt, and Vickers (2001) claim that

²In this paper, we focus on the direct link between profitability of innovation and incentives to innovate. However, another strand of the literature also emphasized that competition is a way for shareholders to discipline firms' managers (Aghion, Dewatripont, and Rey 1999a, Aghion, Dewatripont, and Rey 1999b).

their model allows to generally address the link between competition and innovation. More precisely, they describe their parameter of demand α as follows: *“Although α is ostensibly a taste parameter, we think of it as proxying the absence of institutional, legal and regulatory impediments to entering directly into a rival firm’s market by offering a similar product. Under this interpretation, α reflects, in particular, the influence of anti-trust policy”*.

This interpretation of a structural inverted-U relationship between competition and innovation is debatable. Boutin (2012a) shows with a very simple framework that different taste parameters, that increase rivalry between firms, have a different impact on the flow of innovation. Thus, there is no mapping between measures of competition based on taste parameters, or firms’ profits, and the rate of innovation. Moreover, even if there was such a mapping, there is no public policy that addresses differentiation, nor, as such, price cost-margins. Competition policy luckily does not intend to influence citizens’ opinions about consumer goods. It aims at ensuring that markets work as well as they can, given customers’ tastes and firms’ intrinsic technical constraints. The antitrust enforcer could, for instance, ask what is the influence of foreclosure on the rate of innovation. A duopoly game parametrized by a taste parameter would only be useful here if one could deduce what change of a taste parameter corresponds to the exclusion of a competitor.

Boutin (2012a) describes why attempts to model the influence of competition between firms by taste parameters or, empirically, by Lerner index or price cost margins, cannot provide useful insights for policy. The very wide gap between the literature on "competition and innovation" and what competition policy really aims at has also been described by Shapiro (2011). Therefore, one has to directly address each type of common abuse on its own merits, for instance as Goettler and Gordon (2011) do for foreclosure and Nocke (2007) for collusion. These two examples also show that looking for a structural link between competition and innovation is a quixotic effort: while collusion reduces innovation, foreclosure could in certain situations increase it (even though this does not compensate the increase of prices for consumers).³

In this paper we propose to directly analyze the issue of mergers on the dynamics of innovation. The relevance of the question has been pointed out, among others by Katz and Shelanski (2005), Katz and Shelanski (2007) and Shapiro (2011). It goes beyond the traditional discussion on whether the innovativeness of a market calls for a more or less lenient approach to merger control and tries to qualify, in a fully dynamic model, the parameters that matter for the assessment. Because it focuses on the dynamics of a specific industry, it is naturally related to earlier dynamic industry simulations, such as Pakes and McGuire (1994) and Ericson and Pakes (1995), which were surveyed by Doraszelski and Pakes (2007). However, our paper does not focus on market structure, but rather, on the endogenous process of step-by-step innovation. Therefore, it is more closely related to Aghion, Harris, Howitt, and Vickers (2001) and Goettler and Gordon (2011). However, it differs from these two strands of literature in several important ways. This paper focuses on a discrete time oligopoly without entry. This is a pure step-by-step innovation model without leapfrogging. Moreover, contrary to Goettler and Gordon (2011), we do not treat goods as pure durable goods. These modelling choices derive from factual and theoretical reasons, which are explicitly explained in the relevant parts of the paper. In many instances, we

³See a more general discussion on the link with Goettler and Gordon (2011) in section 4.3

will refer to the example of the HDD sector that we have mentioned before. Nevertheless, the analysis developed in this paper and its intuitions apply to many other sectors. Moreover, the model is flexible enough to reflect stylized features of other markets.

Analyzing mergers requires setting up a flexible model of demand for an arbitrary large number of asymmetric firms, owning several lines of products. Then in this model we also allow for investment in quality and characterize the dynamic equilibrium under pure Markov strategies. This can be used to numerically compute the equilibrium and the distribution of states. We do this in section 1. Then, section 2 presents descriptive statistics and shows that the distribution of states has a dramatic influence on the outcome of innovation. This could not be observed in previous research, which applied static frameworks. If some states are very innovative but extremely unlikely, they do not contribute to the average level of innovation. Importantly, we find that a duopoly is, on average, very un-innovative. Then, we discuss the welfare effect of mergers in section 3. Overall, this model allows drawing general conclusions, in particular, as regards the effects of mergers to duopolies, which are always detrimental to consumers because the dynamic effect adds harm to the classical static price increase. Moreover, this model can be calibrated on a case by case basis to reflect the factual context of a given merger and provides evidence as regards the likely effect of the merger on innovation and its aggregate effect on consumer welfare.

1 Model

1.1 Model of demand: logit with competitive fringe

We model N firms competing in a product market and facing demand derived from a random utility model (Mc Fadden 1978). Logit demand is a simple, flexible and parsimonious functional form and, for this reason, it is very often used in applied industrial organization. Moreover, random utility models also allow linking demand with consumer surplus, which is key for the analysis of the welfare effect of mergers. Customers value (vertical) quality, as well as other product characteristics. They also have a disutility to pay. Overall, the utility of a customer i buying product j is:

$$U_{ij} = \tilde{k}_i L - \sigma p_i + \xi_i + \epsilon_{ij}, \quad (1.1.1)$$

where L is the taste for quality, \tilde{k}_i is the technological level of product i , p_i the price of product i , σ a scaling parameter of the disutility to pay and ξ_i a fixed parameter that captures the mean utility value of the fixed characteristics of product i apart from the technological level and prices. Then, ϵ_{ij} is the idiosyncratic component of the utility of customer j when he buys product i . The vector (ϵ_{ij}) is assumed to be independently and uniformly distributed following a Gompertz distribution. This form of random utility is a simple way to generate demand for vertically differentiated products while accounting for persistent asymmetries between products through the fixed effect ξ .

For the purpose of finding equilibrium prices for a given pattern of technologies, we assume

that firms can have several products characterized by potentially different technological levels. The total number of products is $Nprod$. We also assume that there exist a competitive fringe, o , marketing a good at zero price, whose technological level is at most K levels behind the most advanced of all the products $i \in \{1, \dots, Nprod\}$ and at least one level behind the least advanced. This competitive fringe plays the role of the outside good and ensures that the demands addressed to the various firms do not depend on their absolute technological levels, but only on their relative ones. This is necessary to generate stationary demand, given that we want to focus on stationary equilibria and distributions of states. Moreover, it also reflects the stylized fact that there normally exist some products of inferior technology which are sold by competitors at a much lower price. This could be due to patent expiry, for instance, or simply a result of imitation.⁴

We denote the technological level of the fringe by k_o and define $k_i = \tilde{k}_i - k_o$. Then, when we normalize the size of the market to 1, the market share of good i is the probability that customers choose product i :

$$s_i = \frac{e^{k_i L - \sigma p_i + \xi_i}}{1 + \sum_{m \in \{1, \dots, Nprod\}} e^{k_m L - \sigma p_m + \xi_m}} \quad (1.1.2)$$

The profit of firm f , that own products \mathcal{I}_f , is:

$$\Pi_f = \sum_{m \in \mathcal{I}_f} p_m s_m$$

It follows that the first order condition for the price of product i belonging to firm f is:

$$p_i^* = \frac{1}{\sigma} + \sum_{m \in \mathcal{I}_f} p_m^* s_m^* \quad (1.1.3)$$

Even though there exist no analytical solution to equation 1.1.3, this can be easily solved for any set of parameter $L, (k_i), \sigma$, using the natural contraction mapping, starting from $\frac{1}{\sigma}$:

$$p_i^{n+1} = \frac{1}{\sigma} + \sum_{m \in \mathcal{I}_f} p_m^n s_m^n$$

1.2 Dynamic game

1.2.1 Innovation

Firms can invest to increase the (vertical) qualities of their products. In this context, from a period to another, each product keeps all its other characteristics, captured by ξ_i , but its quality could increase. One could see the products in our model as brands and ξ_i as their time invariant reputation. We assume that this process of innovation takes place in discrete time. Boutin (2012a) shows that the choice of a continuous time model makes the game more tractable but, for purely technical reasons, rules out static strategic interactions, and, more generally, any effect related to simultaneous innovations. This is a general issue with continuous time models, which are not adequate to capture important heuristics (see, for instance, Fudenberg and Tirole 1985).

⁴An alternative way to make demand stationary is to allow firms to continue selling products from previous generations. However, this does not create the type of constraint that is generated by the competitive fringe.

We focus on the case of a stable oligopoly with a ladder of innovation. The same firms interact over a large number of periods and innovate gradually to improve their products' qualities. Therefore, there is no entry, or leapfrogging, i.e., firms have to climb each step of the ladder. We believe that this corresponds to reality in technology markets. It is normally necessary for competitors to patiently climb a ladder. For instance, both for HDD and for CPUs, a large driver of quality improvement is related to density and this is acquired through gradual improvements in technology. Moreover, entry only happens in the long term and is normally associated with a disruptive technology, such as, for example, LCD for TV or flash memory for HDD. Even then, entry is gradual and associated with a (patient) series of improvements. For example, given the barrier to entry created by the number of patents required to produce a modern HDD, it is practically impossible for a new actor to enter into the HDD market. Entry here could only occur, for instance, from a firm producing SSD. Only when, through gradual innovations, the capacity of SSD increased, SSD would in the long run become a competitor of HDD in a "mass storage" market. As we exclude entry and leapfrogging, we abstract from this long term emergence of new players with new technologies. Our model applies to the medium term of industries, where one established technology, controlled by a given set of actors, improves sequentially. This is the relevant time dimension for competition policy and merger control.

We also assume that firms cannot differ of more than K technological levels. This is a necessary requirement to solve the game numerically. Firm f can invest in period t to increase the technological level of all its products by a given increment. The cost of R&D is assumed to be quadratic in the probability of success: f has to invest $\frac{\gamma n_f^2}{2}$ to succeed with probability n_f .⁵

For a given number of N firms and a maximum spread of technological levels K , we denote by Σ the set of states and by Ω the set of all the possible outputs of innovation from one period to another. For example, with two firms and a maximum gap of one technological level, we have:⁶

$$\Sigma = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

We limit ourselves to pure strategy Markov equilibria, where all the players play pure Markov strategies. Thus, we can write the value function of firm f in state s in period t as:

$$V_{f,s}^t = \max_{n_{f,s}^t} \left\{ \pi_{f,s} - \frac{\gamma n_{f,s}^t{}^2}{2} + \delta \left(\sum_{\omega \in \Omega} \mathbb{P}(\omega|s, \mathcal{N}_s^t) V_{f, \mathcal{T}(s, \omega)}^{t+1} \right) \right\} \quad (1.2.1)$$

where:

$$\mathbb{P}(\omega|s, \mathcal{N}_s^t) = \prod_{f' \in \mathcal{F}} \left\{ n_{f',s}^t \omega_{f'} (1 - n_{f',s}^t)^{(1 - \omega_{f'})} \right\}$$

is the probability of outcome ω in state s , with the profile of actions $\mathcal{N}_s^t = \left(n_{f,s}^t \right)_f$ and $\mathcal{T}(s, \omega)$

⁵The game described here is a generalization of Boutin (2012b).

⁶Note that all the products of each firm have the same technological levels, so Σ and Ω solely depend on the number of firms, irrespective of the number of products each of them owns.

is the state resulting from the outcome ω in state s .

Focusing on a pure strategy Markov equilibrium, we can infer the optimal action of firm f by derivating $V_{f,s}^t$ with respect to $n_{f,s}^t$. We then get

$$n_{f,s}^t = \frac{\delta}{\gamma} \left(\sum_{\omega \in \Omega} \frac{\partial \mathbb{P}(\omega|s, \mathcal{N}_s^t)}{\partial n_{f,s}^t} V_{f, \mathcal{T}(s, \omega)}^{t+1} \right) \quad (1.2.2)$$

where:

$$\frac{\partial \mathbb{P}(\omega|s, \mathcal{N}_s^t)}{\partial n_{f,s}^t} = (1 - 2\omega_f) \underbrace{\prod_{f' \in \mathcal{F}_{-f}} \left\{ n_{f',s}^t \omega_{f'} (1 - n_{f',s}^t) (1 - \omega_{f'}) \right\}}_{P_{-f}(\omega|s, \mathcal{N}_s^t)}$$

where F_{-f} classically refers to the set of the remaining firms. For the rest of the paper, we denote:

$$\begin{aligned} P_{-f}(\omega|s, \mathcal{N}_s^t) &= \mathbb{P}(\omega|s, \mathcal{N}_s^t, \omega_f) \\ &= \prod_{f' \in \mathcal{F}_{-f}} \left\{ n_{f',s}^t \omega_{f'} (1 - n_{f',s}^t) (1 - \omega_{f'}) \right\} \end{aligned}$$

The derivative of the objective function is linear and the second derivative ($-\gamma$) unambiguously negative. Therefore, $n_{f,s}^t$ is a global maximum. For internal consistency reasons, probabilities of success can neither be negative, nor larger than one. Because of the convexity of the objective function, the optimal action is therefore $n_{f,s}^t = 0$ in case $\frac{\delta}{\gamma} \left(\sum_{\omega \in \Omega} \frac{\partial \mathbb{P}(\omega|s, \mathcal{N}_s^t)}{\partial n_{f,s}^t} V_{f, \mathcal{T}(s, \omega)}^{t+1} \right) \leq 0$ and, conversely, $n_{f,s}^t = 1$ in case $\frac{\delta}{\gamma} \left(\sum_{\omega \in \Omega} \frac{\partial \mathbb{P}(\omega|s, \mathcal{N}_s^t)}{\partial n_{f,s}^t} V_{f, \mathcal{T}(s, \omega)}^{t+1} \right) \geq 1$. With the parameters we choose, the constraints never bind. However, in the algorithm presented further down, these constraints have to be taken into account. To simplify the exposition, we however, focus on the interior solution in this section. Moreover, the profits do not change over time. Therefore, value functions and actions are also constant over time. To simplify the exposition, we also drop all the time indexes in the remaining of this article.

To understand the intuitions of the model, let us focus again on the example with two symmetric firms and one maximum technological gap. These firms can either be laggards (-1) or leaders (1) in the $(1, 2)$ or $(2, 1)$ unlevelled states or neck-to-neck (0) in the levelled case $(1, 1)$. The equilibrium is characterized by $n_{-1}^*, n_0^*, n_1^*, V_{-1}^*, V_0^*, V_1^*$, which is the solution of:

$$\begin{cases} \gamma n_{-1}^* &= \delta(1 - n_1^*) (V_0^* - V_{-1}^*) \\ \gamma n_1^* &= \delta n_{-1}^* (V_1^* - V_0^*) \\ \gamma n_0^* &= \delta((1 - n_0^*) (V_1^* - V_0^*) + n_0^* (V_0^* - V_{-1}^*)) \end{cases}$$

and:

$$\begin{cases} (1 - \delta)V_{-1}^* &= \Pi_{-1} - \frac{\gamma}{2} n_{-1}^{*2} + \delta(1 - n_1^*) n_{-1}^* (V_0^* - V_{-1}^*) \\ (1 - \delta)V_1^* &= \Pi_1 - \frac{\gamma}{2} n_1^{*2} - \delta(1 - n_1^*) n_{-1}^* (V_1^* - V_0^*) \\ (1 - \delta)V_0^* &= \Pi_0 - \frac{\gamma}{2} n_0^{*2} + \delta n_0^* (1 - n_0^*) (V_1^* - V_0^* - (V_0^* - V_{-1}^*)) \end{cases}$$

Therefore, the efforts of firms to innovate are directly determined by their expected returns from innovation. For instance, for a laggard, the return from innovation is $V_0^* - V_{-1}^*$ if the leader fails to innovate and zero otherwise (even if they both succeed, the laggard remains behind). In expected term, this return is then $(1 - n_1^*) (V_0^* - V_{-1}^*)$. For the leader, the return is associated

with losing its leadership. For neck-and-neck firms, it is determined as a weighted average of the value from moving ahead ($V_1^* - V_0^*$) and from not being left behind ($V_0^* - V_{-1}^*$). The weight is determined by the probability that the other firm succeeds (n_0^*). Moreover, the flow of value is the current profit minus the current R&D costs, plus the expected discounted return.

Similarly, as in the example with two symmetric firms, it is generally useful to rewrite (1.2.2) in the following way:

$$n_{f,s} = \frac{\delta}{\gamma} \left\{ \sum_{\omega \in \Omega} P_{-f}(\omega | s, \mathcal{N}_s^t) \left(V_{f, \mathcal{T}(s, 1, \omega_{-f})} - V_{f, \mathcal{T}(s, 0, \omega_{-f})} \right) \right\} \quad (1.2.3)$$

where $\omega = (\omega_f, \omega_{-f})$ has been separated into the component of the outcome for firm f (ω_f) and the vector of outcomes for the remaining firms (ω_{-f}).

