Central European Science Journals
w w w . c e s j . c o m

Central European Journal of Physics
Central European Science Journals
CEJP 2(3) 2004 467–491
Review article

Neutrino Oscillations and the Early Universe

Daniela P. Kirilova*†

[1] *Institute of Astronomy, Bulgarian Academy of Sciences, blvd. Tsarigradsko Shosse 72, Sofia, Bulgaria*

[2] *Physique Theorique, Universite Libre de Bruxelles, Boulevard du Triomphe, 1050 Bruxelles, Belgium*

[3] *Abdus Salam Internatonal Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy*

Received 13 January 13 2004; accepted 12 March 2004

Abstract: The observational and theoretical status of neutrino oscillations in connection with solar and atmospheric neutrino anomalies is presented briefly. The effect of neutrino oscillations on the evolution of the early Universe is discussed in detail. A short review is given of the standard Big Bang Nucleosynthesis (BBN) and the influence of resonant and non-resonant neutrino oscillations on active neutrinos and on primordial synthesis of He-4. BBN cosmological constraints on neutrino oscillation parameters are discussed.

Keywords: Neutrino anomalies, neutrino oscillations, cosmological role of neutrino mass and mixings, BBN constraints on neutrino mixing
PACS (2000): 14.60.Pq, 26.35.+c, 98.80.-k, 95.35.+d

1 Introduction

Neutrino - a neutral weakly interacting particle, is of extreme interest for Physics and Astrophysics. It is a key to the investigation of weak interactions and the physics beyond the standard electroweak model. On the other hand, being a very weakly interacting particle, and hence having a uniquely great penetrating capability, neutrinos carry precious information for the astrophysical processes in the most dense regions of the star cores and from the very early evolutionary stages of the Universe. Therefore, an understanding of neutrino characteristics is of great importance.

* Regular Associate of Abdus Salam ICTP
† Email: Daniela.Kirilova@cern.ch; dani@libra.astro.bas.bg

The contemporary particle physics theory neither requires nor forbids a nonzero neutrino mass. Neutrinos are assumed massless in the standard model of particle physics. In the more general case of non-zero neutrino masses, the weak neutrino eigenstates may be a linear combination of the mass eigenstates, which means that transitions between neutrinos with different types (flavours), the so called *neutrino oscillations* are possible. Neutrino oscillations and their role in resolving the solar neutrino puzzle were first proposed by B. Pontecorvo (see [1]) and after 45 years they continue to be the theme of leading experimental and theoretical research.

In recent years positive indications for neutrino oscillations were obtained at the greatest neutrino experiments (evidence for solar neutrino oscillations: Homestake, Kamiokande, SuperKamioKa, Gallex, SAGE, SNO; evidence for atmospheric neutrino oscillations: Super-KamioKa, Macro, Soudan 2, IMB; evidence for neutrino oscillations in terrestrial experiments: LSND, KamLAND, K2K) (see refs. [2, 3, 4]). Each of these neutrino anomalies, namely the solar neutrino problem, atmospheric neutrino anomaly and the positive results of terrestrial LSND, K2K and KamLAND experiments may be resolved by the phenomenon of neutrino oscillations. These results have a great significance since any experimental evidence for neutrino masses or mixing is a signal of new physics (NP), that is, physics beyond the standard model of electroweak interactions.

On the other hand, neutrino oscillations affect early Universe evolution by affecting expansion rate, neutrino densities and neutrino energy spectrum, neutrino - antineutrino asymmetry, thus influencing the neutrino involved processes, as for example cosmological nucleosynthesis, structure formation, etc. Cosmological nucleosynthesis, traditionally called Big Bang Nucleosynthesis (BBN) explains very successfully the data on the primordial abundances of the light elements D, He-3, He-4 and Li-7, and is traditionally used as a probe of the conditions of the early Universe, due to the high accuracy of the theoretically predicted abundances of light elements and to the good accuracy of their primordial values inferred from observations. Hence, BBN is a powerful probe for NP, just as neutrino oscillations.

Most stringent constraints on neutrino oscillations parameters are obtained from BBN considerations. In particular, LMA and LOW active-sterile solar oscillation solutions and atmospheric active-sterile solutions were excluded many years before the global analysis of experimental neutrino data pointed to the preference of flavour oscillations for solving these neutrino anomalies.

In the following we will present a brief introductory review of the solar and atmospheric neutrino anomalies and then discuss in more detail the role of neutrino oscillations in the early Universe and the cosmological constraints on oscillation parameters, following from BBN.

1.1 Neutrino oscillations

The basic idea of *neutrino oscillations* is that left-handed mass eigenstates ν_i are distinct from the left-handed flavour eigenstates ν_f:

$$\nu_i = U_{if}\ \nu_f \qquad (f = e, \mu, \tau).$$

Then in the simple two-neutrino oscillation case in vacuum, the probability to find after a time interval t a given neutrino type in an initially homogeneous neutrino beam of the same type is: $P_{ff} = 1 - \sin^2 2\vartheta \sin^2(\delta m^2 t/4E)$, where δm^2 denoting the neutrino squared mass difference and ϑ denoting the oscillations mixing angle are the oscillation parameters, E is the neutrino energy. i.e. the flavour composition changes with time.

The medium distinguishes between different neutrino types due to different interactions [5, 6]. This leads to different average potentials V_f for different neutrino types.

$$V_f = Q \pm L$$

where $f = e, \mu, \tau$, $Q = -bET^4/(\delta m^2 M_W^2)$, $L = -aET^3 L^\alpha/(\delta m^2)$, L^α is determined by the fermion asymmetries of the plasma, a and b are positive constants different for the different neutrino types, $-L$ corresponds to the neutrino and $+L$ to the antineutrino case. The sterile neutrino does not feel the medium, hence $V_s = 0$.

The effects of the medium can be hidden in δm^2 and ϑ. Namely, the matter mixing angle in the adiabatic case is expressed through the vacuum oscillation parameters and the characteristics of the medium, such as its density and temperature. For the early Universe, the following relation holds:

$$\sin^2\vartheta_m = \sin^2\vartheta/[\sin^2\vartheta + (Q \mp L - \cos 2\vartheta)^2],$$

Although in general the medium suppresses oscillations by decreasing their amplitude, there also exists a possibility of enhanced oscillation transfer in case a resonant condition between the parameters of the medium and the oscillation parameters holds:

$$Q \mp L = \cos 2\vartheta.$$

Then the mixing in matter becomes maximal, independently of the value of the vacuum mixing angle, i.e. resonant transfer takes place.

At high temperature of the early Universe, for a lepton asymmetry of the order of the baryon asymmetry, $Q > L$. So for $\delta m^2 < 0$ resonance is possible both for neutrino and antineutrino. At low temperature, however, $L > Q$, and as can be seen from the resonant condition, if $\delta m^2 > 0$, a resonance in the neutrino ensemble can take place, while for $\delta m^2 < 0$, the resonance is possible only for the antineutrinos.

Both the non-resonant and resonant oscillation cases are interesting from a cosmological point of view and from the viewpoint of the discussed neutrino anomalies.

2 Neutrino anomalies and neutrino oscillation experiments

2.1 Solar neutrino deficit

According to contemporary astrophysical understanding, the Sun is a Main Sequence star at the stage of hydrogen burning. It produces an intense flux of electron neutrinos as a result of its nuclear reactions generating the solar energy. Due to its weak interaction with matter, the solar neutrino reaching the Earth comes from the very deep solar core and carries valuable information about stellar structure and its evolution. Hence, the detection of the neutrino from the Sun has been recognized as a task of great importance as early as the fifties, when Davis started a radiochemical experiment, designed to detect neutrinos from the Sun, in the golden mine at Homestake. The solar neutrinos also present the unique opportunity for investigation of the neutrino properties like neutrino mass and mixing, because the Sun is at a very large distance from the Earth, and also because the solar density varies strongly from the center to the surface and thus offers interesting conditions for the penetration of neutrino through layers with different density and thickness, and finally because during night neutrinos penetrate through the Earth to reach the detectors.

Since the first attempts to measure solar neutrinos, there have been different types of solar neutrino experiments, using Cl, Ga and H_2O and D_2O as targets for measuring electron neutrino from the Sun. The measured fluxes of solar neutrinos in these solar experiments (using different detection methods [§] and sensitive to different energy ranges) are in qualitative agreement with the assumption that Sun burns due to nuclear reactions in its core. However, *all the data of the solar neutrino experiments point to a considerably lower neutrino flux than expected according to the standard Solar Model. Furthermore, the suppression is different in various experiments, sensitive to different energy range.* This problem is called *solar neutrino anomaly.*

Despite the continuous improvements of the Solar Model [7, 8] and the predicted neutrino fluxes in the last 40 years, the discrepancies between the observations and the predictions of the model persist. Depending on the energy the measured fluxes differ by a factor 0.3 to 0.6 of the predicted values. The recent measurement of elastic scattering, neutral currents and charged currents processes at SNO experiment [9] provide $\sim 5\,\sigma$ signal for neutrino flavour transitions that is not strongly dependent on the Solar Model. Hence, there remain less doubts about the Solar Model prediction capability.

Thus, in case we exclude the possibility that most of the leading solar neutrino experiments are wrong, the experimental data points more and more convincingly to the necessity of new neutrino physics.

Neutrino oscillations are capable of explaining the observational data and its discrepancies with the predictions of the Solar Model: the electron neutrino, produced in the solar core, undergoes transformations into other flavours while penetrating through the

[§] There exist radiochemical experiments like GALLEX, SAGE and Homestake and electron experiments like Kamiokande, SuperKamioka and SNO.

