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Multi-Output Profit Efficiency
and Directional Distance Functions

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Abstract

We extend a recently developed DEA methodology for cost efficiency analysis towards profit efficiency settings. This establishes a novel DEA toolkit for profit efficiency assessments in situations with multiple inputs and multiple outputs. A distinguishing feature of our methodology is that it assumes output-specific production technologies. In addition, the methodology accounts for the use of joint inputs, and explicitly includes information on the allocation of inputs to individual outputs. We also establish a dual relationship between our multi-output profit inefficiency measure and a technical inefficiency measure that takes the form of a multi-output directional distance function. Finally,

we demonstrate the empirical usefulness of our methodology by an empirical application to a large service company.

1 Introduction

Production processes that generate multiple outputs are typically characterized by jointly used inputs, i.e. inputs that simultaneously benefit different outputs. These joint inputs give rise to economies of scope, which actually form a prime economic motivation for Decision Making Units (DMUs) to produce more than one output. In the current paper, we establish a methodology for multi-output profit efficiency evaluation that explicitly accounts for jointly used inputs. In particular, our methodology distinguishes between joint inputs and inputs that are allocated to specific outputs.

The method that we develop fits within the popular Data Envelopment Analysis (DEA; after Charnes, Cooper and Rhodes (1978)) approach to productive efficiency measurement. This DEA approach is intrinsically nonparametric, which means that it does not require a parametric/functional specification of the (typically unknown) production technology. It "lets the data speak for themselves" by solely using technological information that is directly revealed by the observed production units. It then reconstructs the production possibility sets by (only) assuming standard production axioms (such as monotonicity and convexity).[1] A DMU's efficiency is measured as the distance of the corresponding input-output combination to the efficient frontier of this empirical production set. Typically, a DMU's efficiency can be computed by simple linear programming. Its nonparametric nature and its easy computation largely explain DEA's widespread use as an analytical research instrument and decision-support tool.

Recently, Cherchye et al (2013, 2014a) introduced a novel DEA methodology to

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[1]See Fare, Grosskopf and Lovell (1994), Cooper, Seiford and Zhu (2004), Cooper, Seiford and Tone (2007), Fried, Lovell and Schmidt (2008) and Cook and Seiford (2009) for extensive reviews of DEA. From an economic perspective, DEA itself is rooted in the structural approach to modeling efficient production behavior that was initiated by Afriat (1972), Hanoch and Rothschild (1972), Diewert and Parkan (1983) and Varian (1984). Given the explicit economic motivation of our following analysis, our contribution also fits in this tradition of structural efficiency analysis.

analyze cost efficiency in multi-output settings. The methodology assumes output-specific production technologies, accounts for joint inputs in the production process, and incorporates specific information on how inputs are allocated to individual outputs. These authors have also shown that their cost efficiency measure evaluated at shadow prices is dually equivalent to a specific multi-output version of the Debreu (1951) - Farell (1957) measure of (radial) input efficiency. This is an attractive feature, as DEA practitioners often use this Debreu-Farrell measure for evaluating the technical efficiency of a DMU's input use (when assuming a fixed output).

The current paper extends this methodology for multi-output efficiency assessments to profit efficiency settings. We strongly believe this provides a significant and relevant addition to the existing cost efficiency framework. In many practical settings, profit efficiency is considered to be the best suited criterion for evaluating the performance of productive activities. In addition, by its very definition cost efficiency is a necessary condition for profit efficiency. Profit efficiency evaluations are generally more stringent than cost efficiency evaluations. As a result, they can signal additional sources of inefficiency and, thus, potential performance improvements. In this respect, as we will indicate, an appealing feature of our multi-output approach is that it also allows us to allocate a DMU's aggregate profit inefficiency to individual outputs. This helps to better identify specific output production processes where substantial profit efficiency gains are possible, which can usefully assist DMU managers to direct their performance improvement actions in an effective way (i.e. primarily towards outputs that are characterized by considerable inefficiency).

In developing our profit efficiency methodology, we also start from output-specific technologies and distinguish between joint inputs and output-specific inputs in the process of multi-output production. Next, we will show that our profit inefficiency measure under shadow prices has a dual representation as a directional distance func-

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tion. We believe this is an interesting property, as directional distance functions have become increasingly popular as a technical inefficiency measure that simultaneously includes outputs produced and inputs used. Basically, this duality result extends the one of Chambers, Chung and Fare (1998) towards our specific multi-output setting. A particular feature of our analysis here is that we explicitly account for output-specific technologies with jointly used inputs in establishing the duality relationship.

The rest of the paper is structured as follows. Section 2 introduces some necessary notation and terminology. Section 3 introduces our method for multi-output profit inefficiency measurement. Section 4 establishes the dual representation of our profit inefficiency measure as a directional distance function. Section 5 shows the practical usefulness of our method through an application to a large service company. Section 6 concludes.

2 Preliminaries

The distinction between inputs and outputs becomes less relevant in profit efficiency analysis. Therefore, to simplify notation it will often be convenient to work with "netputs" in our following exposition. As we will explain, netput vectors simultaneously capture inputs used (as negative components) and outputs produced (as positive components). We will define this netput concept for our specific setting with joint and output-specific inputs. In turn, this will allow us to introduce our notion of output-specific technologies and, correspondingly, our particular concept of multi-output profit.

Netputs and multi-output technologies. We consider a production technology that uses N inputs to produce M outputs, which we represent by the vectors

$\mathbf{X} = (x^1, \ldots, x^N)' \in \mathbb{R}_+^N$ and $\mathbf{Y} = (y^1, \ldots, y^M)' \in \mathbb{R}_+^M$, respectively. Our method distinguishes between joint and output-specific inputs.

- *Output-specific* inputs are allocated to individual outputs m, i.e. they specifically benefit the production process of (only) the m-th output. In our formal analysis, we will use $\alpha_k^m \in [0, 1]$ (with $\sum_{m=1}^M \alpha_k^m = 1$) to represent the fraction of the k-th output-specific input quantity that is allocated to output m.