Then, the effort of firm f is directly influenced by the weighted average benefits from innovating, the weights being the conditional probabilities of the outcomes.⁷ This weighted average directly depends on the actions of the other firms as their efforts make some outcomes more or less likely.⁸ The equilibrium is then given by equations (1.2.1) and (1.2.3). Even in the simpler case, with two firms and two technological levels, it is not possible to find a closed form solution to this equilibrium. However, as in Boutin (2012b), it is possible to find a numerical solution through an iterative process. Starting from an economy without innovation, where $V_{f,s} = \frac{\pi_{f,s}}{1-\delta}$ and $n_{f,s} = 0$, we then iteratively:

1. update $n_{f,s}$ using equation (1.2.3)⁹,
2. update $V_{f,s}$ using equation (1.2.1) with the $n_{f,s}$ updated in step 1.

Even though this process is generally not a contraction mapping, it converges fairly quickly and the simulations never show that the solution depends on the initial point. This finding is in line with the literature (see, for instance, Doraszelski and Pakes 2007). Last, for the range of parameters we consider, constraints never bind.

1.2.2 Distribution of states

It is then relevant to examine the ergodicity of the process of states. If the process is ergodic, one can indeed focus on the (unique) stationary distribution of states and on the aggregate average innovation, as in Aghion, Harris, Howitt, and Vickers (2001) and Aghion, Bloom, Blundell, Griffith, and Howitt (2005). To understand the issue of ergodicity, it is useful to once more first focus on the simple example of two symmetric firms with one maximum technological gap, which is also the framework of Aghion, Bloom, Blundell, Griffith, and Howitt (2005). Then,

⁷It is generally the case that $V_{f, \mathcal{T}(s, 1, \omega_{-f})}$ is larger than $V_{f, \mathcal{T}(s, 0, \omega_{-f})}$: innovation generally increases profits.

⁸Such strategic interactions cannot be found in continuous time models. There, simultaneous innovations are of second order: If firm f has a density of probability of success n_f and firm f' a density $n_{f'}$, they succeed between t and $t + dt$ with probability $n_f n_{f'} dt^2$, which is negligible. As a consequence, in continuous time models, the action of one firm is only indirectly influenced by the actions of others through value functions. With two firms and two technological states, Boutin (2012b) shows that neglecting the direct strategic interactions leads to very different heuristics.

⁹It is necessary to take the constraints $0 \leq n_{f,s} \leq 1$ into account in this process.

there are in substance two states: *levelled* (L) or *unlevelled* (U), and we have:

$$\begin{aligned}
\mathbb{P}\{s_t = L\} &= P_{L \rightarrow L} \mathbb{P}\{s_{t-1} = L\} + P_{U \rightarrow L} \mathbb{P}\{s_{t-1} = U\} \\
&= (1 - P_{L \rightarrow U}) \mathbb{P}\{s_{t-1} = L\} + P_{U \rightarrow L} (1 - \mathbb{P}\{s_{t-1} = L\}) \\
&= P_{U \rightarrow L} + (1 - (P_{L \rightarrow U} + P_{U \rightarrow L})) \mathbb{P}\{s_{t-1} = L\} \\
&= \frac{P_{U \rightarrow L}}{P_{L \rightarrow U} + P_{U \rightarrow L}} \left(1 - (1 - (P_{L \rightarrow U} + P_{U \rightarrow L}))^{t+1}\right) \\
&\quad + (1 - (P_{L \rightarrow U} + P_{U \rightarrow L}))^t \mathbb{P}\{s_0 = L\}
\end{aligned}$$

Then, as long as $P_{L \rightarrow U}$ and $P_{U \rightarrow L}$ are not both equal to 0 or 1, the distribution of states converges to a stationary distribution:

$$\mathbb{P}\{s_t = L\} \xrightarrow{t \rightarrow \infty} \frac{P_{U \rightarrow L}}{P_{L \rightarrow U} + P_{U \rightarrow L}}$$

Moreover, if one exits much faster from any given state, as compared to others, there is a much smaller probability to be in this state. Therefore, the imbalance between the levels of innovation in different states is reflected in the distribution of states. If one state is much more innovative than the other states, the stationary probability to be in this state is small.

Even though our general model is more complicated, the same intuitions apply. A stationary distribution is in the eigen space corresponding to 1 of the transpose of the Markov transition matrix. Each row of this matrix sums to 1 and clearly 1 is an eigen value. Then, this is a general result that for any finite state Markov chain, there exists a stationary distribution. In the example before, if there is no transition, any distribution is stable. Otherwise, if transition happens with probability 1, the uniform distribution is stationary. The mere existence of a stationary distribution is of limited interest. What really matter is whether the system is ergodic (or globally stable). Then, the distribution always converges to the only stationary distribution and the initial distribution of states has no persistent influence.

As the simple example indicates, there might be a problem if the system is periodic or if there is no transition from some states to others. In the first case, there are cycles of states while, in the second case, states are completely separated. However, this is not the case in practice here. In our model, the probability to come closer to the perfectly levelled state is typically strictly positive. Therefore, each state is connected to the perfectly levelled state in at maximum K steps. This means that, starting from any point, any two chains meet with positive probability after at least K periods and all states are connected with each other. The process s_t is then ergodic (see Theorem 4.3.18, Stachurski 2009). The intuition behind this result is that the transition matrix after a maximum $2K$ steps contains only strictly positive elements, which are by definition bounded by 1. If we call M the transition matrix of the (stationary) equilibrium of the dynamic game, the ergodic distribution of states, p^* , can be found by solving the following system (see Section 4.3.1, Stachurski 2009):

$$(I - M + B)'p^* = b$$

where I is the identity, B a matrix of ones and b a vector of ones.

2 Market structure and the dynamics of innovation

Before turning to the analysis of the effects of mergers on consumer welfare, it is useful to first understand the impact of the number of firms on the distribution of states. We first run two series of simulations for symmetric firms, for a growing number of firms and technological states. A first series of results are shown in Table 1, which shows the expected rate of innovation in the industry. This is a similar approach to Aghion, Harris, Howitt, and Vickers (2001) and Aghion, Bloom, Blundell, Griffith, and Howitt (2005). In this example, the number of firms always increases the expected number of innovations in the market and there is a sharp drop in innovation from three to two firms. As we will show in section 3, this finding is extremely robust. The influence of the maximum number of technological gaps is non monotonic.

Table 1: Expected innovation

	number of firms				
	2	3	4	5	6
2 tech. states	0.0990	0.3449	0.4143	0.4566	0.4893
3 tech. states	0.1523	0.3290	0.4433	0.5001	0.5359
4 tech. states	0.1037	0.2068	0.2917	0.3561	0.4057

Note: All firms are symmetric, with $\xi = 0$ and one product each. The other parameters are: $\delta = 0.9$, $\gamma = 5$, $L = 1$, $\sigma = 1$.

However, all innovations do not have the same impact on the market. While some innovations push the technological frontier, others could simply result in catching-up, which can be seen as pure costly replication, at least from a technological point of view. The fact that in our example the expected innovation is always increasing with the number of firms could be explained by pure replication rather than by pushing the technological frontier. Therefore, it is interesting to single out the expected rate of innovations for the market, i.e. the ones that actually push the technological frontier. We do this in Table 2. The number of firms then also has a non-monotonic influence on the speed at which the technological frontier is pushed. As we will show later, this is a scale effect: with more firms, each firm is smaller and therefore has smaller returns to innovation. This mitigates, and in some cases even outweighs, the effect of increased competition for a larger number of firms.

Table 2: Expected innovation for the market

	number of firms				
	2	3	4	5	6
2 tech. states	0.0581	0.1637	0.1679	0.1658	0.1624
3 tech. states	0.0909	0.1731	0.2064	0.2100	0.2057
4 tech. states	0.0639	0.1176	0.1536	0.1745	0.1862

Note: All firms are symmetric, with $\xi = 0$ and one product each. The other parameters are: $\delta = 0.9$, $\gamma = 5$, $L = 1$, $\sigma = 1$.

A very striking result in both Tables 1 and 2 is the sharp drop of innovation for three firms

as compared to two firms. This drop is of a very significant magnitude. Because we focus on firms' rivalry, our model is such that there is no innovation under monopoly. Therefore, the drop of innovation from 3 to 2 firms is at least of the same absolute order of magnitude than the drop resulting from a duopoly to a monopoly. It is crucial to understand the reason for this drop because it will have direct consequences on the impact of mergers to duopoly. This drop is a direct consequence of the influence of market structure on the distribution of states. To illustrate this, we first focus on the difference between two and three firms, with one maximum technological gap. Table 3 shows the probability to be in different states, as well as the expected innovation in each of the given states. In this table, we put together symmetric cases (for instance we denote both states 1,0 and 0,1 as 1,0). The main conclusion is that the distribution of states matters a lot.

Table 3: Comparison of the innovation with 2 and 3 firms (2 technological levels)

	2 firms				3 firms			
		Prob.	Innov.	Mkt. Innov.		Prob.	Innov.	Mkt. Innov.
levelled	1,1	0.07	0.74	0.60	1,1,1	0.05	0.71	0.55
unlevelled	1,0	0.93	0.05	0.02	1,1,0	0.49	0.30	0.24
					1,0,0	0.46	0.36	0.04

Note: All firms are symmetric, with $\xi = 0$ and one product each. The other parameters are: $\delta = 0.9$, $\gamma = 5$, $L = 1$, $\sigma = 1$.

More levelled states are generally more innovative. The levelled state with two players and two technological levels is probably the most innovative state of all market structures. However, in this case, the two remaining states are unlevelled and, typically, not very innovative. As a result, firms exit from the levelled state much faster than from the unlevelled states. As there is a very small steady state probability to be in the levelled state, the average innovation is small in this market structure.

The situation is very different with three firms and two technological levels. In every state, there exist at least two firms which are neck and neck with each other. Either two firms lag behind and want to catch-up, or one firm lags behind and the two others want to escape competition from each other. For size reasons, no state is as innovative as the levelled state with two firms. However, the unlevelled states with three firms are more innovative than the unlevelled states with two firms. Last, the innovation rates are much more balanced between states. For all these reasons, innovation is overall greater with three firms than with two.

Similar intuitions apply with a larger number of technological states. Table 4 summarizes the results in a way that is comparable to Table 3, but this time for 3 technological states. It shows that for both 2 and 3 firms, only some states are significant. These are the least innovative ones. However, for three firms, the distribution is less unbalanced and the significant states, i.e. the least innovative ones, are still fairly innovative. The first two significant types of states (3,1,1 and 3,3,1) are such that two firms are neck and neck. This makes them more innovative than their equivalent with two firms (3,1). The last significant type of state (3,2,1) is more evenly distributed than its equivalent with two firms (3,2 and the fringe), which makes the state more likely. Overall, the structure with two firms is typically much less innovative than any other and

Table 4: Comparison of the innovation with 2 and 3 firms (3 technological levels)

2 firms				3 firms			
	Prob.	Innov.	Mkt. Innov.		Prob.	Innov.	Mkt. Innov.
1,1	0.00	0.77	0.62	1,1,1	0.00	0.75	0.58
2,1	0.00	0.91	0.83	2,1,1	0.00	0.83	0.61
2,2	0.01	0.87	0.68	2,2,1	0.00	0.81	0.60
3,1	0.90	0.09	0.03	2,2,2	0.00	0.82	0.61
				3,1,1	0.67	0.20	0.05
				3,3,1	0.06	0.64	0.53
				3,2,1	0.26	0.57	0.38
3,2	0.09	0.68	0.60	3,2,2	0.01	0.77	0.53
				3,3,2	0.00	0.77	0.58

Note: All firms are symmetric, with $\xi = 0$ and one product each. The other parameters are: $\delta = 0.9$, $\gamma = 5$, $L = 1$, $\sigma = 1$.

this might indicate that mergers to duopoly are, in this context, very detrimental to innovation.

These intuitions are confirmed by a comprehensive set of simulations. The most interesting ones are reported in the appendix. We summarize the two main observations regarding the impact of market structure on innovation in the following Observation 2.1.

Observation 2.1. *Regarding the impact of market structure on innovation, we find:*

1. *Aggregate innovation is driven by the least innovative states*
2. *Duopolies are ceteris paribus the least innovative market structures*

3 Consumer welfare effects of mergers

It is of course difficult to conclude from this exercise that markets with a larger number of firms are, generally, less innovative. While this argument is valid *ceteris paribus*, it is likely that market structure is determined jointly by other factors that also affect innovation. It is, for instance, likely that there exist more firms in markets with a higher level of demand. More generally, Boutin (2012a) argues that comparing innovation in different markets is not very informative. As explained earlier, markets differ with respect to many characteristics in a way that makes it difficult to identify the impact of a single factor. However, when mergers occur between two firms, market structure more or less changes *ceteris paribus*. Even though mergers can be triggered by changes in structural parameters, these changes are likely to be continuous over time and the merger itself introduces a discontinuity. Mergers are therefore rare instances where market structure changes very drastically while other parameters stay constant, at least in the short run. Whilst the focus of merger control is normally to predict the short-term impact of a given merger on prices, it is generally because the prices are seen as the main direct proxy of consumer welfare. However, in the case of mergers in innovative markets, as discussed before, product quality is also a driver of consumer welfare. The two issues can be analyzed independently. Nevertheless, it is preferable to combine the two aspects to be able to directly assess consumer welfare.

3.1 Consumer welfare

In our analysis, we voluntarily focus on the analysis of consumer welfare only as this is the standard on which the leading antitrust agencies in the world analyze mergers. The U.S. horizontal merger guidelines, for instance, mention that “*the Agencies normally evaluate mergers based on their impact on customers*” (U.S. Department of Justice and Federal Trade Commission 2010), while the European Commission’s horizontal merger guidelines refer to “*Effective competition brings benefits to consumers, such as low prices, high quality products, a wide selection of goods and services, and innovation. Through its control of mergers, the Commission prevents mergers that would be likely to deprive customers of these benefits by significantly increasing the market power of firms*” (European Commission 2004).

Due to the micro-foundation of the logit demand, we can directly express consumer’s conditional mean utility in time t as the *inclusive value*:

$$\mathbb{E}_t \{U_t\} = \ln \left(\sum_{m \in \{1, \dots, N_{prod}\}} e^{\tilde{k}_m^t L - \sigma p_m^t + \xi_m} \right) \quad (3.1.1)$$

$$= \ln \left(\sum_{m \in \{1, \dots, N_{prod}\}} e^{k_m^t L - \sigma p_m^t + \xi_m} \right) + \tilde{k}_0^t \quad (3.1.2)$$

This conditional mean utility depends on the technological profile \mathbf{k}^t and therefore on the technological state s_t , on the profile of prices, \mathbf{p}^t , and on the technological level of the fringe \tilde{k}_0^t . Because the technological frontier can be pushed every period, consumers’ utility tends to increase over time. Therefore, there exists no such thing as $\mathbb{E}\{U\}$. However, we can compute two complementary measures of consumers’ utility. The first one is the expected value of the left part of equation 3.1.2, abstracting from the fringe:

$$EU = \mathbb{E} \left\{ \ln \left(\sum_{m \in \{1, \dots, N_{prod}\}} e^{k_m^t L - \sigma p_m^t + \xi_m} \right) \right\} \quad (3.1.3)$$

EU represents the utility of buying from the incumbents, compared to the fringe. It solely depends on the distribution of states and is stationary. We refer to EU as the *static utility*.

Moreover, even though the absolute technological level is not stationary, the pace at which it advances is stationary. Therefore, we can define the expected increase of utility from one period to another:

$$\begin{aligned} \partial EU &= \mathbb{E} \{U_{t+1} - U_t\} \\ &= \mathbb{E} \{\mathbb{E}_t \{U_{t+1}\} - U_t\} \end{aligned} \quad (3.1.4)$$

We refer to ∂EU as the *dynamic utility*. A merger changes both ∂EU and EU . If we refer to EU^{pre} and EU^{post} as the pre and post merger static utilities, then $\Delta EU = EU^{post} - EU^{pre}$ is the impact of the merger on static utility. It will typically be negative, mostly because mergers increase prices. Similarly, $\Delta \partial EU = \partial EU^{post} - \partial EU^{pre}$ is the impact of the merger on dynamic

utility. It can be positive or negative depending on whether the merger increases rivalry between firms or not, and on whether the new firm is more efficient in innovation and has a larger scale. It is particularly interesting to compare $\Delta\partial EU$ and ΔEU . For instance, if a merger increases dynamic utility but decreases static utility (because it increases prices), the proportion between the two is a first order indication of how many periods are necessary to make the merger increase consumer welfare, in expected terms.

3.2 Mergers

As explained earlier, when two firms merge, this normally does not lead to the disappearance of one of the two firms (or of its assets, products, etc.). Thus, mergers are different from a simple change in the number of firms, as considered in the previous section. This is Perry and Porter's (1985) seminal answer to Salant, Switzer, and Reynolds (1983). In our model of demand, mergers are best modelled as a change of ownership of brands. If each firm, for instance, had one brand pre-merger, the merged entity then has two brands and jointly decides their prices. This is the approach taken in this paper. It dates back to Deneckere and Davidson (1985).

