Quantity Adjustment Costs and Price Stickiness*

by

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1. Introduction

Price stickiness is often modelled as the consequence of costly price adjustment mechanisms. Sheshinski and Weiss (1977), for instance, base their argument on the idea that these costs are associated with the transmission of price information to consumers: it is costly to print price catalogs, but it is also costly to sell at outdated prices when there is inflation. See also Mussa (1981) or Rotemberg (1982a), (1982b) for similar arguments and models.

We derive a similar result from costly quantity adjustment mechanisms, which seem more natural: variations in production capacities clearly bring along some costs, as do simple day-to-day variations, since part of the equipment may have to be shut down, or on the contrary, reactivated, involving some fixed costs; there may also be temporary production losses due to learning, after new equipment has been installed, etc. See Eisner and Strotz (1963), Lucas (1967), Hayashi (1982).

We develop a simple model of an oligopolistic industry in which each firm chooses, à la Cournot, its current and future production levels, demand being given; firms face adjustment costs when they vary production over time. This behavior leads to an aggregate price equation which exhibits lags in price adjustments; it is only when the number of firms in the industry gets large, that costs and demand shifts are transmitted without delay.

An interesting feature of the model is that the optimal decision of each producer is more sluggish than the optimal aggregate behavior of the industry; in the limit, when the number of firms is large, the response of the
industry to changes in costs or demand conditions is immediate, while each producer's response remains sticky.

2. The model

We consider an industry consisting of n firms, producing a homogeneous commodity. Each firm i (i = 1,2,...,n) chooses production levels $q_{it}$, $t=1,2,\ldots,\infty$ to maximize the discounted value of all future profit streams $\pi_{it}$

$$\sum_{t=0}^{\infty} (1+r)^{-t} \pi_{it}$$

where

$$\pi_{it} = p_t q_{it} - \gamma_{it}q_{it} - [c(q_{it}-q_{i,t-1})+1/2 d(q_{it}-q_{i,t-1})^2]$$

In these two expressions, $r$ is a time discount rate, $p_t$ is the price of the good; marginal production costs $\gamma_{it}$ are constant, while the term between brackets represents costs associated with variations of production levels or capacities; to the usual linear term $c(q_{it}-q_{i,t-1})$, we add quadratic costs $1/2 d(q_{it}-q_{i,t-1})^2$.

We assume the inverse demand function to be linear and given by

$$p_t = A_t - b(q_{1t} + q_{2t} + \ldots + q_{nt})$$

Each firm's optimization problem will thus be quadratic.\(^1\)

\(^1\) Introducing uncertainty would not change the conclusions, since we can make use of the certainty equivalence theorem.
We finally assume that firms face competition à la Cournot and we look for a Nash equilibrium in quantities; the necessary and sufficient conditions for such an equilibrium can be written:

\[ \frac{\partial \pi_{it}}{\partial q_{it}} + (1+r)^{-1} \frac{\partial \pi_{i,t+1}}{\partial q_{it}} = 0, \quad t \geq 0, \]

or

\[ (2.4) \quad p_t - bq_{it} - \gamma_{it} - c - d(q_{it} - q_{i,t-1}) + (1+r)^{-1}[c + d(q_{i,t+1} - q_{it})] = 0, \quad t \geq 0. \]

Aggregating (2.4) over firms, leads to

\[ n[A_t - bQ_t - c - (1+r)^{-1}c] - [b + d + (1+r)^{-1}d]Q_t + dQ_{t-1} + d(1+r)^{-1}Q_{t+1} = 0, \quad t \geq 0, \]

or

\[ (2.5) \quad B_t - aQ_t + Q_{t-1} + (1+r)^{-1}Q_{t+1} = 0, \quad t \geq 0 \]

with \( a = 1 + (1+r)^{-1} + (n+1)b/d \) and \( B_t = n[A_t - \gamma_t - c + (1+r)^{-1}c]/d \).

\( Q_t = \Sigma q_{it} \) represents total production and \( \gamma_t = \Sigma \gamma_{it}/n \), the average marginal production cost.

3. The dynamics of industry behavior

(2.5) is a difference equation which shows that the aggregate production decision \( Q_t \) made at date \( t \), depends on decisions made in the past (\( Q_{t-1} \)) and in the future (\( Q_{t+1} \)). Adjustment costs enter thus through past as well as through future variations of production levels.
To solve the second order difference equation (3.1), we look for a solution of the form  

\[ Q_t = \alpha (Q_{t-1} + x_t). \]  

Then, \( Q_{t+1} = \alpha (Q_t + x_{t+1}) = \alpha^2 (Q_{t-1} + x_t) + \alpha x_{t+1}; \) substituting these expressions into (2.5) leads to  

\[ B_t + (Q_{t-1} + x_t) [ (1+r)^{-1} \alpha^2 - \alpha + 1 ] - x_t + \alpha (1+r)^{-1} x_{t+1} = 0. \]