- *Joint* inputs are not allocated to specific outputs but are simultaneously used in the production process of *all* the outputs. Clearly, these joint inputs generate interdependencies between the production processes of different outputs.[2]

We will represent the allocation of inputs to outputs by means of a vector $\mathbf{A}^m \in \mathbb{R}_+^N$ for each output m, for which the entries are defined as (with $\alpha_k^m \in [0, 1]$ and $\sum_{m=1}^M \alpha_k^m = 1$)

$$
(\mathbf{A}^m)_k = \begin{cases} 1 & \text{if input } k \text{ is joint and used to produce output } m, \\ \alpha_k^m & \text{if input } k \text{ is output-specific and used to produce output } m, \\ 0 & \text{if input } k \text{ is not used to produce output } m. \end{cases}
$$

Then, each vector $\mathbf{A}^m$ defines the input vector $\mathbf{X}^m = \mathbf{A}^m \odot \mathbf{X}$, which thus contains the input quantities used in the production process of output m.[3]

As indicated above, we can often simplify our notation by working with netputs, which simultaneously stand for outputs and inputs. Specifically, we use $\mathbf{Z} = \begin{bmatrix} \mathbf{Y} \\ -\mathbf{X} \end{bmatrix} \in \mathbb{R}^{M+N}$ to denote the aggregate netput vector. In a similar vein, $\mathbf{Z}^m =$

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[2]See Cherchye, De Rock and Walheer (2015) for the introduction of sub-joint inputs. These inputs play a similar role as joint inputs, but only for a *subset* of (instead of all) outputs. It is straightforward to include this third type of inputs in our methodology, but for the ease of the exposition we abstract from this in the current paper.

[3]The symbol $\odot$ stands for the Hadamard (or element-by-element) product.

$$
\begin{bmatrix} y^m \\ -\mathbf{X}^m \end{bmatrix} \in \mathbb{R}_+^{1+N} \text{ represents the netput vector that is specific to output } m.
$$

Our multi-output analysis will involve a specific representation of each output m's production technology. This technology defines the output-specific production possibility set

$$
T^m = \{\mathbf{Z}^m \in \mathbb{R}^{1+N} \mid \mathbf{Z}^m \text{ is technically feasible}\},
$$

which contains all the combinations of output-specific and joint inputs (in $\mathbf{X}^m$) that can produce the output quantity y^m.

Prices and profits. To define profit, we use $\mathbf{P}_x = (p_x^1, \ldots, p_x^N)' \in \mathbb{R}_+^N$ for input prices and $\mathbf{P}_y = (p_y^1, \ldots, p_y^M) \in \mathbb{R}_+^M$ for output prices. Correspondingly, the netput price vector is given as $\mathbf{P} = \begin{bmatrix} \mathbf{P}_y \\ \mathbf{P}_x \end{bmatrix} \in \mathbb{R}_+^{M+N}$.

To incorporate our distinction between output-specific and joint inputs, we make use of output-specific input prices $\mathbf{P}_x^m \in \mathbb{R}_+^N$. First, for output-specific inputs, these prices coincide with the actual prices, i.e.

$$
(\mathbf{P}_x^m)_k = (\mathbf{P}_x)_k \text{ for } k \text{ an output-specific input.}
$$

Next, following Cherchye et al (2013, 2014a) we make use of output-specific prices $(\mathbf{P}_x^m)_k$ for every joint input k. Essentially, these prices $(\mathbf{P}_x^m)_k$ capture the fractions of the aggregate input price $(\mathbf{P}_x)_k$ that are allocated to individual outputs m. Efficient production requires the output-specific prices $(\mathbf{P}_x^m)_k$ to add up to the aggregate DMU-level prices, i.e. they must satisfy

$$
\sum_{m=1}^{M} (\mathbf{P}_x^m)_k = (\mathbf{P}_x)_k \text{ for } k \text{ a joint input.}
$$

As explained in detail by Cherchye, De Rock and Vermeulen (2008), these output-specific prices have a similar interpretation as Lindahl prices for public goods. Specifically, Pareto efficient provision of public goods equally requires these Lindahl prices to sum up to the aggregate prices.

Taken together, the output-specific price vector $\mathbf{P}^m$ for each output m is given as

$$\mathbf{P}^m = \begin{bmatrix} p_y^m \\ \mathbf{P}_x^m \end{bmatrix}.$$

Correspondingly, for every output m we can define the output-specific profit

$$\pi^m = \mathbf{P}^{m\prime}\mathbf{Z}^m.$$

In turn, by summing these output-specific profit, we obtain the aggregate profit[4]

$$\pi = \sum_{m=1}^{M} \pi^m = \sum_{m=1}^{M} \mathbf{P}^{m\prime}\mathbf{Z}^m = \mathbf{P}'\mathbf{Z}.$$

The last equality also shows that summing the profit levels associated with individual netputs $\mathbf{Z}^m$ yields, by construction, the DMU's profit level defined in terms of the aggregate netput $\mathbf{Z}$. Given this, we will work with the sum profit $\sum_{m=1}^{M} \mathbf{P}^{m\prime}\mathbf{Z}^m$ in what follows, without explicitly considering $\mathbf{P}'\mathbf{Z}$.

3 Multi-output profit efficiency

In practice, the true production technology is typically unknown. Therefore, in empirical efficiency evaluations, we need to reconstruct the production possibilities from

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[4]To obtain the last equality we use that
$\sum_{m=1}^{M} \mathbf{P}^{m\prime}\mathbf{Z}^m = \sum_{m=1}^{M}(p_y^m y^m) - \sum_{m=1}^{M}(\mathbf{P}_x^{m\prime}\mathbf{X}^m) = \mathbf{P}_y'\mathbf{Y} - \mathbf{P}_x'\mathbf{X} = \mathbf{P}'\mathbf{Z}.$

a set of T observed DMUs. In what follows, we assume a setting in which we observe, for each DMU t, the netput vectors $\mathbf{Z}_t^1, \ldots, \mathbf{Z}_t^M$, which contain the joint and output-specific inputs, as well as the resulting outputs.