In our context, firms have no incentive to shut down a brand. Therefore, we assume that the new merged entity keeps all its initial products. For the purpose of the simulations in this paper, we simply assume that, pre-merger, each firm has one product, or brand, and the merged firm then has two brands post-merger. One could imagine a wide variety of other set-ups, for instance two one brand firms merging to face a two brand competitor. The model is flexible enough to accommodate any of these variations. So far, the variants we have simulated did not change the broad message of the paper.

The incentives regarding the post-merger organisation of research are more complex. Pre-merger, each firm has one R&D center, which benefits all the products of the firm. Post-merger, the merged entity might merge its two R&D centers and serve its two brands from it. We call this option *full mergers*. At first glance, this is more efficient. However, products of the two brands might not be compatible and might require dedicated lines of research. Moreover, merging R&D centers probably requires significant fixed costs. Therefore, the two brands might alternatively be forced or choose to keep their R&D independent. We call this option *conglomerate mergers*. At first sight, full mergers are more efficient for the society as there is no duplication of R&D. However, full mergers also limit the incentives of the merged entities to invest in R&D as one innovation suffices to serve the two products sold by the firm. Overall, full mergers do not necessarily lead to more innovation than conglomerate mergers.¹⁰

Last, the impact of mergers on innovation depends not only on the number of initial firms, but also on their market positions at the start. Some mergers create a dominant actor, who will persistently be a leader facing little threat that his competitors will catch-up. Some others, starting from asymmetric market structures, will create a more powerful rival to the previ-

¹⁰We note that there exists a third option for the merged entity, at least in case research is entirely fungible as in the full merger. The merged entity could keep its two R&D centers, put them in competition and use any innovation for all of its brands. This would be even more efficient in case n is small enough. However, in our repeated environment, this would first require full competition between R&D centers and then full disclosure of all relevant hard and soft information once one R&D center succeeded in order to put the two research centers on an equal footing for the subsequent innovations. This does not seem entirely credible from an organizational point of view. Therefore, we discard this option.

ously dominant actor. The analysis of mergers requires taking into account all these different situations.

To arrive to a comprehensive view of the effects of mergers on innovation and welfare, we simulate a large number of mergers. Then we summarise the findings in a series of observations. We make the following simulations:

1. We simulate the merger of 2 firms, from a pool ranging between 3 and 6 firms
2. For each number of initial firms (3,4,5,6), we simulate mergers where:
 - (a) starting from a symmetric market shares (e.g. with three firms with market shares of 33%), two symmetric firms merge, which creates an asymmetric market structure (*mergers to asymmetry*);
 - (b) starting from an asymmetric market structure, where one firm is twice larger than the others (e.g. with three firms, one firm has 50% market share and two other have 25% each), two of the smaller firms merge, which brings more symmetry to the market (*mergers to symmetry*)¹¹
3. For each initial market structure and the type of merger in terms of symmetry, we simulate full-mergers, as well as conglomerate mergers, for a wide range of structural parameters.

4 Results

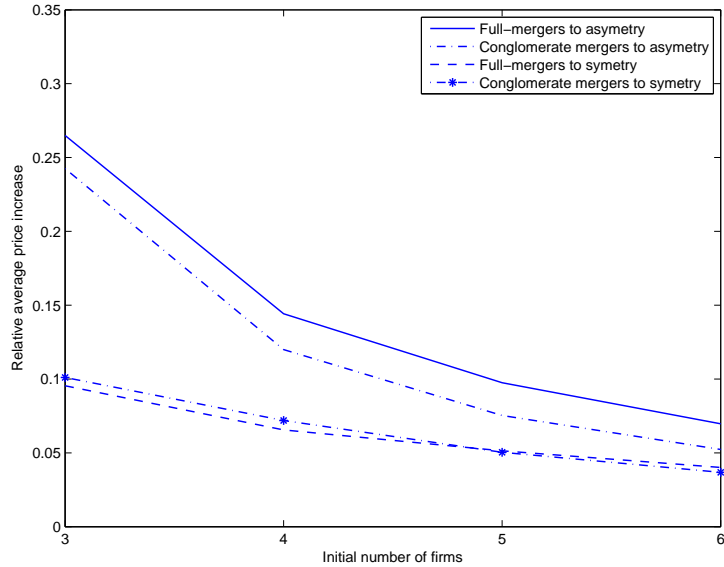
4.1 Influence of market structure

We now turn to the results of a first series of simulations. We first focus on the impact of market structure. Therefore, for a set of structural parameters $\delta = 0.85$, $\gamma = 10$, $L = 1$ and $\sigma = 1$, we simulate mergers of two firms out of an original number of 3 to 6. The mergers we simulate can be either mergers to asymmetry or symmetry (in the sense described earlier). They can be either full-mergers or conglomerate mergers.

We first focus in Figure 1 on the impact of market structure on the price effect of mergers, which is the most classical standard for merger analysis. All the mergers we consider take place in a Bertrand differentiated context. Given that there are no efficiencies in terms of marginal costs, these mergers all increase prices. This feature has been first shown by Farrell and Shapiro (1990) for Cournot competition with assets, but this is also true in our context. However, price increases can be more or less significant. Prices increase more when the initial number of firms was smaller as well as when the merged entity becomes dominant (*mergers to asymmetry*). For example, mergers to duopolies have a larger price effect than mergers from 4 to 3 firms. These effects can be found in any static model and they naturally also appear in our dynamic model. Moreover, mergers to asymmetry give rise to a clear market leader who tends to be persistently a technological leader (because of its larger market share, its returns to innovation are larger). This increases further the tendency of mergers to asymmetry to increase prices more than mergers to symmetry as the technological leader sells a better product with a price

¹¹As explained earlier, asymmetric market structures are generated using the fixed effect ξ . More precisely, we choose for the large firm i ξ_i that gives it a twice larger market share and such that $\xi_i + \sum_{j \neq i} \xi_j = 0$.

Figure 1: Price effect of mergers



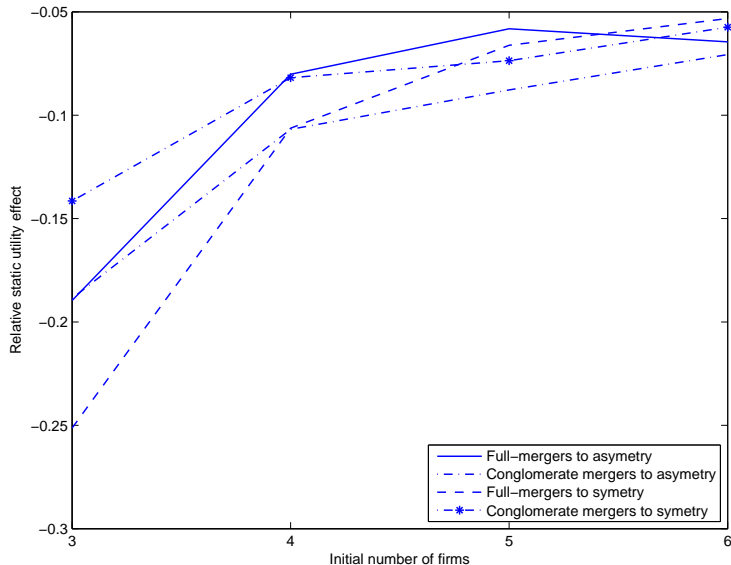
Note: We simulate mergers between two firms out of an initial number of 3, 4, 5 or 6 firms. Mergers can be mergers to symmetry or to asymmetry and can be either conglomerate mergers or full mergers. For each merger, we solve the model pre and post-merger for the following parameters: $\delta = 0.85$, $\gamma = 10$, $L = 1$ and $\sigma = 1$. We compute the average price pre-merger, P^{pre} , as well as post-merger, P^{post} . Then, we plot the relative average price increase $DP = (P^{post} - P^{pre})/P^{pre}$ for each type for different numbers of initial firms.

premium. Last, full mergers make the merged entity more efficient. In mergers to asymmetry, this translates into an even larger persistence of the merged entity in the technologically more advanced states, compared to conglomerate mergers. Therefore, price effects are larger for full mergers to asymmetry than for conglomerate mergers to asymmetry. For mergers to symmetry, the fact that the merged firms are more efficient to innovate in the case of full-mergers might limit the price increase of full-mergers compared to conglomerate mergers. However, this effect only dominates when the gap in the initial asymmetric market was significant enough, i.e. in our simulations when the initial number of firms is limited.

Naturally, higher prices when associated with higher quality do not necessarily harm consumers. It is therefore necessary to focus directly on the utility effects of mergers, as in Figures 2, 3 and 4. The market structures considered in this section are always rather symmetric. It is apparent in our model that fringe firms, which have a much smaller market share than their competitors and are very unlikely to become market leaders, innovate very little and contribute marginally to consumers' utility. Therefore, to infer the likely effects of a merger in a real situation, one should mostly take into account the non-fringe firms. The model is simple enough to confirm this intuition with calibrated simulations in particular cases.

Figure 2 focuses on the effect of mergers on static utility, i.e. the utility consumers derive from buying from the oligopoly rather than from the fringe. It is first apparent that there is less of a systematic difference between full and conglomerate mergers for utility than for prices. Higher prices post full mergers are indeed partly related to a larger persistence of unlevelled

Figure 2: Static utility effect of mergers



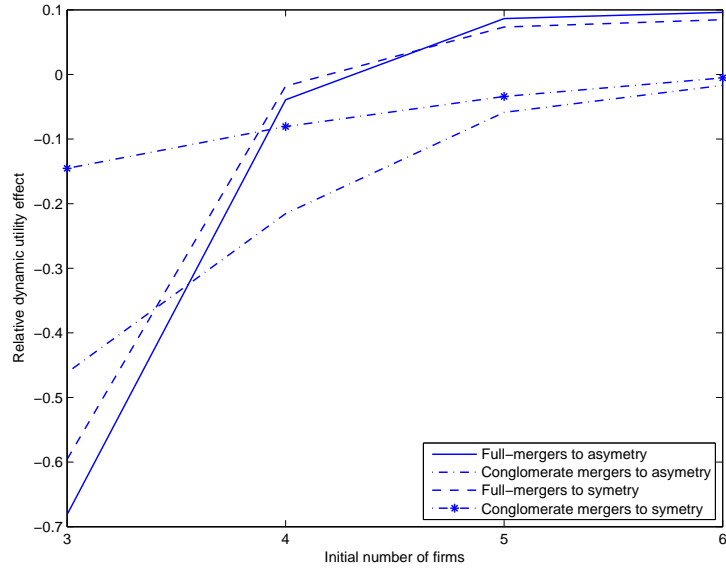
Note: We simulate mergers between two firms out of an initial number of 3, 4, 5 or 6 firms. Mergers can be mergers to symmetry or to asymmetry and can be either conglomerate mergers or full mergers. For each merger, we solve the model pre and post-merger for the following parameters: $\delta = 0.85$, $\gamma = 10$, $L = 1$ and $\sigma = 1$. For each merger, we compute pre and post-merger static utilities EU^{pre} and EU^{post} as given by equation 3.1.3. Then, we plot $EU^{post} - EU^{pre}$ (static effect).

states. Then, higher prices are partly compensated by higher quality offerings by the merged entity. Nevertheless, the residual utility left to consumers for higher quality after prices increases are rather limited: the merged entity captures most of the benefits of higher quality through prices. Moreover, generally speaking, the static effect is larger starting from a smaller number of firms, which is quite intuitive.

In terms of dynamic utility, i.e. the pace at which customer’s utility increases, the picture is more contrasted, as shown in Figure 3. As expected, mergers to duopoly have a very detrimental effect on the benefits customers derive from innovation. Full-mergers lead to a situation very close to the duopolies presented in Section 2. Even though they are less efficient from a purely technological point of view, conglomerate mergers lead to less harm to consumers, from a dynamic perspective. The merged entity takes investment decisions in innovation jointly for its two lines of products. However, the outcomes of innovation still correspond to independent uncertain processes. This leads to more balanced distribution of states, the merged entity being less often a clear leader for all of its products than in case of a full merger. Because mergers to asymmetry lead to more persistence of leadership for the merged entity, they are even more harmful than mergers to symmetry. For mergers to three or more firms, the picture is drastically different. Because they are more efficient, full mergers are less harmful and can even be beneficial, from a dynamic perspective. Conversely, conglomerate mergers are always harmful. Overall, efficiency gains for full mergers can either give rise to an efficiency offense for mergers to duopolies or to an efficiency defence for mergers to more than three firms.

It is particularly interesting to compare the magnitude of the static and dynamic effects,

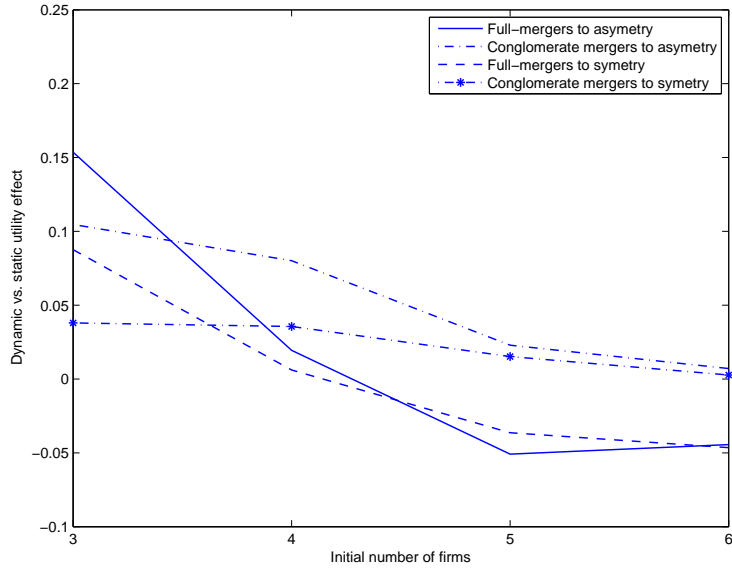
Figure 3: Dynamic utility effect of mergers



Note: We simulate mergers between two firms out of an initial number of 3, 4, 5 or 6 firms. Mergers can be mergers to symmetry or to asymmetry and can be either conglomerate mergers or full mergers. For each merger, we solve the model pre and post-merger for the following parameters: $\delta = 0.85$, $\gamma = 10$, $L = 1$ and $\sigma = 1$. For each merger, we compute pre and post-merger dynamic utilities ∂EU^{pre} and ∂EU^{post} as given by equation 3.1.4. Then, we plot $\partial EU^{post} - \partial EU^{pre}$ (*dynamic effect*).

as this is done in Figure 4. Even though the dynamic effects of mergers could compensate the initial loss in some cases, it is apparent that the magnitude is rather small. Dynamic gains only represent at best a very small share of the initial loss. It would require a very large number of periods to reverse the utility effect of mergers, such that the dynamic efficiency defence would need to be complemented by other efficiency gains. Conversely, dynamic losses can represent a considerable share of the initial static losses. The 15% in the full-mergers to asymmetric duopoly means in practice that the initial harm is double every 5 periods.

Figure 4: Dynamic vs. static utility effect of mergers



Note: We simulate mergers between two firms out of an initial number of 3, 4, 5 or 6 firms. Mergers can be mergers to symmetry or to asymmetry and can be either conglomerate mergers or full mergers. For each merger, we solve the model pre and post-merger for the following parameters: $\delta = 0.85$, $\gamma = 10$, $L = 1$ and $\sigma = 1$. For each merger, we compute pre and post-merger static utilities EU^{pre} and EU^{post} as well as dynamic utilities ∂EU^{pre} and ∂EU^{post} as given by equations 3.1.3 and 3.1.4. Then, we plot $(\partial EU^{post} - \partial EU^{pre}) / (EU^{post} - EU^{pre})$ (*dynamic vs. statics*).

4.2 Influence of structural parameters

4.2.1 Prices

We now turn to the effects of structural parameters, first focusing on prices. As already explained, all mergers we consider increase prices. However, mergers have a higher price effect when consumers' disutility to pay, σ , is lower. This is a classical result for any differentiated Bertrand model: lower σ generally leads to lower price elasticities, which can be exploited by merging firms. These effects can be found in any static model and naturally also appears in our dynamic model.

