On the efficiency of retaliation rules
An application to GATT

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August 1995

Abstract
There are several articles in the General Agreement on Tariffs and Trade which constitute escape clauses and provide safeguards; the most important ones are contained in Articles XII, XVIII and XIX. The paper is concerned with Article XIX which allows a country to retaliate or to be compensated for discriminatory measures. Evidence shows, however, that this article has rarely been used. We give theoretical reasons for which this has been the case. We also show that the retaliation mechanism which is built into the rule prevents tariff wars, but does not lead to free trade. We provide an alternative retaliation rule which yields free trade as the unique equilibrium of the tariff game.

Published Annales d’Economie et de Statistique 47 (1997), 51-63

1The authors are grateful to Jean Gabszewicz, André Sapir, T.N. Srinivasan and Jean Waelbroeck for helpful comments and discussions, as well as to three referees whose. This version was completed while the second author was visiting the University of Brussels and CORE. Financial support from the Belgian Government under contract PAI 26 is gratefully acknowledged.
1 Introduction

The General Agreement on Tariffs and Trade (GATT) contained several articles which constituted escape clauses and provided safeguards, in case of unexpected events; the most important ones were Articles XII, XVIII and XIX.\(^2\) Both Articles XII and XVIII allowed developing and developed countries, respectively, to take action in order to deal with unfavorable balance of payments problems, while Article XIX permitted emergency action against sudden surges of imports of a particular commodity.\(^3\)

Articles XII and XVIII seemed, by and large, to be accepted, while Article XIX which permitted governments to raise tariffs to protect their producers, was the object of much controversy. Action permitted under Article XIX was assumed to be temporary, non-discriminatory and subject to compensation or retaliation,\(^4\) though the precise form of it was far from being clearly stated. The article provided that if the consultations between the injured and the exporting country did not result in agreement, the importing country could nevertheless take action but then “the affected contracting parties shall be free (...) to suspend (...) such equivalent concessions or other obligations under this agreement (...) which contracting parties do not disapprove.” And interestingly enough, this is the only article which allowed for compensation, while countries resorting to Articles XII and XVIII did not need to

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\(^2\)See Alan Wolff (1983) and Augustine Tan (1989) for other safeguards.

\(^3\)The main provisions of Article XIX were the following: “If, as a result of unforeseen developments (...), any product is being imported (...) in such increased quantities and under such conditions as to cause or threaten serious injury to domestic producers (...) or directly competitive products, the contracting party shall be free, in respect of such product, and to the extent and for such time as may be necessary to prevent or remedy such injury, to suspend the obligation in whole or in part (...). Before any contracting party should take action (...) it shall give notice in writing to the contracting parties.”

compensate their trading partners.\textsuperscript{5}

The evidence shows that Article XIX has been rarely used.\textsuperscript{6} Its applications affected a relatively small part of international trade,\textsuperscript{7} it has “rarely if ever, been interpreted in a way that would appear consistent with the text,” [Sampson (1987), p.143] and has been the object of some controversy. In many cases\textsuperscript{8} the exporting country did not seem to receive compensation while retaliation was used even less frequently.\textsuperscript{9}

In practice, countries which have invoked Article XIX, used various protective measures, like tariff increases, quotas, orderly market arrangements and, mostly, voluntary export restraints. In his analysis Perez-Lopez (1989) suggests that tariff surcharges may be less stringent than quantitative restrictions; they act on the price mechanism, can usually be made less discriminatory and less difficult to administer, and also avoid rents generated by quotas and VER’s. Moreover, quotas and VER’s hurt only consumers in the importing country who, as mentioned by Brian Hindley (1987, p. 694), had no chance to complain to the GATT.\textsuperscript{10}