Sun and/or the cosmic space and till it reaches the terrestrial detectors of electron neutrinos. Hence, the registered electron neutrino flux is reduced in comparison to the flux produced in the Sun core ¶.

There existed different types of solutions for solar neutrino oscillations: Small Mixing Angle (SMA) and Large Mixing Angle (LMA), depending on the mixing angles at around $\delta m^2 \sim 10^{-5}$ eV^2 and LOW and vacuum oscillation solutions corresponding to very small $\delta m^2 \leq 10^{-7}$ eV^2 mass differences and maximum mixing. The present solar neutrino data definitely prefers flavour oscillation solutions to active-sterile ones (as was pointed out first from cosmology considerations - see section 4), and in the light of the recent results of the terrestrial experiment KamLAND, the LMA solution is split into two sub-regions, and is the chosen one. The analysis of the recent results of the SNO salt phase has further decreased the errors of determination of oscillation parameters. High LMA is disfavoured at 3σ level and the best fit point values for the low LMA solution are $\delta m^2 = 7.1 \times 10^{-5}$ eV^2 and $\tan^2\theta = 0.42$. For more details see [10, 11, 12, 13, 14, 15, 16].

2.2 Atmospheric neutrino anomaly

A continuous isotropic flux of cosmic rays, consisting of protons and heavy nuclei, is bombarding the Earth's atmosphere. As a result of its interactions with the atmospheric particles pions and kaons are produced, which decay and produce muon and electron neutrinos with a wide energy range. The theoretical prediction for the ratio of the muon to the electron flux is $r = (\nu_\mu + \bar{\nu_\mu})/(\nu_e + \bar{\nu_e}) \sim 2$ for energies less than 1 GeV. Besides, identical up-coming and down-coming fluxes are expected due to the isotropy of the cosmic rays flux and due to the spherical symmetry of the Earth's atmosphere. Any deviation from these predictions is an indication of new neutrino physics.

The underground neutrino experiments SuperKamioka, Soudan 2 and Macro, as well as the earlier experiments IMB and Kamiokande, have *measured the ratio r considerably lower than the expected one* (see for example [17]). This discrepancy, the so called *atmospheric neutrino anomaly* is known already for more than 10 years. Besides, a dependence of the muon neutrino deficit on the zenith angle and distortion of the energy spectrum is observed.

The experimental data can be explained in terms of neutrino oscillations, namely by the transition of the muon neutrino into another type. The latest data analysis indicates the $\nu_\mu \leftrightarrow \nu_\tau$ channel as the dominant one. The oscillations into sterile neutrino are not favoured, because of the absence of suppression of oscillations by the medium, expected in the sterile case at high energies. The best fit oscillation parameters for the available data are nearly maximal mixing and $\delta m^2 \sim (2.6 \pm 0.4) \times 10^{-3}$ eV^2. For more detail see [18] and references therein.

¶ Except SNO measurements, which can detect all flavour neutrinos, not only electron ones.

2.3 Laboratory experiments

Besides these two astrophysical indications for neutrino oscillations and non-zero neutrino mass, there exist also laboratory experiments, the so-called terrestrial experiments LSND [19], K2K [20] and KamLAND [21], whose data have given an indication for oscillations, too.

The short baseline Los Alamos Liquid Scintillation Neutrino Detector (LSND) experiment has registered appearance of electron antineutrino in a flux of muon antineutrino. This anomaly might be interpreted as $\nu_\mu \leftrightarrow \nu_e$ oscillations with $\delta m^2 = O(1\ \ \mathrm{eV}^2)$ and $\sin^2 2\theta = O(0.003)$.

In case LSND result is confirmed‖ an addition of a light singlet neutrino (sterile neutrino ν_s) is required, because three different mass differences, needed for the explanation of the solar, atmospheric and LSND anomaly require 4 different neutrino masses. This simple extension already has difficulties, because oscillations into purely sterile neutrinos do not fit neither the atmospheric nor solar neutrino data. Both 2+2 and 3+1 oscillation schemes have problems. Active-sterile oscillations are strongly restricted by cosmological considerations as well (for a review on 2 oscillation constraints see [23] and for 4 neutrino oscillation schemes see [24, 25, 26].

K2K, a long baseline neutrino experiment, has probed the δm^2 region explored by atmospheric neutrinos. It has measured muon neutrino deficit in a beam coming from KEK to Kamiokande. The distance is 250 km and $E \sim 1.3$ GeV. 56 events were observed instead of the expected 80 ± 6 in case without oscillations. A hint of energy spectrum distortion is also indicated by the analysis.

The results are consistent with SuperKamioka atmospheric data and confirm atmospheric neutrino solution.

KamLAND (Kamioka Liquid Scintillator Anti-Neutrino Detector) experiment, with the use of reactor antineutrinos, explored the region of oscillation parameters relevant for the solar neutrinos. It has measured electron antineutrino deficit in the flux of antineutrinos coming from reactors at ~ 180 km distance. In the context of two-flavour neutrino oscillations, the KamLAND results single out LMA solution, as the oscillation solution to the solar neutrino problem. The previously allowed LMA region is further reduced by its results (see [21]). So, the KamLAND result appears to confirm, in a totally independent and completely terrestrial manner, that solar neutrino deficit is indeed due to neutrino oscillations, which was suspected in many solar neutrino experiments over the last 40 years.

KamLAND reactor and KEK accelerator experiments strongly contributed to the reduction of the allowed range of mass differences for solving the solar and atmospheric neutrino anomalies. Their results mark the beginning of the precision epoch in determinations of neutrino characteristics.

The results of the neutrino experiments confirm non-zero neutrino mass and mixing.

‖ The LSND result was not confirmed by KARMEN. It will be tested in future by the ongoing MiniBoom experiment at Fermilab [22].

Non-zero neutrino mass is also cosmologically welcome, as it may play the role of the hot dark matter component essential for the successful structure formation (see the next section). The standard model (SM) of particle physics, $SU_c(3) \times SU_W(2) \times U_Y(1)$, does not predict non-zero neutrino mass and mixing. In order to explain the smallness of the neutrino mass differences, new physics beyond SM is required. Hence, neutrino data gathered at neutrino experiments and the cosmological considerations concerning neutrino mass and mixing, discussed below, point the way towards this NP - hopefully the true unified theory of elementary particles.

3 Neutrinos in the early universe

After the photons of the microwave background radiation, neutrinos are the most abundant particles in the Universe [**]. Hence, in case they have non-zero mass they may contribute considerably to the total energy density of the Universe. From the requirement that the neutrino density should not exceed the matter density, and assuming that the Universe is older than the Earth, an upper bound on the neutrino mass was derived – "Gerstein-Zeldovich" limit (see [27]). The contemporary version of it reads

$$\Sigma m_{\nu_f} \leq 94 eV \Omega_m h^2 = 15 eV,$$

where Ω_m is the matter density in terms of the critical density and h is the dimensionless Hubble parameter. In deriving this limit $\Omega_m < 0.3$ and $h = 0.7$ is assumed in accordance with contemporary astronomical data. Hence, the neutrino mass of any neutrino flavour should be less than about 5 eV.

Much stronger limits on the neutrino mass may be obtained accounting for the considerable role of neutrinos in other important cosmological processes, like the primordial nucleosynthesis and the formation of large scale structure of the Universe, the formation of the cosmic microwave background radiation, etc.

3.1 Large scale structure and neutrino

The mass of the visible (radiating) matter in the Universe is at most 0.01 of the total mass deduced from its gravitational effect. The remaining 0.99 consists of the so-called *Dark Matter* of the Universe (DM). Only a negligible portion of this DM may be in the form of invisible baryons, i.e. 0.04 of the total mass, i.e. DM is mainly non-baryonic. Massive neutrinos with a mass of a few eV could naturally be the candidates for DM component in clusters of galaxies [28].

On the other hand, according to the accepted contemporary theory, structures in the Universe are a result of gravitational instabilities of overdensity perturbations. These perturbations result from the initial microscopical perturbations generated at the inflationary stage, which have been inflated during the exponential expansion. Neutrinos

[**] Their present day number density is $n_\nu = n_{\bar{\nu}} \sim 56$ cm^{-3} for each neutrino flavour.

cluster more efficiently in larger potential wells. If they play the role of DM, the mass-to-light ratio should increase with scale. That behavior was not confirmed by observations. Moreover, to be in agreement with the sizes of the structures observed today, it was found necessary to speed up the growth of the perturbations, which is naturally achieved by the presence of non-relativistic DM at the epoch of perturbations growth, which should be the main DM component. So, massive neutrinos are not the main DM component in galaxies and clusters.

Still, the precise analysis of the recent microwave background anisotropy data and structure data at large red-shifts imply the necessity of some admixture of hot DM which can be naturally provided by light neutrinos with $\Sigma m_{\nu_f} < 2.2$ eV or $m_{\nu_f} < 0.73$ eV (see for example [29, 30]). Recent WMAP measurements (see [31]) together with the LSS analysis has improved the limit: $\Sigma m_{\nu_f} < 0.69$ eV and hence, $m_{\nu_f} < 0.23$ eV. See also a recent analysis of the CMB, LSS and X-ray galaxy cluster data [32], which give the bounds for the preferred non-zero neutrino mass : $\Sigma m_{\nu_f} \sim 0.64$ eV or $m_{\nu_f} \sim 0.21$ eV per neutrino.