However, the dynamic nature of our model has greater consequences on the role played by L . A larger L leads to a larger distance between the incumbents and the fringe in any technological state. It increases the prices in any state and this can be exploited by the merging firms. However, the dynamic nature of the model also affects the effect of L on expected prices. A stronger preference for vertical quality increases the average price in unlevelled states. Therefore, if a merger brings more technological leadership, it will increase the price further. On the contrary, if the merger brings more rivalry, this will mitigate the direct effect of L on prices as markets are less likely to be levelled after the merger. Because of the direct effect of L on the distance to the fringe, stronger taste for quality is always positively related to the price effect of mergers. However, this effect is stronger in the case of mergers to asymmetry, where unlevelled markets are more likely, than for mergers to symmetry, where levelled markets are more likely. Last,

conglomerate mergers have the same effect as full mergers in static terms. However, they lead to less leadership in the case of mergers to asymmetry and bring less rivalry in the case of mergers to symmetry. Therefore, price effects of conglomerate mergers are closer to the more classical static effect. We can summarize the effects of mergers on prices as follows:

Observation 4.1. *Regarding the impact of mergers on expected prices, we find that prices increase more if:*

1. *The concentration of the industry increases more. This is the case if*
 - *The merger starts from a smaller number of firms*
 - *The merger leads to more asymmetry*
2. *σ is smaller*
3. *L is larger and this effect is the strongest for full mergers to asymmetry.*

4.2.2 Consumer welfare

As already shown, higher prices when associated with higher quality do not necessarily harm consumers. For instance, L also directly impacts consumers' utility and this should be taken into account in the welfare analysis of the effect of L . Similarly, a smaller σ leads to higher prices and larger price effect of mergers. However, it is also indicative of a smaller disutility to pay from consumer. To look directly at the impact of structural parameters on the welfare effect of mergers, we will first focus on mergers to duopolies, then on mergers to three firms and, last, on mergers to more than four firms. The following conclusions that we draw from our model are based on many simulations. Supporting figures are presented in the appendix, in particular Figures 5 to 24.

We first focus on mergers to duopolies, for which we summarize our results in Observation 4.2. As already described, mergers to duopoly very significantly increase prices and lead to more leadership persistence. This technological persistence is more pronounced in the case of mergers to asymmetry, as well as in the case of full mergers. Indeed, conglomerate mergers force the merged entity to innovate on its two lines of research to acquire full technological leadership. The remaining firm also innovates and has high incentives to do so both when the market is fully levelled and when only one branch of the merged entity is a technological leader. Then, overall, the likelihood that the merged entity is a full technological leader is smaller in the case of a conglomerate merger than in the case of a full merger. Therefore, full mergers are worse for customers than conglomerate mergers and mergers to asymmetry than mergers to symmetry. This is true both from a static and from a dynamic perspective.

Observation 4.2. *Regarding the impact of mergers to duopoly on utility, we find:*

1. *Mergers to duopoly significantly decrease static utility*
 - *Mergers to asymmetry harm customers more than mergers to symmetry*
 - *Full mergers harm customers more than conglomerate mergers*
 - *Relative to the initial static utility, mergers are more detrimental for smaller values of L*
2. *Mergers to duopoly very significantly decrease dynamic utility*
 - *Mergers to asymmetry harm customers more than mergers to symmetry*
 - *Full mergers harm customers more than conglomerate mergers*
 - *Relative to the initial static utility, mergers are more detrimental for larger values of L*
3. *The dynamic loss can represent a significant portion of the static loss.*

However, from a static perspective, i.e. as compared to the fringe, technological leadership translates into better quality. This mitigates the price increases. Therefore, in relative terms, mergers decrease static utility more when quality matters less (L is small). This is exactly the opposite for dynamic utility. Innovation, and therefore the dynamic utility, falls more in relative terms (post merger compared to pre merger) when quality matters more. Overall, the dynamic losses are a larger share of the static losses when quality matters more. This can add very significant harm for consumers. In some cases, in little more than a year, the static initial harm is doubled due to the dynamic effect.

From the firms' perspective, full mergers are more efficient, as the same R&D effort benefits more products. It is generally a debate whether these type of efficiencies are passed-through to customers or whether they only benefit firms. However, the case of full mergers to duopoly shows that this increased efficiency ultimately harms customers even further. This is a new type of *efficiency offence*. Nevertheless, as we will show, in less concentrated market structures, efficiency ultimately benefits customers. The issues of price increases and innovation should not be treated sequentially, but rather be analysed in a more integrated manner.

We then turn to the case of mergers to three firms, which is an intermediate case between mergers to duopoly and mergers to a larger number of firms. A summary of the results can be found in Observation 4.3. Price effects of mergers to three firms are generally less severe than the ones of mergers to duopolies. Moreover, they lead to less persistence in technological leadership. While mergers to three firms still very significantly reduce static utility, the dynamic effect, even though still detrimental to consumers, is much less severe than for mergers to duopolies. Because mergers to asymmetry bring more persistence of technological leadership than mergers to symmetry, they are more detrimental, both from a static and from a dynamic perspective.

As for mergers to duopoly, full mergers are more efficient from the firms' perspectives. Because there always exists at least the rivalry between the two non-merging firms, full mergers are therefore less detrimental than conglomerate mergers, even though they are all detrimental

Observation 4.3. *Regarding the impact of mergers to three firms, or above, on consumers' utility, we find:*

1. *Merger to three significantly decrease static utility*
 - *Mergers to asymmetry harm customers more than mergers to symmetry*
 - *Full mergers to asymmetry harm customers less than conglomerate mergers to asymmetry*
 - *Full mergers from symmetry harm customers more than conglomerate mergers to from symmetry*
2. *Mergers to three firms significantly decrease dynamic utility*
 - *Mergers to asymmetry harm customers more than mergers to symmetry*
 - *Full mergers harm customers less than conglomerate mergers*
 - *Relative to the initial static utility, mergers are more detrimental for larger values of L*
3. *The dynamic loss represents a small portion of the static loss.*

to consumers from a dynamic perspective. Moreover, the more consumers value quality, the more detrimental are mergers from a dynamic perspective. Mergers to asymmetry tend to create a more persistent technological leader. However, this technological leader is more efficient in bringing innovations to the market in the case of full mergers and therefore full mergers to asymmetry are less detrimental than conglomerate mergers from a dynamic perspective. This is the opposite for mergers to symmetry: conglomerate mergers preserve more of the symmetry between players and harm customers less than full mergers, from a dynamic perspective.

Observation 4.4. *Regarding the impact of mergers to four firms, or above, on consumers' utility, we find:*

1. *Mergers to four firms or more significantly decrease static utility*
 - *Conglomerate mergers decrease utility more than full mergers*
 - *Conglomerate mergers to asymmetry decrease utility more than conglomerate mergers to symmetry*
 - *Full mergers to symmetry decrease utility more than full mergers to asymmetry for large values of L , while this can be reversed for small values of L*
 - *Relative to the initial static utility, mergers are more detrimental for smaller values of L*
2. *Full mergers bring dynamic efficiencies that benefit customers*
3. *Conglomerate mergers decrease dynamic utility*
4. *The dynamic gains or losses represent a small portion of the static loss.*

The results for mergers to more than four firms are presented in Observation 4.4. The static effect of mergers to more than four firms is in line with the previous intuitions, even though the magnitude of consumer harm is naturally smaller. When mergers leave a sufficient number of

firms in the market, in this case at least four, full mergers bring dynamic benefits to consumers. Conversely, conglomerate mergers even in this case continue to decrease utility, from a dynamic perspective. However, in both cases, the dynamic gains or losses represent a small portion of the static losses. When firms put together all, or significant parts, of their R&D, innovation could give grounds for clearing a merger, on the basis of an *efficiency defence*. However, for a merger to be beneficial to customers after a reasonable period of time, this source of efficiencies would have to be complemented by other efficiencies that benefit consumers, such as the more classical marginal cost reductions.

4.3 Link with Goettler and Gordon (2011)

The type of logit demand with an outside good used in this paper is very standard in modern applied industrial organization. However, it departs from Goettler and Gordon (2011), who, while analysing the related issue of the link between foreclosure and innovation have emphasized the durability of goods, which drives a large part of their results. In Goettler and Gordon (2011), durability has two main consequences. First, durability means that firms are always facing competition from the stock of products previously acquired by customers and they have to innovate to foster demand. Second, firms can dynamically price discriminate among customers and, typically, derive larger profits from innovating.

The fact that innovation fosters new demand is in our point of view a very valid claim. Taking this factor into account to make the model more realistic would also require to take into account other important features of high tech industries. First, durability is a matter of degree. While consumer electronics, for instance, are definitely not perishable, they do not provide a constant utility forever either. The first reason for that is that they break down at some point. To come back to the example of HDD, Hard Disks are not only pieces of electronic engineering. They are also advanced pieces of mechanical engineering. They normally need to be replaced because they break down and not only because there exists hard drives saving more energy or offering better capacity. Moreover, consumer electronics are part of a system which evolves as a whole. For example, a computer, with given characteristics, is likely to provide a decreasing relative utility over time as applications evolve and require more speed, memory, resolution, etc. In the same vein, increases in quality of some components of the system induces the need to upgrade others. As HDD is part of this system, the need for storage also increases over time. Therefore, demand for HDD can also be fostered by other innovations. Last, markets expand and as far as new consumers are concerned, competition remains the main driver of innovation, as we show. Goettler and Gordon (2011) mention that 85% of CPUs purchases are replacements of old CPUs. This still means a growth of 15% every year, i.e. the market doubles approximately every 5 years. Goettler and Gordon (2011) also point out that a CPU is, on average, "upgraded" every 3.3 years to show that demand is driven by innovation. Overall, the effect of market growth on demand seems comparable to the effect of innovation.

Adding durability to our model and taking into account growth in overall demand and degradation would add a significant level of complexity to our model.¹² Moreover, it is very unlikely to change the main message of our paper. Even in this context, mergers significantly

¹²A growing, non-stationary, demand would for instance lead to non-stationary strategies.

reduce firm's rivalry. Therefore, mergers will be detrimental to innovation as long as the scale effect does not offset this loss of rivalry. This is all the more true that it is not the durability itself that generates the counterintuitive result of Goettler and Gordon (2011) but the possibility it creates of dynamic pricing.

Goettler and Gordon's (2011) observation 1 show that dynamic pricing significantly increases firms' profits. This effect is strongest for the monopoly, which is much better at dynamically price discriminating captive customers (as any demand not served today for a monopolist remains to be served tomorrow, while it is partly addressed today by competitors in duopolies). This drives the very counterintuitive result that a monopoly is more innovative than a duopoly. Without this aspect of dynamic price discrimination, the duopoly is more innovative (see observation 2). In a nutshell, the industry profits are much larger with a monopoly, due to dynamic price discrimination. Therefore, innovation is much more profitable with a monopoly than with a duopoly and this effect marginally dominates the intuitive effect of loss of any rivalry moving from a duopoly to a monopoly our paper focuses on. However, when firms efficiently dynamically price discriminate, they leave customers with little utility. Therefore, the benefits of innovation are then mostly captured by firms. Even in Goettler and Gordon's (2011) framework, where monopolies innovate more, monopolies are unambiguously harmful to customers.

Moreover, the ability of monopolies producing durable goods to price discriminate is at the core of one of the oldest disputes in microeconomics. On one side, the so-called Pacman conjecture states that a monopoly is able to "eat down" the demand curve and extract all consumer surpluses. On the other side, the so-called Coase conjecture states that patient consumers can always wait to buy and a monopoly cannot commit to high prices. These two arguments have only been re-conciliated relatively recently by von der Fehr and Kuhn (1995). While the Goettler and Gordon's (2011) model clearly bends towards the Pacman conjecture, von der Fehr and Kuhn (1995) show that this is not something one should necessarily expect from a theoretical point of view.

There are two possible explanations of Goettler and Gordon's (2011) results. The first one is that this model specifically has one sub-game perfect equilibrium in prices. The second one is that the algorithm selects one particular equilibrium among many. For instance, focusing on the limit of finite time games could possibly select equilibria which are unfavorable to patient consumers. It is only in the first situation that Goettler and Gordon's (2011) counterintuitive results are robust, even though they could still be model specific. In the second case, the robustness of the results would require a very costly investigation of all the possible equilibria, for instance using homotopy methods.

In any event, only very extreme forms of dynamic price discrimination can overturn the results we show in the paper. The scope for dynamic price discrimination is dramatically larger for monopolies than for duopolies, as shown in Goettler and Gordon (2011). The effects stemming from the loss of rivalry between firms we show in this paper for mergers to duopolies are therefore unlikely to be offset because of dynamic pricing. This is all the more true for mergers to a larger number of firms, where the scope for dynamic pricing is likely to be marginal. Therefore, our results are very likely to be valid with durable goods, even in a model where firms are able to exploit customers' by dynamic prices.

4.4 Link with Mermelstein et al. (2014)

Mermelstein, Nocke, Satterthwaite, and Whinston (2014) also focuses on the relationship between mergers and investment, but from a different angle. It first models a duopoly (or a monopoly) with price competition, linear demand and assets. Therefore, investments consist of physical production capacity and aim at lowering marginal costs of production. The model, and its prediction, rather applies to homogenous basic industries, while our paper describes better more innovative sectors focusing on product quality. Second, and more importantly, it focuses on the extent to which external growth, i.e. mergers, are substitute strategies to internal growth, i.e. investment in assets. In other words, to reduce their marginal costs, firms can either buy new assets or merge with the other firm. Then, whether merger control allows them to merge or not changes their optimal internal investment strategies.

Mermelstein, Nocke, Satterthwaite, and Whinston's (2014) framework assumes that mergers create very significant synergies (in Farrell and Shapiro's (1990) sense) of 9% in the central simulations. Moreover, it is assumed that a merger to monopoly is immediately followed by the emergence of a potential entrant. Last, greenfield investments by this potential entrant are only marginally more expensive than brownfield investments by incumbents. Despite these very favourable assumptions, there are still, from consumers' point of view, very little reasons to welcome these mergers to monopoly. For this reason, the optimal policies, as well as myopic policies are still to *"basically allow no merger"*. Mermelstein, Nocke, Satterthwaite, and Whinston (2014) so far does not extend to more realistic market structures and mergers to less concentrated markets structures where internal growth could modify optimal policies. However, it already confirms two of the main intuitions of our paper. The first one is that composition effects are crucial to take into account to properly analyse the performance of dynamic oligopolies. The second one is that mergers (and therefore merger policy) has a large influence on firms' investment strategies (and in Mermelstein, Nocke, Satterthwaite, and Whinston (2014) both brownfield investments by incumbents and greenfield investments by entrants).

Conclusions

In this paper, we develop a model of oligopoly with step-by-step innovation. This allows us to go beyond the simple Schumpeter hypothesis on innovation. We show that, in this context, the least innovative states of an industry are the ones that drive the average performance of the industry. For an industry to be innovative, it is necessary to have enough rivalry between firms in any state. This prevents persistence of the technological states that bring little innovation to consumers. A very strong result of this model is that, as far as market structure is concerned, duopolies are extremely un-innovative.

It almost directly stems from this observation that mergers to duopoly are unambiguously detrimental to consumers. They induce large prices and persistence of technological leadership, that leads to less innovation and, ultimately, less value for consumers. This is even more the case for mergers that decrease symmetry and for full mergers. Mergers to three firms have, overall, a small dynamic impact. At first order, their impact is therefore driven mostly by price increases. However, it would take larger marginal costs decrease than predicted in a static

framework to make mergers to three firms acceptable. Conversely, full mergers to more than four firms potentially bring dynamic benefits to consumers. However, the dynamic gains would only outweigh the static loss in a time horizon that exceeds the one considered as relevant by antitrust agencies. This source of efficiencies is therefore generally not sufficient and would have to be complemented by other benefits to consumers, such as the ones stemming from marginal cost reductions. In addition to these general intuitions, the model is simple enough to be adjusted and calibrated for different innovative industries where product quality matters.

The fact that symmetry and rivalry are key drivers of the welfare effects of mergers had already been shown in a static framework. For instance, Vergé (2010) analyzes the welfare effects of mergers with remedies in a Cournot model with assets. In Farrell and Shapiro's (1990) sense, these mergers are without synergies, i.e. the only source of efficiencies are the reshuffling of production between the various assets (which is already the most common source of short term efficiencies). In this context, only mergers that increase symmetry after acceptable remedies should be accepted and this is never the case of mergers to duopoly. Symmetry also matters for mergers in homogenous Bertrand with capacity constraints (Bertrand-Edgeworth). Then, the support of mixed strategies is determined by the capacity of the largest player.¹³ Therefore, mergers that decrease symmetry, in the sense that the largest player in terms of capacity becomes larger, are likely to lead to significantly higher prices. An example of such an assessment can be found in Buettner, Federico, Kühn, and Magos (2013). However, our paper sheds lights on a new area that had not been explored by the literature so far: the link between mergers and the persistence of technological leadership, leading to low aggregate rates of innovation. This is of particular relevance for innovative industries where the issue of capacity constraints, already explored in the literature, is largely irrelevant.

