All-optical controlled switching between time-periodic square waves in diode lasers with delayed feedback

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We investigate the square-wave (SW) self-modulation output of an edge-emitting diode laser subject to polarization rotated optical feedback in detail, both experimentally and theoretically. Our experimental results show that the 2τ-periodic SW, where τ is the delay of the feedback, coexists with other SW oscillations of shorter periods. We have found that these new SWs are specific harmonics of the fundamental one and their periods are $P_n \simeq 2\tau/(1+2n)$, where n is an integer. Numerical simulations and analytical studies of laser rate equations confirm the multistability of SW solutions. By adding a weak conventional optical feedback, we show that the switching between the different periodic SWs can be easily controlled. The delay of this feedback control is the key parameter determining the harmonic that is stabilized. Numerical simulations corroborate the effectiveness of our experimental control scheme.

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Over the past few years, nonlinear delay dynamics have found a particularly prolific area of research in the field of photonic devices [1–3]. When part of the light emitted by a semiconductor laser is reflected back into the laser after a substantial delay, a rich variety of pulsating outputs can be observed. Among the diverse setups studied experimentally, the laser subject to polarization rotated optical feedback (PROF) is particularly attractive [4–10]. It has found a variety of applications stemming from all-optical production of high frequency pulses [11] and from its chaotic behavior for random bit generation [12]. Moreover, its rate equations can be analyzed in more detail than the traditional single polarization feedback [7]. In this system, the emitted TE mode is rotated toward the TM mode and reinjected into the laser, while the emitted TM mode is not fed back into the laser. The two polarization modes, however, interact through the carrier density. If the delay τ of the feedback is relatively large compared to the time constants of the laser, it has been shown that the laser exhibits periodic square-wave (SW) oscillations with a period close to twice the delay [7]. In this Letter, we show experimentally and numerically that stable SW oscillations of period $P_n \simeq 2\tau/(1+2n)$ ($n = 0, 1, 2, \ldots$) coexist with the main 2τ-periodic regime for the same values of the parameters. All-optical switching between these different time-periodic states is then realized by using a second weak conventional optical feedback (COF) which acts on both polarization modes. Multistate switching devices are needed for the next generation of all-optical computing and communication [13]. The multistability phenomenon is observed for many nonlinear systems and the transition of complex multistability to controlled monostability has raised a lot of attention [14,15]. For Vertical-Cavity Surface-Emitting Lasers (VCSELs) with the PROF setup, it was recently shown that a second weak feedback allows generation of robust SWs with respect to parameter changes [10]. By adding this second feedback with a delay time θ, we experimentally demonstrate that at fixed θ, we may switch from the main P_0 state to a specific P_n state by simply increasing the feedback amplitude. Furthermore, the value of n depends on how close θ is to τ. If $|\theta - \tau| \to 0$, higher order harmonic SWs sequentially appear (n increases) until fast sustained relaxation oscillations dominate.

The experimental setup is illustrated schematically in Fig. 1. We have used a JDSU 50 mW single mode Fabry-Perot edge-emitting laser diode (LD) stabilized in temperature at 19.00°C. It has a nominal wavelength of 834.9 nm and a solitary threshold of 27.10 mA. For all data presented in this Letter, the LD is driven with a current of 100.00 mA and operates in a single longitudinal mode. The external cavity consists of two feedback loops. The first feedback (from Mir1) rotates the polarization of the TE mode and reinjects it into the laser TM mode (PROF). It is this feedback that generates the SWs. The second feedback (from Mir2) is conventional and reflects the light of the two polarization modes (COF). This feedback is weak and is used to switch between different SWs. The beam emerging from the LD is collimated by a lens (CL) and passes through a nonpolarizing beam splitter (BS1, $T = 70\%$). The transmitted beam enters a Faraday rotator (FAR ROT) which is a Faraday isolator

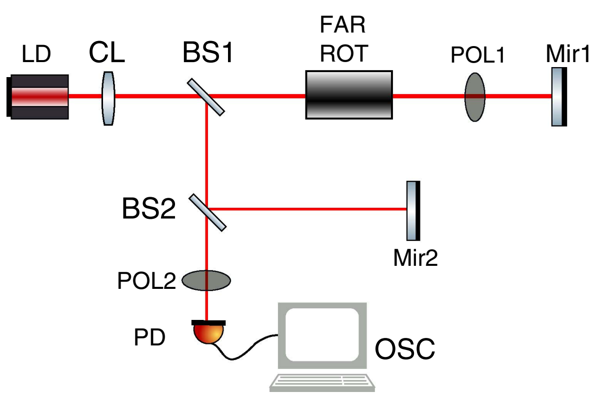

Fig. 1. Experimental setup: abbreviations are defined in the text.