It is remarkable that such mass value is in accordance with the picture of oscillation models, which predict oscillations between nearly degenerate neutrinos, with negligibly different masses (with mass difference $\sim 10^{-5}$ eV in case of solar neutrino anomaly and ~ 0.001 eV in case of the atmospheric anomaly). Each of these degenerate neutrino types, if they have masses of the order of ~ 0.2 eV, can successfully play the role of the hot DM.

For comparison, the laboratory bound on electron neutrino mass from Tritium β-decay experiments is $m_{\nu_f} < 2.2$ eV. Given the small mass differences indicated by solar and atmospheric neutrino data, this bound applies to each neutrino eigenstate, i.e. $\Sigma m_{\nu_f} < 6.6$ eV [33]. So, the cosmological bounds on neutrino masses at present are more restrictive than the laboratory constraints. For recent reviews of laboratory measurements of neutrino masses and also their cosmological and astrophysical constraints see [34, 35, 36, 37].

3.2 BBN and neutrino

One of the most interesting events in the early Universe is the primordial nucleosynthesis of the light elements. The idea for the production of elements through nuclear reactions in the hot plasma during the early evolutionary stage of the Universe belongs to George Gamow and was proposed and developed in the 1930s and 1940s [38]. In the subsequent 70 years this idea has developed into an elegant and well-known theory, namely, theory of the cosmological nucleosynthesis (Big Bang Nucleosynthesis), explaining successfully the data on the abundances of D, He-3, He-4 and Li-7 (see for example [39], and references therein).

Recently BBN theory has been improved: nuclear data have been re-examined, all nuclear reactions rates involving ~ 100 processes have been updated, new processes included, estimates of the weak rates improved. Also the statistical uncertainty of observational determination of the light elements has been improved. Finally, thanks to the precise determination of the baryon density in CMB anisotropy measurements, it was used as an input in BBN. The present status of BBN after the recent measurements of CMB anisotropies by WMAP experiment is presented in detail in [40, 41, 42, 43].

Based on the excellent agreement between CMB and BBN predictions and the observational data, we now believe that the physical processes, typical for the BBN epoch, are determined to a great precision. Hence, BBN is the most powerful probe for new physics, like the physics predicting neutrino oscillations and non-zero neutrino mass.

According to the Standard BBN model (SBBN) the cosmological nucleosynthesis proceeds when the temperature of the plasma falls down to 1 MeV, when the weak processes, governing the neutron-proton transitions become comparable with the expansion rate. As a result the neutron-to-proton ratio freezes out at temperatures below 0.7 MeV. This ratio enters in the rapid nuclear reactions which follow, leading to the synthesis of D and other light elements formed in the first hundred seconds after the Big Bang.

Only at $T < 80$ keV, a temperature well below the D binding energy ~ 2.2 MeV, the building of complex nuclei became possible, the first step being: $n+p = D+\gamma$. At higher T deuterons quickly photo-dissociated because of the large photon-to-baryon ratio η^{-1}.

In SBBN, η is actually the only parameter. Most sensitive to η among the light elements is D, therefore it was considered till recently the best baryometer. The observational η value preferred today is the one obtained on the basis of measurements of D in high redshift QSO Absorption Line Systems of [44], namely $\eta \sim (6.1 \pm 0.5) \times 10^{-10}$ and η obtained from the CMB anisotropy measurements $\eta = (6.1 \pm 0.25) \times 10^{-10}$ [31]. Present day CMB measurements of η are considered tighter than that of the BBN, so η_{CMB} is used as an input for BBN calculations.

According to the standard BBN during the early hot and dense epoch of the Universe only D, ^{3}He, ^{4}He, ^{7}Li were synthesized in considerable amounts. ^{4}He has the highest binding energy among the light nuclides, hence D and ^{3}He were rapidly burned into it. ^{4}He was most abundantly produced. The production of heavier elements was hindered by the rapid decrease of the Universe density with the cooling of the Universe, growing Coulomb barriers and the absence of a stable mass 5 nuclide. The latter were formed much later in stars.

He-3 and Li-7 have a complex post BBN evolution. They are both created and destroyed in stars, hence are unreliable as a cosmological probes. Besides both their theoretically calculated and observational values suffer from large uncertainties. Although D had a clear post BBN chemical evolution, i.e., it is believed to have been only destroyed after BBN, because it has the lowest binding energy among the light nuclides, still has large theoretical and observational uncertainties, up to 10%: $D/H = (2.6 \pm 0.4) \times 10^{-5}$ [44]. Besides, there is a significant dispersion among the derived D abundances at low metalicity Z, a fact suggesting either the existence of systematic errors or a revision of our concepts about the post BBN evolution of D.

On the contrary, Y_p, predicted by BBN, is calculated with great precision (see [45, 39, 41, 40, 42]). The theoretical uncertainty is less than 0.1% ($|\delta Y_p| < 0.0002$) within a wide range of values of the baryon-to-photon ratio η. The predicted He-4 value is in relatively good agreement with the observational data for He-4 and is consistent with the abundances of the other light elements. Contemporary helium values, inferred from astrophysical observational data, are 0.238–0.245 [46, 47]. Although there exist some tension between the two different measurements giving different He values, and also between the observed He values and the predicted ones, using η indicated either by D measurements or by CMB, taking the central Helium value of the 2 measurements 0.238 and assuming the systematic error of 0.005 reestablishes the agreement both between different helium measurements and also between observed and the predicted He-4 values. Thus there is an agreement at the 2σ level and the uncertainty is only around 2%.

Measurements of primordial helium from CMB data are possible. Hopefully, future Planck CMB measurements will be capable of determining the helium mass fraction within $\delta Y \sim 0.01$ in a completely independent way (see [48]). Primordial helium value is also in a good accordance with the initial helium content, necessary for the successful star evolution modelling [49, 50].

In conclusion, ^{4}He is the most abundantly produced ($\sim$ 24% by mass), most precisely measured (uncertainty $\sim$ 2%) and most precisely calculated element (uncertainty $\sim$ 0.1%). This fact and its relatively simple chemical evolution make it the preferred element for probing non-standard physics. Particularly, for the analysis of the oscillations effect on BBN as well, He-4 is the traditionally used element. Therefore, we shall discuss it in more detail below.

According to the standard cosmological nucleosynthesis (SBBN), He-4 primordial yield essentially depends on the freezing of the reactions interconverting neutrons and protons:

$$\nu_e + n \leftrightarrow p + e^-, \quad e^+ + n \leftrightarrow p + \tilde{\nu_e},$$

which maintain the equilibrium of nucleons at high temperature ($T > 1MeV$) $n/p \sim \exp(-\Delta m/T)$, where $\Delta m = m_n - m_p$, T is the temperature $T = T_\gamma = T_e = T_\nu$ prior to electro-positron annihilation. For the radiation dominated epoch $\rho \sim \rho_\gamma + \rho_\nu + \rho_e = g_{eff} T^4$. The n/p ratio freeze-out occurs when in the process of expansion the rates of these weak processes $\Gamma_w \sim G_F^2 E_\nu^2 N_\nu$ become comparable and less than the expansion rate $H(t) = 8\pi G\rho/3 \sim \sqrt{g_{eff}}\ T^2$. Since the temperature of freezing is $T_f \sim g_{eff}^{1/6}$ the neutron-to proton frozen ratio $(n/p)_f$ is sensitive to g_{eff}.

Further evolution of the neutron-to-proton ratio is due to the neutron decays that proceed until the effective synthesis of D begins. Almost all available neutrons are sucked into He-4. So, the primordially produced mass fraction of He-4, to a good approximation, is

$$Y_p(\text{He-4}) \sim 2(n/p)_f/(1 + n/p)_f \exp(-t/\tau_n).$$

Hence, He-4 that is produced, is a strong function of the effective number of relativistic degrees of freedom at BBN epoch, $g_{eff} = 10.75 + 7/4\delta N_s$, and neutron mean lifetime τ_n, which parameterizes the weak interaction strength. He-4 depends weakly on the nucleon-to photon ratio η. It depends also on the electron-neutrino spectrum and on the neutrino-antineutrino asymmetry, which enter through Γ_w. In the standard BBN model three neutrino flavors ($\delta N_\nu = 0$), zero lepton asymmetry and equilibrium neutrino number densities and spectrum distribution are postulated:

$$n_{\nu_e}(E) = (1 + \exp(E/T))^{-1}.$$

Due to its strong dependence on g_{eff}, the He-4 abundance is used to constrain the number of the relativistic during BBN particles [51], usually parameterized by δN_ν. For a discussion of BBN constraints on δN_ν see for example [52]. The present BBN upper bounds on δN_ν depending on the concrete analysis vary in the range $\delta N_\nu < 0.1$ - $\delta N_\nu < 0.7$ [53, 41, 40]. The value we consider reliable enough for putting cosmological constraints on new physics parameters, corresponding to $\sim 3\%$ overproduction of He-4 is $\delta N_\nu < 0.64$.

BBN constraint is in agreement with the constraints based on LSS and WMAP data [54].

The dependence of the primordial abundances on the density and on the nucleon kinetics was used also for constraining massive stable neutrinos and decaying massive neutrinos [55, 56, 57]. The current status of these constraints is presented in [58].

For more details on neutrino role in cosmology see [36, 59] and the references therein. In the following we will concentrate mainly on BBN and neutrino oscillations.