In the current section, we will additionally assume that the empirical analyst also observes the associated netput price vector $\mathbf{P}_t = \begin{bmatrix} \mathbf{P}_{y,t} \\ \mathbf{P}_{x,t} \end{bmatrix} \in \mathbb{R}_+^{M+N}$. At this point, two remarks are in order. First, the assumption of observed prices is often restrictive in empirical settings. In the next section, we will show how we can relax this assumption by using shadow prices. Second, throughout we will assume that we do not have any information about the output-specific prices for the joint inputs, which typically holds true in practical applications (including our own application in Section 5). However, it is worth to indicate that, if extra information on output-specific prices were available, it would actually be fairly easy to integrate this information in our profit efficiency analysis.

Taken together, we assume that we observe a data set

$$S = \{(\mathbf{Z}_t^1, \ldots, \mathbf{Z}_t^M, \mathbf{P}_t) \mid t = 1, \ldots, T\}.$$

Empirical efficiency criterion. Following a nonparametric approach, we reconstruct the production possibilities while avoiding (non verifiable) parametric assumptions regarding the DMUs' technologies. In our profit efficiency analysis, we (only) use the following minimalistic prior regarding the production possibility sets.

Axiom T1 (observability means feasibility): Observing the netputs $\mathbf{Z}_t^1, \ldots, \mathbf{Z}_t^M$ implies for all $m = 1, \ldots, M$ that $\mathbf{Z}^m \in T^m$.

This axiom has a very natural interpretation. Basically, it says that what we

observe is certainly feasible. Or, if we observe the netput vector $\mathbf{Z}_t^m = \begin{bmatrix} y_t^m \\ -\mathbf{X}_t^m \end{bmatrix}$,

then we conclude that the input $\mathbf{X}_t^m$ can effectively produce the output y_t^m.[5]

Adopting our above notation of the previous section, we let $\mathbf{P}_t^m = \begin{bmatrix} p_{y,t}^m \\ \mathbf{P}_{x,t}^m \end{bmatrix}$ rep-

resent the m-th output-specific prices for DMU t, with the subvector $\mathbf{P}_{x,t}^m$ containing

the prices for the output-specific and joint inputs as characterized in Section 2. Then,

building on Axiom T1, we obtain our empirical condition for profit efficient produc-

tion behavior.

Definition 1 (Profit efficiency): *Let* $S = \{(\mathbf{Z}_t^1, \ldots, \mathbf{Z}_t^M, \mathbf{P}_t) \mid t = 1, \ldots, T\}$ *be a*

data set. Then, DMU t is profit efficient if there exist, for all outputs m, output-

specific price vectors $\mathbf{P}_t^m = \begin{bmatrix} p_{y,t}^m \\ \mathbf{P}_{x,t}^m \end{bmatrix} \in \mathbb{R}_+^{1+N}$, *such that*

(i) $(\mathbf{P}_{x,t}^m)_k = (\mathbf{P}_{x,t})_k$ *for output-specific inputs k,*

(ii) $\sum_{m=1}^M (\mathbf{P}_{x,t}^m)_k = (\mathbf{P}_{x,t})_k$ *for joint inputs k,*

(iii) $\mathbf{P}_t^{m'} \mathbf{Z}_t^m \geq \mathbf{P}_t^{m'} \mathbf{Z}_s^m$ *for all observations $s = 1, \ldots, T$.*

In words, this definition states that DMU t is profit efficient if, for the input and

output prices that apply to t (captured by $\mathbf{P}_t^m$ for every output m), there does not

exist another observed DMU s (with netput vector $\mathbf{Z}_s^m$) that attains a larger profit.

As such, given our multi-output setting, we have a separate profit efficiency criterion

for each different output m.

While we do observe the aggregate prices $\mathbf{P}_t$, we typically do not observe the

output-specific prices $\mathbf{P}_t^m$ because of jointly used inputs (i.e., for a joint input k, we

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[5]Essentially, this axiom excludes measurement errors. Importantly, however, it is fairly easy to extend our methodology to account for measurement problems. For compactness, we will not discuss this question here, but refer to Cherchye et al (2013) for a detailed treatment.

do not observe the price fraction $(\mathbf{P}_{x,t}^m)_k$ that is borne by m). Therefore, the criterion in Definition 1 (only) requires that there exists *at least one* possible specification of these prices that makes the observed behavior of DMU t consistent with profit efficiency. As soon as such a specification exists, we conclude that profit efficient behavior cannot be rejected given the information that is available (contained in the data set S).

Measuring profit efficiency. In practice, if a DMU t does not meet the profit efficiency criterion in Definition 1, we quantify the degree of profit inefficiency as the extent to which actual profit deviates from maximum profit. In what follows, we will introduce a method to measure profit inefficiency in our multi-output framework. In doing so, we will adapt the "directional" profit efficiency framework of Chambers, Chung and Fare (1998) to our particular setting.

As a first step, we define, for each output m, the profit function

$$\pi_t^m(\mathbf{P}_t^m) = \max_{s \in \{1,...,T\}} \left(\mathbf{P}_t^{m'} \mathbf{Z}_s^m \right),$$

which gives the maximum attainable profit over the observed set S for the prices $\mathbf{P}_t^m$ that apply to DMU t. Correspondingly, when summing over all outputs m, we obtain the aggregate profit function

$$\pi_t(\mathbf{P}_t^1, ..., \mathbf{P}_t^M) = \sum_{m=1}^{M} \pi_t^m(\mathbf{P}_t^m).$$