These results are interesting for several reasons. First, they confirm that mergers can drastically change the incentives of firms to innovate, as well as the benefits customers derive from it. Second, this paper shows that the issue of efficiencies in merger control is even less straightforward in a dynamic setting. Merger control normally rightfully focuses on marginal cost reductions, which are direct drivers of prices. This paper shows that in innovative markets a static analysis is likely to underestimate the level of marginal costs savings required for clearing a merger to duopoly. As marginal costs and innovation are not directly related, it is even possible that there exists no marginal cost saving that can tip the balance in favour of such mergers. We keep this issue for future research. Conversely, as full mergers to more than four firms bring dynamic benefits to consumers, a static framework could overestimate the marginal cost reductions required to clear a merger.

Moreover, even though the issue of marginal cost savings is already interesting, the one of so-called fixed cost savings is even more topical. In a static framework, fixed cost saving are never passed-through to customers. However, it is often argued that they should be taken into account in a dynamic setup. This paper shows that this argument deserves to be better characterised. Full mergers are in any respect more efficient than conglomerate mergers. However, while this efficiency could be passed-through to customers for mergers to four firms or more, this is not

¹³The lower bound of the rationalizable strategies is the price that makes the largest player just indifferent between just undercutting all the other firms playing the lower bound and pricing the higher bound, i.e. the monopoly price on the residual demand when all the other players play the lower bound.

the case for mergers to three firms. Moreover, in the case of mergers to duopoly, these savings in R&D cannot even be qualified as efficiencies because they directly and significantly harm consumers. Fixed cost savings in this context give rise to an *efficiency offense*.

Last, this issue of fixed cost savings relates to the more general issue of the relevance of firms' profits for innovation. We have argued that antitrust agencies rightfully focus on consumer welfare. This is the mandate they received and this is the relevant one as firms' profits are not fungible with consumers' utilities. However, as already mentioned, if firms are financially constrained, the level of firms' profits (and therefore potentially fixed costs savings) directly affects the ability of firms to innovate and therefore impacts customers' utility as well. There is empirical evidence that financially constrained firms invest less in R&D (Aghion, Askenazy, Berman, Cetto, and Eymard 2012). This issue of financial constraints is the highest in our research agenda.

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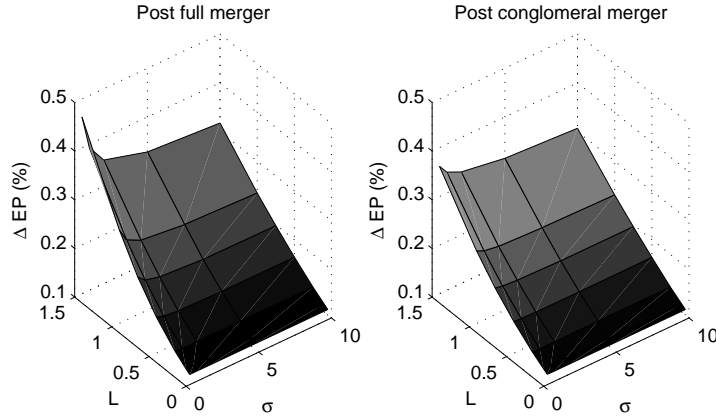
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A Price effect of mergers

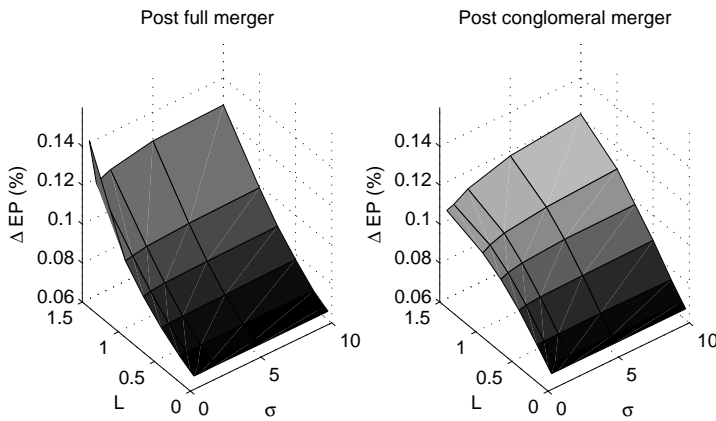
A.1 Mergers to duopoly

Figure 5: Price effect of mergers to duopolies to asymmetry



Note: We simulate mergers of two out of three symmetric firms with initial market shares of 33%, using the model described in Section 1. For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For each merger, we compute the average price pre-merger, P^{pre} , as well as post-merger, P^{post} . Then, we plot the relative average price increase $DP = (P^{post} - P^{pre})/P^{pre}$ for both types of mergers for the different parameters.

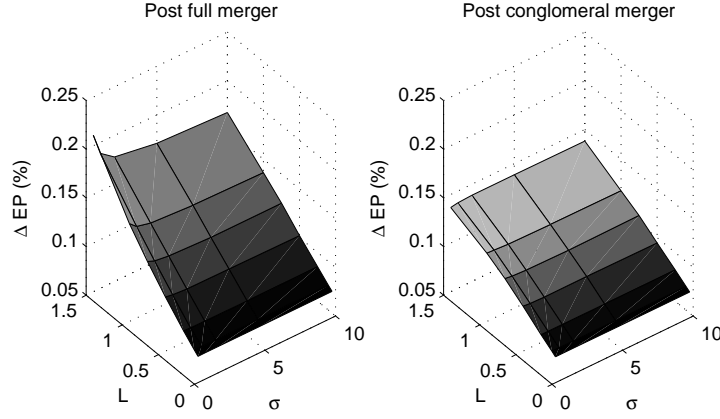
Figure 6: Price effect of mergers to duopolies to symmetry



Note: We simulate mergers of two firms with 25% initial market shares facing a firm with 50% initial market share, using the model described in Section 1. For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For each merger, we compute the average price pre-merger, P^{pre} , as well as post-merger, P^{post} . Then, we plot the relative average price increase $DP = (P^{post} - P^{pre})/P^{pre}$ for both types of mergers for the different parameters.

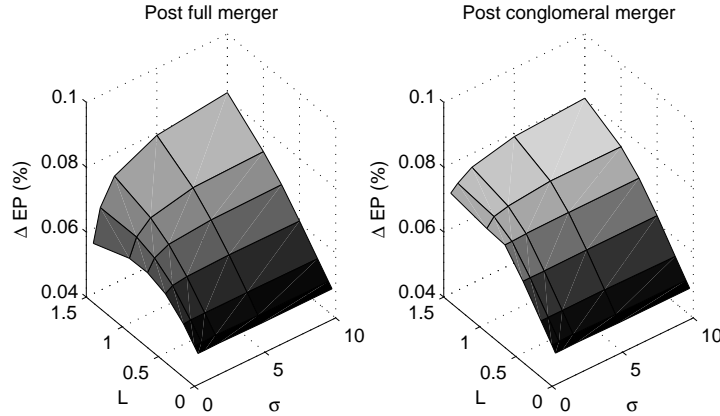
A.2 Mergers to three firms

Figure 7: Price effect of mergers to three firms to asymmetry



Note: We simulate mergers of two out of four symmetric firms with initial market shares of 25%, using the model described in Section 1. For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For each merger, we compute the average price pre-merger, P^{pre} , as well as post-merger, P^{post} . Then, we plot the relative average price increase $DP = (P^{post} - P^{pre})/P^{pre}$ for both types of mergers for the different parameters.

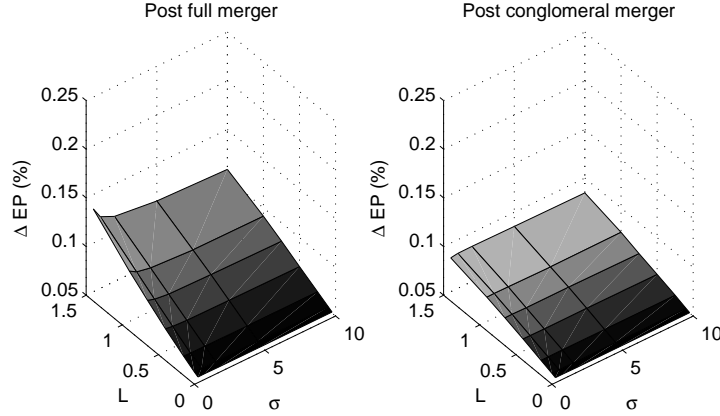
Figure 8: Price effect of mergers to three firms to symmetry



Note: We simulate mergers of two out of four firms using the model described in Section 1. Before the merger, there exists a market leader with an initial market shares of 40%, while the remaining 3 firms, including the merging firms, have an initial market share of 20%. For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For each merger, we compute the average price pre-merger, P^{pre} , as well as post-merger, P^{post} . Then, we plot the relative average price increase $DP = (P^{post} - P^{pre})/P^{pre}$ for both types of mergers for the different parameters.

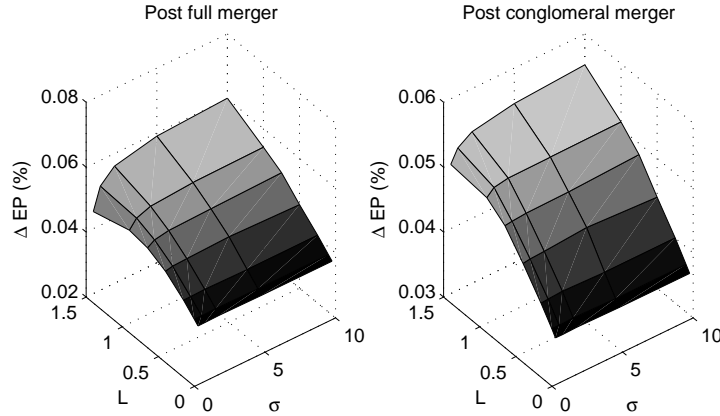
A.3 Mergers to four firms

Figure 9: Price effect of mergers to four firms to asymmetry



Note: We simulate mergers of two out of five symmetric firms with initial market shares of 20%, using the model described in Section 1. For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For each merger, we compute the average price pre-merger, P^{pre} , as well as post-merger, P^{post} . Then, we plot the relative average price increase $DP = (P^{post} - P^{pre})/P^{pre}$ for both types of mergers for the different parameters.

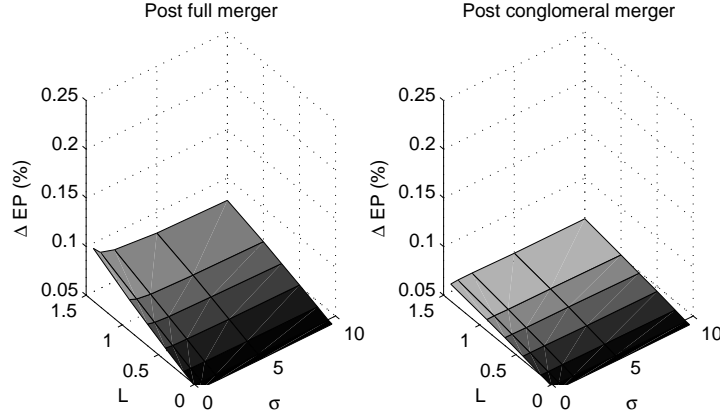
Figure 10: Price effect of mergers to four firms to symmetry



Note: We simulate mergers of two out of five firms using the model described in Section 1. Before the merger, there exists a market leader with an initial market shares of 33%, while the remaining 4 firms, including the merging firms, have an initial market share of 16.5%. For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For each merger, we compute the average price pre-merger, P^{pre} , as well as post-merger, P^{post} . Then, we plot the relative average price increase $DP = (P^{post} - P^{pre})/P^{pre}$ for both types of mergers for the different parameters.

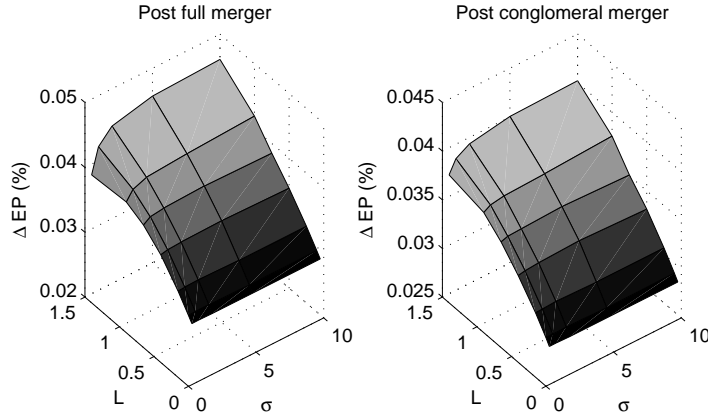
A.4 Mergers to five firms

Figure 11: Price effect of mergers to five firms to asymmetry



Note: We simulate mergers of two out of six symmetric firms with initial market shares of 16.5%, using the model described in Section 1. For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For each merger, we compute the average price pre-merger, P^{pre} , as well as post-merger, P^{post} . Then, we plot the relative average price increase $DP = (P^{post} - P^{pre})/P^{pre}$ for both types of mergers for the different parameters.

Figure 12: Price effect of mergers to five firms to symmetry

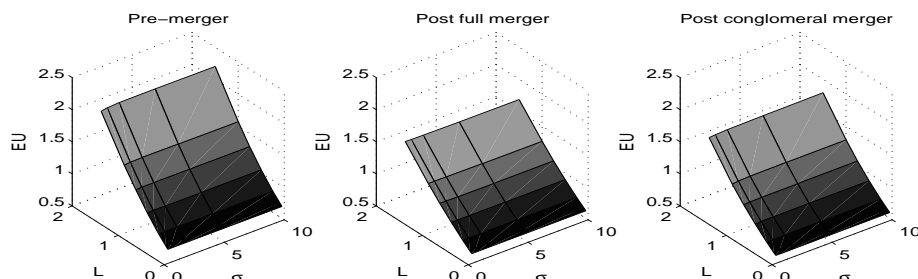


Note: We simulate mergers of two out of five six using the model described in Section 1. Before the merger, there exists a market leader with an initial market shares of 28.6%, while the remaining 5 firms, including the merging firms, have an initial market share of 14.3%. For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For each merger, we compute the average price pre-merger, P^{pre} , as well as post-merger, P^{post} . Then, we plot the relative average price increase $DP = (P^{post} - P^{pre})/P^{pre}$ for both types of mergers for the different parameters.

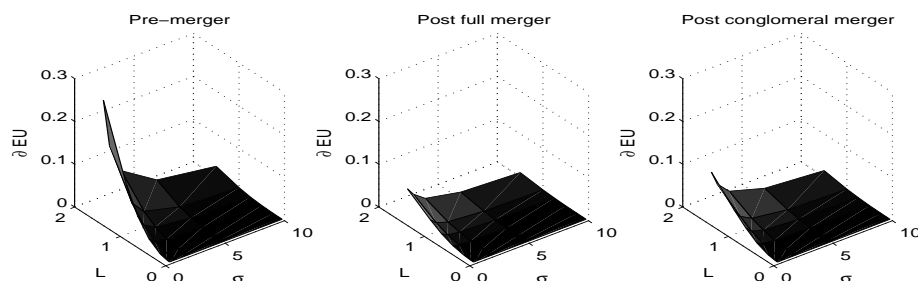
B Utility effect of mergers to duopoly

Figure 13: Pre and post-merger utility for mergers to duopoly

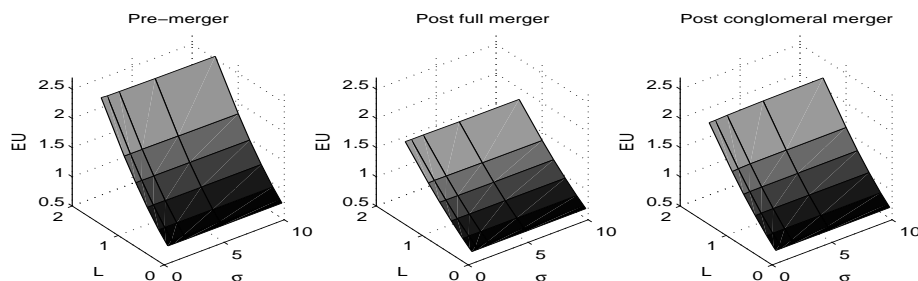
Static utility for mergers to asymmetry



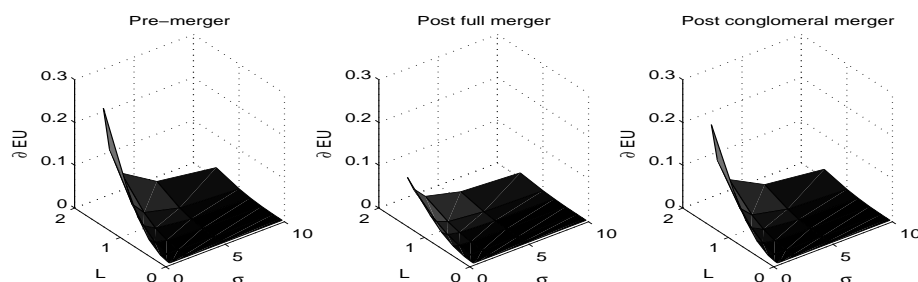
Dynamic utility for mergers to asymmetry