with its polarizers removed. The beam's polarization rotates 45° and exits through a rotatable linear polarizer (POL1) with its transmission axis set at 45° from vertical (which is the TE orientation of LD). A highly reflecting mirror (Mir1) reflects the beam, which is then rotated an additional 45° on the return pass through the rotator. This creates a horizontally polarized TM beam that is reinjected into LD. This specific setup prevents multiple round trips since the polarizer placed after the rotator imposes a unidirectional feedback from the TE to the TM mode. The signal reflected from the beam splitter (BS1) strikes a second nonpolarizing beam splitter (BS2, $T = 50\%$). Part of the light reflected from BS2 is fed back to the laser cavity by a reflecting mirror (Mir2) and by the two beam splitters. This second feedback loop has a much smaller feedback rate because of reflections on beam splitters BS1 and BS2 ($R_1 = 30\%$ and $R_2 = 50\%$, respectively). The signal transmitted from BS2 is sent to a photodetector (PD) with a bandwidth of 2.4 GHz, connected with a 4 GHz real-time oscilloscope (OSC) (or a grating based optical spectrum analyzer). A polarizer (POL2) placed before PD allows us to select the mode to be analyzed.

The length of the PROF loop is 75 cm, implying a delay (round trip time) of $\tau = 5$ ns. Its components need to be carefully aligned to obtain a strong feedback. We typically have a round trip power transmission around 36%. Without the second feedback [i.e., without the second beam splitter (BS2)], we observe a SW self-modulation with a period of 10.4 ns (slightly longer than 2τ, called P_0 state) consistent with [7]. This is shown in the top panel of Fig. 2, where we plot the measured time traces of the TE and the TM modes. The two polarization modes are clearly in antiphase. For lower feedback rates we also obtain the so-called complex SWs documented in [16]. These waveforms are a combination of square and pulsating forms, as illustrated in Fig. 2(b). We have analyzed their properties (coexistence, number of pulses and period) which agree with the conclusions in [16]. Coexisting with the large amplitude simple and complex 2τ-periodic SWs, we have found higher frequency SWs. They were obtained randomly by switching the feedback on and off. Their periods are exactly 3, 5 or 7 times shorter than the 2τ-periodic ones (P_1, P_2, and P_3 states, respectively). Figure 2(c) shows the P_1 state. These harmonics exhibit the same antiphase property as the P_0 SWs and have been observed for long intervals of time. Because of the coexistence of many regimes for the same operating conditions, it is experimentally difficult to select a specific P_n SW, especially for the high frequency ones. To control the selection of specific harmonics and the switching among them, we introduce a low COF. We compare the round trip power transmissions of the COF and the PROF loops by separately measuring the power coming back through BS1 for each feedback. Our results indicate that the COF is ~55 times weaker than the PROF. This setup allows us to switch from the P_0 SW to a P_n state and back again simply by switching on and off the COF. Of particular interest is that the delay of the COF, θ, controls the value of n, i.e., to which harmonic the system switches. When $|\theta - \tau| \to 0$, higher order harmonic SWs sequentially appear (n increases) until fast sustained relaxation oscillations dominate, as shown in Fig. 3. We only present time traces of the TM mode since the TE mode is in antiphase, as in Fig. 2, and does not provide additional information. With our setup, we are able to stabilize P_n states for $n = 1$ to 5 (three cases

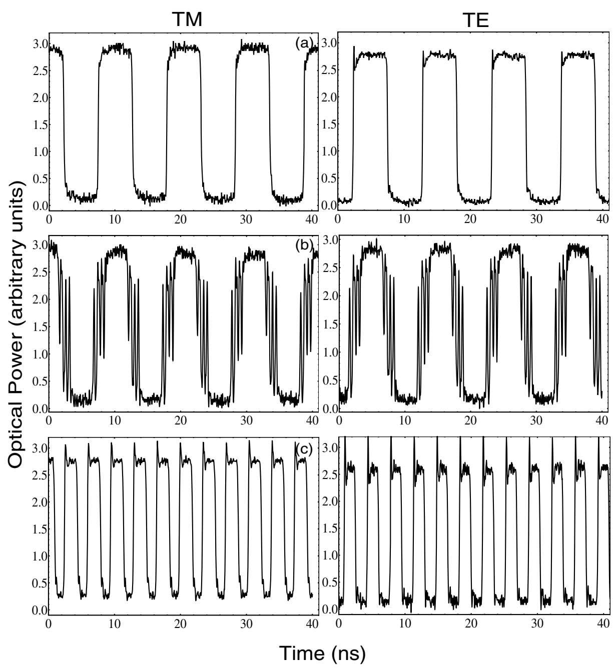

Fig. 2. Experimental time series of different waveforms coexisting under strong PROF (without COF). Left and right, TM and TE mode, respectively. (a) SWs of period 10.4 ns (P_0). (b) complex SWs (SW with short pulses), same period. (c) SWs of period 3.5 ns (P_1).