In the sequel, we will focus on the profit inefficiency measure[6]

$$PE_t^C(\mathbf{P}_t^1, ..., \mathbf{P}_t^M) = \frac{\sum_{m=1}^M \pi_t^m(\mathbf{P}_t^m) - \sum_{m=1}^M \left(\mathbf{P}_t^{m'} \mathbf{Z}_t^m\right)}{\sum_{m=1}^M \mathbf{P}_t^{m'} \mathbf{g}_{\mathbf{z}^m}},$$

which we see as a natural translation of Chambers et al's "directional" profit inefficiency measure to our specific multi-output setting. In this definition, each $\mathbf{g}_{\mathbf{z}^m}$ represents the directional distance vector for the output m.[7] Equivalently, we can also express it as $\mathbf{g}_{\mathbf{z}^m} = \begin{bmatrix} g_{y^m} \\ \mathbf{g}_{\mathbf{x}^m} \end{bmatrix}$, with $g_{y^m} \in \mathbb{R}$ and $\mathbf{g}_{\mathbf{x}^m} \in \mathbb{R}^N$ defining the output and input directions, respectively. In practice, these directional vectors are chosen by the empirical analyst prior to the actual efficiency evaluation (see our own application in Section 5 for example specifications of $\mathbf{g}_{\mathbf{z}^m}$). Clearly, $PE_t^C(\mathbf{P}_t^1, ..., \mathbf{P}_t^M) = 0$ reveals profit efficiency, while higher values PE_t^C indicate a greater degree of profit inefficient behavior.

In general, the value of the measure $PE_t^C(\mathbf{P}_t^1, ..., \mathbf{P}_t^M)$ will depend on the output-specific input prices that are used to evaluate the joint inputs (and contained in $(\mathbf{P}_t^1, ..., \mathbf{P}_t^M)$). As indicated above, these prices are typically not known by the empirical analyst. In what follows, we will choose prices that minimize the value of the profit inefficiency measure PE_t^C for DMU t under evaluation, i.e. we solve

$$PE_t^C = \min_{\mathbf{P}_t^1, ..., \mathbf{P}_t^M \in \mathbb{R}_+^{1+N}} PE_t^C(\mathbf{P}_t^1, ..., \mathbf{P}_t^M),$$

where each output-specific price vector $\mathbf{P}_t^m$ is subject to the conditions outlined in

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[6]We assume that the denominator $(\sum_{m=1}^M \mathbf{P}_t^{m'} \mathbf{g}_{\mathbf{z}^m})$ is positive. For the shadow profit inefficiency measure $\widehat{PE}_t^C$ that we introduce below, this is guaranteed by the normalization constraint $\sum_{m=1}^M \widehat{\mathbf{P}}_t^{m'} \mathbf{g}_{\mathbf{z}^m} = 1$.

[7]In DEA applications, the directional vectors are often DMU-specific, i.e. we have $\mathbf{g}_{\mathbf{z}^m} = \mathbf{g}_{\mathbf{z}_t^m}$. It is common in the literature to drop the subscript t for compactness.

Definition 1. Intuitively, by minimizing the profit inefficiency, we actually choose "most favorable" prices $\mathbf{P}_t^m$ for DMU t under evaluation. In other words, we evaluate DMU t in the best possible light, which gives this DMU the benefit of the doubt in the absence of true price information. Attractively, this falls in line with usual DEA efficiency analysis, which typically can be given a similar benefit-of-the-doubt interpretation.[8]

We conclude that DMU t meets our empirical profit efficiency criterion in Definition 1 if and only if $PE_t^C = 0$. In that case, there effectively does exist a specification of the prices $\mathbf{P}_t^m$ that makes the observed production behavior profit maximizing over the data set S. By contrast, profit inefficiency occurs if $PE_t^C > 0$, with higher values revealing a greater degree of profit inefficiency.

As a final remark, we note that the measure PE_t^C can be computed by means of linear programming. The associated program has a structure that is formally analogous to the one of (**LP-1**) that we present below.[9] Given this direct similarity, and for the sake of compactness, we do not report it here.

4 Shadow prices and duality

In the previous section, we have assumed that the empirical analyst knows the netput price vector $\mathbf{P}_t$ for every DMU t. In practical applications, however, reliable price information is often not available. In such a case, we can conduct efficiency analysis with endogenously defined shadow prices. In what follows, we will apply this shadow

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[8]See, for example, Cherchye et al (2007) for a detailed discussion of the benefit-of-the-doubt interpretation of common DEA models.

[9]The only difference between the linear program for PE_t^C and the program (**LP-1**) for $\widehat{PE}_t^C$ involves the inclusion of the price information contained in the data set S (whereas (**LP-1**) applies to shadow pricing). This price information is easily included in the form of linear constraints, which obviously does not interfere with the linear programming nature of (**LP-1**).

pricing idea to the multi-output profit efficiency framework set out above. Next, we will show that the resulting profit inefficiency measure (under shadow prices) has a dually equivalent representation as a multi-output directional distance function, which establishes a multi-output version of the original duality result in Chambers, Chung and Fare (1998).

Shadow prices. If we do not observe the true prices that apply to each DMU t, the relevant data set becomes

$$\widehat{S} = \{(\mathbf{Z}_t^1, \ldots, \mathbf{Z}_t^M) \mid t = 1, \ldots, T\}.$$

When only $\widehat{S}$ (instead of S) is given, we are forced to use a weakened version of the efficiency criterion in Definition 1. Specifically, we can (only) check whether there exists at least one feasible "shadow" price specification that supports profit efficiency of the evaluated DMU t.

Definition 2 (Shadow profit efficiency): *Let* $\widehat{S} = \{(\mathbf{Z}_t^1, \ldots, \mathbf{Z}_t^M) \mid t = 1, \ldots, T\}$ *be a data set. Then, DMU t is shadow profit efficient if there exist, for each output m, non-zero output-specific shadow price vectors* $\widehat{\mathbf{P}}_t^m = \begin{bmatrix} \widehat{p}_{y,t}^m \\ \widehat{\mathbf{P}}_{x,t}^m \end{bmatrix} \in \mathbb{R}_+^{1+N}$ *such that* $\widehat{\mathbf{P}}_t^{m\prime} \mathbf{Z}_t^m \geq \widehat{\mathbf{P}}_t^{m\prime} \mathbf{Z}_s^m$ *for all observations* $s = 1, \ldots, T$.

