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Abstract

This article develops a theoretical framework to analyze options for financing infrastructure in developing countries. We build a basic model that gives motivations for using a combination of public finance, private debt and private equity. The model is then extended in a number of ways to examine a variety of factors that are important for developing countries when considering financing choices. We focus in particular on key institutional weaknesses that are often important for infrastructure investment. Overall, we show that such weaknesses can be key in determining financing choices, but that they do not all push in the same direction. Financing schemes must therefore be adapted to consider the institutional limitations that are most pertinent in any given context.

JEL classifications: G32, G38, H54, O16
Keywords: Finance, Development, Infrastructure

1 Introduction

Estimates of global infrastructure needs turn around 4% of global GDP or about US$4 trillion per year. For developing countries, the relative importance of the needs is much higher and for some countries can reach over 15% of their GDP. For Asia, the Asian Development Bank (ADB) estimates annual needs at 6 to 6.5% of GDP or US$800 billion to US$1 trillion up to 2020 at least (Bhattacharyay, 2010). For Latin America, the Inter-American Development

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Bank (IDB) estimates needs at, at least, 5.2% of GDP or around US$250 billion, more than double the current level (Inter-American Development Bank, 2013). For Sub-Saharan Africa, the African Development Bank (AfDB) reports annual expenditure needs of 12 to 16 % of GDP, about two thirds in capital expenditure and one third in operation and maintenance of the assets (African Development Bank, 2010). This is about US$75 to $100 billion annually, more than twice current expenditures. Adding up the various sources and accounting for the demands on infrastructure of the growing concerns for climate change, suggest that developing and emerging economies could need as much US$1.5 trillion/year of infrastructure investments up to 2030.

The OECD estimates that these countries only spend currently about US$800-900 billion on infrastructure per year. This leaves a very significant financing gap. Most of the current financing is from domestic sources, including the public sector (55-75%). Official development financing accounts for 5 to 8%. Many of the countries and many of the donors have been hoping for over 25 years now that the private sector will become a major source of financing for the sector. So far, it finances a significant share of about 20-30% of the current infrastructure investment levels (OECD, 2013). This contribution is important but below expectations. Moreover, it suffers from a huge variance. In SSA for instance, over 50% of countries do not have access to any significant private financing and an equivalent observation could be made for some of the countries in Asia and Latin America.

There is a huge need for extra infrastructure investment in many developing countries (Foster and Briceño-Garmendia, 2009; Calderon and Servén, 2010). Traditionally, infrastructure spending in these countries has largely been financed by national governments, but fiscal constraints and competing spending priorities limit the expansion of public financing. Increasing attention is therefore turning to the potential role of private financing through public-private partnerships (PPPs). However, potential problems including an apparent premium on private finance mean that it is unclear whether a large expansion of PPPs is feasible or desirable (Engel, Fischer and Galetovic, 2013).

What the data tells us, in sum, is that, for the foreseeable future, the public sector will continue to be a major source of investment financing of the sector. Moreover, considering that in the poorest countries a large share of the population lives in poverty, it is very likely that many public services, including infrastructure, will continue to be financed from subsidies rather than user fees. Competition between infrastructure, health and education ministries for access to general tax revenue will thus continue to be strong for countries with limited fiscal capacity. This is precisely one of the reasons why many development actors continue

to push for a continuous effort to increase the scope for private financing in infrastructure beyond the relative success achieved in the last 25 years.

There are, however, a few things we have learned with experience that needs to be factored in the assessment of the real scope for public-private partnerships (PPP) in the financing of infrastructure. Potential problems including an apparent premium on private finance mean that it is unclear whether a large expansion of PPPs is feasible or even desirable (Engel, Fischer and Galetovic, 2013). It may thus be useful to step back and rethink the decision making process that needs to be followed to assess the scope and limits of PPPs, internalizing the main lessons of experience in the context of developing and emerging economies. This is the main purpose of this paper.

Our overarching question is 'when should developing countries use private finance, and when should they use public finance', focusing on the decision at the project level. In order to answer this question, we analyse factors influencing the availability and value of private financing, distinguishing between corporate equity and debt-based project finance. In particular, we consider the core motivations for using each of these types of private finance, and then analyse how these may be affected by the developing and emerging economies context. This implies an explicit recognition of the importance of various types of risks as well as of the specific level of distortions introduced by tax financing options but most importantly the importance of the challenges encountered in dealing with the financing side of the business.

Analysis of the relative merits of public versus private infrastructure ownership dates back to the literature on privatization.[1] Typically the focus of this literature was on infrastructure that had already been built, and hence financing issues were not considered explicitly. The policy world then took over the debate and lead it into a combination of ideological positions and empiricism on the relative merits of public and private provision of public services. Most of this literature is largely free from theoretical support. It is only more recently that an important theoretical literature has developed in an effort to understand the costs and benefits of public-private partnerships. Iossa and Martimort (2014) and Engel, Fischer and Galetovic (2014) provide good syntheses of these models, which typically emphasize the extra incentives for effort that come from bundling infrastructure construction with its operation.

A common issue raised by the economic research so far has been that financing details remained secondary, and when considered, the essential line between equity and debt and the differences in costs between the various source of financing was often blurred. And yet,

[1] See, for example, Vickers and Yarrow (1991) and Martimort (2006) for surveys of the theoretical literature.

they are among the main concerns of the project finance literature focusing on how to design infrastructure deals (as nicely summarized by Gatti (2013), for instance). Thus, whilst the economic literature provides important insights on the optimal unbundling and assignment of the existing production activities and the project finance literature is remarkably pragmatic in picking among various financing options and in assigning risks, neither source helps in understanding whether additional financing should be supplied by private lenders or by government accounting jointly for the details of the financing options, risks concerns and fiscal constraints.

However, financing typically remains secondary in these models, and when it is considered the line between equity and debt is often blurred. Thus, whilst the models provide important insights as to when private management should finance infrastructure construction, they help little in understanding whether additional financing should be supplied by private lenders or government.

A major contribution of this paper is to build a framework which allows us to address this issue. Our model considers how infrastructure investment should be split between private debt, private equity and public investment. We thus combine insights from the economic literature with those from corporate finance, which typically do not consider the role of public investment (Tirole, 2010). In the process, we tackle a major policy concern recently summarized by Standard and Poor (2014). We show that, under certain standard assumptions, decisions on these financing sources are not separable. In particular, whilst private debt project finance can co-exist alongside public investment, it is facilitated by private equity investment. Governments may therefore wish to transfer management to private equity even when this is detrimental ex-post, precisely because it allows for a greater amount of project related debt to be raised.

The second contribution of the paper is then to use this framework to analyse how a variety of factors important in developing countries may influence the source of financing used. Focus is given to a number of institutional weaknesses commonly found in developing countries, including limited government commitment and limited accountability. The flexibility of our framework allows us to synthesize the results of a number of papers as well as generate new results. The analysis is both normative and positive, considering the actions of both a benevolent government and one whose priorities differ from those of their citizens.

Finally, considering these multiple factors in the same framework allows us to draw together a narrative of which factors are likely to be dominant in developing countries' financing decisions. We show that 'weak instituions' do not in general push for or against private finance, but instead different weaknesses push in different directions. The financing scheme a

government chooses is therefore likely to be based on which institutional weakness is most salient.