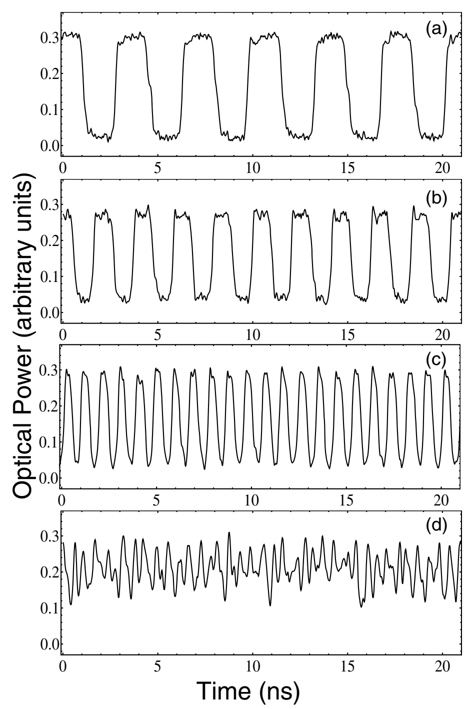

Fig. 3. Experimental time series of the TM mode for different delays θ of the COF loop. (a) $\theta = 3.5$ ns, P_1 state; (b) $\theta = 4.1$ ns, P_2 state; (c) $\theta = 4.7$ ns, P_5 state; (d) $\theta = 4.9$ ns, fast sustained relaxation oscillations.

are illustrated in Fig. 3). This control is very efficient since the selected harmonic SW remains the only stable waveform even though the COF strength is weak.

The behavior of a diode laser subject to PROF is modeled mathematically by three dimensionless rate equations for the slow varying envelope of the electrical fields of the TE and TM modes denoted by E_1 and E_2, respectively and the carrier density N (see [7,17]). The dimensionless formulation of the original equations is derived in detail in [17]. With the extra COF setup, these equations are given by

$$\frac{dE_1}{ds} = (1 + i\alpha)NE_1 + \gamma E_1(s - \theta), \tag{1}$$

$$\frac{dE_2}{ds} = -i\Omega E_2 + (1 + i\alpha)k(N - \beta)E_2 + \eta\sqrt{k}E_1(s - \tau) + \gamma E_2(s - \theta), \tag{2}$$

$$T\frac{dN}{ds} = P - N - (1 + 2N)[|E_1|^2 + |E_2|^2]. \tag{3}$$

In these equations, α is the linewidth enhancement factor, T is the ratio of carrier to cavity lifetimes, P is the pump parameter above threshold and Ω is the frequency detuning between the two polarization modes. k is the ratio of the gain coefficients of the TM and TE modes and $\beta = (1 - k)/(2k)$. η and γ are the amplitudes of the polarization rotated feedback and of the weak conventional feedback, respectively. Time s is measured in units of the cavity lifetime. From now on, τ and θ refer to the dimensionless delays.

If $\gamma = 0$, there exists a unique mixed-mode steady-state that admits Hopf bifurcations to SW oscillations [17]. For τ sufficiently large, these Hopf bifurcations are characterized by their frequencies $\omega_n = (1 + 2n)\pi/\tau$ $(n = 0, 1, \ldots)$ and are located close to the critical point [17]

$$\eta = \sqrt{\frac{(1 - k)^2 + (2\Omega + \alpha(1 - k))^2}{4k}}. \tag{4}$$

Previous numerical simulations have shown that the 2τ-periodic SWs resulting from the first Hopf bifurcation are stable in a large parameter region [7,16,17]. But the harmonics, $2\tau/(2n + 1)$-periodic SWs, have never been reported. By carefully choosing the initial condition of Eqs. (1)–(3), we have found that it is possible to stabilize these coexisting SWs. In our simulations we have used typical values of the parameters: $\alpha = 2$, $k = 0.8$, $\tau = 2000$, $P = 2$, $T = 150$, $\Omega = 0$. With these values and $\gamma = 0$, the critical point for the onset of SWs, Eq. (4), is $\eta = 0.25$. We have then obtained multistable P_n states for $n = 0$ to 7 in the range $\eta = 0.24$–0.28. The two polarization modes oscillate in antiphase as observed in the experiments. It is interesting to note that the analytical construction of the SWs in the limit of large τ proposed in [7] and extended in [17] holds for the P_n SWs. This construction neglects the fast transitions between the plateaus of the SW and concentrates on the constant plateaus by solving a nonlinear map. The mapping does not depend on the length of the plateaus but rather assumes that one delay before an upper plateau, the system exhibited a lower plateau, and *vice versa*. This assumption remains true for all P_n $(n = 0, 1, \ldots)$ and the previous results on the stability and intensity of the plateaus can be extended for these P_n SWs. These analytical predictions have been verified by our numerical simulations.