In this case, we can choose the shadow price vector $\widehat{\mathbf{P}}_t^m$ freely (except from the non-zero and nonnegativity constraints). Implicitly, the shadow prices $(\widehat{\mathbf{P}}_{x,t}^m)_k$ for the joint inputs k define the (aggregate) DMU prices $(\widehat{\mathbf{P}}_{x,t})_k = \sum_{m=1}^M (\widehat{\mathbf{P}}_{x,t}^m)_k$. Next, we note that shadow prices for the output-specific inputs can be different for different outputs, i.e. for output-specific inputs k we can have $(\widehat{\mathbf{P}}_{x,t}^m)_k \neq (\widehat{\mathbf{P}}_{x,t}^{m\prime})_k$ when $m \neq m\prime$.[10]

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[10]In principle, of course, one can impose the constraint that $(\widehat{\mathbf{P}}_{x,t}^m)_k = (\widehat{\mathbf{P}}_{x,t}^{m\prime})_k$ for some output-

Following our reasoning of the previous section, we can evaluate our shadow profit efficiency criterion by the following efficiency measure, which endogenizes the (shadow) price selection in the efficiency evaluation process:

$$\widehat{PE}_t^C = \min_{\widehat{\mathbf{P}}_t^1,\ldots,\widehat{\mathbf{P}}_t^M \in \mathbb{R}_+^{1+N}} \frac{\sum_{m=1}^M \pi_t^m(\widehat{\mathbf{P}}_t^m) - \sum_{m=1}^M \left(\widehat{\mathbf{P}}_t^{m\prime} \mathbf{Z}_t^m\right)}{\sum_{m=1}^M \widehat{\mathbf{P}}_t^{m\prime} \mathbf{g}_{\mathbf{z}^m}}.$$

Similar to before, $\widehat{PE}_t^C$ selects the most favorable netput price vectors $\widehat{\mathbf{P}}_t^m$ to evaluate DMU t's shadow profit efficiency, which effectively applies the benefit-the-doubt pricing in the absence of full price information. It is easy to verify that DMU t satisfies the shadow profit efficiency criterion in Definition 2 if and only if $\widehat{PE}_t^C = 0$, which reveals that there exists at least one possible specification of the shadow price vectors $\widehat{\mathbf{P}}_t^m$ under which DMU t is profit maximizing over the data set $\widehat{S}$.

To operationalize the measure $\widehat{PE}_t^C$, we need to normalize the denominator. In what follows, we will use

$$\sum_{m=1}^M \widehat{\mathbf{P}}_t^{m\prime} \mathbf{g}_{\mathbf{z}^m} = 1.$$

Then, we can formulate our (shadow) profit inefficiency measure $\widehat{PE}_t^C$ as solving

specific input k (and $m \neq m'$), which obtains a stronger efficiency criterion. We refer to Cherchye, De Rock and Hennebel (2014b) for an exploration of such a stronger criterion in a multi-output cost efficiency setting that is formally close to the profit efficiency setting that we consider here.

the linear program (**LP-1**)

$$\widehat{PE}_t^C = \min_{\substack{\widehat{\pi}_t^1,\ldots,\widehat{\pi}_t^M \in \mathbb{R}, \\ \widehat{\mathbf{P}}_t^1,\ldots,\widehat{\mathbf{P}}_t^M \in \mathbb{R}_+^{1+N}}} \sum_{m=1}^{M} \widehat{\pi}_t^m - \sum_{m=1}^{M} \left(\widehat{\mathbf{P}}_t^{m\prime} \mathbf{Z}_t^m \right)$$

s.t.

$$\forall m \in \{1,\ldots,M\} : \widehat{\pi}_t^m \geq \widehat{\mathbf{P}}_t^{m\prime} \mathbf{Z}_s^m \text{ for all } s \in \{1,\ldots,T\},$$

$$\sum_{m=1}^{M} \widehat{\mathbf{P}}_t^{m\prime} \mathbf{g}_{\mathbf{z}^m} = 1,$$

where each $\widehat{\pi}_t^m$ represents $\pi_t^m(\widehat{\mathbf{P}}_t^m)$, i.e. the maximum attainable profit (over the data set $\widehat{S}$) in the production of output m given the output-specific prices $\widehat{\mathbf{P}}_t^m$ that apply to the evaluated DMU t.

As a final note, apart from the "aggregate" profit inefficiency measure $\widehat{PE}_t^C$, we can also define profit inefficiency measures $\widehat{PE}_t^{C,m}$ that are specific to individual outputs m. In particular, let $\widehat{\pi}_t^{m*}$ and $\widehat{\mathbf{P}}_t^{m*}$ solve the above linear problem. Then, we can use

$$\widehat{PE}_t^{C,m} = \frac{\widehat{\pi}_t^{m*} - \widehat{\mathbf{P}}_t^{m*\prime} \mathbf{Z}_t^m}{\widehat{\mathbf{P}}_t^{m*\prime} \mathbf{g}_{\mathbf{z}^m}}$$

Clearly, for $\widehat{PE}_t^C = 0$ we will have $\widehat{PE}_t^{C,m} = 0$ for all m. However, if $\widehat{PE}_t^C > 0$, the measures $\widehat{PE}_t^{C,m}$ allow us to allocate DMU t's profit inefficiency to specific outputs. We will illustrate this feature in our empirical application in Section 5.

Dual representation. Interestingly, our shadow profit inefficiency measure has a dual representation as a multi-output version of the directional distance function introduced by Chambers, Chung and Fare (1998). We believe this is an appealing property as directional distance functions are frequently used in DEA technical efficiency evaluations that simultaneously account for inputs used and outputs produced.