4 BBN with neutrino oscillations

The influence of neutrino oscillations depends on the type of oscillations: oscillation channels, resonant transitions and the degree of equilibrium of oscillating neutrinos (see the review of [23] and the references therein). The effect of *flavour* neutrino oscillations on BBN is negligible if different flavour neutrinos are in thermal equilibrium and with vanishing chemical potentials [60]. However, *active-sterile* oscillations may have considerable influence because they affect both the expansion rate through exciting additional neutrino types and the weak interactions rate due to shifting neutrino densities and energy spectrum from BBN equilibrium values.

On the other hand although solar and atmospheric neutrino anomalies can be explained without a sterile neutrino, and definitely do not allow active-sterile neutrino oscillations as a dominant channel, some subdominant admixture of steriles is not only allowed, but also desirable [61]. Hence, it is interesting to discuss the cosmological effects of active-sterile neutrino oscillations, and to provide cosmological limits to oscillation parameters, which can be helpful in constraining the possibilities of ν_s admixture.

The presence of neutrino oscillations invalidates BBN assumptions about three neutrino flavours, zero lepton asymmetry, equilibrium neutrino energy distribution, thus directly influencing the kinetics of nucleons during the weak freeze-out.

4.1 Neutrino oscillation effects

Qualitatively, neutrino oscillation effects which considerably influence neutrino involved

processes in the Universe are
(a) Excitation of additional degrees of freedom: This leads to faster expansion of the universe, $H(t) \sim g_{eff}^{1/2}$, earlier n/p-freezing, $T_f \sim (g_{eff})^{1/6}$, at times when neutrons were more abundant [60]

$$n/p \sim \exp(-(m_n - m_p)/T_f)$$

This effect gives up to 5% ^{4}He overproduction (if one additional neutrino type is brought into equilibrium by oscillations, $\delta N_s = 1$).
(b) Distortion of the neutrino spectrum: The effect of oscillations may be much stronger than $\delta N_s = 1$ in case of oscillations effective after ν decoupling, proceeding between partially populated sterile neutrino state $0 \le \delta N_s < 1$ and electron neutrino [62, 63, 64]. The non-equilibrium initial condition, for most of the oscillations parameters of the model, leads to considerable and continuous deviations from the equilibrium ν_e spectrum (spectrum distortion).

A recent study of the momentum dependent kinetic equations for oscillating neutrinos before decoupling, showed that even in that case kinetic equilibrium may be strongly broken for some oscillation parameters, especially in the resonant case [26].

Since the oscillation rate depends on energy according to $\Gamma \sim \delta m^2/E$ the low energy neutrinos start to oscillate first, and the more energetic neutrinos later. Hence, the neutrino energy spectrum $n_\nu(E)$ may strongly deviate from its equilibrium form (see Fig.1), if the oscillations proceed between non-equilibrium neutrino states.

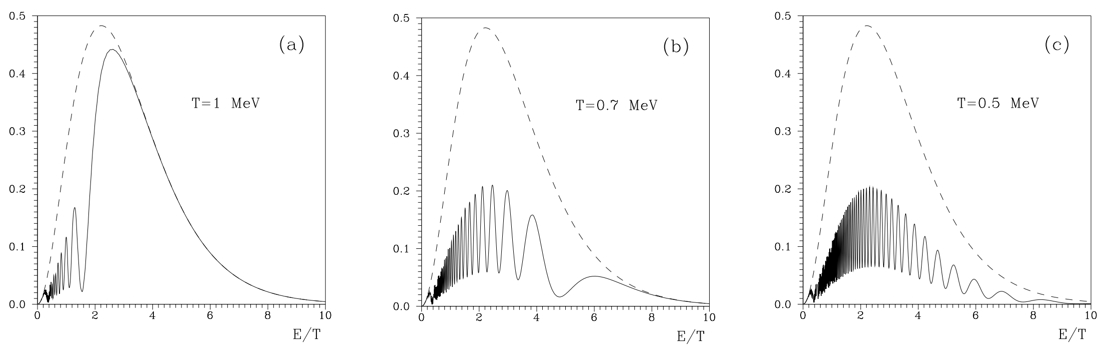

Fig. 1 The figures illustrate the degree of distortion of the electron neutrino energy spectrum $x^2\rho_{LL}(x)$, where $x = E/T$, caused by oscillations with mass difference $|\delta m^2| = 10^{-7}$ eV^2 and mixing $\sin^2 2\vartheta = 0.1$ for $\delta N_s = 0$. The evolution of the spectrum through the period of nucleon freezing is presented at characteristic temperatures 1, 0.7 and 0.5 MeV. The dashed curve gives the equilibrium spectrum at the given temperature.

The distortion leads to both a *depletion of the active neutrino number density* N_ν:

$$N_\nu \sim \int \mathrm{d}E E^2 n_\nu(E)$$

and a decrease of the Γ_w. Thus spectrum distortion influences the nucleon kinetics, causing an earlier n/p-freezing and an overproduction of ^{4}He yield.

The spectrum distortion may also cause underproduction of He-4, when due to oscillations the energy of the greater part of the neutrinos becomes smaller than the threshold for the reaction $\tilde{\nu}_e + p \to n + e^+$ and the n/p_f-ratio decreases leading to a decrease of

He-4. However this effect is a minor one. Hence, the total effect is an overproduction of He-4.

The spectrum distortion is the greatest, if the sterile state is empty at the start of oscillations, $\delta N_s = 0$. It decreases with the increase of the degree of population of the sterile state at the onset of oscillations (see [64] as illustrated in the following figures (Fig.2).

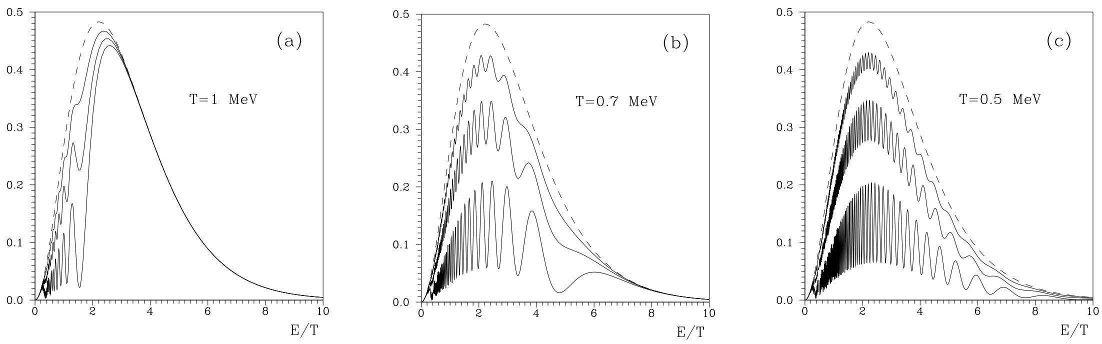

Fig. 2 The figures illustrate the spectrum distortion at different degrees of population of the steriles, namely $\delta N_s = 0$ (lower curve), $\delta N_s = 0.5$ and $\delta N_s = 0.8$ (upper curve). The dashed curve gives the equilibrium spectrum for comparison. It is obvious that the distortion of the spectrum is considerable and with time, involves the whole neutrino ensemble.

Spectrum distortion effect may be considerable both for the vacuum oscillations [62] and oscillations in a medium [63], and also both for the non-resonant [65] and resonant oscillations case [66].

(c) Production of neutrino-antineutrino asymmetry: Neutrino-antineutrino asymmetry may be generated during the resonant transfer of neutrinos [67, 68, 63, 69]. Dynamically produced asymmetry exerts a back effect on the oscillating neutrino and may change its oscillation pattern. It influences evolution of the number density of ν and $\bar{\nu}$, their spectrum distortion and the oscillation pattern, all playing important roles in $n-p$-kinetics.

Even when its value is not sufficiently high to have a direct kinetic effect on the synthesis of light elements, i.e. even when $L << 0.01$ it has an indirect effect on BBN [63, 70, 66]. This dynamically produced asymmetry suppresses oscillations at small mixing angles, leading to less overproduction of He-4 compared to the case without the account of asymmetry growth (see Fig.3), and hence alleviating BBN constraints on oscillation parameters.

For initial asymmetry slightly higher than the baryon one ($L \sim 10^{-6}$), L can also enhance oscillation transfers, leading to an increased overproduction of He-4. However, for naturally small initial $L \sim$ baryon asymmetry, the asymmetry effect is a subdominant one.

Hence, the spectrum distortion effect is the dominant one and for a wide range of oscillation parameters it is large during the period of nucleon freezing and hence affects primordial nucleosynthesis. The rough calculations not accounting for spectrum distortion effects may underestimate oscillations effect on BBN even by several orders of

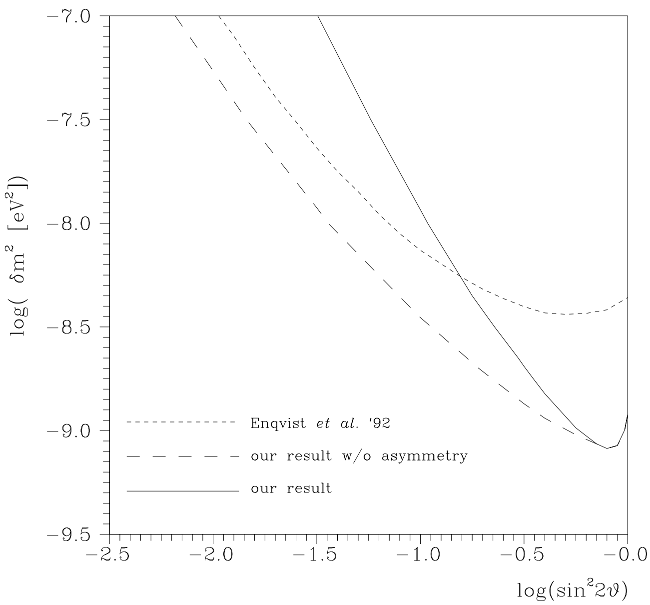

Fig. 3 On the $\delta m^2 - \vartheta$ plane isohelium contour $Y_p = 0.24$ is plotted. The long dashed curve presents the same Y_p without the account of the asymmetry growth, while the small dashed curve presents the results of previous study, where both the spectrum distortion and the asymmetry growth were ignored.

magnitude of δm^2 [70].