By and large, Article XIX was described as ”the Achilles heel of multilateral trade negotiations for the past fifteen years and (...) for the years to

\begin{footnotesize}
\textsuperscript{5}See Table 1 in Jorge Perez-Lopez (1989), p.57 and p.62.
\textsuperscript{6}Sampson (1987) counts 132 cases treated during the forty years of GATT’s existence, mainly by the United States, Australia, Canada and the European Community. Bhagwati (1976) mentions that a large number of industries in the U.S. applied for the escape clause; however, most of the requests were turned down.
\textsuperscript{7}Less than one per cent in 1980, as observed by Sampson (1987), p.146.
\textsuperscript{8}105 out of 132, according to Table 19.3 in Sampson (1987).
\textsuperscript{9}Between 1947 and 1978 compensation was paid in 17 cases and retaliation was used only 4 times [Wolff (1983), p. 379], among which the well-known 1976 U.S.-Canada dispute over beef meat.
\textsuperscript{10}There are other criticisms against VER’s, like “All VER’s are voluntary, but some are more voluntary than the others” and “When you want protection without being protectionist, thank the Lord for VER’s” [Martin Wolf (1989)].
\end{footnotesize}
come” [Sampson (1987), p. 143] ; it “holds the key to success or failure of the Uruguay Round” [Sampson (1987), p. 144]. Though many trade specialists [Bhagwati (1976), Wolff (1983), Sampson (1987), among others] called for a reform of Article XIX, there was no widespread demand for its suppression. Its existence and the possibility to resorting to it served as a safety valve in negotiations where every country was asked to reduce its trade (and non-trade) barriers. The Punta del Este Declaration, which gave birth to the Uruguay Round stated that “a comprehensive agreement on safeguards is of particular importance to the strengthening of the GATT system and to progress in the multi-lateral trade negotiations.” [Tan (1989), p.331].

The Uruguay Round redrafted Article XIX in several respects. The most important revisions are as follows. First, a country may apply for a safeguard measure only following an investigation by the competent authorities of that country, though in critical circumstances a provisional safeguard measure can be taken, which should however not exceed 200 days. Secondly, the new safeguard rule insists on the definition of “serious injury:” the surging import should represent a significant overall impairment in the position of a major proportion of a domestic industry; the serious injury should be based on facts and not on allegations, conjectures or remote possibilities; a report should be drafted and made public. Moreover, if quantitative restrictions are used, they should not reduce the quantity of imports below the level reached during a recent period. Finally, a normal time limit of four years (which can be extended to a maximum of eight years) is imposed.

The new treaty of the World Trade Organization has however kept the idea of compensatory measures. A country using a safeguard measure shall “endeavour to maintain a substantially equivalent level of concessions and other obligations (...). To achieve this objective, the members concerned
may agree on any adequate means of trade compensation for the adverse
effects of the measure."

By and large, the Uruguay Round has taken into account most of the
criticisms that had been addressed against Article XIX, but the basic idea
remains: a country may take action, but has to make concessions to its
trading partners, that are “equivalent” to their loss in exports.

In this paper, we take up the issues raised by Article XIX and its revision;\textsuperscript{11} in particular, we are interested in analyzing situations in which a country may
retaliate to protective measures taken by one of its trade partners. The es-
cape clause, stated in Article XIX provides such a framework. As we already
mentioned, it has rarely been invoked and seems to constitute a good safe-
guard for preventing tariff wars. We offer additional theoretical reasons for
which the escape clause has only seldom been used. We then suggest chang-
ing the working of the retaliation rule to create a framework from which free
trade emerges as the unique equilibrium.

The paper is organized as follows. In Section 2 we introduce the model;
we also formalize the compensating mechanism and show that the built-in
retaliation mechanism prevents countries from initiating a tariff war; that is,
no pair of tariffs can give rise to a tariff war. However, under this rule a tariff
equilibrium fails to exist. In Section 3, we modify the retaliation rule and
and characterize the compensating mechanism so that it yields free trade as
a unique equilibrium of the associated tariff game.