This directional distance function representation appears from the dual version of our linear program (**LP-1**). Specifically, let λ_s^m represent the dual variables for the first constraint (for each output m and DMU s) and β the dual variable for the second constraint of that program. Then, the dual can be written as (**LP-2**)

$$\widehat{PE}_t^C = \max_{\lambda_s^1, \ldots, \lambda_s^M, \beta \in \mathbb{R}_+} \beta$$

$$\forall m \in \{1, \ldots, M\} : \sum_{s=1}^T \lambda_s^m \mathbf{Z}_s^m \geq \mathbf{Z}_t^m + \beta \mathbf{g}_{\mathbf{Z}^m},$$

$$\forall m \in \{1, \ldots, M\} : \sum_{s=1}^T \lambda_s^m = 1.$$

To interpret $\widehat{PE}_t^C$ as a multi-output version of the directional function, we first note that in the case of a single output m, Chambers et al's original version of the general directional distance function is defined as

$$\vec{D}(\mathbf{Z}_t^m; \mathbf{g}_{\mathbf{Z}^m}) = \max \left\{ \beta | \, (\mathbf{Z}_t^m + \beta \mathbf{g}_{\mathbf{Z}^m}) \in T^m \right\}.$$

As a natural extension towards our framework with output-specific technologies, we can define the multi-output version of this distance function as

$$\vec{D}(\mathbf{Z}_t^1, \ldots, \mathbf{Z}_t^M; \mathbf{g}_{\mathbf{Z}^1}, \ldots, \mathbf{g}_{\mathbf{Z}^M}) = \max \left\{ \beta | \forall m \in \{1, \ldots, M\} : (\mathbf{Z}_t^m + \beta \mathbf{g}_{\mathbf{Z}^m}) \in T^m \right\}.$$

Then, it is easy to see that we obtain

$$\widehat{PE}_t^C = \vec{D}(\mathbf{Z}_t^1, \ldots, \mathbf{Z}_t^M; \mathbf{g}_{\mathbf{Z}^1}, \ldots, \mathbf{g}_{\mathbf{Z}^M}),$$

if we define the production possibility set of output m as

$$T^m = \{\mathbf{Z}^m \mid \mathbf{Z}^m \leq \sum_{s=1}^{T} \lambda_s^m \mathbf{Z}_s^m, \sum_{s=1}^{T} \lambda_s^m = 1, \lambda_s^m \geq 0\},$$

i.e. the convex monotone hull of the observed netput vectors $\mathbf{Z}_s^m$. Actually, this convex monotone hull of observed netput vectors is often used as an (empirical) production possibility set in practical DEA analysis. Banker, Charnes and Cooper (1984) first proposed this technology specification in the DEA literature.[11] A distinguishing feature of our framework is that it uses this specification to construct a production possibility set for each different output m. This follows naturally from our particular set-up, which explicitly considers output-specific production technologies (while accounting for interdependencies through joint inputs).

Summarizing, we conclude that our shadow profit inefficiency measure can also be represented as a multi-output technical inefficiency measure. In particular, it can be characterized as a multi-output directional distance function defined for output-specific technologies that are convex and monotone. A specific feature of this characterization is that it accounts for joint input use in the process of multi-output production.

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[11] Banker, Charnes and Cooper (1984) show that we obtain the convex monotone hull as a DEA-type technology approximation if we add the technology assumptions convexity and monotonicity to our Axiom 1. In words, monotonicity implies that the outputs and inputs are freely (or strongly) disposable; i.e. producing less outputs cannot lead to use more inputs and using more inputs never reduces the outputs. It also implies that marginal rates of substitution/transformation (between inputs, outputs, and inputs and outputs) are nowhere negative or, in other words, there is no congestion. Next, convexity says that convex combinations of feasible netput vectors are themselves also technically feasible. This implies that marginal rates of substitution/transformation (between inputs, output and inputs and outputs) are nowhere increasing. The fact that the dual representation of our shadow profit inefficiency measure implies a production set that is convex and monotone follows from the result that these technology properties are essentially "irrelevant" for profit efficiency analysis (i.e. imposing the properties will not interfere with the profit efficiency results). See, for example, Varian (1984) for a detailed discussion of this last point.

5 Application

We illustrate the empirical usefulness of our (shadow) profit efficiency method by an application to the input and output data that were also analyzed by Cherchye et al (2013) in their original study. These authors specifically focused on multi-output cost efficiency. Thus, we complement this first study by assessing the profit efficiency of the same DMUs. In the following, we will first discuss the specificities of our data. Subsequently, we present the findings of our empirical analysis. After showing our results for DMUs' aggregate profit inefficiency, we also consider output-specific profit inefficiencies.

The data. Our data set contains input and output information for 290 offices (DMUs) of a large European service company. Each DMU uses 7 inputs, i.e. three types of labor (x^1, x^2 and x^3), three types of transport (x^4, x^5 and x^6) and other over-head cost (x^7), for the production of 7 outputs. Thus, we have $N = 7$ and $M = 7$. The service company uses an "activity-based costing system", which allows us to allocate the first 6 inputs to the specific outputs. That is, adopting the terminology of Section 2, the three types of labor and transport are output-specific, i.e. they can be allocated to the 7 individual outputs. The other overhead cost is modeled as a joint input, which simultaneously benefits the production of all 7 outputs. We refer to Cherchye et al (2013) for more detailed information on the input and output data that we use.[12]

Because we have no data on input and output prices, we conduct a shadow profit efficiency analysis by using the methodology that we presented in Section 4. That is, we evaluate each DMU's profit efficiency in the best possible light by using "most

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[12]Cherchye et al (2013) also explain that confidentiality and strict non-disclosure agreements prohibit us from providing more details on the nature and operations of the service company under study.

favorable" input and output prices. In doing so, we do impose some restrictions on the shadow prices for the inputs. In particular, we use the restrictions that were also used by Cherchye et al (2013) for the same data, and which have been defined in consultation with the management of the service company.