Static utility for mergers to symmetry

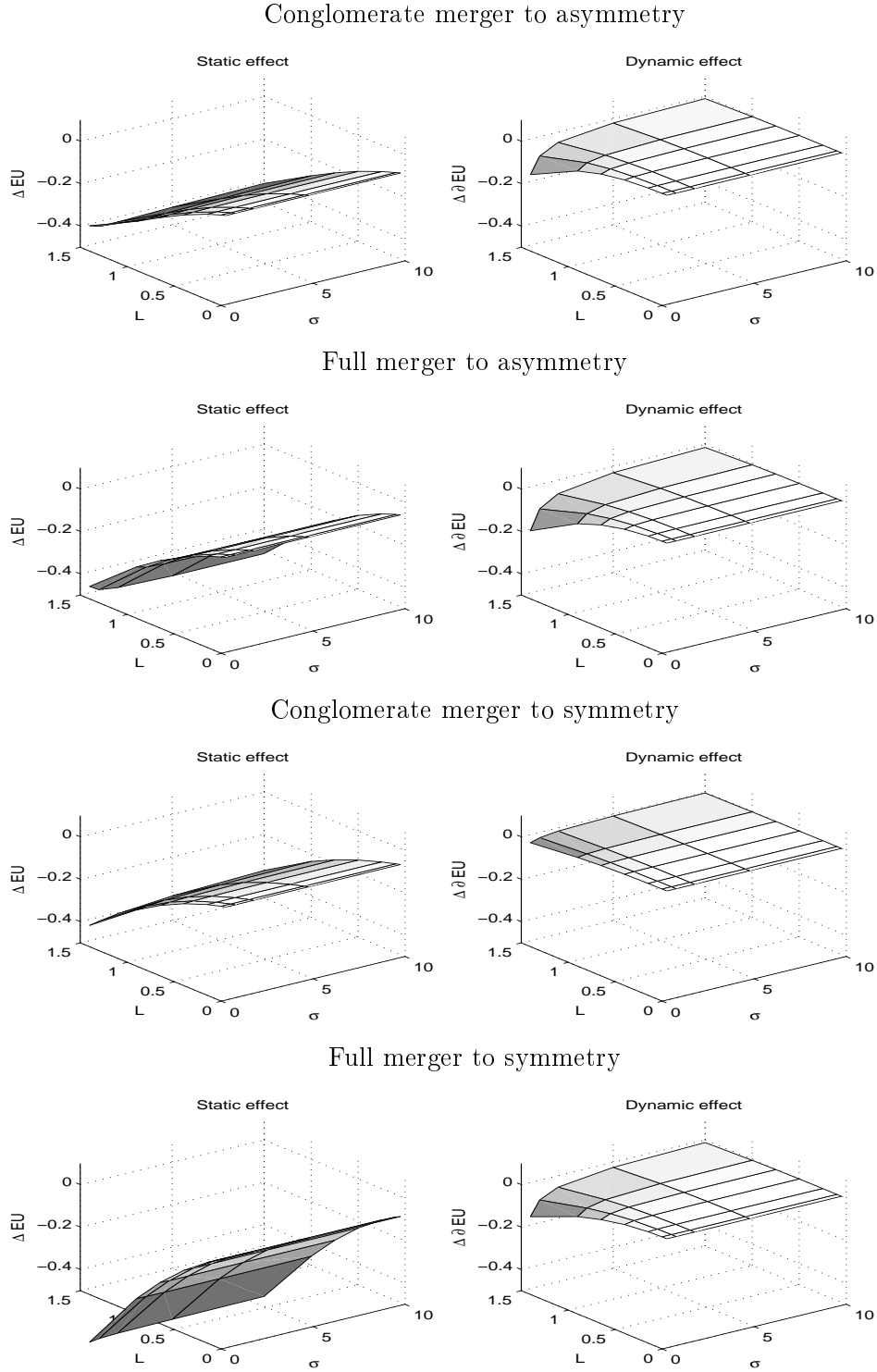


Dynamic utility for mergers to symmetry



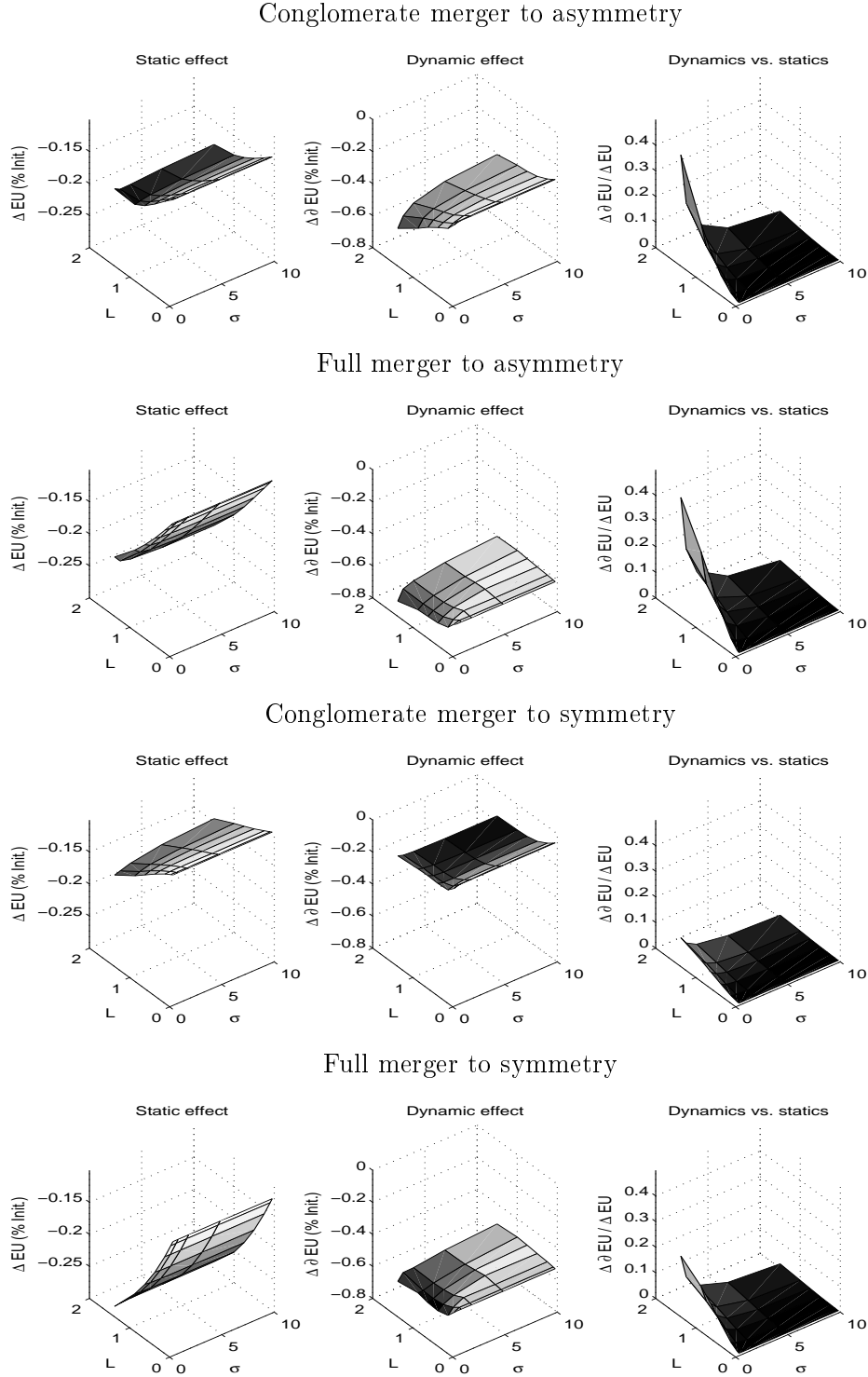
Note: We simulate mergers of two out of three firms using the model described in Section 1. For mergers to asymmetry, all firms have initial market shares of 33%, while, for mergers to symmetry, a market leader with an initial market shares of 50% faces a merger of its two symmetric competitors with initial market shares of 25%. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. For each merger, we compute EU and ∂EU as given by equations 3.1.3 and 3.1.4.

Figure 14: Absolute static vs. dynamics effects of mergers to duopolies



Note: We simulate mergers of two out of three firms using the model described in Section 1. For mergers to asymmetry, all firms have initial market shares of 33%, while, for mergers to symmetry, a market leader with an initial market shares of 50% faces a merger of its two symmetric competitors with initial market shares of 25%. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. For each merger, we compute pre and post-merger static utilities EU^{pre} and EU^{post} as well as dynamic utilities ∂EU^{pre} and ∂EU^{post} as given by equations 3.1.3 and 3.1.4. We plot $EU^{post} - EU^{pre}$ (*static effect*) and $\partial EU^{post} - \partial EU^{pre}$ (*dynamic effect*).

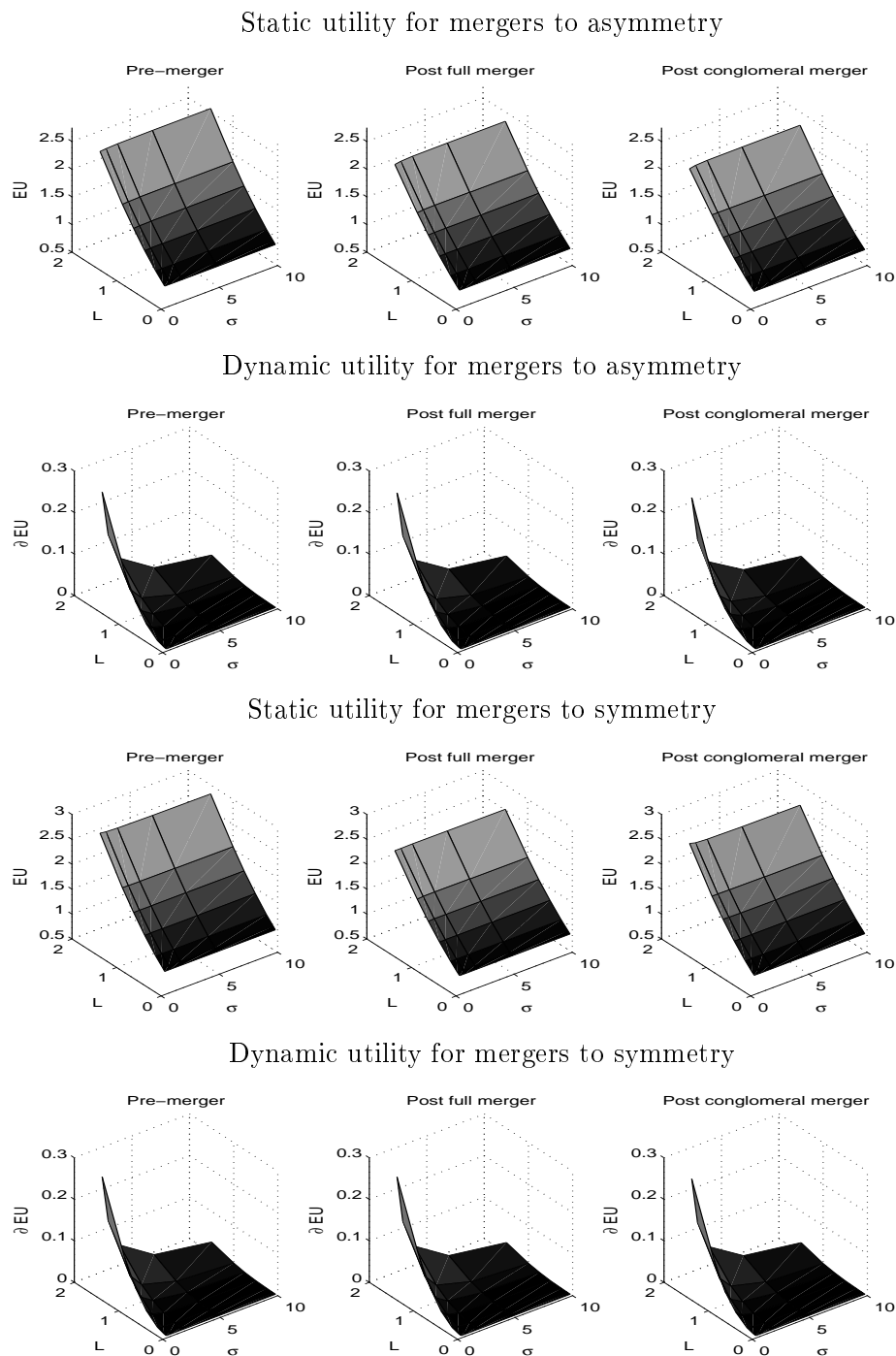
Figure 15: Relative static vs. dynamics effects of mergers to duopolies



Note: We simulate mergers of two out of three firms using the model described in Section 1. For mergers to asymmetry, all firms have initial market shares of 33%, while, for mergers to symmetry, a market leader with an initial market shares of 50% faces a merger of its two symmetric competitors with initial market shares of 25%. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. For each merger, we compute pre and post-merger static utilities EU^{pre} and EU^{post} as well as dynamic utilities ∂EU^{pre} and ∂EU^{post} as given by equations 3.1.3 and 3.1.4. We plot $(EU^{post} - EU^{pre})/EU^{pre}$ (*static effect*), $(\partial EU^{post} - \partial EU^{pre})/\partial EU^{pre}$ (*dynamic effect*) and $(\partial EU^{post} - \partial EU^{pre})/(EU^{post} - EU^{pre})$ (*dynamic vs. statics*).

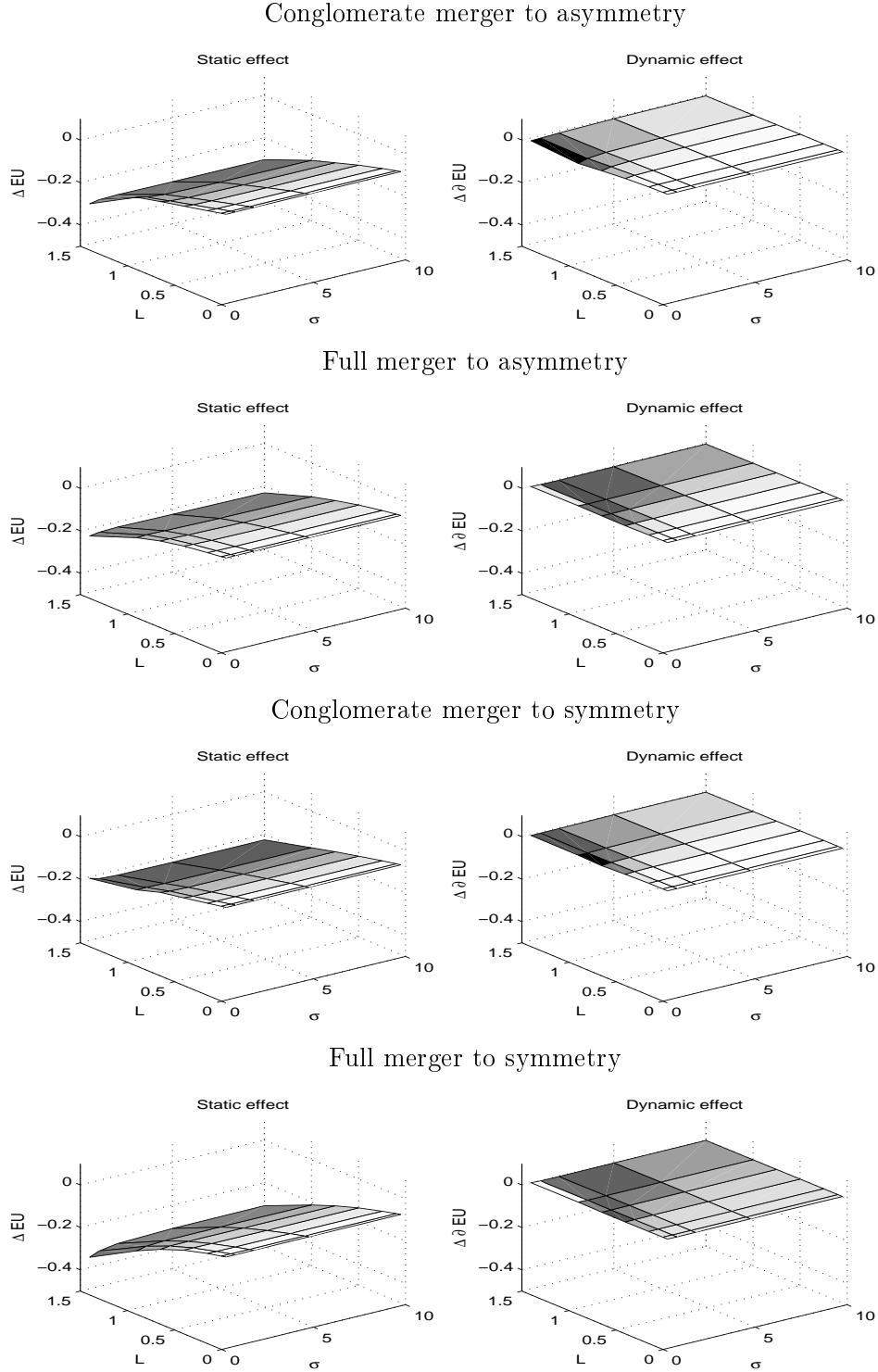
C Utility effect of mergers to three firms

Figure 16: Pre and post-merger utility for mergers to three firms



Note: We simulate mergers of two out of four firms using the model described in Section 1. For mergers to asymmetry, all firms have initial market shares of 25%, while, for mergers to symmetry, a market leader with an initial market shares of 40% faces a merger of two of its symmetric competitors with initial market shares of 20%. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. For each merger, we compute EU and ∂EU as given by equations 3.1.3 and 3.1.4.