We proceed in the following manner. The next section sets up the basic model and shows that, under strict assumptions, governments should use the cheapest form of available finance, and prices are unaffected by the financing decision. In Section 3, we then introduce what we argue to be the key economic motivations for private finance: government's inability to transfer profits from the firm ex-post, and their inability to contract on costs ex-ante. We show that these two factors imply it can be be optimal for a combination of all three types of finance to be used, and we then draw out some comparative statics. Section 4 considers a number of extensions to the model and alternative assumptions that allow us to understand how a range of factors important for developing countries may influence their financing decision. Finally, we conclude by drawing together the analysis and deriving implications for how institutional weaknesses are likely to impact upon developing country infrastructure financing.

2 Model and financing equivalence benchmark

We begin by setting up the core model. We then make a series of stringent assumptions which lead to an 'irrelevance result', allowing us to understand the assumptions at work in the results later in the paper.

2.1 Core model

Our model has two players, the government and the firm, and takes place over two periods. In period 1 investment decisions are made by the government and the firm. In period 2, the infrastructure is productive, collecting revenue and paying any operating costs.

We consider a single infrastructure project which requires an initial investment I in period 1. We distinguish between three types of financing - private equity, I_E, government financing, I_G and project related debt, I_D - which together make up the total investment, i.e. :

$$I = I_G + I_D + I_E \tag{1}$$

Public financing is taken from the general government budget, which has an opportunity cost of $1 + \lambda_t$ in period t. In other words, any dollar not spent on this infrastructure project will be spent on something else that benefits society an amount $1 + \lambda_t$ - this could be lower taxation,

an additional infrastructure project or spending in another sector.[2]

Private debt is raised from external financiers, who demand an expected rate of return of $1+i_D$. Similarly, private equity demands a rate of return of $1+i_E$. In the following model, the key distinction between the two private financing sources is that we assume equity investors can contract with the firm's manager on effort undertaken, whereas debt lenders cannot. This distinction does not always quite match reality, in that some lenders may have control rights or monitoring technologies that allow them to influence managers, and equally some outside equity investors may have no control over managerial behavior. One should therefore not necessarily take a literal interpretation when considering results regarding debt/equity ratios below, but instead consider equity as representing the part of external finance which has relatively more influence on the firm's behavior. Since monitoring a firm and contracting with their manager involves transactional costs, it is reasonable to assume that finance with such an ability comes at a premium.[3]

In the second period, the firm makes a profit Π given by the following profit function:

$$\Pi = pq - C + t \tag{2}$$

where C is the firm's cost, p is the price, $q = q(p)$ is the quantity consumed and t is the transfer from the government. The price p and transfer t are set by the government, whilst q is given according to a consumer surplus function $S(q)$, such that $p = S'(q)$ and $S''(q) < 0$. In project finance, this is equivalent to the demand forecast used to estimate a firm's potential revenues.

To keep the model simple, we assume that C takes a binary distribution. In particular, $C = \underline{c}$ with probability e and $C = \overline{c}$ with probability $1 - e$, where $\overline{c} > \underline{c}$ and e is the effort made by the management of the firm. Effort gives the manager of the firm a dis-utility of $\kappa(e)$, with $\kappa'(e) > 0$ and $\kappa''(e) > 0$ for $e > e_0$, where $e_0 > 0$ is such that $\kappa(e_0) = 0$.[4] Moreover, we assume that the cost function κ is such that the manager will always exert effort in the

[2]Typically, λ is assumed to stem from the distortionary effect of taxation, but it could also be higher or lower than such distortions if political economy motivations result in sub-optimal taxation.

[3]Another major difference between equity and debt is that typically lenders are repaid before shareholders. We indeed make such an assumption in our analysis, but, as demonstrated by Tirole (2010, p.134), such a division of payments stems naturally from the assumption that equity investors have more influence over marginal profit.

[4]Typically moral hazard models are such that $e_0 = 0$, but we set a positive value here to ensure there is some probability of low operational costs even when the manager is not incentivized. Alternatively, it would be equivalent to assume $\kappa(0) = 0$ and the probability of low operational costs is $e + e_0$.

range $[e_0, 1)$.

We assume that e is not directly observable, but can be effectively contracted upon by any actor who can contracts on cost.[5] We further assume that in period 2 the firm has a limited liability constraint, such that it will close down production if it expects to make negative profits.

In period 1, the government sets I_G and makes promises regarding the future price and transfer such that the firm is willing to finance the remaining $I - I_G$. It is then the firm who decides upon how this private finance is split between debt (I_D) and equity (I_E) through deciding how the potential profits will be split between investors.[6] In particular, debt-holders are promised π_D, and equity investors receive any remaining profit $\pi_E = \Pi - \pi_D$.

Let the expected profit made by debt investors and equity investors be Π_D and Π_E respectively. We assume that the expected rates of return earlier defined correspond to the opportunity cost of investing - hence for the debtholder the opportunity cost of investing an amount I_D is $(1 + i_D)I_D$ and for the equity investor the opportunity cost of investing an amount I_E is $(1 + i_E)I_E$. The expected profits of debt and equity investors can therefore be written as follows:

$$\Pi_D = \pi_D \mathbb{P}[\Pi \geq \pi_D] + \mathbb{E}[\Pi | \Pi < \pi_D]\mathbb{P}[\Pi < \pi_D] - (1 + i_D)I_D \tag{3}$$

$$\Pi_E = \mathbb{E}[\Pi - \pi_D | \Pi \geq \pi_D]\mathbb{P}[\Pi \geq \pi_D] - (1 + i_E)I_E \tag{4}$$

Both types of investors have a participation constraint, i.E. $\Pi_D \geq 0$ and $\Pi_E \geq 0$. We assume that the government has all the bargaining power with respect to private investors since private investors compete with one another.[7]

We assume that both the government and investors are risk neutral. Our results would be very similar if we assumed parties to be equally risk averse. Other models of public-private partnerships model private investors as being risk-averse whilst the government is risk neutral, but at least two reasons suggest to us it is not clear such an assumption is reasonable for developing countries. First, international investors may be more able to hedge

[5] We thus essentially assume that the manager does not have a binding limited liability constraint, though allowing for such would not substantially change the analysis below.

[6] The assumption that the firm chooses how much debt to issue is standard in the literature and indeed typically reflects reality. Increasing concern however has been made that investors may choose to issue too much debt because they do not pay the full costs of potential financial distress. Jamison, Mandy and Sappington (2014) analyze such a scenario and derive the regulator's optimal response.

[7] In reality, private investors may have some bargaining power due to limited competition - if included in the model, this would act in a very similar way to increasing the cost of private investment.

the risks involved in a large infrastructure project than national governments. Second, credit constrained developing country governments will be forced to pass on risk to consumers or taxpayers, who may well be more risk averse than private investors. Overall therefore, we prefer not to make an assumption either way as to which party is more risk averse.

The social welfare function is then given as follows:

$$W = \Pi_D + \Pi_E - (1 + \lambda_1)I_G + \delta\left(S(q) - qp + \sigma(e) - (1 + \lambda_2)t\right) \tag{5}$$

where $\sigma(e)$ captures any additional (negative or positive) effect of cost-reducing effort. For instance, $\sigma(e)$ may be an increasing function of e if effort includes training staff who may go on to work elsewhere, but may be decreasing if reducing costs is associated with lowering quality or increasing pollution.

Social welfare in period 2 is discounted at a rate δ compared to welfare in period 1. The government discounts at this rate, and also takes into account the fact that the opportunity cost of public funds may be different in the two periods. Let us define the government's effective interest rate to therefore be i_G, which is given by the following expression:

$$1 + i_G = \frac{1 + \lambda_1}{\delta(1 + \lambda_2)} \tag{6}$$

In this way, the government is indifferent between 1\$ today and $(1+i_G)$\$ tomorrow, and hence i_G is comparable to the interest rates used for private finance, i_E and i_D.