\textsuperscript{11}For simplicity, we still refer to the agreement on safeguards as “Article XIX.”
2 The model

There are two countries denoted by \(i, j\), \(i \neq j\), and one representative consumer in each country. Country \(i\) exports commodity \(i\) and imports commodity \(j\), on which the government of \(i\) can impose a tariff \(t_i\). Both representative consumers behave competitively: each of them takes prices \((p_i, p_j)\) and tariffs \((t_i, t_j)\) as given and maximizes utility subject to a budget constraint, where the tariff levied is returned to him under the form of a lump sum transfer \(T_i\). Country \(i\) is endowed with \(w_i\) units of commodity \(i\): it exports \(e_i\) units of \(i\) and imports \(m_i\) units of \(j\). From this competitive mechanism, it is possible to derive indirect utility functions, which are used as payoff functions depending on the tariffs \((t_i, t_j)\); each government chooses tariffs to maximize its own payoff function.

The countries’ payoff functions

The preferences of the representative consumer in country \(i\) are given by a Cobb-Douglas utility function:\(^{12}\)

\[
\alpha_i \log(w_i - e_i) + (1 - \alpha_i) \log m_i,
\]

where \(0 < \alpha_i < 1.\)^{13} Consumery \(i\) chooses his bundle \((w_i - e_i, m_i)\) by solving

\[
\max \{\alpha_i \log(w_i - e_i) + (1 - \alpha_i) \log m_i\}
\]

\(^{12}\)The Cobb-Douglas specification (or any CES specification) forces both countries to trade; this seems to be a minimal requirement: in a two-country setting, either they both trade, or none of them does. The assumption that each country is endowed in one of the commodities is made to simplify the calculations; if both countries were endowed in both commodities, one would have to distinguish several “regimes” according to which country exports which commodity (commodities).

\(^{13}\)All the derivations are made for country \(i\); the expressions for \(j\) can be obtained by interchanging \(i\) and \(j\).
subject to
\[ p_j(1 + t_i)m_i = p_i e_i + T_i, \]

where the constraint imposes trade balance between the two countries: expenditure (the value of tariff-ridden imports) should be equal to income (the value of exports plus the tariff revenue). Note that the lump sum transfer \( T_i \) is equal to \( p_j t_i m_i \), but when consumer \( i \) makes his decision, \( T_i \) is considered as given. It is straightforward to compute the optimal consumption bundle of \( i \),

\[
\begin{align*}
  w_i - e_i & = \frac{\alpha_i(1 + t_i)w_i}{(1 + \alpha_i t_i)} \quad (1) \\
  m_i & = \frac{p_i(1 - \alpha_i)w_i}{p_j(1 + \alpha_i t_i)}. \quad (2)
\end{align*}
\]

Market clearing conditions imply:
\[ m_i = e_j. \quad (3) \]

This makes it possible to compute the ratio of the prices generated by the pair of tariffs \((t_i, t_j)\):
\[
\frac{p_j}{p_i} = \frac{w_i(1 - \alpha_i)(1 + \alpha_j t_j)}{w_j(1 - \alpha_j)(1 + \alpha_i t_i)}. \quad (4)
\]

Substituting this price ratio in (1) and (2) yields the bundle of country \( i \):
\[
\begin{align*}
  w_i - e_i & = w_i \frac{\alpha_i(1 + t_i)}{1 + \alpha_i t_i}; \quad (5) \\
  m_i & = \frac{1 - \alpha_i}{1 + \alpha_j t_j}. \quad (6)
\end{align*}
\]

Using (5) and (6), we can compute the utility levels generated by the pair \((t_i, t_j)\):
\[
\begin{align*}
  u_i(t_i, t_j) & = \alpha_i \log \frac{1 + t_i}{1 + \alpha_i t_i} + (1 - \alpha_i) \log \frac{1}{1 + \alpha_j t_j} + c_i, \quad (7)
\end{align*}
\]
where the value of $c_i = \alpha_i \log \alpha_i w_i + (1 - \alpha_i) \log(1 - \alpha_j) w_j$ is independent of the tariffs. Similar calculations for country $j$ yield:

$$u_j(t_i, t_j) = \alpha_j \log \frac{1 + t_j}{1 + \alpha_j t_j} + (1 - \alpha_j) \log \frac{1}{1 + \alpha_i t_i} + c_j,$$

where the constant $c_j$ is again independent of tariffs.