The term between brackets \( (1+r)^{-1} \alpha^2 - \alpha + 1 \) is a quadratic equation in \( \alpha \) which has two positive real roots, the smallest of which, equal to  

\[ \alpha = \frac{a(1+r) - [a^2(1+r)^2 - 4(1+r)]^{1/2}}{2}, \]  

is smaller than 1, since \( a > 1 + (1+r)^{-1}. \)

Setting \( \alpha \) equal to that value, equation (2.5) can be rewritten  

\[ x_t = \alpha (1+r)^{-1} x_{t+1} + B_t. \]

Assuming that \( A_t \) (and \( B_t \)) grow at a smaller rate than \( (1+r)^{-1}, \) the unique bounded solution of (3.3) is given by  

\[ x_t = \sum_{k=0} \alpha (1+r)^{-1}^k B_{t+k}, \]

Replacing now \( x_t \) by (3.1) the solution of (2.5) is  

\[ Q_t = \alpha Q_{t-1} + \alpha \sum_{k=0} \alpha (1+r)^{-1}^k B_{t+k}. \]

\[^2\text{See e.g. Sargent (1987), 399-401 for the solution method}\]
Equation (3.5) shows that, as long as $\alpha \neq 0$ (and this will always be the case, if parameters $r$, $b$ and $d$ are non-zero), total production will be sticky and will, through $B_t$, depend on expected demand shifts and on costs.

It is now easy to derive the industry price equation; using the inverse demand function $p_t = A_t - bQ_t$, (3.5) can be written

\[
(3.6) \quad p_t = \alpha p_{t-1} + \{A_t - \alpha A_{t-1} - b\alpha \sum_{k=0}^{\infty} \alpha^k(1+r)^{-k}B_{t+k}\},
\]

showing that also prices are sticky.

The price equation (3.6) has thus been obtained under the assumption of costly adjustments of quantities, while usually such a behavior is obtained under the less realistic assumption of costly adjustment of prices.

It is interesting to examine the relation between price stickiness and concentration in the industry. This can be done by using equation $(1+r)^{-1}a^2 - a\alpha + 1 = 0$ and differentiating it with respect to $\alpha$. We obtain:

\[
[-a + 2\alpha(1+r)^{-1}] \partial \alpha / \partial n = \alpha \partial a / \partial n = ab/d > 0.
\]

For $\alpha < 1$, the term between brackets $-a + 2\alpha(1+r)^{-1} < 0$ since $a > 1 + (1+r)^{-1} > 2\alpha(1+r)^{-1}$; hence, $\partial \alpha / \partial n$ must be negative and $\alpha$ is increasing with concentration. This result is however strongly linked to the assumption of quadratic adjustment costs, and, as shown in Ginsburgh and Michel (1988), the reverse may hold for other types of adjustment costs.
4. The dynamics of individual producers' behavior

In the Cournot-Nash equilibrium which is assumed here, each producer takes other producers' decisions as given; producer \( i \) considers \( Q_{it} = \sum_{j \neq i} Q_{jt} \) as given; from (2.4), and using the inverse demand equation \( p_t = A_t - b(Q_{it} + q_{it}) \), we get

\[
(4.1) \quad B_{it}^* - a^* q_{it} + q_{i,t-1} + (1+r)^{-1} q_{i,t+1} = 0
\]

with \( a^* = 1 + (1+r)^{-1} + 2b/d \) and

\[
B_{it}^* = \frac{A_t - bQ_{it} - \gamma_{it} - c_t + (1+r)^{-1} c_{t+1}}{d}.
\]

To solve (4.1), one proceeds exactly like in the case of (2.5), and the resulting equation, corresponding to (3.5) is

\[
(4.2) \quad q_{it} = \alpha^* q_{i,t-1} + \alpha^* \sum_{k=0}^\infty [\alpha^*(1+r)^{-1}]^k B_{t+k}^*.
\]

where \( \alpha^* \) is the smallest (and < 1) root of \( (1+r)^{-1} \alpha^* - a^* \alpha^* + 1 - 0. \)

It is easy to check that the smallest root of this equation is decreasing in \( a^* \); since \( a^* < a, \alpha^* > \alpha \) showing that the behavior of each individual producer is more sticky than the overall behavior. Note also that this difference increases with the number of producers, so that for large \( n \), aggregate production will adjust very quickly to cost and demand conditions, despite the fact that each individual producer will keep adjusting with some delay.

\[3\] It can be verified that the aggregation of (4.2) over individuals leads to (3.5).
5. Concluding comments

We have shown that, under some specific, but widely used assumptions, adjustment costs on quantities lead to aggregate price and supply inertia, similar to what had been shown to hold under the assumption of costly price adjustments. We also show that the sluggishness present in individual producers’ decisions is modified when the aggregate production decision is considered; this illustrates once again the difficulty of interpreting aggregate relations, and trying to use them to infer properties of the behavior of individual units.
References