In period 2, first effort is made by the manager, and then the operational cost is revealed to all parties. The timeline is given in Figure 1.

2.2 Benchmark assumptions and result

We now make a number of stringent assumptions which enable us to arrive at a benchmark result regarding optimal financing:

Assumption 1. Unlimited transfers: There are no limits on transfers between the firm and the government.

Assumption 2. Contractable cost: We assume that operational costs C are observable and can be contracted on by both the government and private equity investors.

Assumption 3. Government commitment: In period 1, the government can commit to the transfers and prices that will occur in period 2.

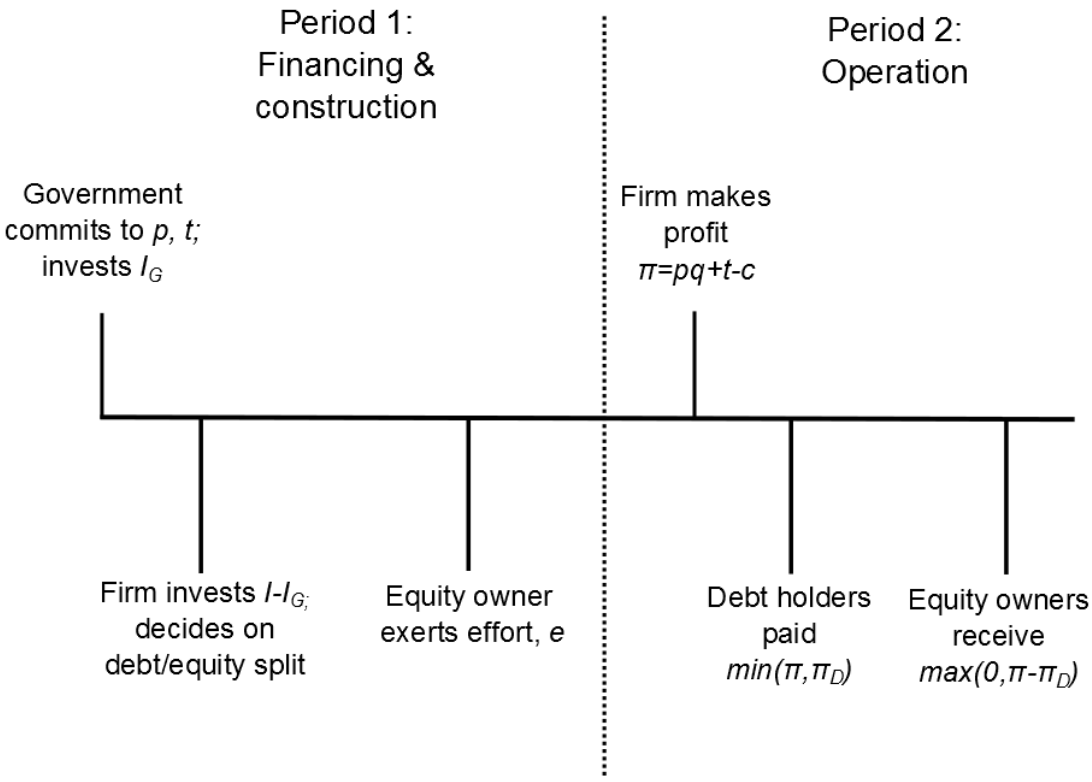

Figure 1: Timeline

Assumption 4. Benevolent government: The government maximizes social welfare.

We can now solve the model to find the optimal financing scheme chosen by the government. This is given in the following proposition:

Proposition 1. Given assumptions (1) to (4), the project will be financed by the cheapest available financing source.

The intuition behind this result is relatively straightforward. First, it is clear to see that the government will set I_E and I_D such that private investors make zero profit in equilibrium - i.e. their participation constraints will be binding. This is because, even though profits appear in the government's welfare function, at the margin profits come from either increasing the government's investment (I_G) or their transfers to the firm (t), and both sources of finance have

an opportunity cost greater than 1. Any increase in private investors profits then comes at a cost of λ_1 or λ_2 to social welfare, and hence the government will set profits at the minimum.

A similar argument exists to understand the core of the proposition. By Assumption 1, any money saved by the government in investment costs is essentially lost through lower future transfers from the firm. The logic here is essentially the same as set out in Engel et al. (2013) and Vining and Boardman (2014) - current government 'savings' that stem from PPPs are illusory to the extent that future government revenues are forfeited. Moreover, private equity holds no advantage over government investment when it comes to contracting on cost (Assumption 2), and hence if private equity is more expensive there is no reason to use it. The socially optimal outcome is therefore to simply use the cheapest source of finance available, and since the government can commit to such a policy (Assumption 3) and is benevolent (Assumption 4), this is the solution it will impliment.

Hence, if there are no limits on the amount of financing available from each source, then simply the cheapest source will be used. For instance, in the case where $i_G < i_D < i_E$, the project would be financed publicly. However, it may be that there are limits on the amount of funding available from each source (if, for example, there are budget caps on public spending). In this case, when $i_G < i_D < i_E$, as much government funding as is available will be used to finance the project. If this is not sufficient to cover the investment, private debt will then be used. Finally, if need be, equity will be used to close the remaining financing gap.

Finally, note that prices will not depend on the type of finance used or the fixed cost. In particular, they will be set according to the following equation:

$$\eta(p) = \frac{\lambda_2}{1 + \lambda_2} \tag{7}$$

where $\eta(p)$ is the price elasticity of demand at price p. This is the standard Ramsey pricing formula which ensures that the distortion caused by pricing is equal to the opportunity cost of public funds.

3 Motivations for private financing

In many contexts, it is indeed the case that the cost of private finance is above the government's discount rate, i.e. $i_G < i_D < i_E$. Why then do we see private financing being used to finance infrastructure investment? In this section, we set out two basic motivations for using private financing which each correspond to a certain type of financing source. In particular, we explore

how private debt may be used to reap the benefits of prices set above cost-recovery levels, and how private equity may be useful when it is better able to contract on cost than the government. For the remainder of the paper we assume that $i_G < i_D < i_E$.

3.1 Motivation for private debt: No transfers during operation

In this sub-section, we continue to hold assumptions (2)-(4), but we replace Assumption 1 with the following:

Assumption 1′ . No transfers during operation: It is impossible for transfers to be made between the firm and the government in period 2.

This assumption reflects the fact that governments frequently set up institutional structures that make it costly for transfers to be made between the infrastructure firm and the government. Indeed, it is often modeled as an important impact of privatization and, when ownership remains public, limiting government transfers is often a key part of 'corporatization' programs designed to improve performance (Martimort, 2006).

One potential reason why such a restriction may be put in place is that otherwise interest groups such as firm owners or service users may be able to successfully lobby for (inefficient) public subsidies (Vining and Boardman, 2014). Evidence of this can be seen from the fact that, when transfers are relatively easy, infrastructure is often subsidized by the government even though such subsidies are frequently inefficient and regressive (see, for example, Komives, Foster, Halpern and Wodon (2005)). Note that an alternative assumption that would lead to very similar results is that only by outsourcing revenue collection to the private sector can the government commit to certain price levels in advance, which is close to the assumption made in Auriol and Picard (2009, 2013). In other words, we could exchange Assumption 1′ for one that says that the government can more easily commit to a price if the profit is going to pay back private debt than if it is going to pay the government.