The retaliation mechanism

We now turn to the description of the retaliation mechanism [See also Thursby and Jensen (1983)]. Suppose that both countries face previously set tariffs $t_i, t_j$ and country $i$ raises its tariffs by $\tau_i$. Then, country $j$ retaliates by raising its tariff from $t_j$ to $t_j + \mu_j(t_i, t_j, \tau_i)$, where $\mu_j(t_i, t_j, \tau_i)$ represents country’s $j$ retaliation function.

We denote by $I_i(\mu_j)$ the set of tariff pairs $(t_i, t_j)$, which, given $\mu_j$, prevent country $i$ from initiating the tariff war:

$$I_i(\mu_j) = \{(t_i, t_j) \mid u_i(t_i + \tau_i, t_j + \mu_j(t_i, t_j, \tau_i)) \leq u_i(t_i, t_j) \text{ for all } \tau_i \geq 0\}.$$

Similarly, let $I_j(\mu_i)$ be the set of tariff pairs $(t_i, t_j)$, which, given $\mu_i$, prevent country $j$ from initiating the tariff war:

$$I_j(\mu_i) = \{(t_i, t_j) \mid u_j(t_i + \mu_i(t_i, t_j, \tau_j), t_j + \tau_j) \leq u_j(t_i, t_j) \text{ for all } \tau_j \geq 0\}.$$

Finally, let the No-Tariff-War-Region, $NTWR(\mu_i, \mu_j)$ be the set of tariff pairs $(t_i, t_j)$ which, given $\mu_i$ and $\mu_j$, prevent both countries from initiating the tariff war:

$$NTWR(\mu_i, \mu_j) = I_i(\mu_j) \cap I_j(\mu_i).$$
The GATT compensating mechanism

Article XIX calls for a compensation to the exporting country in case the importing country raises its tariff. However, it does not specify which form the compensation should take; the revised version merely says that the exporting country “may agree on any adequate means of trade compensation for the adverse effects of the measure.” Bhagwati (1976, p.1008) suggests two types of compensation: either the importing country grants a tariff concession on another commodity or the exporting country withdraws a previous concession. An alternative interpretation would be to let the affected country retaliate so as to preserve the welfare level it enjoyed prior to the increase of the restriction by the other country. We assume the following scheme: whenever a country raises its tariff, the trading partner is entitled to retaliate in such a way that the net value of its trade with the first country is unchanged. This interpretation is consistent with the revised version of Article XIX, though it is not the only possible interpretation.\footnote{Note that the revised version is in terms of quantities and not values of imports: “if a quantitative restriction is used, such a measure shall not reduce the quantity of imports below the level of a recent period which shall be the average of imports in the last three representative years for which statistics are available.”}

Formally, let $t_i, t_j$ and $p_i, p_j$ be the pre-move tariffs and the associated equilibrium prices, respectively. In response to country $i$ raising its tariff to $t_i^n = t_i + \tau_i$ country $j$ raises its tariff by the value of the retaliation function denoted $\mu_j^G(t_i, t_j, \tau_i)$. Our interpretation of compensation requires that:

$$p_j(m_i^n - m_i) = p_i(m_j^n - m_j), \quad (9)$$

where $m_i^n$ (respectively $m_i$) represents imports of $i$ after (respectively before) the move; the left-hand side (respectively the right-hand side) of (9) repres-
sents the change of the value of i’s (respectively j’s) imports. Note that the prices used in this evaluation are the pre-move prices; an obvious alternative would be to use post-move prices. Using (2) and (4), it is easy to check that:

\[ p_j m_i = p_i m_j. \] (10)

Then (9) and (10) imply:

\[ \frac{p_j}{p_i} = \frac{m^n_j}{m^n_i}. \] (11)

Using (4) again, we obtain expressions of the post-move imports for any given changes in tariffs \( \tau_i \) and \( \tau_j \):

\[ m^n_i = \frac{w_j(1 - \alpha_j)}{1 + \alpha_j(t_j + \mu_j^G(t_i, t_j, \tau_j))}, \]

\[ m^n_j = \frac{w_i(1 - \alpha_i)}{1 + \alpha_i(t_i + \tau_i)}. \]