4.2 Production of He-4 in the presence of neutrino oscillations

The case of oscillations effective before electron neutrino freezing, was considered both analytically [71, 72] and numerically [73] accounting precisely for a) and partially for b) (namely, estimating the depletion of the neutrino number densities, assuming equilibrium neutrino energy spectrum).

Analytical description was found in the case of very small mixing angles and 'large' mass differences $\delta m^2 > 10^{-6}$ eV^2, and for the case without spectrum distortion effects [36, 59, 26].

The non-equilibrium picture of neutrino oscillation effects (a)-(c) is hard to describe analytically. For non-equilibrium neutrino oscillations effective after active neutrino decoupling, i.e. for $(\delta m^2/eV^2)\sin^4 2\theta < 10^{-7}$, the spectrum distortion effect was shown to play a considerable role. For that case a complete self-consistent numerical analysis of the kinetics of the oscillating neutrinos, the nucleons freeze-out and the asymmetry evolution was provided [63, 65, 66]. Kinetic equations for neutrino density matrix and neutron number densities in *momentum space* [63, 74] were used to make a proper precise account for spectrum distortion effect, neutrino depletion and neutrino asymmetry at each neutrino momentum.

The production of the primordial ^{4}He, Y_p in the presence of $\nu_e \leftrightarrow \nu_s$ oscillations, effective after ν_e decoupling, was calculated. The numerical analysis was provided for the temperature interval [0.3 MeV, 2 MeV]. He-4 production was calculated both in the non-resonant [65] and resonant [66] oscillation cases (see also [75]).

Primordial helium is considerably overproduced in the presence of active-sterile neutrino oscillations due to the effects (a)-(c). The kinetic effect of neutrino oscillations δN_{kin} due to spectrum distortion usually comprises a major portion of the total effect, i.e. it plays the dominant role in the overproduction of ^{4}He. It can be larger than the one corresponding to an additional degree of freedom. The overproduction is maximal for the case of initially empty ν_s state $\delta N_s = 0$ [75].

In the *non-resonant case* the effect of oscillations is proportional to the oscillation parameters. It becomes very small (less than 1%) for small mixings: as small as $\sin^2 2\theta = 0.1$ for $\delta m^2 = 10^{-7} eV^2$, and for small mass differences: $\delta m^2 < 10^{-10}$ eV^2 at maximal mixing. The effect is maximal at maximal mixing for a given mass difference.

Y_p^{max} increase with δm^2 till $\delta m^2 = 10^{-7}$ eV^2 in our model. Further increase of the mass differences requires a decrease of the maximal mixing angle considered, such that oscillations remain effective after ν_e decoupling ($\sin^4 2\vartheta \leq 10^{-7} \delta m^{-2}$). Therefore, for higher δm^2 in the discussed oscillation model $\vartheta < \pi/4$, and Y_p^{max} decreases with further increase of δm^2 beyond $\delta m^2 \sim 10^{-7}$eV^2 (see Fig.4).

In the *resonant oscillation case*, however, for a given δm^2 there exists some resonant mixing angle, at which the oscillation effects are enhanced by the medium due to the MSW effect (see [5, 6]): hence, the overproduction of He-4 is greater than that corresponding to the maximal vacuum mixing angle. If $N_s = 0$, then $\delta N_{kin}^{max} > 1$ for $\delta m^2 > 10^{-9}$ eV^2.

He-4 overproduction in the resonant case can be up to 31.8%, while in the non-resonant case, up to 13.8% (see Fig.4). Thus the maximum overproduction of ^{4}He corresponds to an increase of the neutrino effective degrees of freedom $\delta N_{kin}^{max} \sim 6$.

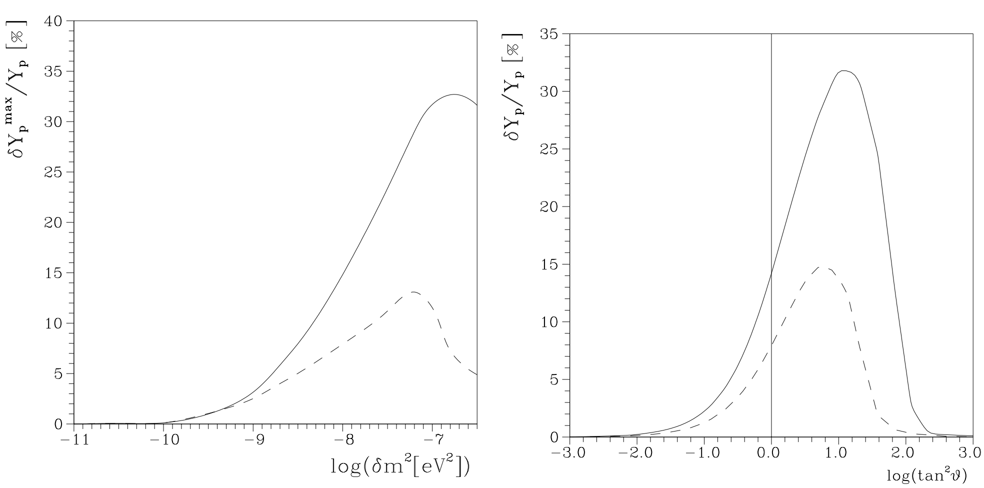

Fig. 4 In the figure on the left, the maximal relative increase in the primordial ^{4}He as a function of neutrino mass differences: $\delta Y_p^{max}/Y_p = \delta Y_p^{osc}/Y_p(\delta m^2)$ is shown for $\vartheta = \pi/4$ in the non-resonant case, and for the resonant mixing angles in the resonant case (upper curve). In the figure on the right, maximum primordial ^{4}He abundance for the resonant and the non-resonant oscillation case is shown as a function of the neutrino mixing angle at $\delta m^2 = 10^{-8}$ eV^2 (lower curve) and $\delta m^2 = 10^{-7}$ eV^2.

4.3 Cosmological constraints on oscillation parameters

Cosmological Constraints: $\delta N_s = 0$ Case

Observational data on primordial He-4 puts stringent limits on the allowed active-sterile oscillation parameters. BBN limits were first derived in the pioneering works [71, 72, 76, 73] under the assumption of kinetic equilibrium (neutrinos were described by a single momentum state with thermal average energy). These constraints accounted for the effects a) and partially for b). These constraints have been recently updated [36, 26, 77]. In the non-resonant case they can be approximated by

$$(\delta m^2_{\nu_e\nu_s}/eV^2)\sin^4 2\theta^{\nu_e\nu_s} = 3.16.10^{-5}(\delta N_\nu)^2$$
$$(\delta m^2_{\nu_\mu\nu_s}/eV^2)\sin^4 2\theta^{\nu_\mu\nu_s} = 1.74.10^{-5}(\delta N_\nu)^2$$

assuming kinetic equilibrium and using stationary point approximation. The bounds are reasonably accurate for large mass differences in case of efficient repopulation of active neutrinos. For the exact constraints in the resonant case see [26].

However, as discussed in the previous subsection, for lower mass differences, when sterile neutrino production takes place after active neutrino freezing, the repopulation of active neutrino becomes slow and hence, kinetic equilibrium may be strongly broken due to active-sterile oscillations. The spectrum distortion of ν_e may be large, leading to strong influence on nucleon kinetics (effects (b) and (c) described earlier). The analysis of such oscillations with precise kinetic accounting for the effects (b) and (c) lead to stringent constraints on oscillation parameters for small mass differences. We will discuss below these BBN constraints [65, 66, 78, 75].

We assume the uncertainty of observational helium-4 to be $\delta Y_p/Y_p^s < 3\%$ in accordance with observations of helium and also with resent WMAP constraints on additional relativistic degrees of freedom [54, 40, 41]. Then the range of cosmologically excluded electron-sterile oscillations parameters is situated above the 3% contour shown in Fig.5:

The analytical fits to the exact constraints are:

$$\delta m^2(\sin^2 2\vartheta)^4 \le 1.5\times 10^{-9}\mathrm{eV}^2 \quad \delta m^2 > 0$$
$$|\delta m^2| < 8.2\times 10^{-10}\mathrm{eV}^2 \quad \delta m^2 < 0, \ \text{large} \ \vartheta,$$

Due to the precise account of the kinetic effects of oscillations, these constraints are nearly an order of magnitude stronger at large mixings than other numerical calculations [73] of 2-neutrino mixing and constraining the mass differences greater than that derived for oscillations effective before the electron neutrino freeze-out [36, 26]. In the resonant case, due to the proper accounting of the asymmetry generated in oscillations, they are less restrictive at small mixings [††] than the constraints [73].