If the project has no private finance, then the government must set prices so that they exactly cover operational costs - i.e. such that $pq = c$. This follows directly from Assumption 1′ - since there are no transfers between the firm and the government in the operational phase, the government cannot subsidize the firm, nor can it receive a transfer from the firm. It can therefore not recover any of the investment costs from the users.[8]

[8]The logic would be similar had we assumed transfers were possible but subject to user capture. The government may want to receive a transfer from the firm in order to essentially pay for some of the capital investment the government made, but users may be able to force prices to be lower than optimal.

If the project does have private finance, the government can commit to setting prices above the level where they cover only operational costs. In particular, the government will commit to a price such that the firm can pays its operational costs and repay the private debt that was issued. In this way, therefore, the government essentially uses private project financing in order to ensure that service users contribute towards the investment cost. Hence, even if public debt is cheaper than private debt ($i_G < i_D$), governments may still prefer to use private debt in order to ensure that users contribute towards the cost of investment.[9] The precise result is given in the following proposition:

Proposition 2. Optimal financing depends on the opportunity cost of public funds in the following way:

1. If the opportunity cost of public funds is lower than the discounted private interest rate premium, i.e. $\lambda_1 < \frac{i_D - i_G}{1+i_G}$, then the project will be financed by the government as much as possible.

2. If the opportunity cost of public funds is higher than the discounted private interest rate premium, i.e. $\lambda_1 > \frac{i_D - i_G}{1+i_G}$, and anticipated operation costs are (at least sometimes) sufficiently low, then the project will use private debt financing. The amount of private debt financing used will be increasing in the opportunity cost of public funds and decreasing in the operational costs.

The intuition behind this proposition is that, if we rule out transfers from the firm to the government, private finance is now the only way the government can get users to contribute towards investment costs. In other words, private finance is used as a commitment device to enable user prices to be higher in the future. This comes from the assumption that the government cannot commit to users repaying the government for public investment - that essentially the government is concerned that users will force a renegotiation such that prices are lowered and the government does not receive a direct return on its public investment.

Proposition 2 then says that whether the government uses private finance will depend on the cost of public funds in period 1 relative to the effective cost of debt. If the opportunity cost of public funds is relatively low, then public finance is still the cheapest way to finance the infrastructure. However, if the opportunity cost of public funds is relatively high, then

[9] In the model in this paper, we do not distinguish between service users and those that benefit from government spending not used on infrastructure. In reality, the government may be limited in it's ability to transfer value from service users to others through taxation, and hence this provides an additional motivation to charge prices above cost recovery.

the government will wish the users of the infrastructure to contribute to the investment costs. They will therefore use private finance. The lower the operational costs, the more the amount of private financed used, as there is more potential to set a price substantially above operational cost-recovery.

If private financing is used, then the government would like to promise a price p^*, which is given according to the formula

$$\eta(p^*) = \frac{\lambda_1}{1+\lambda_1} - \frac{i_D - i_G}{(1+\lambda_1)(1+i_G)} \tag{8}$$

Similarly to the price set when transfers were available, this price level aims to balance the distortion caused by setting high prices with the opportunity cost of public funds. Note that, if λ_1 is not much larger than λ_2, the price is lower than in the previous section due to the fact that revenues are not recouped directly by the government, but instead through private debt investment, which comes at a cost. In other words, setting prices involves balancing a trade-off between users and the government budget. Higher prices lead to lower consumer surplus, but more money available for the government. In the previous section, when transfers were allowed, this money came directly from the firm. In this section, there are no direct transfers, so instead high prices only benefit the government through enabling more private finance and hence less government investment in period 1. Since private finance is costly, this therefore reduces the benefits of the government setting higher prices.

Since the firm has a limited liability constraint, it cannot make negative profits in period 2, and hence the government cannot set a price below cost recovery levels. Hence the price will be the maximum of p^* and the price which ensures $pq = c$. Private debt financing will only be used when the price will be above cost recovery levels some of the time, i.e. $p^*q^* \geq \underline{c}$

If this price level is always above cost recovery levels, then the amount of private investment will be as follows:

$$I_D = \frac{1}{1+i_D}\left(p^*q^* - e\underline{c} - (1-e)\overline{c}\right) \tag{9}$$

where q^* is the quantity demanded when $p = p^*$.

This proposition leads us to three comparative statics results. First, less private debt will be used when the cost of this debt (i_D) is higher, for straightforward reasons. Second, private financing will be increasing in the opportunity cost of public funds. This is because, as public funds become more valuable, the government would prefer to have the investment

paid for by users rather than out of the government budget, and the manner in which it can do so is through private debt. The logic here is similar to the result produced in Auriol and Picard (2013). Third, private financing will be decreasing in the expected operational cost of the infrastructure firm, $\mathbb{E}[C]$. This is because higher operational costs decrease the potential profits of the firm, and hence the expected return of private investors.

3.2 Motivation for private equity: Non-contractible cost

We now explore motivations as to why private equity investment may be used alongside project finance lending. In particular, we have assumed that the key distinction between the two private financing sources is that equity investors can effectively contract with the firm's manager on effort undertaken, whereas debt lenders cannot. Up until now, Assumption 2 has given us that the government can also essentially contract on effort with the firm, and hence there has been no need to employ private equity. We now however replace Assumption 2 with the following assumption:

Assumption 2′ . Non-contractible cost: The government can observe cost, but cannot contract on cost or commit to prices that depend on cost.

This assumption reflects the fact that there are some operational costs born by the firm on which the government cannot contract, but on which private equity investors can. This is a common assumption made in the literature on public private partnerships, often implicitly by assuming that the owner of the private equity is the same agent as the manager making the effort decision (Iossa and Martimort, 2012, 2014; Engel et al., 2014). If all costs are of this nature, this essentially limits the government to implementing 'price cap' style regulation.[10]

This difference in contracting abilities may come about for at least three potential reasons. First, comparative expertise may mean private investors are more knowledgeable about the sector, have more contracting capacity or a better monitoring technology (Dewatripont and Legros, 2005; Moszoro, 2014). Second, restrictions on government contracting, such as standardized civil servant pay scales, may prevent the government from implementing the optimal cost-dependent contract. Third, the government may not be able to tie its hands as easily as the private sector, since, for instance, it may hold more power over the judiciary. Hence,

[10]Note, however, that our analysis is still consistent with situations where the firm's revenue is a function of some costs, such as something like 'cost-plus regulation, we simply require that there are some fraction of the firm's costs that remain outside of the contract.

even if the government could write a contract based on such costs, such a contract may not be credible.

It follows from this assumption that, without employing private equity, the government cannot induce the firm to make effort to lower costs. The effect of this depends on whether or not private financing is used. If financing is entirely public, then the expected price will be higher, since the government will set prices at cost recovery levels. If financing is partly private, then the firm's expected profitability will be lower, and hence less private finance will be raised and the government will need to invest more.

The government may therefore want private equity investment in order to induce cost reduction. The following proposition gives us a description of how much private equity will be invested:

Proposition 3. Private equity will only be used alongside private debt. If private equity is used, then the amount of private equity and private debt are increasing in the potential cost savings, $\overline{c} - \underline{c}$.