Substituting this in (10) leads to:

\[ \frac{1 + \alpha_j t_j}{1 + \alpha_i t_i} = \frac{1 + \alpha_j(t_j + \mu_j^G(t_i, t_j, \tau_i))}{1 + \alpha_i(t_i + \tau_i)} \]

or

\[ \mu_j^G(t_i, t_j, \tau_j) = \frac{\tau_i \alpha_i (1 + \alpha_j t_j)}{\alpha_j (1 + \alpha_i t_i)}. \] (12)

Thus, the retaliation function of \( j \) is linear in \( \tau_i \). Then per-unit retaliation, denoted \( \bar{\mu}_j^G(t_i, t_j) \) and derived from the compensating mechanism, is given by:

\[ \bar{\mu}_j^G(t_i, t_j) = \frac{\alpha_i (1 + \alpha_j t_j)}{\alpha_j (1 + \alpha_i t_i)}. \] (13)

Similarly, if country \( j \) raises its tariff to \( t^n_j = t_j + \tau_j \), country \( i \) retaliates by raising its tariff by \( \mu_i^G(t_i, t_j, \tau_j) \). In this case we derive:

\[ \mu_i^G(t_i, t_j, \tau_j) = \frac{\tau_j \alpha_j (1 + \alpha_i t_i)}{\alpha_i (1 + \alpha_j t_j)}. \] (14)
Similarly, the per-unit retaliation function of country $i$ is given by:

$$
\tilde{\mu}_G^i(t_i, t_j) = \frac{\alpha_j(1 + \alpha_i t_i)}{\alpha_i(1 + \alpha_j t_j)}.
$$

(15)

Note that both per unit retaliation functions are linear with respect to tariff changes of the other country and are inverse to each other:

$$
\tilde{\mu}_G^i(t_i, t_j) \cdot \tilde{\mu}_G^j(t_i, t_j) = 1.
$$

It is easy to see that under the GATT compensating mechanism $(\mu_G^i, \mu_G^j)$ defined by (9), the No-Tariff-War-Region coincides with the set of all non-negative tariff pairs, so that we can state:

**Proposition 2.1.** $NTWR(\mu_G^i, \mu_G^j) = \mathbb{R}^2_+.$

**Proof.** See Appendix.

This proposition shows that, given “historically” determined tariffs $(t_i, t_j)$, the compensating mechanism is useful in preventing tariff wars: countries choose not to increase tariffs, though Article XIX allows them to do so. However, this does not solve the issue of optimal or equilibrium tariffs. In the next section, we address the question of existence and characterization of compensating mechanisms which would not only avoid tariff wars, but also yield free trade as an equilibrium of the associated retaliation game.

### 3 Free trade as a unique equilibrium under a modified compensating mechanism

In this section we show that it is possible to select a rule even from a set of linear retaliation functions which avoids tariff wars and leads to free trade as the unique equilibrium of a tariff game, that we formally define now. Let
$M$ be the set of continuous retaliation functions $\mu: \mathbb{R}_+^2 \times \mathbb{R} \to \mathbb{R}_+$ which are, for each country, function of three variables: the two existing nonnegative tariffs and the tariff adjustment of the other country (which could be positive or negative).

**Definition.** Let the two retaliation functions $\mu_i, \mu_j \in M$ be given. Define by $\Gamma(\mu_i, \mu_j)$ the non-cooperative game with two players $i, j$, where the strategy set of each player is the set of nonnegative numbers $\mathbb{R}_+$. The payoff function of $i$ is given by:

$$\Pi_i(t_i, t_j) = \begin{cases} u_i(t_i, t_j) & \text{if } (t_i, t_j) \in NTWR(\mu_i, \mu_j) \\ 0 & \text{if } (t_i, t_j) \notin NTWR(\mu_i, \mu_j) \end{cases}$$

and the payoff function of $j$, $\Pi_j(t_i, t_j)$, is determined in a similar way.

The set of equilibria of $\Gamma(\mu_i, \mu_j)$ is denoted by $E(\Gamma(\mu_i, \mu_j))$.

