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DEMOCRACY AND DYNAMIC WELFARE OPTIMA

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We show that there exists a rule of deciding on random election dates, such that the solution of a welfare optimum computed on an infinite horizon, coincides with short-run economic policies chosen by a democratically elected government.

A dynamic social welfare optimum can be defined as a solution of the following infinite horizon optimal control problem:

$$\max \sum_{t=0}^{\infty} (1+\rho)^{-t} V(x_t, u_t), \quad \text{subject to} \tag{1a}$$

$$x_{t+1} - x_t = f(x_t, u_t), \qquad t = 0, 1, \ldots, \tag{2}$$

$$x_0 \ \text{ given.} \tag{3}$$

$V(.)$ is an instantaneous welfare function, x_t and u_t are vectors of state and control variables, ρ is the social discount rate and the evolution of the system is described by (2).

Let $\{\bar{x}_t, \bar{u}_t\}$, $0 \leqslant t \leqslant \infty$ be an optimal solution of (1a)–(3). Except if we pick correctly the terminal condition $x_T = \bar{x}_T$, the solution of the finite time horizon problem, truncated at $t = T < \infty$, will be different from $\{\bar{x}_t, \bar{u}_t\}$, $0 \leqslant t \leqslant \infty$.

Let us reinterpret the finite time horizon problem as one in which a democratically elected administration seeks to maximize the welfare of its electors; elections occur every T years and $V(x_t, u_t)$ is the effect in T, the legal election date, of what happened during year t; $V(x_t, u_t)$ can also be interpreted as an aggregate voting function, which relates votes for the incumbent administration to economic events [see, e.g., Nordhaus (1975)]. Assume finally, though this is not essential to our argument, that voters have a memory which decays at a certain rate μ. The short-term problem of the administration which looks for reelection in year T is

$$\max \sum_{t=0}^{T-1} (1+\mu)^{t+1-T} V(x_t, u_t), \tag{1b}$$

subject to (2)–(3).

Our purpose is to show that it is possible to define a stochastic rule of choosing the election date T in such a way that the solutions of problems (1a)–(3) and (1b)–(3) coincide. In other words, the decisions made by a benevolent dictator (welfare maximum) and those made in a democratic setting are the same.

Proposition. *There exists a rule of deciding on random election dates, such that the welfare optimum coincides with the solution chosen by a democratically elected government. This rule will depend only on the social discount rate, and is independent of the memory decay rate.*

Proof. Consider an institutional setting in which every year θ ($\theta = 1, 2, \ldots$), the administration resorts to and follows one of the two possible outcomes of a random process: 'an election is organized in year θ' or 'no election is organized in year θ'. Let p_θ be the probability of the first outcome with

$$p_\theta \geq 0, \qquad \sum_{\theta=1}^{\infty} p_\theta = 1. \tag{4}$$

The administration seeks to maximize the expected number of votes

$$\sum_{\theta=1}^{\infty} p_\theta \sum_{t=0}^{\theta-1} (1+\mu)^{t-\theta+1} V(x_t, u_t), \tag{5}$$

subject to (2)–(3).

Assume $V(x_t, u_t)$ is bounded; then the series defined by (5) converges and we can write it alternatively as

$$\sum_{\theta=1}^{\infty} \sum_{t=0}^{\theta-1} (1+\mu)^{t-\theta+1} p_\theta V(x_t, u_t)$$

$$= \sum_{t=0}^{\infty} \sum_{\theta=t+1}^{\infty} (1+\mu)^{t-\theta+1} p_\theta V(x_t, u_t) \sum_{t=0}^{\infty} \pi_t V(x_t, u_t) \tag{6}$$

with

$$\pi_t = \sum_{\theta=t+1}^{\infty} (1+\mu)^{t-\theta+1} p_\theta . \tag{7}$$

π_t is a 'memory weighted' probability of survival of the ruling administration.

Comparing (1a) and (6), it is seen that the proposition is verified for probabilities p_θ satisfying (4) and (6), with $\pi_t = \alpha(1+\rho)^{-t}$, $\alpha > 0$; then the solutions maximizing (1a) or (5) coincide.

Look for probabilities of the form

$$p_\theta = (1-\sigma)\sigma^\theta \quad \text{with} \quad 0 < \sigma < 1$$

and find values for α and σ such that

$$\alpha(1+\rho)^{-t} = \pi_t = (1-\sigma) \sum_{\theta=t+1}^{\infty} (1+\mu)^{t+1-\theta}\sigma^{\theta} .$$

Now

$$\alpha(1+\rho)^{-t} = (1-\sigma)\sigma^{t+1} \sum_{\theta=0}^{\infty} \sigma^\theta (1+\mu)^{-\theta}$$

$$= (\sigma(1-\sigma)(1+\mu)/(1+\mu-\sigma))\sigma^t .$$

This equality will hold for

$$\alpha = \sigma(1-\sigma)(1+\mu)/(1+\mu-\sigma) > 0 \quad \text{and} \quad \sigma = (1+\rho)^{-1} .$$

Hence $p_\theta = \rho(1+\rho)^{-\theta-1}$ and is independent of μ. Q.E.D.

One noticeable property of this random electoral process is that the probabilities are independent of the memory decay rate, a parameter the value of which is very hard to assess; moreover, it does not matter whether voters are myopic (i.e., do not evaluate the post-election consequences of pre-electoral policies) or cast strategic votes [see, e.g., McRae (1977)].

Note that even for high values of ρ, say 10%, the probability of elections after one year is less than 0.1 and p_θ decreases afterwards. The probability of elections occuring before 7.5 years is less than 0.5; this means that, on average, governments will last longer than the usual three to five years and will not be any more than under the present system prevented from conducting long-term economic policies.

References

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