If private equity is used, then the equity investor will induce the following level of effort:

$$\kappa'(e) = \overline{c} - \underline{c} - \frac{i_E - i_D}{1 + i_E} e\kappa''(e) \tag{10}$$

We can see that, if there is no price premium for equity over debt, then the effort induced will be such that the marginal cost of effort ($\kappa'(e)$) is equal to the marginal benefit ($\overline{c} - \underline{c}$). However, when equity is more expensive than debt, **effort will be lower than optimal since inducing effort requires financing with equity rather than debt**. Given a price level p set by the government, the amount of equity and debt invested will then be given according to the following equations:

$$I_E = \frac{1}{1 + i_E}(e\kappa'(e) - \kappa(e)) \tag{11}$$

$$I_D = \frac{1}{1 + i_D}\left(pq - \overline{c} + \frac{i_E - i_D}{1 + i_E} e^2\kappa''(e)\right) \tag{12}$$

We can therefore see that, even though equity is more expensive than debt, a substantial portion of the expected profits may be given to equity. This is necessary in order to insure cost saving effort is undertaken, and hence mimics the result of Dewatripont and Legros (2005). They note that you need to not have too much 'outsider' financing in order to ensure effort is induced. The expected profitability of the firm is at least $e(\overline{c} - \underline{c}) - \kappa(e)$, since the

government cannot promise a price below that at which $pq = \overline{c}$. Hence we can see that at least some profit is promised to debt-holders, rather than equity investors, and this gives us the first part of the proposition.

Proposition 3 therefore tells us how the amount of private finance will vary if private equity is used. The government does however face a choice over whether to use private equity or not, the result of which is given in the following proposition:

Proposition 4. The relative payoff of using private equity is greater when there are more positive externalities of cost reduction and lower when there are more negative externalities of cost-reduction.

This result is similar to those given in Martimort and Pouyet (2008) and Iossa and Martimort (2014), where there is a correspondence between δ in their papers and $\sigma'(e)$ in ours. Using private equity finance, which is equivalent to bundling in their models, increases cost-reducing effort. Whether or not this increases total welfare depends partly on the size and sign of the externalities of such effort.

4 Contextual factors affecting financing

The previous section laid out potentially key reasons why governments may use private finance, and allowed us to generate some basic comparative statics that gave us insight as to when private financing would be more likely to be used, and how the quantity of such finance might vary. We now treat this as a baseline, in that in each following subsection we start off with assumptions 1′, 2′, 3 and 4. We then further relax one of these assumptions and/or extend the model in some way to demonstrate other important factors that may influence a government's decision of how much (if any) private finance to use. Particular focus is given to the factors which are likely to be most important for developing countries.

4.1 Limited government commitment

Let us first relax the assumption that the government can commit to prices in advance, Assumption 3, and replace it with the following:

Assumption 3′. Limited commitment: With probability $1-\rho$, the government will uphold the price which it committed to, but with probability ρ it will renege upon this commitment and set the ex-post welfare maximizing price.

Suppose that the cost of defaulting on loans is sufficiently high that it is not ex-post optimal for the government to reduce the price such that default occurs.[11] In this case, the government will simply reduce the price such that no rewards go to equity. We then arrive at the following proposition:

Proposition 5. Both total private financing and equity investment is decreasing in the risk of government equity expropriation, ρ.

This is a similar result to Spiegel and Spulber (1994), Moore et al. (2014) and other articles who show that regualted firms will issue more debt relative to equity when faced with the threat of government expropriation of equity. Since there is a lower amount of equity invested, investors will incentivize the manager to induce less effort, and costs will be higher in expectation. Since prices are unchanged, this therefore reduces the expected profitability of the firm and hence the total amount of private investment.

We have abstracted in this subsection from the incentives of the government to renege. Other models, such as Wren-Lewis (2013) and Danau and Vinella (2014) consider that the government's decision on whether or not to renege depends on the amount the government stands to gain. Indeed, Danau and Vinella (2014) show that, when this is case, this creates an upper bound on the amount of private finance invested. This thus provides another reason why limited commitment pushes against private finance (and particularly equity), which could be modeled here by having ρ depend on the benefit of reneging.

4.2 Exogenous risk

So far, we have assumed that the uncertainty in the firm's profitability relates to the types of cost that the firm has some control over. However, in reality, much profitability uncertainty stems from other sources, both on the cost and demand side, particularly for developing countries.

In order to model the effect of such risks on private financing, let's suppose that there's a probability $\gamma \in [0, 1/2)$ that costs are increased by an amount $\tilde{c}$, and with probability γ costs are reduced by the same amount. Expected costs are therefore unaffected by γ, but their variance is increasing in γ.

[11] Such an assumption is standard in the literature - see, for example, Spiegel and Spulber (1994) and Moore, Straub and Dethier (2014) - and may be justified for a number of reasons. Of course, it may also be that there is a risk of government reneging resulting in default - such a risk would simply reduce the amount of private finance overall.

Now suppose that $\tilde{c}$ is sufficiently large such that $\tilde{c} + \underline{c} > p^*q^*$. Since this creates the possibility that the firm might go bankrupt, we need to adjust assumption 1′ somewhat:

Assumption 1″. Almost no transfers: It is impossible for transfers to be made between the firm and the government in period 2, except to prevent loan default.

In particular, we assume that, if costs are sufficiently high that the firm will default on its loans, the government will bail out the firm such that the loans are repaid. This then leads to the following proposition:

Proposition 6. Both total private financing and equity investment is decreasing in exogenous risk, γ.

The logic follows exactly as in the previous subsection with limited government commitment. With probability γ, equity will not receive a return even if effort has succeeded, as only debt is bailed out. As a result, less effort is undertaken and the total expected profitability of the firm falls.

4.3 Asymmetric information

In the model of the previous section, the fact that cost was non-contractible (Assumption 2) gave us a motivation for using private equity. However, we assumed that, ex-post, the government knew as much as the firm did regarding cost. In reality, the firm is often better informed than the government as to its cost level, and hence many models of infrastructure regulation assume asymmetric information between the firm and government. This is likely to be particularly the case in developing countries, where limited regulatory capacity decreases the ability of a government regulator uncovering the firm's true cost. In order to understand the implications such an information asymmetry has for finance, we replace Assumption 2 with the following assumption:

Assumption 2″. Non-observable cost: The government observes cost with probability ζ, but can never contract on cost or commit to prices that depend on cost.

We are assuming that, whilst the government does not observe cost, the private equity investor does. Indeed, the assumption that information asymmetries are lower for those that benefit from residual profits is common. Such differences in asymmetric information could be micro-founded by exploiting the fact that, due to Assumption 1′, the private equity investor has a greater incentive to learn the underlying cost than the government.

We thus arrive at the following proposition:

Proposition 7. The range of parameters for which private financing is optimal are increasing in the asymmetry of information that favours the firm, $1 - \zeta$.

The logic behind this proposition is the following. When private finance is used, price does not depend on cost, and hence social welfare is unchanged in ζ in this case. When no private finance is used, the price will be $\underline{p}$ with probability $e_0\zeta$ and $\overline{p}$ otherwise. Hence the social welfare of using public finance is increasing in ζ. This is similar to the result given in Auriol and Picard (2013), who argue that public management is less desirable when there are large information asymmetries, as this produces a large information rent that the government cannot recoup. Finally, the effect will be enhanced if we combine asymmetric information with the limited commitment assumption (Assumption 3′), as the government will presumably not be able to expropriate when it does not know whether equity will make a return or not.

4.4 Cross-subsidies and cream skimming

We have so far implicitly assumed that the decision on how to finance one infrastructure project can be treated independently from any other infrastructure projects. In reality, however, it may not be appropriate to consider infrastructure finance in this way, for at least two reasons. First, infrastructure projects are often variable in size - roads may vary in the number of towns they serve, whilst distribution networks may cover more or less of the unconnected. Second, cross-subsides within firms are often used, frequently in the form of universal service obligations or universal pricing restrictions. This is a particularly important issue in developing countries since the high opportunity cost of public funds may make such subsidies the second-best way to fund network expansion (Laffont and N'Gbo, 2000).