Note that we assume the payoffs for each player to be zero outside the No-Tariff-War-Region. One may argue [Johnson (1953)] that a country, initiating a tariff war, can improve its welfare. In our setup, however, none of the tariff pairs outside $NTWR$ is sustainable. Indeed, consider a pair of tariffs $(t_i, t_j)$ and assume that country $i$ can benefit by raising its own tariff while foreseeing retaliation by $j$, as allowed by the compensating mechanism. It will be shown that there exists an optimal increase, $\tau_i$, which optimizes the after-retaliation utility of country $i$. Then, either country $j$ does not initiate its own war, or there is an optimal tariff increase, $\tau_j$, which optimizes the after-retaliation utility of country $j$. It can be shown that the process would terminate after a finite number of steps; this means that countries entered the No-Tariff-War-Region. Thus, even if the countries are originally located outside $NTWR$, they eventually end up there, implying that we may restrict our attention to $NTWR$. 

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It is important to point out that Proposition 2.1 and the result of Kennan and Riezman (1988) imply that under the GATT compensating rule \((\mu_i^G, \mu_j^G)\), the set of equilibria of the game \(\Gamma(\mu_i^G, \mu_j^G)\) is empty. As we indicated above, our aim is to modify the GATT mechanism so that the resulting game has an equilibrium which is free trade.

Consider the class of retaliation functions \(\Lambda \subset M\) that are independent of the existing tariffs and linear in tariff adjustments of the other country. Thus, if the retaliation functions \(\mu_i(t_i, t_j, \tau_j)\) and \(\mu_j(t_i, t_j, \tau_i)\) belong to \(\Lambda\), there exist nonnegative numbers \(\lambda_i\) and \(\lambda_j\) such that:

\[
\begin{align*}
\mu_i(t_i, t_j, \tau_j) &= \lambda_i \tau_j \\
\mu_j(t_i, t_j, \tau_i) &= \lambda_j \tau_i.
\end{align*}
\]

We first show that for any function in \(\Lambda\), there always exists a pair of tariffs for which no country wants to initiate a tariff war and, hence, the set of tariff pairs yielding a positive payoff for each country in the game \(\Gamma(\mu_i, \mu_j)\) is nonempty:

**Proposition 3.1.** For every pair \((\mu_i, \mu_j) \in \Lambda\), \(NTWR(\mu_i, \mu_j)\) is non-empty.

**Proof.** See Appendix.

In the next proposition, we derive our main result on the characterization of a linear mechanism which yields free trade as an equilibrium of the game:

**Proposition 3.2.** Free trade is an equilibrium of the tariff game \(\Gamma(\mu_i, \mu_j)\), if and only if \(\mu_i(t_i, t_j, \tau_j) = \lambda_i^* \tau_j\) and \(\mu_j(t_i, t_j, \tau_i) = \lambda_j^* \tau_i\), where \(\lambda_i^* = \alpha_j / \alpha_i\) and \(\lambda_j^* = \alpha_i / \alpha_j\).

**Proof.** See Appendix.
Note that, unlike the GATT compensating mechanism \((\mu^G_i, \mu^G_j)\), the per-unit retaliation rule specified in Proposition 3.2 depends only on the ratio of the budget shares of the domestic commodities in both countries. This rule is simpler than the rule implied by Article XIX and does not require more information on the countries’ preferences.

4 Conclusions

We have provided some theoretical reasons for the relative infrequent use of the retaliation rule provided by Article XIX of the GATT Treaty. Indeed, a country starting a tariff war will eventually hurt itself and, therefore decides not to alter its tariffs.

The correct application of the compensating mechanism (i.e., the determination of the allowed level of retaliation) requires to be able to calculate the “post tariff move” trades. We develop a rule which has two advantages over the existing one. First, it is easier to implement since the level of retaliation can be computed on the basis of budget shares only. Second, in contrast to the current compensating mechanism, our retaliation rule yields free trade as an equilibrium of the associated tariff game.
5 Appendix