Multi-output profit efficiency. To compute the shadow profit inefficiency measure $\widehat{PE}^C$, we first need to specify the directional vector $\mathbf{g_{Z}}^m$ for each output m. To demonstrate the versatility of our approach, we will consider three different directional vectors. These three directional vectors are the most popular ones in applied DEA analysis and, as we will explain below, imply alternative interpretations of the observed degree of profit inefficiency. The first two directional vectors are:

- $\mathbf{g_{Z}}^m = (y^m, \mathbf{0})$ for each output m, which measures profit inefficiency in terms of proportional output increase, and

- $\mathbf{g_{Z}}^m = (0, \mathbf{X}^m)$ for each output m, which measures profit inefficiency in terms of proportional input reduction.

An attractive feature of these directional vectors is that they imply shadow profit inefficiency measures that have a dual representation in terms of the input and output oriented Debreu (1951) - Farrell (1957) technical efficiency measures, respectively.[13]

The third specification of the directional vector is:

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[13]In particular, $\mathbf{g_{Z}}^m = (y^m, \mathbf{0})$ obtains (a multi-output version of) the Debreu-Farrell output measure (DF_O) minus one as the outcome of our program (**LP-2**), i.e. $\widehat{PE}^C = DF_O - 1$ (where $DF_O \geq 1$ and $DF_O = 1$ indicates efficiency). Similarly, $\mathbf{g_{Z}}^m = (0, \mathbf{X}^m)$ obtains one minus (a multi-output version of) the Debreu-Farrell input measure (DF_I) as the outcome of (**LP-2**), i.e. $\widehat{PE}^C = 1 - DF_I$ (where $DF_I \leq 1$ and $DF_I = 1$ indicates efficiency). See, for example, Chambers, Chung and Fare (1998) and Fare and Grosskopf (2000) for a detailed discussion on the relations between directional distance functions and Debreu-Farrell efficiency measures (including dual representations). Using our results in Section 4, we can extend these authors' arguments to our particular multi-output setting.

- $\mathbf{g}_{\mathbf{z}^m} = (y^m, \mathbf{X}^m)$ for each output m, which measures profit inefficiency in terms of equiproportional output increase and input reduction.

We opt for using this additional specification because it simultaneously considers output and input improvements in the DMUs' efficiency assessment. Basically, in its dual form, the resulting profit inefficiency measure combines the Debreu-Farrell input and output efficiency measures in a single metric.

Table 1 summarize our results for these three directional vectors. $\widehat{PE}^{C,E}$ refers to profit inefficiency with equiproportionate output increase and input reduction, $\widehat{PE}^{C,I}$ to profit inefficiency with input reduction only, and $\widehat{PE}^{C,O}$ to profit inefficiency with output increase only. We provide summary statistics on the distribution of the inefficiency measures as well as information on the number of efficient DMUs (in absolute and relative terms).[14]

Interestingly, we observe quite some variation in profit inefficiency across the DMUs in our sample, for all three directional vectors under consideration. For the measure $\widehat{PE}^{C,E}$ the mean profit inefficiency amounts to 15.9%. This implies that, on average, DMUs should equiproportionally reduce inputs and expand outputs by 15.9% to attain shadow profit efficiency. Next, for the measure $\widehat{PE}^{C,I}$, we find a mean inefficiency of 26.9%. Thus, if output is kept fixed, we need an average input reduction of 26.9% to achieve efficiency.[15] Finally, the mean value of $\widehat{PE}^{C,O}$ equals 31.1%, which signals that profit efficiency requires an average output expansion of 31.1% when inputs are fixed at their given level.

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[14]It can be verified that, for a given DMU, the value of $\widehat{PE}^{C,E}$ can never exceed the values of $\widehat{PE}^{C,O}$ and $\widehat{PE}^{C,I}$, by the very construction of these measures. This definitional property also appears from the results in Table 1.

[15]These results for $\widehat{PE}^{C,I}$ are directly comparable with the cost efficiency results in the column "Basic" in Panel A of Table 2 in Cherchye et al (2013), which equally measure (in casu cost) efficiency in terms of proportional input reduction. As we can expect a priori, the efficiency scores based on profit maximizing behavior are lower due to the more stringent nature of the underlying efficiency criterion.

As a final remark, note that the numbers of efficient offices is the same for the three directional vectors. This could be expected a priori, as the benchmark of shadow profit efficient behavior is the same for the three exercises. The basic difference between these exercises relates to the way in which we measure the degree of inefficiency (i.e. the directional vectors), which indicates alternative possible strategies to "remedy" observed profit inefficiencies (i.e. through (only) input reduction, (only) output expansion or simultaneous input reduction and output expansion).

	$\widehat{PE}^{C,E}$	$\widehat{PE}^{C,I}$	$\widehat{PE}^{C,O}$
Min	0	0	0
Mean	0.159	0.269	0.369
Median	0.149	0.283	0.311
Max	0.649	0.790	1
St. dev.	0.12	0.13	0.15
#Efficient	33	33	33
%Efficient	11.38	11.38	11.38

Table 1: Multi-output profit efficiencies

Output-specific multi-output profit efficiency. As indicated in Section 4, we can allocate a DMU's aggregate profit inefficiency to individual outputs by computing output-specific profit inefficiencies. We believe this provides useful management input as it helps to better identify specific output production processes where substantial profit efficiency gains are possible. By using this information, DMU managers can direct their performance improvement actions in a more effective way.

We will illustrate this practice for the (aggregate) profit inefficiency measure $\widehat{PE}^{C,E}$. Table 2 summarizes our results.[16] In that table, each $\widehat{PE}^{C,m}$ gives the profit inefficiency specific to output m ($m = 1, ..., 7$). These measures have the same interpretation as the aggregate measure $\widehat{PE}^{C,E}$, but now the equiproportionate input

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[16]The results for the measures $\widehat{PE}^{C,I}$ and $\widehat{PE}^{C,O}$ are reported in Tables 5 and 6 in the Appendix. The interpretation of these tables is directly analogous to the one of Table 2.

reduction and output expansion specifically applies to the m-th output production process.