Within our framework, let us consider the following situation. Suppose that there is a core infrastructure project that has the characteristics already set out. Now suppose that, there is an additional area of size μ where the operational costs are $\mu\hat{c}$, the benefit of infrastructure provision is $\mu S(q)$ and the cost of investment is μI. We assume that this area is high cost, and in particular that $\hat{c} > p^*q^*$, but that investment in infrastructure in this area nonetheless increases social welfare. We make the same assumptions regarding financing this infrastructure as with the core project, but we allow cross-subsidies to be made between the two projects.

Proposition 8. The optimal amount of private finance is decreasing in the area which potentially benefits from cross-subsidization, μ.

The logic behind this result is as follows. One of the main motivations for private financing is that this allows the government to make use of the firm's potential profits when transfers are not possible. If, however, there exists the possibility of cross-subsidization, then this provides an alternative output for potential profits. If distributional concerns are sufficiently high (for example, if the difference between $\hat{c}$ and p^*q^* is large), then the welfare gains of using them for this purpose may be larger than those from reducing government investment.

Given this result, a potential concern of 'cream-skimming' arises when considering infrastructure finance. In particular, actors may be tempted to design infrastructure projects in such a way that the amount of private finance available is maximized (i.e. by reducing μ). However, to the extent that this reduces the potential profits available for cross-subsidization in the future, the benefit of doing so is short-lived.

4.5 Government non-benevolence

Up until now we have always assumed that the government maximizes social welfare. However, not only is such an assumption frequently unrealistic, but government non-benevolence is used often as an argument in the debate on infrastructure finance. In particular, we consider two potential arguments in the frame of our model:

1. Government non-benevolence may result in the wrong financing choice
2. Government non-benevolence may result in the wrong project choice

In order to understand the actions of a non-benevolent government, we replace Assumption 4 with the following:

Assumption 4′. Non-benevolent government: The government may overvalue certain projects relative to the cost of public funds and may discount the future more than socially optimal.

In particular, we assume that the government has the following objective function:

$$V = -(1 + \lambda_1)I_G + \tilde{\delta}(S(q) - qp + \sigma(e)) \tag{13}$$

where $\tilde{\delta}$ combines both the government's discounting and their over-valuation of the project (and hence may be bigger or smaller than δ). We can consider $\tilde{\delta}$ to be the government's

bias. $\tilde{\delta} = \delta$ represents an unbiased government. $\tilde{\delta} < \delta$ represents a 'short-termist' or 'over-pessimistic' government, or one who diminishes the welfare of the infrastructure compared to others. $\tilde{\delta} > \delta$ then may represent a government who exhibits favouritism to the potential infrastructure users, or who is 'over-optimistic' when it comes to the project's value.

This then leads to the following proposition

Proposition 9. Private financing is decreasing in the amount with which the government inflates the project's value relative to current public spending, δ.

This result is similar to those found by Maskin and Tirole (2008) and Vining and Boardman (2014), who argue that government's who only care about the short-term will wish to use private finance because this shifts the costs of the project into the future. Maskin and Tirole (2008, Section 6) note that this may be a particular concern if citizens are not aware of the size of these future costs, because citizens will then be unaware that the government is acting non-benevolently. One potential way in which future costs may be hidden is through the potential that the government will bail out the firm when in financial distress (Danau and Vinella, 2014).

Since we have previously presumed a benevolent government, we have so far concentrated on infrastructure projects that are valuable to invest in. However, government non-benevolence raises the possibility that, not only might the government choose the wrong financing scheme, but it might choose the wrong projects. In order to analyze how such a possibility is influenced by the financing options available, let us now consider a project where, potentially, it is not always valuable to invest in the project. In order to do so, let us replace Assumption 2 with the following assumption:

Assumption 2‴. Non-observable cost: Operational costs are determined ex-ante, and are revealed to the government and potential investors, but not the public.

In particular, we now assume that the cost C is not affected by effort, but is given exogenously ex-ante.[12] Moreover, we assume that the project is such that it is socially beneficial to undertake the project if operational costs will be low, i.e. $C = \underline{c}$, but not if operational costs will be high, $C = \overline{c}$. This is similar to the assumptions made in Maskin and Tirole (2008) and Martimort and Pouyet (2008), who also consider the risks of government capture by special interest groups.

[12]This therefore rules out any role for equity in what follows. Alternatively, we could add in an additional cost parameter and retain the effort from the previous model, but doing so complicates our analysis without producing any additional insights.

A benevolent government will therefore choose to undertake the project if and only if costs are revealed to be low. On the other hand, a non-benevolent government may make two kinds of socially damaging decisions. First, governments who are 'over-optimistic' or exhibit favouritism towards infrastructure users - i.e. when $\tilde{\delta} - \delta$ is large - may invest in projects even when they have a high cost ($C = \overline{c}$). Second, governments who are short-termist - where $\tilde{\delta}$ is substantially below δ - may not invest in projects even when costs are low ($C = \underline{c}$). Since the electorate does not observe cost, it cannot (immediately) punish such non-benevolence, as the government can simply claim that costs are low or high respectively.

One way citizens can potentially restrict government non-benevolence is to limit their financing options. Maskin and Tirole (2008), for instance, consider the possibility of a cap on public infrastructure spending. Equally, we could consider a cap on the amount of private investment, which might be implemented through capping the price which could be charged by the firm. These possibilities lead to the following proposition:

Proposition 10. Limits on infrastructure financing can improve welfare in the following situations:

1. If the government inflates projects' values, i.e. $\tilde{\delta} > \delta$, then a cap on the use of public finance can improve both the government's financing choice and project choice.

2. If the government overly discounts projects' values, i.e. $\tilde{\delta} < \delta$, then a cap on private financing can improve the government's financing choice, but not its project choice.

The logic behind the project choice part of statement 1 is the idea that forcing infrastructure finance to be private exploits the information brought by the private sector, which is explored in Maskin and Tirole (2008). Since the amount of private finance depends on the expected operational costs, the public can use the private sector's choice as to which projects to finance when they do not trust the government's. The same logic does not hold when $\delta < 1$, because here the problem is that the government does not invest in projects that it should, and hence caps are ineffective at solving this problem. Nonetheless, they are useful to the extent that, for projects that the government does choose to invest in, they may prevent excessive private financing.

Restricting private financing may be particularly useful if the government can collude with the firm to extract profits corruptly, as explored by Martimort and Pouyet (2008). In particular, the asymmetric information on the future operational costs allows the government to under-estimate the firm's potential profits. By stating to the public that $C = \overline{c}$, private

lendors will only be expected to invest $\frac{1}{1+i_D}(p^*q^* - \bar{c})$, and hence will receive an excessive return that can be split between the firm and the non-benevolent government. Overall, therefore, knowing the precise kind of government non-benevolence is important in determining whether they are more or less likely to use private finance.

5 Conclusions

Our analysis has shown that a range of factors relevant for developing countries may influence how infrastructure is financed. A summary of a selection of these can be found in Table 5. The table describes the comparative statics results assuming that at least some of each financing source will be used. The table also helps us to understand how the value of each financing source changes as a function of context, and thus gives an indication as to whether the government will choose to use the funding source at all.

When it comes to private debt, the basic trade-off is between saving public money and increasing the expected prices consumers will face. In this regard, all the factors with a $\downarrow$ in the private debt column of Table 5 pushes against using private debt, because they effectively decreases the amount of money the government will save for a given price. Additionally, greater information asymmetries push in favor of using private debt since they increase the expected price consumers will face even without private finance.