[††] Note that the asymmetry generated by oscillations suppresses oscillations at small mixing angles, reflecting in less overproduction of He-4 in comparison with the case where asymmetry generation was neglected

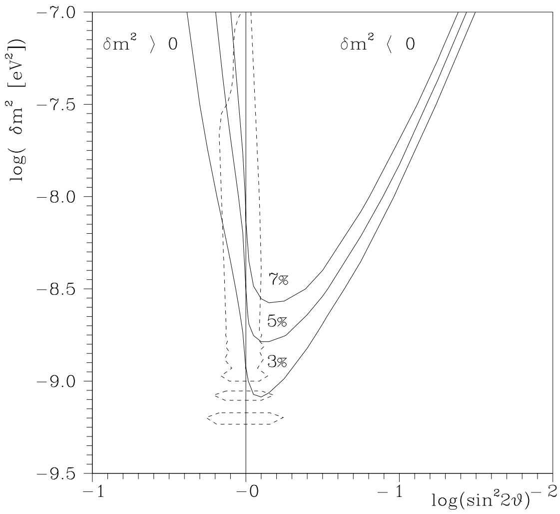

Fig. 5 The combined iso-helium contours for the non-resonant and the resonant case, for $\delta Y_p = (Y_{osc} - Y_p)/Y_p = 3\%, 5\%, 7\%$ [78]. The dashed curves show LOW sterile solution.

The cosmological constraints exclude almost completely sterile LOW solution to the solar neutrino problem, besides the sterile LMA solution and sterile atmospheric solution, excluded in previous works. This result is consistent with the global analysis [14, 4, 16, 10, 76, 79, 15] of the data from neutrino oscillation experiments KamLAND, SNO, SuperKamiokande, GALLEX+GNO, SAGE and Chlorine, which do not favour $\nu_e \leftrightarrow \nu_s$ solutions.

These cosmological constraints should be generalized for the case of 4-neutrino mixing[‡‡], since the effect of mixing between active neutrinos on the BBN constraints of electron-sterile oscillations has been proved important in the resonant oscillation case [26].

The cosmological constraints can be relaxed assuming even higher than the 0.007 systematic error. However, even for 0.25 isohelium contour the constraints on LOW sterile solution are not removed, but just relaxed.

Small relic fermion asymmetry $L << 0.1$, however, can suppress oscillations with small mass differences, thus removing BBN constraints for the corresponding oscillation parameters. For the oscillations effective after neutrino decoupling we have found that $L \sim 10^{-5}$ is large enough to suppress oscillations and remove BBN constraints [78]. Hence we expect that it is possible to remove the cosmological constraints only for rather small δm^2, as long as asymmetry larger than 0.1 is not allowed. Such asymmetry in ν_e sector changes unacceptably the kinetics of nucleons and the He-4 yield. While in $\nu_{\mu,\tau}$ sectors it is constrained to the same level on the basis of redistribution of the asymmetries due to oscillations [80].

Cosmological Constraints: $\delta N_s \neq 0$ Case

Sterile neutrinos ν_s may be present at the onset of BBN epoch — they may be produced in GUT models, in models with large extra dimensions, Manyfold Universe models, mir-

[‡‡] as was already done for the case of fast oscillations proceeding before neutrino decoupling in [26]

ror matter models, or in $\nu_{\mu,\tau} \leftrightarrow \nu_s$ oscillations in 4-neutrino mixing schemes. Hence, the degree of population of ν_s may be different depending on the ν_s production model. Therefore, it is interesting to study the distortion of ν_e energy spectrum due to oscillations $\nu_e \leftrightarrow \nu_s$, and its influence on BBN for different degree of population of the initially present sterile neutrinos $0 \leq \delta N_s \leq 1$. Y_p, for different δN_s values and different sets of oscillation parameters $Y_p(\delta N_s, \delta m^2, \sin^2 2\vartheta)$ has been calculated [64].

$\delta N_s \neq 0$ present before $\nu_{\mu,\tau} \leftrightarrow \nu_s$ just leads to an increase of the total energy density of the Universe, and it is straightforward to re-scale the existing constraints. In the $\nu_e \leftrightarrow \nu_s$ oscillations case, however, the presence of ν_s at the onset of oscillations also influences the kinetic effects of $\nu_e \leftrightarrow \nu_s$ on BBN. Larger δN_s decreases the kinetic effects, because the element of initial non-equilibrium between the active and the sterile states is less expressed (see the dashed curves in Fig.6).

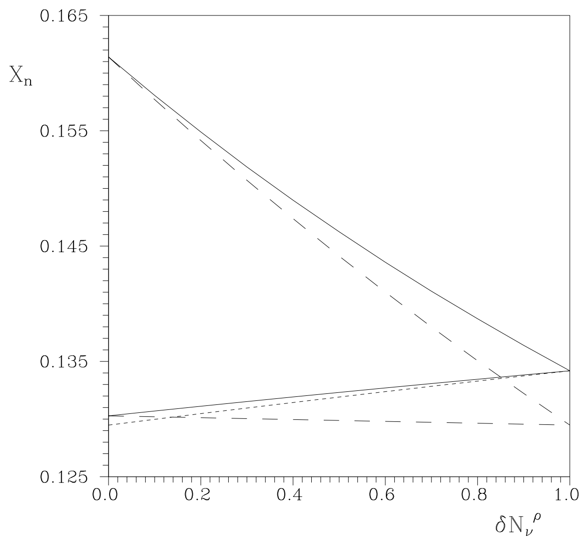

Fig. 6 The solid curves show frozen neutron number density relative to nucleons $X_n^f = N_n^f/N_{nuc}$ as a function of the sterile neutrino initial population, at $\delta m = \pm 10^{-7}$ eV^2, $\sin^2 2\theta = 10^{-1}$. The dashed curves represent the kinetic effect, while the dotted curve shows the effect of energy density increase. The upper curves correspond to the resonant case and the lower the non-resonant one.

Neutrino spectrum distortion effect is very strong even when there is a considerable population of the sterile neutrino state before the beginning of the electron–sterile oscillations. It always gives positive δN_{kin}, which for a large range of initial sterile population values, is larger than 1. The kinetic effects are the strongest for $\delta N_s = 0$, they disappear for $\delta N_s = 1$, when ν_e and ν_s states are in equilibrium, and the total effect reduces to the SBBN with an additional neutrino (Fig.6).

We have calculated the cosmological constraints corresponding to 3% He overproduction and sterile state initial population in the range $0 \leq \delta N_s \leq 0.54$. Higher δN_s correspond to higher overproduction of He, which we consider not very relevant to the available observational data of He-4 abundance and hence, we do not discuss it here. In the next figure we present the cosmological constraints corresponding to $\delta N_s = 0.5$ in comparison with the constraint $\delta N_s = 0$ discussed previously.

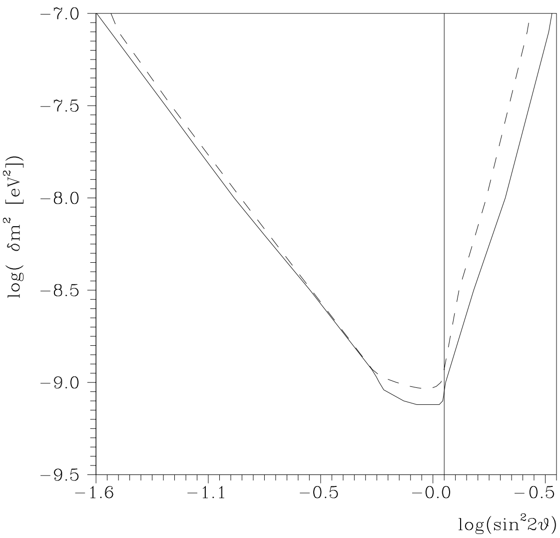

Fig. 7 The dashed contours present BBN constraints corresponding to $\delta Y_p = (Y_{osc} - Y_p)/Y_p = 3\%$ for $\delta N_s = 0$, the solid — $\delta N_s = 0.5$.

Our numerical analysis has shown that up to $\delta N_s = 0.54$ the cosmological constraints are slightly altered but remain stringent, as before. So, the presence of non-zero initial population of the sterile state does not change considerably the existing stringent cosmological constraints on active-sterile neutrino oscillations.

In conclusion, BBN provides the most stringent constraints on active-sterile neutrino oscillations if lepton asymmetry is naturally small (i.e. $L \sim B$), and thus restricts the fraction of the sterile neutrinos in the neutrino anomalies. Besides, as discussed previously, cosmology puts yet the most stringent upper limit to neutrino masses. Hence, it provides a valuable precision probe of new neutrino physics.

5 Conclusion

During the last quarter of the 20th century strong evidence for non-zero neutrino masses and mixings has been provided from various neutrino oscillation experiments (astrophysical: solar neutrino experiments and atmospheric neutrino experiments and terrestrial: LSND, K2K and KamLAND). It has been shown that solar and atmospheric neutrino anomalies can be resolved in terms of flavour neutrino oscillations. Active-sterile oscillation solutions were disfavoured as a dominant channel in solar neutrino and in atmospheric neutrino solutions by cosmological considerations, as well as by recent analysis of the data from neutrino oscillations experiments. The results of KEK, KamLAND and SNO experiments strongly contributed to the reduction of the allowed range of oscillation parameters for solving the neutrino anomalies and marked the beginning of the precision determinations of neutrino characteristics.