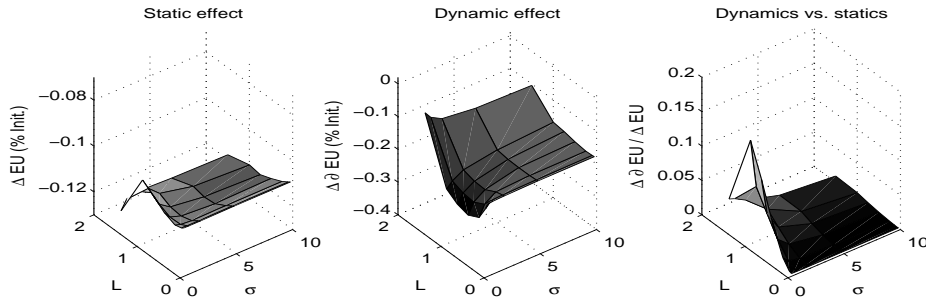
Figure 17: Absolute statics vs. dynamics effects of mergers to three firms



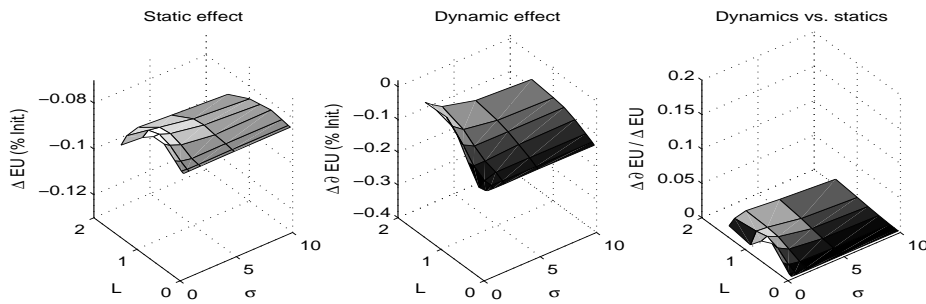
Note: We simulate mergers of two out of four firms using the model described in Section 1. For mergers to asymmetry, all firms have initial market shares of 25%, while, for mergers to symmetry, a market leader with an initial market shares of 40% faces a merger of two of its symmetric competitors with initial market shares of 20%. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. For each merger, we compute pre and post-merger static utilities EU^{pre} and EU^{post} as well as dynamic utilities ∂EU^{pre} and ∂EU^{post} as given by equations 3.1.3 and 3.1.4. We plot $EU^{post} - EU^{pre}$ (*static effect*) and $\partial EU^{post} - \partial EU^{pre}$ (*dynamic effect*).

Figure 18: Relative statics vs. dynamics effects of mergers to three firms

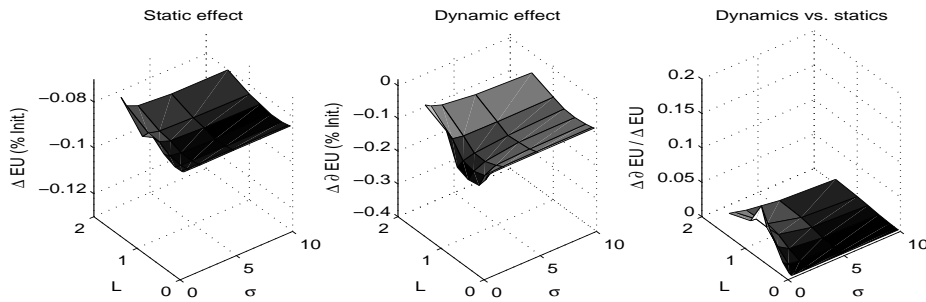
Conglomerate merger to asymmetry



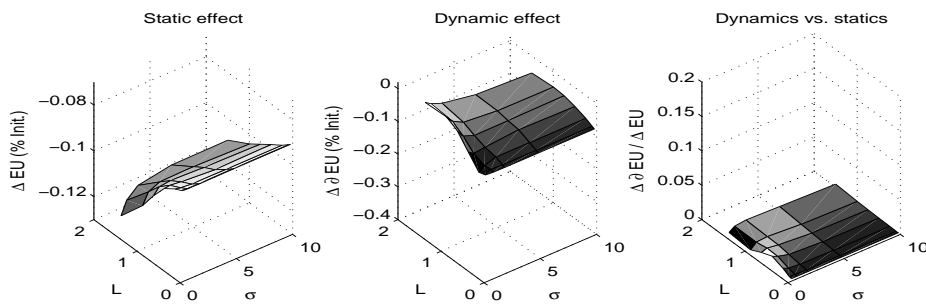
Full merger to asymmetry



Conglomerate merger to symmetry



Full merger to symmetry

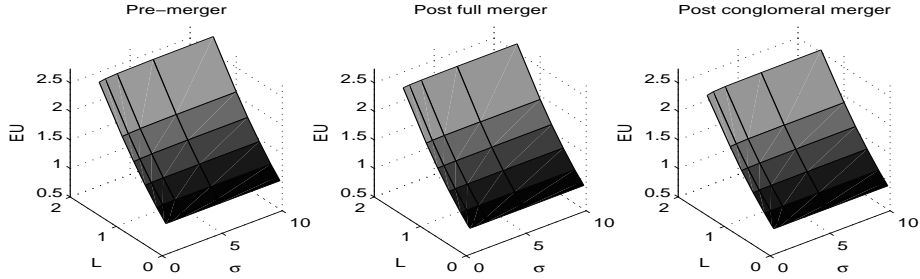


Note: We simulate mergers of two out of four firms using the model described in Section 1. For mergers to asymmetry, all firms have initial market shares of 25%, while, for mergers to symmetry, a market leader with an initial market shares of 40% faces a merger of two of its symmetric competitors with initial market shares of 20%. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. For each merger, we compute pre and post-merger static utilities EU^{pre} and EU^{post} as well as dynamic utilities ∂EU^{pre} and ∂EU^{post} as given by equations 3.1.3 and 3.1.4. We plot $(EU^{post} - EU^{pre})/EU^{pre}$ (*static effect*), $(\partial EU^{post} - \partial EU^{pre})/\partial EU^{pre}$ (*dynamic effect*) and $(\partial EU^{post} - \partial EU^{pre})/(EU^{post} - EU^{pre})$ (*dynamic vs. statics*).

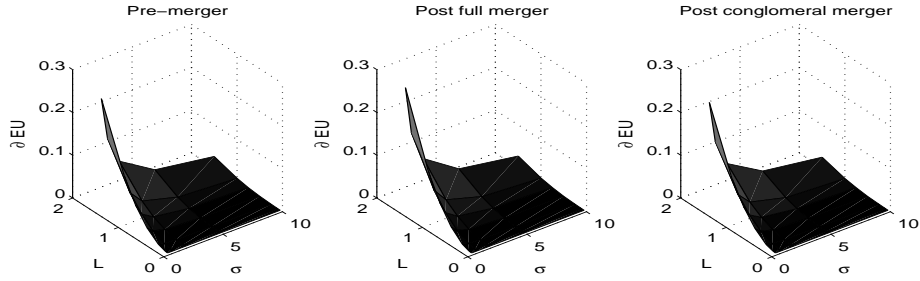
D Utility effects of mergers to four firms

Figure 19: Pre and post-merger utility for mergers to four firms

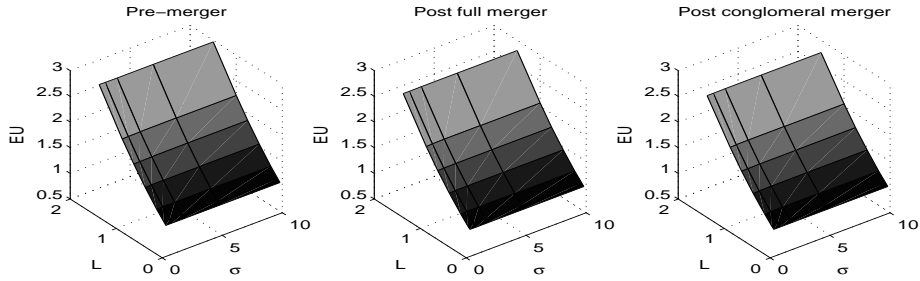
Static utility for mergers to asymmetry



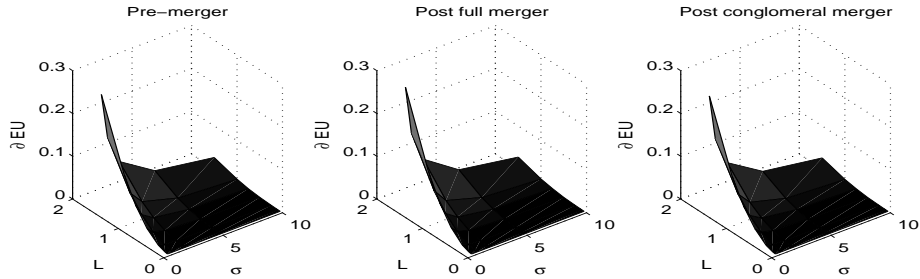
Dynamic utility for mergers to asymmetry



Static utility for mergers to symmetry

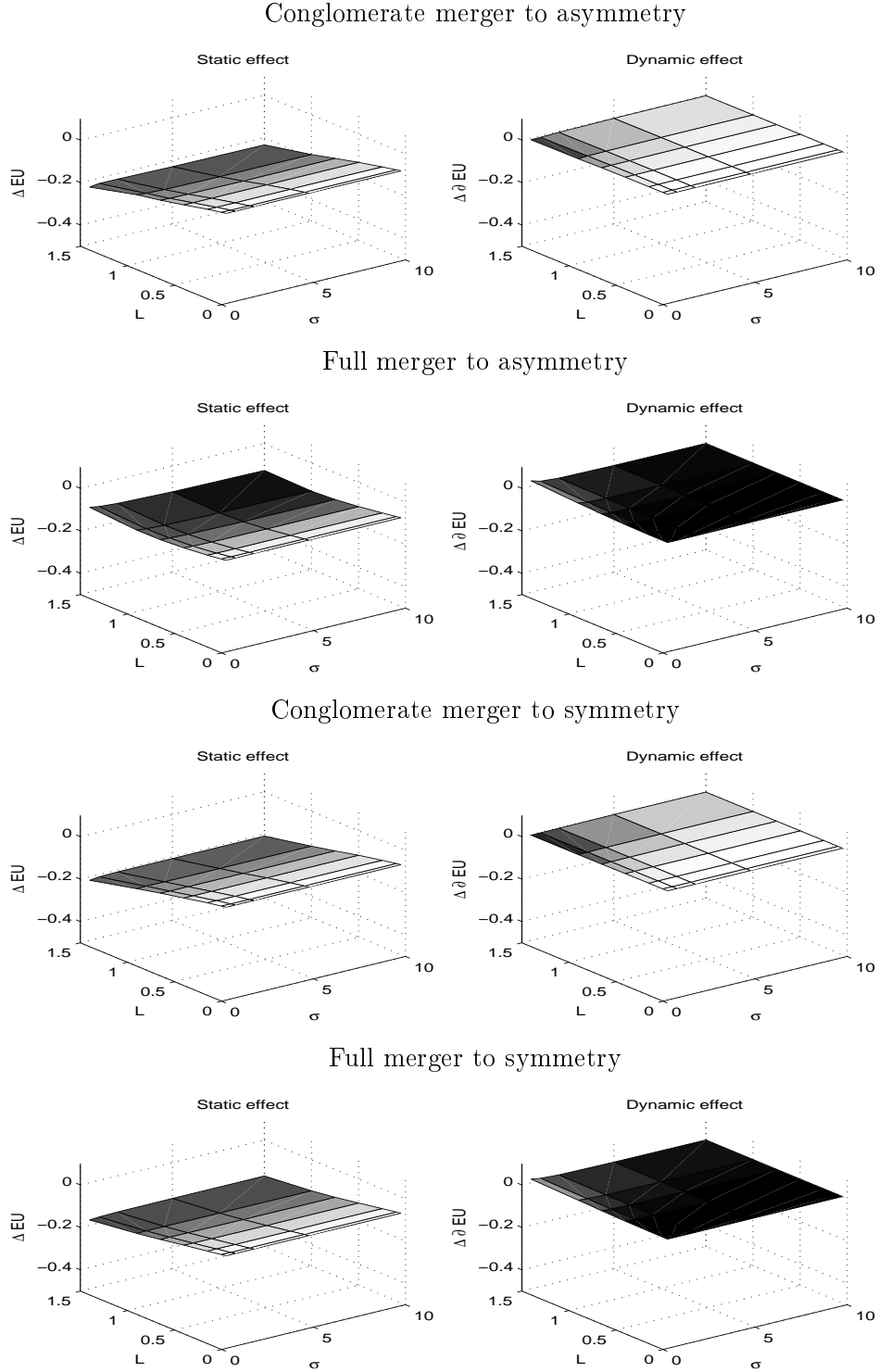


Dynamic utility for mergers to symmetry



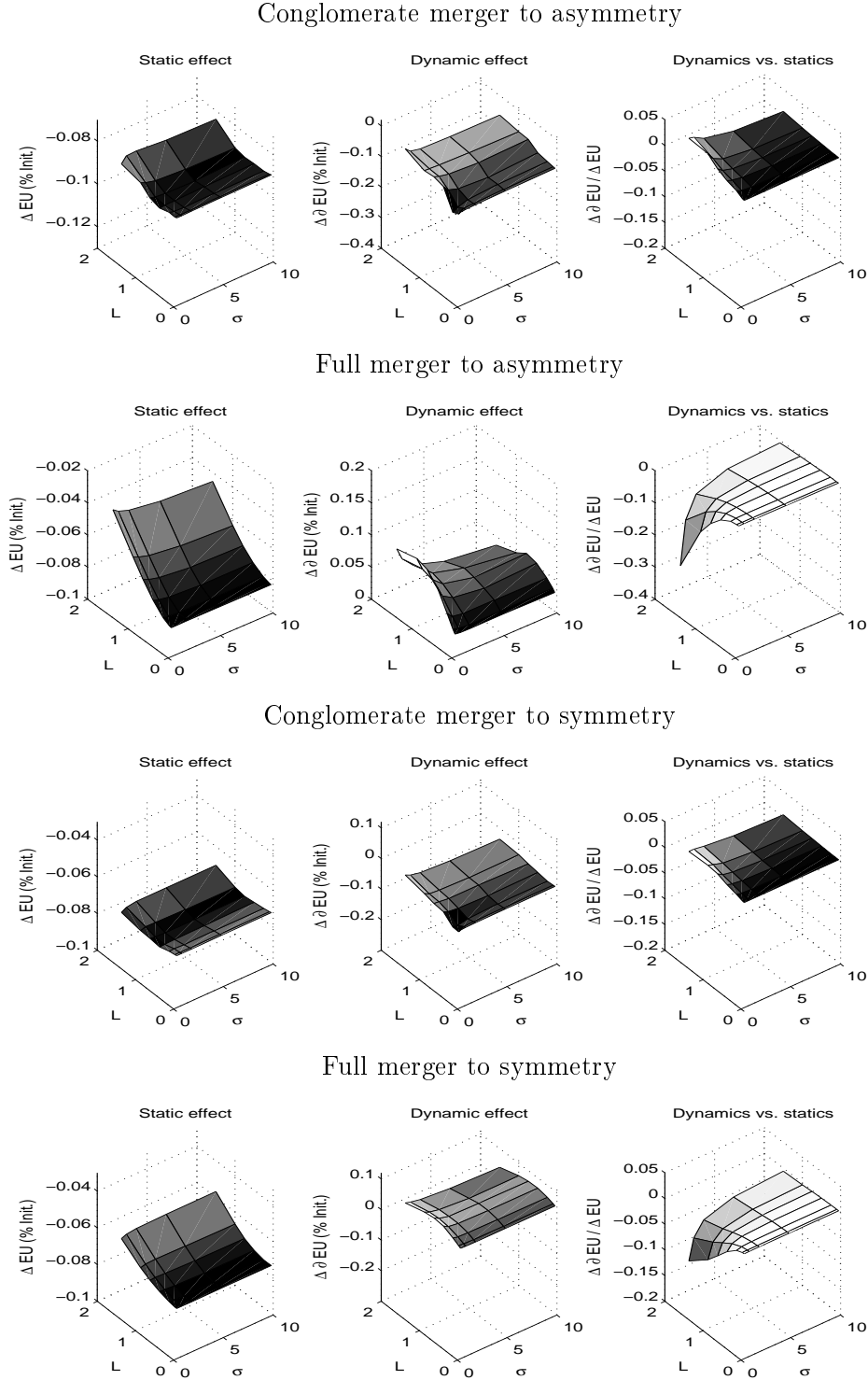
Note: We simulate mergers of two out of five firms using the model described in Section 1. For mergers to asymmetry, all firms have initial market shares of 20%, while, for mergers to symmetry, a market leader with an initial market shares of 33% faces a merger of two of its symmetric competitors with initial market shares of 16.5%. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. For each merger, we compute EU and ∂EU as given by equations 3.1.3 and 3.1.4.