Proof of Proposition 2.1. Let the pair of tariffs \((t_i, t_j)\) be given. Suppose that country \(i\) has raised its tariff by \(\tau_i\) and country \(j\) retaliated by invoking the compensating mechanism. Then, we can derive the utility level \(u_i(t_i + \tau_i, t_j + \mu_j^G(t_i, t_j))\) of country \(i\). Its derivative with respect to \(\tau_i\) is equal to:

\[
\alpha_i(1 - \alpha_i)\left(\frac{1}{1 + t_i + \tau_i(1 + \alpha_i(t_i + \tau_i))} - \frac{1}{1 + \alpha_i \tau_i}\right).
\]

It is easy to see that the value of this expression is negative for all values \(t_i\) and \(t_j\), so that country \(i\) looses if it starts a tariff war. The same holds for country \(j\) and we conclude that under the GATT retaliation rule neither of the countries would raise its tariffs. □

Proof of Proposition 3.1. Let the pair of retaliation functions \((\mu_i, \mu_j)\) ∈ \(\Lambda\) be given. Then, there exist two positive numbers \(\lambda_i, \lambda_j\) such that \(\mu_i(t_i, t_j, \tau_j) = \lambda_i \tau_j\) and \(\mu_j(t_i, t_j, \tau_i) = \lambda_j \tau_i\). We now show that the set of tariff pairs \(I_i(\mu_j)\) which would prevent country \(i\) from initiating a tariff war is given by:

\[
I_i(\mu_j) = \{(t_i, t_j) ∈ \mathbb{R}_+^2 \mid t_j ≤ \max[0, h_i(t_i)]\}
\]

where

\[
h_i(t_i) = \frac{\alpha_i \alpha_j \lambda_j t_i^2 + (1 + \alpha_i) \alpha_j \lambda_j t_i - \alpha_i + \alpha_j \lambda_j}{\alpha_i \alpha_j}.
\]

Indeed, consider the sign of the derivative of the payoff function of country \(i\) with respect to the raise in its own tariff:

\[
\text{sgn}\left\{\frac{\partial u_i(t_i + \tau_i, t_j + \tau_j \lambda_j)}{\partial \tau_i}\right\} = \]

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\[
\text{sgn}\left\{ \alpha_i \left[ \frac{1}{1 + \tau_i t_i} - \frac{\alpha_i}{1 + \alpha_i t_i + \tau_i \alpha_i} \right] - (1 - \alpha_i) \frac{\alpha_j \lambda_j}{1 + \alpha_j t_j + \tau_i \alpha_j \lambda_j} \right\} = \\
\text{sgn}\left\{ A_0 + A_1 \tau_i + A_2 \tau_i^2 \right\}
\]

where \( A_0 = \alpha_i (1 + \alpha_j t_j) - \alpha_j \lambda_j (1 + t_i)(1 + \alpha_i t_i) \), \( A_1 = -\alpha_j \lambda_j (1 + 2 \alpha_i t_i) < 0 \) and \( A_2 = -\alpha_i \alpha_j \lambda_j < 0 \). The peak of the parabola \( f(\tau_i) = A_0 + A_1 \tau_i + A_2 \tau_i^2 \) is at \( \tau_i = -A_1/2A_2 < 0 \). Thus, the function \( f(\cdot) \) takes nonnegative values for positive \( \tau_i \) if and only if \( A_0 > 0 \), implying that country \( i \) would not initiate the tariff war only if \( A_0 \leq 0 \). That is, \((t_i, t_j) \in I_i(\mu_j)\) if and only if \( t_j \leq \max[0, h_i(t_i)]\).

Similar examination of the region \( I_j(\mu_i) \) would imply that:

\[
I_j(\mu_i) = \{(t_i, t_j) \in \mathbb{R}_+^2 \mid t_i \leq \max[0, h_j(t_j)]\}
\]

where

\[
h_j(t_j) = \frac{\alpha_i \alpha_j \lambda_i t_j^2 + (1 + \alpha_j) \alpha_i \lambda_i t_j - \alpha_j + \alpha_i \lambda_i}{\alpha_i \alpha_j}.
\]

The intersection of the two regions \( I_i(\mu_j) \) and \( I_j(\mu_i) \) is therefore given by:

\[
NTWR(\mu_i, \mu_j) = \{(t_i, t_j) \in \mathbb{R}_+^2 \mid h_j^{-1}(t_i) \leq t_j \leq \max[h_i(t_i), 0]\}.
\]