The essential role of neutrino oscillations for processes in the early Universe was realized. Stringent cosmological constraints on the oscillation parameters were obtained on the basis of BBN considerations. They cannot be relaxed considerably if exaggerated He-4 abundance is assumed. Moreover, even assuming unnaturally large lepton asymmetry ($B << L < 0.1$), they are only relaxed, not removed. In the natural case of small L of the order of the baryon asymmetry, BBN provides the most stringent constraints on active-sterile neutrino oscillation parameters, and restricts the fraction of the sterile neutrinos in the neutrino anomalies. Besides, cosmological analysis of CMB, LSS and BBN data provide the most stringent upper limit for neutrino masses.

Non-zero neutrino masses and mixings, required by the experimental data and cosmology, require new physics beyond the standard electroweak model. The scale of this NP is inversely proportional to the neutrino masses and appears to be of the order of the Grand Unification scale. Hence, the neutrino oscillation experiments and cosmology point the way towards the Grand Unified Theory of elementary particles.

Acknowledgements

I am glad to thank the referees for useful suggestions. I am grateful to M. Chizhov for the comments on the draft version and for the overall help. It is my pleasure to thank O. Atanacković-Vukmanović and the organizing committee of the NCYA Conference for the hospitality, and I. Tkachev and the organizing committee of CAPP2003 for the hospitality and the financial support of my participation.

The author is a regular associate of ICTP, Trieste, Italy. This work is supported in part by the Belgian Federal Government (office for scientific affairs grant and IAP 5/27)

References

[1] B. Pontecorvo: "Mesonium and Antimesonium", *Sov. Phys. JETP*, Vol. 6, (1958), pp. 429–431.

[2] M. C. Gonzalez-Garcia, Y. Nir: "Neutrino masses and mixing: evidence and implications", *Rev. Mod. Phys.*, Vol. 75, (2003), pp. 345–402.

[3] A. Smirnov: "Neutrino Physics after KamLAND", (2003), hep-ph/0306075

[4] C. Giunti, M. Laveder: "Essential Solar Neutrinos", (2003), hep-ph/0301276.

[5] L. Wolfenstein: "Neutrino oscillations in matter", *Phys. Rev.*, Vol. D 17, (1978), pp. 2369–2374.

[6] S. Mikheyev, A. Smirnov: "Resonance enhancement of oscillations in matter and solar neutrino spectroscopy", *Sov. J. Nucl. Phys.*, Vol. 42, (1985), pp. 913–917; S. Mikheyev, A. Smirnov: "Resonant Amplification of ν Oscillations in Matter and Solar-Neutrino Spectroscopy", *Nuovo Cim.*, Vol. 9C, (1986), pp. 17–26; "Neutrino oscillations in a variable-density medium and ν-bursts due to the gravitational collapse of stars", *Sov. Phys. JETP*, Vol. 64, pp. 4–7.

[7] S. Basu, M. H. Pinsonneault, J. N. Bahcall: "How much do helioseismological inferences depend on the assumed reference model?", *Ap.J.*, Vol. 529, (2000), pp. 1084–1100.

[8] J. N. Bahcall, M. H. Pinsonneault, S. Basu, "Solar Models: Current Epoch and Time Dependences, Neutrinos, and Helioseismological Properties", *Ap. J.*, Vol. 555, (2001), pp. 990–1012.

[9] Q. Ahmad et al.: "Direct Evidence for Neutrino Flavor Transformation from Neutral-Current Interactions in the Sudbury Neutrino Observatory", *Phys. Rev. Lett.*, Vol. 89, (2002), pp. 011301 [6 pages]; nucl-ex/0309004

[10] J. Bahcall, M. Gonzalez-Garcia, C. Pena-Garay : "Solar Neutrinos Before and After KamLAND", *JHEP*, Vol. 02, (2003), pp. 009 [25 pages].

[11] A. Balantekin, H. Yuksel : "Global analysis of solar neutrino and KamLAND data", *J. Phys. G: Nucl. Part. Phys.*, Vol. 29, (2003), pp. 665–682.

[12] A. Bandyopadhyay, S. Choubey, S. Goswami, S.T. Petcov, D.P. Roy: "Constraints on neutrino oscillation parameters from the SNO salt phase data", *Phys. Lett.*, Vol. B 583, (2004), pp. 134–148.

[13] P. Creminelli, G. Signorelli, A. Strumia: "Frequentist analyses of solar neutrino data", *JHEP*, Vol. 05, (2001), p. 052 [17 pages].

[14] P. Holanda , A. Smirnov: "LMA MSW solution of the solar neutrino problem and first KamLAND results", *JCAP*, Vol. 02, (2003), pp. 001 [20 pages]; hep-ph/0309299.

[15] G. Fogli, E. Lisi, A. Palazzo, A.M. Rotunno: "Solar neutrino oscillation parameters after first KamLAND results", *Phys. Rev.*, Vol. D 67, (2003), pp. 073002 [10 pages]; Fogli G. et al. : "Neutrino Oscillations: A Global Analysis", (2003), hep-ph/0310012; hep-ph/0309100.

[16] M. Maltoni, T. Schwetz, M. A. Tortola, J. W. F. Valle: "Status of three-neutrino oscillations after the SNO-salt data", *Phys. Rev.*, Vol. D 68, (2003), p. 113010 [8 pages], hep-ph/0309130 (v.2);
M. Maltony, T. Schwetz, J. Valle: "Combining the first KamLAND results with solar neutrino data", *Phys. Rev.*, Vol. D 67, (2003), pp. 093003 [5 pages];
M. Maltony: "Status of neutrino oscillations I: the tree-neutrino scenario", hep–ph/0401042.

[17] Y. Fukuda et al.: "Evidence for Oscillation of Atmospheric Neutrinos", *Phys. Rev. Lett.*, Vol. 81, (1998), pp. 1562–1567.

[18] C. Giunti : "Status of Neutrino Masses and Mixing", (2003), hep–ph/0309024.

[19] A. Aguilar A. et al.: "Evidence for neutrino oscillations from the observation of $\bar{\nu}_e$ appearance in a $\bar{\nu}_\mu$ beam", *Phys. Rev.*, Vol. D 64, (2001), pp. 112007 [22 pages].

[20] M. Ahn et al.: "Indications of Neutrino Oscillation in a 250 km Long-Baseline Experiment", *Phys. Rev. Lett.*, Vol. 90, (2003), pp. 041801 [5 pages].

[21] K. Eguchi et al.: "First Results from KamLAND: Evidence for Reactor Antineutrino Disappearance", *Phys. Rev. Lett.*, Vol. 90, (2003), pp. 021802 [6 pages].

[22] A. Bazarko : "Status of MiniBooNE (for the BooNE Collaboration)", In: *The proceedings of 31st International Conference on High Energy Physics* (ICHEP 2002), Amsterdam (The Netherlands), 24-31 Jul 2002, Edited by S. Bentvelsen, P. de Jong, J. Koch, E. Laenen. Amsterdam, The Netherlands, North-Holland, pp. 33–36, 2003.

[23] D.P. Kirilova and M.V. Chizhov: "BBN and Cosmological Constraints on Neutrino Oscillation Parameters", In: *Hot Points in Astrophysics*, Dubna (Russia), 2001, pp. 56–66; CERN-TH/2001–020, astro-ph/0108341.

[24] S. Bilenky, C. Giunti, W. Grimus, T. Schwetz: "Four-neutrino mixing and Big-Bang Nucleosynthesis", *Astrop. Phys.*, Vol. 11, (1999), pp. 413–428.

[25] P. Di Bari: "Update on neutrino mixing in the early universe", *Phys. Rev.*, Vol. D 65, (2002), pp. 043509 [14 pages].

[26] A. Dolgov, F. Villante: "BBN bounds on active-sterile neutrino mixing", *Nucl. Phys. B*, (2003), Vol. 679, pp. 261–298.

[27] S. S. Gershtein, Ya. B. Zel'dovich: "Rest Mass of Muonic neutrino and Cosmology", *JETP Letters*, Vol. 4, (1966), pp. 120–122; *Pis'ma ZhETF*, Vol. 4, (1966), pp. 174–177.

[28] R. Cowsik, J. McClleland: "Gravity of Neutrinos of Nonzero Mass in Astrophysics", *Ap. J.* Vol. 180, (1973), pp. 7–10.

[29] M. Fukugita, G.-C. Liu, N. Sugiyama: "Limits on Neutrino Mass from Cosmic Structure Formation", *Phys. Rev. Lett.*, Vol. 84, (2000), pp. 1082–1085; O. Ø. Elgarøy et al. : "New Upper Limit on the Total Neutrino Mass from the 2 Degree Field Galaxy Redshift Survey", *Phys. Rev. Lett.*, Vol. 89, (2002), pp. 061301 [4 pages].

[30] Ø. Elgarøy, O. Lahav: "Upper limits on neutrino masses from the 2dFGRS and WMAP: the role of priors", *JCAP*, Vol. 04, (2003), pp. 004 [24 pages];
S. Hannestad: "Neutrino masses and the number of neutrino species from WMAP and 2dFGRS", *JCAP*, Vol. 05, (2003), pp. 004 [19 pages].

[31] D. Spergel, et al.: "First-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters", *Ap. J. Suppl.*, Vol. 148, (2003), pp. 175–194.

[32] S. Allen, R. Schmidt, S. Bridle: "A preference for a non-zero neutrino mass from cosmological data", *Mon. Not. Roy. Astron. Soc.*, Vol. 346, (2003), pp. 593–600.

[33] V. Barger, T. Weiler, K. Whisnant: "Inferred 4.4 eV upper limits on the muon- and tau-neutrino masses", *Phys. Lett.*, Vol. B 442, (1998), pp. 255–258.