Figure 20: Absolute statics vs. dynamics effects of mergers to four firms



Note: We simulate mergers of two out of five firms using the model described in Section 1. For mergers to asymmetry, all firms have initial market shares of 20%, while, for mergers to symmetry, a market leader with an initial market shares of 33% faces a merger of two of its symmetric competitors with initial market shares of 16.5%. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. For each merger, we compute pre and post-merger static utilities EU^{pre} and EU^{post} as well as dynamic utilities ∂EU^{pre} and ∂EU^{post} as given by equations 3.1.3 and 3.1.4. We plot $EU^{post} - EU^{pre}$ (*static effect*) and $\partial EU^{post} - \partial EU^{pre}$ (*dynamic effect*).

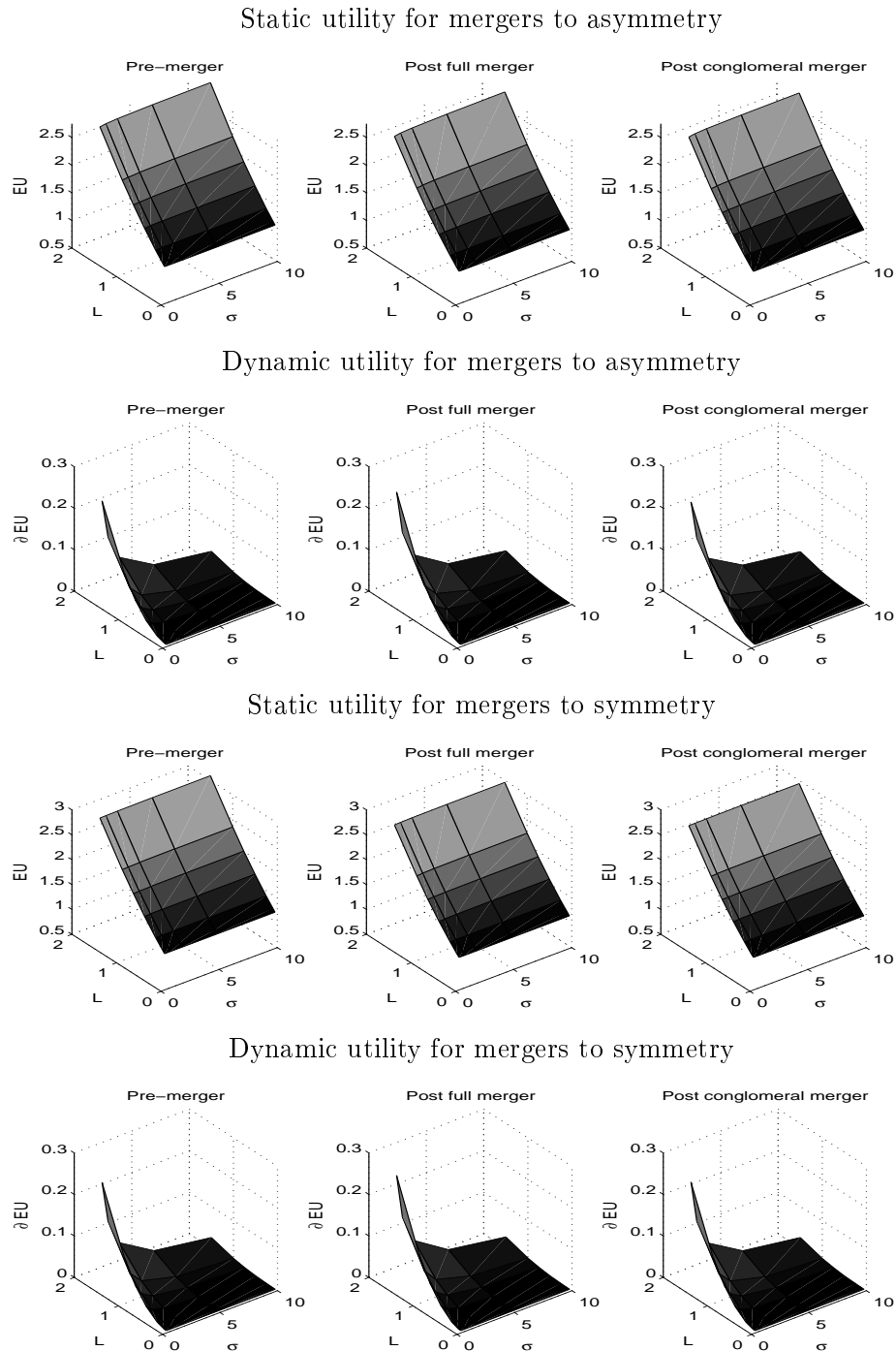
Figure 21: Relative statics vs. dynamics effects of mergers to four firms



Note: We simulate mergers of two out of five firms using the model described in Section 1. For mergers to asymmetry, all firms have initial market shares of 20%, while, for mergers to symmetry, a market leader with an initial market shares of 33% faces a merger of two of its symmetric competitors with initial market shares of 16.5%. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. For each merger, we compute pre and post-merger static utilities EU^{pre} and EU^{post} as well as dynamic utilities ∂EU^{pre} and ∂EU^{post} as given by equations 3.1.3 and 3.1.4. We plot $(EU^{post} - EU^{pre})/EU^{pre}$ (*static effect*), $(\partial EU^{post} - \partial EU^{pre})/\partial EU^{pre}$ (*dynamic effect*) and $(\partial EU^{post} - \partial EU^{pre})/(EU^{post} - EU^{pre})$ (*dynamic vs. statics*).

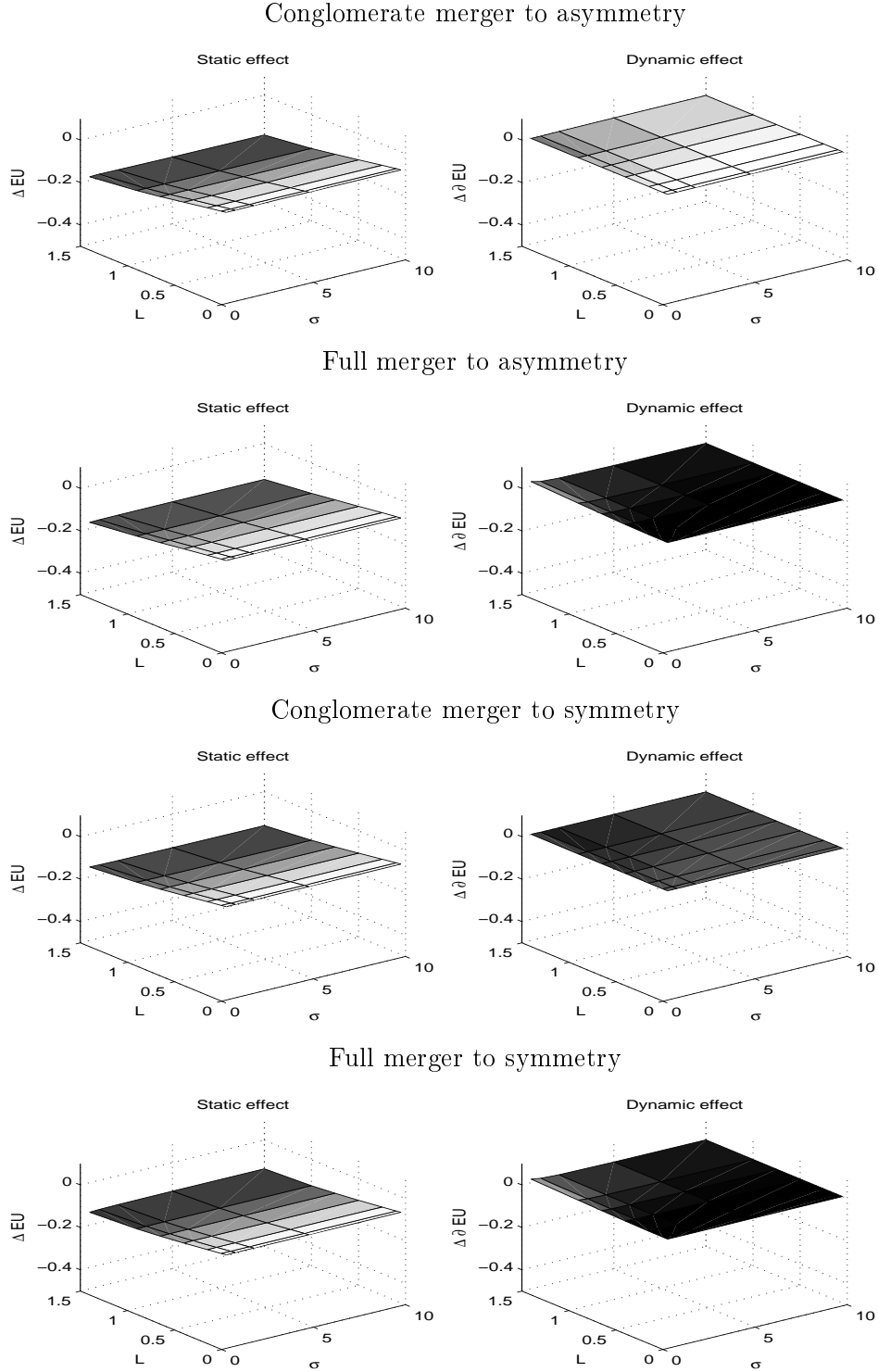
E Utility effects of mergers to five firms

Figure 22: Pre and post-merger utility for mergers to five firms



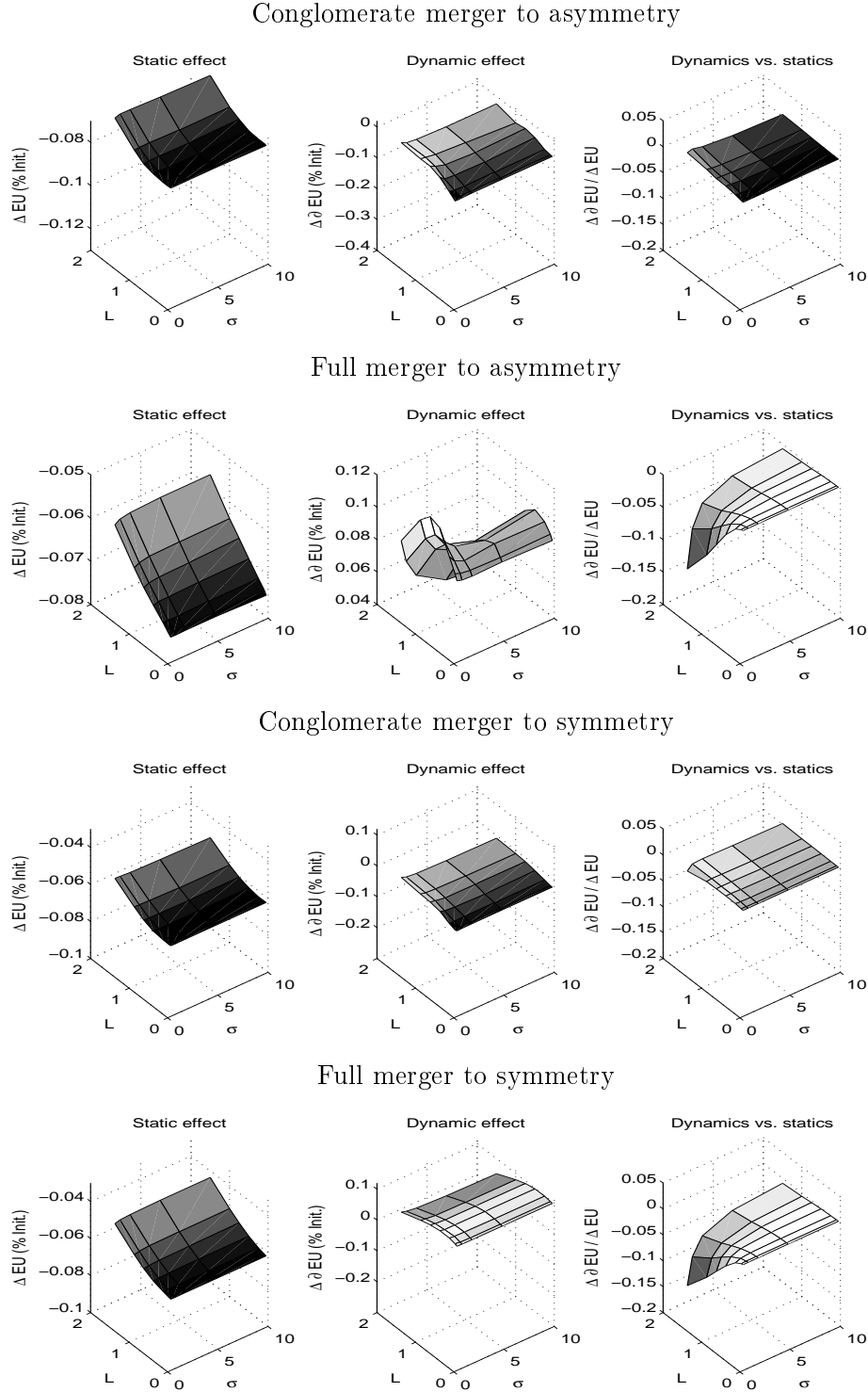
Note: We simulate mergers of two out of six firms using the model described in Section 1. For mergers to asymmetry, all firms have initial market shares of 16.5%, while, for mergers to symmetry, a market leader with an initial market shares of 28.6% faces a merger of two of its symmetric competitors with initial market shares of 14.3%. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. For each merger, we compute EU and ∂EU as given by equations 3.1.3 and 3.1.4.

Figure 23: Absolute statics vs. dynamics effects of mergers to five firms



Note: We simulate mergers of two out of six firms using the model described in Section 1. For mergers to asymmetry, all firms have initial market shares of 16.5%, while, for mergers to symmetry, a market leader with an initial market shares of 28.6% faces a merger of two of its symmetric competitors with initial market shares of 14.3%. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. For each merger, we compute pre and post-merger static utilities EU^{pre} and EU^{post} as well as dynamic utilities ∂EU^{pre} and ∂EU^{post} as given by equations 3.1.3 and 3.1.4. We plot $(EU^{post} - EU^{pre})/EU^{pre}$ (*static effect*), $(\partial EU^{post} - \partial EU^{pre})/\partial EU^{pre}$ (*dynamic effect*) and $(\partial EU^{post} - \partial EU^{pre})/(EU^{post} - EU^{pre})$ (*dynamic vs. statics*).

Figure 24: Relative static vs. dynamics effects of mergers to five firms



Note: We simulate mergers of two out of six firms using the model described in Section 1. For mergers to asymmetry, all firms have initial market shares of 16.5%, while, for mergers to symmetry, a market leader with an initial market shares of 28.6% faces a merger of two of its symmetric competitors with initial market shares of 14.3%. Post-merger, the merged entity can put together all R&D (*full merger*) or not (*conglomerate merger*). For these simulations, we use the following parameters: $\delta = 0.85$, $\gamma = 10$, $L \in \{0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1, 1.5\}$ and $\sigma \in \{0.5, 1, 2, 5, 10\}$. For each merger, we compute pre and post-merger static utilities EU^{pre} and EU^{post} as well as dynamic utilities ∂EU^{pre} and ∂EU^{post} as given by equations 3.1.3 and 3.1.4. We plot $(EU^{post} - EU^{pre})/EU^{pre}$ (*static effect*), $(\partial EU^{post} - \partial EU^{pre})/\partial EU^{pre}$ (*dynamic effect*) and $(\partial EU^{post} - \partial EU^{pre})/(EU^{post} - EU^{pre})$ (*dynamic vs. statics*).