When it comes to private equity, the decision is potentially a trade-off between taking advantage of the potential cost savings equity may induce and any negative externalities created by cost-reducing effort. Such externalities include the fact that private owners may act less sensitively regarding the poor, produce pollution or lower the quality of service. Similarly to debt, all the factors with a $\downarrow$ in the private equity column of Table 5 push against using private equity at all, since they increase the relative cost of any cost-saving effort.

One way to synthesize these findings is through the framework of institutional weaknesses set out in Estache and Wren-Lewis (2009), who consider four limitations that frequently pose problems for infrastructure in developing countries: limited commitment, limited accountability, limited capacity and limited fiscal efficiency. Since our model allows us to consider multiple factors simultaneously, we can to a certain extent create a decision tree for financing based on the relative importance of these institutional weaknesses. Of course, the factors that are important will vary largely across countries, but given developing countries' experiences with infrastructure finance we can speculate as to the following loose hierarchy in decision

Table 1: Factors influencing amount of finance source used

	Public finance	Private debt	Private equity
Cost of public funds	↓	↑	0
Cost of private debt	↑	↓	↑
Cost of private equity	0	↑	↓
Operational costs	↑	↓	0
Potential cost savings	0	↑	↑
Equity expropriation risk	↑	↕	↓
Exogenous risk	↑	↕	↓
Need for cross-subsidies	↑	↓	0
Government discounting	↓	↑	0
Government favoritism	↑	↓	0

Note: The ↑ symbol means that the factor increases the amount of that financing source that will be used, the ↓ symbol means it decreases and the ↕ symbol means the effect is ambiguous. The last two rows correspond to the actions of an unconstrained non-benevolent government, but note that social welfare is improved by promoting financing in the other direction.

making.

If the government suffers from severe limited commitment, it is unlikely to be able to raise private finance at all. Even a more moderate form of limited commitment, where only equity is threatened, will still increase the cost of private financing. Hence only in sectors where institutions are such that expropriation risks are small can serious financing be considered.

The government's accountability may then be the next most important factor in determining financing choice. A government whose incentives vary substantially from social welfare is likely to be relatively uninterested in factors affecting the optimal financing choice. For instance, a government that cares only about the short-term is unlikely to spend a large amount of public money on infrastructure investment. Moreover, a government that strongly favors

particular groups is likely to be limited in the amount of infrastructure it can finance, either by budget caps imposed through checks and balances or a lack of ability to raise extra taxes. In both cases, therefore, private finance offers a potential way out, with the attraction being that the investment is effectively paid for by the users who benefit from it.

If the government is able to access affordable private finance and is relatively benevolent, then other factors affecting the optimal form of financing start to come into play. In particular, the fiscal efficiency of the government is likely to be the next most important factor in determining the financing choice. Most straightforwardly, if the opportunity cost of public funds is very high, then private financing may be attractive even if it comes at a high cost. The important exception to this rule is a large need for cross-subsidization. In this case, there is a risk that private finance my 'cream skim', by investing in the most profitable projects, and thus leave the poorest worst off. In this case, the financing decision of any individual project must take into account the financing plan for the rest of the sector.

Finally, a relatively benevolent government who faces similar prices for public and private finance may then take into account the capacity of the regulator who will oversee the operation of the infrastructure once it is built. If it is reasonable to the expect that the regulator will be very weak, and hence face a large information asymmetry, then there are likely to be potentially large cost reductions which the government could not take advantage of through pricing. This pushes in the direction of private finance, as a means to both increase the probability that costs will be reduced and to take advantage of this lower expected costs.

Overall therefore, we can see that it is certainly not the case that developing countries should in general use more or less private finance than developed countries. Instead, the financing choice will depend to a large extent on the country context, and in particular on the institutional weaknesses that are felt most accutely.

6 Appendix: Proofs of propositions

Proof of Proposition 1. Since the government can contract upon cost, it can effectively decide upon the effort level exerted by the manager. Private equity therefore has no advantage over private debt, and the investors will finance their share of investment using whichever source is cheapest. We assume that this is debt (i.e. that $i_D < i_E$) without loss of generality.

The government will set the transfer level t such that the firm makes no expected profit beyond what is required to reward private investors, i.e. $t = \pi_D + c - pq$. Hence expected

social welfare is

$$\mathbb{E}[W] = \delta\mathbb{E}[S(q) + \lambda_2 qp + (1+\lambda_2)(c+\pi_D)] + \sigma(e) - (1+\lambda_1)I + (1+\lambda_1)\frac{1}{1+i_D}\mathbb{E}[\pi_D] \quad (14)$$

Since the government sets the transfer and the prices, they effectively set the level of private finance irrespective of prices. Hence we can see that they will set π_D to zero if $i_D > i_G$ and to it's maximum level if $i_G > i_D$. Then, differentiating by q and setting to zero, we get

$$0 = p + \lambda_2 p + \lambda_2 q\frac{dp}{dq} \quad (15)$$

If $\eta(p) = -\frac{p}{q}\frac{dq}{dp}$, then we have price given according to equation (7) □

Proof of Proposition 2. Suppose that the government uses some private financing I_D. Such financing will be repaid by setting a price $\overline{p}$ when costs are high ($C = \overline{c}$) and a price $\underline{p}$ when costs are low ($C = \underline{c}$). The expected return on debt is therefore $e(\underline{pq} - \underline{c}) + (1-e)(\overline{pq} - \overline{c})$. Since the cost of debt is i_D, we have

$$(1+i_D)I_D = e(\underline{pq} - \underline{c}) + (1-e)(\overline{pq} - \overline{c}) \quad (16)$$

The government's discounted expected welfare is

$$\begin{aligned}\delta\mathbb{E}[W] =& e(S(\underline{q}) - \underline{qp}) + (1-e)(S(\overline{q}) - \overline{qp}) - (1+\lambda_1)(1+i_G)(I - I_D) + \sigma(e) - (1+\lambda_1)\kappa(e)\\ =& e(S(\underline{q}) - \underline{qp}) + (1-e)(S(\overline{q}) - \overline{qp}) - (1+\lambda_1)(1+i_G)I + \sigma(e) - (1+\lambda_1)\kappa(e)\\ &+ (1+\lambda_1)\frac{1+i_G}{1+i_D}\left(e(\underline{pq} - \underline{c}) + (1-e)(\overline{pq} - \overline{c})\right) \end{aligned} \quad (17)$$

Differentiating by $\underline{q}$ and setting to zero gives

$$0 = -\underline{q}\frac{d\underline{p}}{d\underline{q}} + (1+\lambda_1)\frac{1+i_G}{1+i_D}\left(\underline{p} + \underline{q}\frac{d\underline{p}}{d\underline{q}}\right) \quad (18)$$

i.e.

$$0 = 1 + (1+\lambda_1)\frac{1+i_G}{1+i_D}\left(\underline{\eta} - 1\right) \quad (19)$$

i.e.

$$\underline{\eta} = \frac{\lambda_1}{1+\lambda_1} - \frac{i_D - i_G}{(1+\lambda_1)(1+i_G)} \tag{20}$$

□

Proof of Proposition 3. If private equity is used, the equity investor decides upon both the level of effort induced and the debt-equity ratio. Suppose that equity is promised to receive any residual profits that exist when costs are low (such a division can be proven to be optimal - see Tirole (2010, p.134)). Hence, at this stage, their expected profit is

$$e(pq - \underline{c} - \pi_D) - \kappa(e) \tag{21}$$

Effort will therefore be given by the formula

$$\kappa'(e) = pq - \underline{c} - \pi_D \tag{22}$$

Given this, let us now consider how much debt the firm will issue. Suppose the firm is given an investment requirement $I - I_G$ and price p and can choose how much to finance through debt and how much to finance through equity . Since equity is more expensive than debt, the investor will issue at least as much debt as there is guaranteed profit, i.e. $p - \underline{c} > \pi_D \geq p - \overline{c}$.