Table 2 reveals substantial heterogeneity across the 7 outputs. For example, we find that the production process of output 2 is generally the most inefficient one. It is characterized by the highest average inefficiency score (no less than 95%) and the lowest number of efficient DMUs (3.10 % of our sample). By contrast, the production process of output 1 appears to be most efficient, in terms of both the average inefficiency (only 12.1%) and the number of efficient DMUs (31.03 %). It is useful to relate these observations to the production shares of the 7 outputs that are given in Table 3. Interestingly, the most efficient output 1 has by far the greatest average share, whereas the production share of the least efficient output 2 is virtually zero.

	$\widehat{PE}^{C,E}$	$\widehat{PE}^{C,1}$	$\widehat{PE}^{C,2}$	$\widehat{PE}^{C,3}$	$\widehat{PE}^{C,4}$	$\widehat{PE}^{C,5}$	$\widehat{PE}^{C,6}$	$\widehat{PE}^{C,7}$
Min	0	0	0	0	0	0	0	0
Mean	0.159	0.121	0.950	0.617	0.546	0.393	0.523	0.443
Median	0.149	0.084	1	0.701	0.632	0.390	0.611	0.474
Max	0.649	1	1	1	1	1	1	1
St. dev.	0.12	0.09	0.05	0.13	0.13	0.11	0.13	0.12
#Efficient	33	90	9	29	28	49	27	32
%Efficient	11.38	31.03	3.10	10	9.66	16.90	9.31	11.03

Table 2: Output-specific profit efficiencies

Apart from revealing interesting efficiency patterns at the level of the full sample of DMUs (see table 2), our output-efficiency scores also provide useful information for individual DMUs. We illustrate this by means of Table 4, which shows the inefficiency results for two selected DMUs. DMU 8 is close to profit efficiency in terms of our aggregate efficiency score, whereas DMU 70 exhibits considerable inefficiency in terms of its aggregate score.

The output-specific efficiency scores give a more balanced picture of the efficiency performance of these two DMUs. For example, we find that the high efficiency of

	Share of the total production (%)
Output 1	90.78
Output 2	0
Output 3	6.84
Output 4	0.32
Output 5	0.04
Output 6	0.91
Output 7	1.09
Total	100

Table 3: Share of the total production per output (sample average)

DMU 8 is particularly due to its efficient production of the outputs 1, 2, 3 and 7. However, there is substantial potential to realize efficiency gains in the production of the outputs 4 and 5 (and also, but to a far lesser extent, in the production of output 6). As for DMU 70, we find that the high level of aggregate inefficiency is caused by inefficient production of all the outputs. But, again, we observe substantial heterogeneity across outputs (with output-specific inefficiencies ranging from 13.8% to 77.4%).

We believe these two examples clearly show the usefulness of our output-specific efficiency measures to direct the attention of DMU managers towards individual outputs that are characterized by profit inefficiency. Importantly, this holds not only for DMUs with low efficiency (like DMU 70) but also for DMUs of which the aggregate performance is close to efficient (like DMU 8).

DMUs	$\widehat{PE}^C$	$\widehat{PE}^{C,1}$	$\widehat{PE}^{C,2}$	$\widehat{PE}^{C,3}$	$\widehat{PE}^{C,4}$	$\widehat{PE}^{C,5}$	$\widehat{PE}^{C,6}$	$\widehat{PE}^{C,7}$
DMU 8	0.061	0	0	0	0.446	0.563	0.050	0
DMU 70	0.218	0.138	0.672	0.282	0.745	0.615	0.774	0.593

Table 4: Output-specific profit efficiencies for selected DMUs

6   Conclusion

We presented a novel DEA toolkit for profit efficiency analysis in the context of multi-output production. A distinguishing feature of our methodology is that it assumes output-specific production technologies. In addition, the methodology accounts for the use of joint inputs, and explicitly includes information on the allocation of inputs to specific outputs.

We have specified a multi-output profit inefficiency measure when prices are observed, as well as a shadow profit inefficiency measure that can be used if prices are unknown. Our framework also allows us to define output-specific profit inefficiency measures, which allocate a DMU's aggregate profit inefficiency to individual outputs. Finally, we established a dual relationship between our multi-output profit inefficiency measure and a technical inefficiency measure that takes the form of a multi-output directional distance function.

We illustrated our methodology by an empirical application to a large European service company. This demonstrated the practical usefulness of our measure for (shadow) profit inefficiency at the aggregate DMU level. Next, we showed that our output-specific profit inefficiency measures provide useful management input. They can identify individual outputs that are characterized by substantial inefficiency, so that performance improvement actions can be directed primarily towards these outputs.

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Appendix

	$\widehat{PE}^C$	$\widehat{PE}^{C,1}$	$\widehat{PE}^{C,2}$	$\widehat{PE}^{C,3}$	$\widehat{PE}^{C,4}$	$\widehat{PE}^{C,5}$	$\widehat{PE}^{C,6}$	$\widehat{PE}^{C,7}$
Min	0	0	0	0	0	0	0	0
Mean	0.269	0.197	0.151	0.633	0.587	0.472	0.576	0.513
Median	0.283	0.162	0	0.795	0.754	0.556	0.732	0.607
Max	0.790	1	1	1	1	1	1	1
#Efficient	33	93	242	51	54	62	46	50
%Efficient	11.38	32.07	83.45	17.59	18.62	21.38	15.86	17.24

Table 5: Output-specific profit inefficiencies for the input reduction direction

	$\widehat{PE}^C$	$\widehat{PE}^{C,1}$	$\widehat{PE}^{C,2}$	$\widehat{PE}^{C,3}$	$\widehat{PE}^{C,4}$	$\widehat{PE}^{C,5}$	$\widehat{PE}^{C,6}$	$\widehat{PE}^{C,7}$
Min	0	0	0	0	0	0	0	0
Mean	0.369	0.272	0.961	0.821	0.817	0.649	0.811	0.755
Median	0.3106	0.179	1	1	1	1.	1	1
Max	1	1	1	1	1	1	1	1
#Efficient	33	85	8	30	28	47	26	30
%Efficient	11.38	29.31	2.76	10.34	9.66	16.20	8.97	10.34

Table 6: Output-specific profit inefficiencies for the output expansion direction