The boundary of \( I_i \) is determined by a quadratic function \( h_i(t_i) \) whereas the boundary of \( I_j \) is determined by an inverse parabola \( h_j^{-1}(t_i) \). Since the function \( h^{-1} \) is of the order of \( \sqrt{t_i} \) for large enough values of \( t_i \), it follows that there exists a positive number \( T \) such that for every \( t_i > T \), there exists a value \( t_j \) such that \((t_i, t_j) \in NTWR(\mu_i, \mu_j)\). Thus the No-Tariff-War-Region is nonempty. \( \Box \)

**Proof of Proposition 3.2.** We proceed in two steps:
(a) a pair \((t_i, t_j)\) constitutes an equilibrium of the retaliation game \(\Gamma(\mu_i, \mu_j)\) if and only if the point \((t_i, t_j)\) is located at the intersection of two curves \(y = h_i(x)\) and \(x = h_j(y)\), or, equivalently, \(t_j = h_i(t_i) = h_j^{-1}(t_i)\),

(b) free trade is an equilibrium of the game \(\Gamma(\mu_i, \mu_j)\) if and only if \(\lambda_i = \lambda_i^*\) and \(\lambda_j = \lambda_j^*\). Moreover, in this case, free trade is the unique equilibrium.

Part (a). Indeed, \(\bar{t}_j = h_i(\bar{t}_i) = h_j^{-1}(\bar{t}_i)\). Observe that \(\partial u_i(t_i, t_j)/\partial t_i\) and \(\partial u_j(t_i, t_j)/\partial t_j\) are positive-valued on \(\mathbb{R}_+^2\). Then \((\bar{t}_i, \bar{t}_j) \not\in E(\Gamma)\) if either there is \(\tilde{t}_j > \bar{t}_j\) such that \((\bar{t}_i, \tilde{t}_j) \in NTWR(\mu_i, \mu_j)\) or there exists \(\hat{t}_i > \bar{t}_i\) such that \((\hat{t}_i, \bar{t}_j) \in NTWR(\mu_i, \mu_j)\). Since both curves \(h_i\) and \(h_j\) are upward-sloping, these two situations cannot occur. This completes the “if” part of the proof.

To show that every point \((t_i, t_j)\) which is not located on the intersection of the curves \(h_i\) and \(h_j\) cannot be an equilibrium, there are a few cases to examine. If \(h_i(t_i) < \max[0, h_j^{-1}(t_i)]\), then there is no \(t_j\) such that \((t_i, t_j)\) belongs to \(NTWR\), and subsequently, to \(E(\Gamma)\). If \(h_i(t_i) > h_j^{-1}(t_i)\), then the positivity of \(\partial u_i(t_i, t_j)/\partial t_i\) and \(\partial u_j(t_i, t_j)/\partial t_j\) implies that no \(t_j < h_i(t_i)\) can be sustained at equilibrium. Also, for \(t_j = h_i(t_i)\), country \(i\) would be better off by moving to the right of \(t_i\) to increase its tariff. The case \(h_i(t_i) > 0 > h_j(t_i)\) is treated similarly.

Part (b). By (a), free trade, characterized by the pair of tariffs \((0, 0)\), is an equilibrium if and only if \(h_i(0) = h_j^{-1}(0) = 0\), yielding \(\lambda_i^* = \alpha_j/\alpha_i\) and \(\lambda_j^* = \alpha_i/\alpha_j\). Then, \(h_i'(0) = (1 + \alpha_i)/\alpha_j > 1 > \alpha_i/(1 + \alpha_j) = (h_j^{-1})'(0)\). Thus, convexity of \(h_i(\cdot)\) and concavity of \(h_j^{-1}(\cdot)\) imply that there is no positive \(t_i\) for which \(h_i(t_i) = h_j^{-1}(t_i)\). It follows, therefore, that free trade is the unique equilibrium under the \((\lambda_i^*, \lambda_j^*)\)-rule. \(\Box\)
6 References