To study the control of the SWs with a weak COF, we take $\gamma = 0.005$ and gradually change θ in the interval $0.7\tau < \theta < 1.3\tau$. We choose $\eta = 0.28$, a value for which the P_0 state is strongly stable without COF for a very large class of initial conditions, see Fig. 4(a). As θ approaches τ, higher order harmonic SWs become sequentially excited as shown in Figs. 4(b) and 4(c). When $\theta = \tau$, fast oscillations are found as it was the case for a VCSEL with the same setup [10]. The switching from P_0 to a given P_n is obtained by changing γ from 0 to 0.005. Our numerical simulations confirm the experimental observations: the weak COF stabilizes a P_n waveform and the value of n depends on the delay θ as illustrated in Fig. 4. The two modes oscillate in antiphase and only the TM intensity is shown. These periodic waveforms are stable in time and there is a good qualitative agreement between our numerical and experimental results.

The experiments shown in Fig. 3 suggest that $2n + 1$ there is an odd integer close to $\tau/|\theta - \tau|$. We have numerically investigated the effect of the feedback control by examining the progressive deformation of the initial P_0 SW. We observe the emergence of a small additional plateau of size close to $|\theta - \tau|$; that gets progressively amplified at each step τ. As time increases, it produces a subdivision of the original plateau of length τ ultimately leading to a SW of period close to one of the P_n states. In

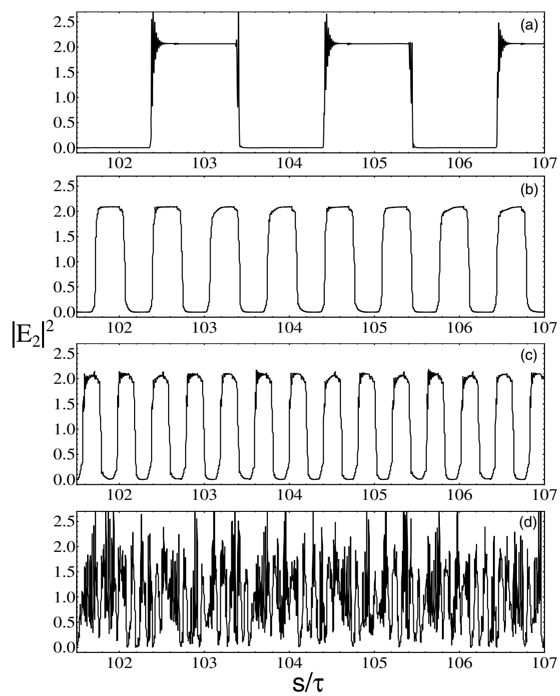

Fig. 4. Numerical time series of the TM mode intensity. Values of the parameters are defined in the text. (a) P_0 SW obtained without COF ($\gamma = 0$) (b) P_1 SW, $\theta = 1.3\tau$ ($\gamma = 0.005$) (c) P_2 SW, $\theta = 1.15\tau$ ($\gamma = 0.005$) (d) Fast oscillations, $\theta = \tau$ ($\gamma = 0.005$).

future work, we plan to improve our model equation and explore in details the relation between P_n and $\tau/|\theta - \tau|$. We didn't observe the coexistence of distinct P_n states for the same value of τ and θ. However, such coexistence could be possible in tiny regions close to the transitions between P_n states.

In this Letter, we have shown numerically and experimentally that stable SWs of period $P_n = 2\tau/(2n+1)$ where n is an integer coexist for a diode laser subject to the PROF setup. The existence and multistability of these waveforms were anticipated by the determination of Hopf bifurcation points of the mixed mode steady-state. The analytical construction of the 2τ-periodic SWs [7,17] extends naturally to these higher harmonic SWs and confirms their linear stability with respect to the relaxation oscillation time-scale. We have introduced a weak COF and demonstrated that it can successfully control the switching between P_0 and P_n. We select n by simply changing the delay of the COF.

We believe that this multiplicity of SWs resulting from nearby Hopf bifurcations is generic to a large class of optical feedback systems exhibiting a large delay. Recently, an opto-electronic oscillator with a large delay in its feedback loop has been studied [18] and the same coexistence of harmonic SW oscillations has been found both experimentally and numerically. The evolution equations are completely different from the laser rate Eqs. (1)–(3). But the common property is the presence of nearby Hopf bifurcation points leading to SWs with frequencies that are multiples of τ^{-1}.

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