The amount remaining for the equity investor to invest is $I - I_G - I_D$. Her overall expected profit is therefore

$$\mathbb{E}[\Pi_E] = e(pq - \underline{c} - \pi_D) - (1+i_E)(I - I_G - I_D) - \kappa(e) \tag{23}$$

Now, $(1+i_D)I_D = e\pi_D + (1-e)(pq - \overline{c})$. From equation (22), we have that $\pi_D = pq - \underline{c} - \kappa'(e)$. Hence substituting this into (23) gives us the investor's profit as a function of effort:

$$e\kappa'(e) - (1+i_E)(I - I_G) - \kappa(e) + \frac{1+i_E}{1+i_D}\left(e(pq - \underline{c} - \kappa'(e)) + (1-e)(pq - \overline{c})\right) \tag{24}$$

Differentiating this by e and setting to zero then gives us that

$$e\kappa''(e) + \frac{1+i_E}{1+i_D}\left(pq - \underline{c} - \kappa'(e) - pq + \overline{c} - e\kappa''(e)\right) = 0 \tag{25}$$

Hence

$$\kappa'(e) = \overline{c} - \underline{c} - \frac{i_E - i_D}{1 + i_E} e\kappa''(e) \tag{26}$$

Since effort does not depend on the price level, prices will be set as before. This gives us the following levels of equity and and debt investment

$$I_E = \frac{1}{1 + i_E}(e\kappa'(e) - \kappa(e)) \tag{27}$$

$$I_D = \frac{1}{1 + i_D}\left(pq - \overline{c} + \frac{i_E - i_D}{1 + i_E} e^2\kappa''(e)\right) \tag{28}$$

Now, from the effort equation, we have

$$\kappa''(e) = \frac{i_E - i_D}{1 + i_E}\kappa''(e) + \frac{i_E - i_D}{1 + i_E} e\kappa'''(e) \tag{29}$$

Hence

$$\frac{1 + i_D}{1 + i_E}\kappa''(e) = \frac{i_E - i_D}{1 + i_E} e\kappa'''(e) \tag{30}$$

Hence $e\kappa'''(e)$ is positive. So both I_D and I_E are increasing in effort, and hence cost savings.

□

Proof of Proposition 4. Without private equity, the firm will simply induce an effort e_0, which is the effort which minimizes the cost function $\kappa(e)$. Necessarily, this will be less than the effort e undertaken with private equity, as the equity investor has an additional incentive to reduce costs.

Suppose $p^*q^* \geq \overline{c}$. Then private debt will be used in both scenarios and the price will not vary depending on financing. One effect of using private equity is that this will increase the amount of total private finance, for this given price level, as shown above. This is clearly advantageous for the government. Another impact is that effort is higher. If $\sigma(e) > \sigma(e_0)$ this effect is also advantageous for the goverment, and so private equity financing will be used. On the other hand, if $\sigma(e) < \sigma(e_0)$, then there is a trade-off between the extra financing and the net negative externality caused by the extra effort.

□

Proof of Proposition 5. Once the investment has been made, the equity owners payoff is now

$$(1-\rho)e(pq-\underline{c}-\pi_D)-\kappa(e) \tag{31}$$

Hence

$$\kappa'(e)=(1-\rho)(pq-\underline{c}-\pi_D) \tag{32}$$

The amount remaining for the equity investor to invest is $I-I_G-I_D$. Her overall expected profit is therefore

$$\mathbb{E}[\Pi_E]=(1-\rho)e(pq-\underline{c}-\pi_D)-(1+i_E)(I-I_G-I_D)-\kappa(e) \tag{33}$$

Now, however, we have $\pi_D=pq-\underline{c}-\kappa'(e)/(1-\rho)$. Substituting in as before gives us the investor's profit as a function of effort:

$$e\kappa'(e)-(1+i_E)(I-I_G)-\kappa(e)+\frac{1+i_E}{1+i_D}\left(e(pq-\underline{c}-\kappa'(e)/(1-\rho))+(1-e)(pq-\overline{c})\right) \tag{34}$$

Differentiating this by e and setting to zero then gives us that

$$e\kappa''(e)+\frac{1+i_E}{1+i_D}\left(pq-\underline{c}-\kappa'(e)/(1-\rho)-pq+\overline{c}-e\kappa''(e)/(1-\rho)\right)=0 \tag{35}$$

Hence

$$\kappa'(e)=(1-\rho)(\overline{c}-\underline{c})-\left(1-(1-\rho)\frac{1+i_D}{1+i_E}\right)e\kappa''(e) \tag{36}$$

Total private financing must be decreasing because price is the same, but average costs are higher.

□

Proof of Proposition 6. See proof of proposition 5. □

Proof of Proposition 8. In order to keep the proof simple, but without loss of generality, we use the model of section 3.1, such that we can treat effort as fixed and the only source of private financing is private debt. Let the price and quantity in the high cost region be p and q

The government's discounted expected welfare is

$$\begin{aligned}\delta\mathbb{E}[W] =& e(S(\underline{q}) - \underline{qp}) + (1-e)(S(\overline{q}) - \overline{qp}) - (1+\lambda_1)(1+i_G)I + \sigma(e) - (1+\lambda_1)\kappa(e) \\ &+ \mu\left(S(q) - pq\right) + (1+\lambda_1)\frac{1+i_G}{1+i_D}\left(e(\underline{pq} - \underline{c}) + (1-e)(\overline{pq} - \overline{c}) + \mu(pq - \hat{c})\right) \end{aligned} \tag{37}$$

Differentiating with respect to quantities again gives us an interior solution such that

$$\eta(\underline{p}) = \eta(\overline{p}) = \eta(p) = \frac{\lambda_1}{1+\lambda_1} - \frac{i_D - i_G}{(1+\lambda_1)(1+i_G)} \tag{38}$$

Hence the amount of private finance is

$$I_D = \frac{1}{1+i_D}\left((1+\mu)p^*q^* - e\underline{c} - (1-e)\overline{c} - \mu\hat{c}\right) \tag{39}$$

Since $\hat{c} > p^*q^*$, we can see that I_D is decreasing in μ.

□

Proof of Proposition 9. By looking at the government's objective function, we can see that the government will behave the same as a benevolent government which has a discount factor of $\frac{1+i_G}{\delta}$. Hence they will set price according to:

$$\eta(p^*) = \frac{\lambda_1}{1+\lambda_1} - \frac{\delta(1+i_D) - 1 + i_G}{(1+\lambda_1)(1+i_G)} \tag{40}$$

Which we can clearly see is decreasing in δ, giving us our result. □

Proof of Proposition 10. As with the proof of Proposition 8 we treat effort as fixed and the only source of private financing is private debt.

The financing parts of part 1 and 2 follows immediately from Proposition 9. For the project choice of part 1, consider that private investment is as follows:

$$I_D = \frac{1}{1+i_D}\left(p^*q^* - C\right) \tag{41}$$

Hence, by limiting I_G to be no more than $I - \frac{1}{1+i_D}\left(p^*q^* - \underline{c}\right)$, citizens can insure only low cost projects are financed.

□

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