[34] S. Bilenky, C. Giunti, J. Grifols, E. Massó: "Absolute values of neutrino masses: status and prospects", *Phys. Rept.*, Vol. 379, (2003), pp. 69–148.

[35] S. Sarkar: "Neutrinos from the Big Bang", In: *Proc. Indian Natl. Sci Acad.*, Vol. 70A, (2004), pp. 163–178;
M. Prakash, J. M. Lattimer, R. F. Sawyer, R. R. Volkas: "Neutrino Propagation in Dense Astrophysical Systems", *Ann. Rev. Nucl. Part. Sci.* Vol. 51, (2001), pp. 295–344.

[36] A. D. Dolgov: "Neutrinos in cosmology", *Phys.Rept.* Vol. 370, (2002), pp. 333-535; "Cosmological Implications of Neutrinos", *Surveys in High Energy Physics*, Vol. 17, (2002), pp. 91–114.

[37] G. Raffelt: "Neutrino Masses in Astroparticle Physics", *New Astron. Rev.*, Vol. 46, (2002), pp. 699–708.

[38] G. Gamow: "Nuclear Transformation and the Origin of the Chemical Elements", *Ohio Journal of Science*, Vol. 35, (1935), 406-413;
"Concerning the Origin of Chemical Elements", *Journal of the Washington Academy of Sciences*, Vol. 32, (1942), 353–355;
"Expanding Universe and the Origin of Elements", *Phys. Rev.*, (1946), Vol. 70, pp. 572–573.

[39] S. Esposito, G. Mangano, G. Miele, O. Pisanti: "Big bang nucleosynthesis: an accurate determination of light element yields", *Nucl. Phys.*, Vol. B 568, (2000), pp. 421–444.

[40] A. Cuoco, F. Iocco, G. Mangano, G. Miele, O. Pisanti, P.D. Serpico: "Present status of primordial nucleosynthesis after WMAP: results from a new BBN code", (2003), astro-ph/0307213.

[41] R. Cyburt, B. Fields, K. Olive: "Primordial nucleosynthesis in light of WMAP", *Phys. Lett.*, Vol. B 567, (2003), pp. 227–234.

[42] A. Coc, E. Vangioni-Flam, P. Descouvemont, A. Adahchour, C. Angulo: "Updated Big Bang Nucleosynthesis Compared with Wilkinson Microwave Anisotropy Probe Observations and the Abundance of Light Elements", *Ap. J.*, Vol. 600, (2004), pp. 544-552.

[43] G. Steigman: "Primordial Nucleosynthesis", (2003), astro-ph/0308511.

[44] D. Kirkman, D. Tytler, N. Suzuki, J.M. O'Meara, D. Lubin: "The cosmological baryon density from the deuterium to hydrogen ratio towards QSO absorption systems: D/H towards Q1243+3047", *Astrophys. J. Suppl.*, Vol. 149, (2003), pp. 1-28.

[45] R. E. Lopez, M. S. Turner: "Precision prediction for the big-bang abundance of primordial ^{4}He", *Phys. Rev.*, Vol. D 59, (1999), pp. 103502 [14 pages].

[46] K. Olive, G. Steigman, E. Skillman: "The Primordial Abundance of ^{4}He: an Update", *Ap. J.*, Vol. 483, (1997), pp. 788–797.

[47] Yu. I. Izotov, T. X. Thuan: "The Primordial Abundance of ^{4}He Revisited, Astrophysical Journal", *Ap. J.*, Vol. 500, (1998), pp. 188–216.

[48] R. Trotta, S. Hansen S.: "Constraining the helium abundance with CMB data", *Phys. Rev.*, Vol. D 69, (2004), pp. 023509 [10 pages]; see also G. Huey, R. Cyburt, B. Wandelt: "Precision Primordial ^{4}He Measurement with CMB Experiments", (2003), astro-ph/0307080.

[49] G. Bono, A. Balbi, S. Cassisi, N. Vittorio, R. Buonanno: "On the Primordial Helium Content: Cosmic Microwave Background and Stellar Constraints", *Ap. J.*, Vol. 568, (2002), pp. 463–469.

[50] S. Cassisi, M. Salaris, A. Irwin: "The initial Helium content of Galactic Globular Cluster stars from the R-parameter: comparison with the CMB constraint", (2003), astro-ph/0301378.

[51] V. Shvartsman: "Density of Relict Particles with Zero Rest Mass in the Universe", *Pisma Zh.Eksp.Teor.Fiz.*, Vol. 9,(1969), pp. 315–317; *JETP Lett.*, Vol. 9, pp. 184–186;
Steigman G. et al.: "A reexamination of the cosmological bound to the number of neutrino flavors", *Phys. Lett.*, Vol. B 176, (1986), pp. 33–38.

[52] E. Lisi, S. Sarkar, F. Villante: "Big bang nucleosynthesis limit on N_ν", *Phys. Rev.*, Vol. D 59, (1999), pp. 123520 [11 pages].

[53] V. Barger, J.P. Kneller, Hye-Sung Lee, D. Marfatia, G. Steigman: "Effective number of neutrinos and baryon asymmetry from BBN and WMAP", *Phys. Lett.*, Vol. B 566, (2003), pp. 8–18.

[54] P. Crotty, J. Lesgourgues, S. Pastor: "Measuring the cosmological background of relativistic particles with the Wilkinson Microwave Anisotropy Probe", *Phys. Rev.*, Vol. D 67, (2003), pp. 123005 [5 pages].

[55] N. Terasawa, K. Sato: "Nonradiative decay of the neutrino and primordial nucleosynthesis", *Phys. Lett.*, Vol. B 185, (1987), pp. 412–416.

[56] A. Dolgov, D. Kirilova: "Nonequilibrium Decays of Light Particles and Primordial Nucleosynthesis", *Int. J. Mod. Phys.*, Vol. A 3, (1988), pp. 267–277.

[57] G. Gyuk, M. Turner: "Relaxing the big-bang bound to the baryon density", *Phys. Rev.,* Vol. D 50, (1994), pp. 6130–6134.

[58] A.D. Dolgov, S.H. Hansen, S. Pastor, D.V. Semikoz: "Unstable massive tau-neutrinos and primordial nucleosynthesis", *Nucl. Phys.*, Vol. B 548, (1999), pp. 385–407.

[59] A. D. Dolgov: "Cosmological Constraints on Neutrino Masses and Mixing", (2003), hep-ph/0306154.

[60] A. D. Dolgov: "Neutrinos in the early Universe", *Sov. J. Nucl. Phys.,* Vol. 33, (1981), pp. 700–706.

[61] P. Holanda, A. Smirnov: "Homestake result, sterile neutrinos and low energy solar neutrino experiments", (2003), hep-ph/0307266.

[62] D. Kirilova: "Neutrino Oscillations and the primordial Nucleosynthesis", (1988), JINR E2–88–301.

[63] D. Kirilova, M. Chizhov: "Nonequilibrium Neutrino Oscillations and Primordial Helium Production", In: *Neutrino96*, Helsinki, 1996, pp. 478–484.

[64] D. Kirilova: "Neutrino Spectrum Distortion Due to Oscillations and its BBN Effect", (2002), *Int. J. Mod. Phys. D* Vol. in press, CERN-TH/2002–209, hep-ph/0209104.

[65] D. Kirilova, M. Chizhov: "Cosmological nucleosynthesis and active-sterile neutrino oscillations with small mass differences: The nonresonant case", *Phys. Rev.*, Vol. D 58, (1998), pp. 073004 [14 pages].

[66] D. Kirilova, M. Chizhov: "Cosmological nucleosynthesis and active-sterile neutrino oscillations with small mass differences: the resonant case", *Nucl. Phys.*, Vol. B 591, (2000), pp. 457-468.

[67] S. P. Mikheyev, A. Yu. Smirnov: "Neutrino oscillations in matter with varying density", in the Proceedings of the Sixth Moriond Workshop "'86 Massive Neutrinos in Astrophysics and in Particle Physics", eds. O. Fackler, J. Trân Thanh Vân, Editions Frontières, 1986, pp. 355-372.

[68] P. Langacker, S.T. Petcov, G. Steigman, S. Toshev: "Implications of the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism of amplification of neutrino oscillations in matter", *Nucl. Phys.*, Vol. B 282, (1987), pp. 589–609.

[69] R. Foot, M. Thomson, R. Volkas: "Large neutrino asymmetries from neutrino oscillations", *Phys. Rev.*, Vol. D 53, (1996), pp. R5349–R5353.

[70] M. Chizhov, D. Kirilova: "On Neutrino-Mixing-Generated Lepton Asymmetry and the Primordial Helium–4 Abundance", (1999), hep–ph/9908525.

[71] R. Barbieri, and A. Dolgov: "Bounds on sterile neutrinos from nucleosynthesis", *Phys. Lett.*, Vol. B 237, (1990), pp. 440–445.

[72] R. Barbieri, A. Dolgov: "Neutrino oscillations in the early universe", *Nucl. Phys.*, Vol. B 349, (1991), pp. 743–753.

[73] K. Enqvist, K. Kainulainen, M. Thomson: "Stringent cosmological bounds on inert neutrino mixing", *Nucl. Phys.*, Vol. B 373, (1992), pp. 498–528.

[74] D. Kirilova, M. Chizhov: "Nonequilibrium neutrino oscillations and primordial production of ^{4}He", *Phys. Lett.*, Vol. B 393, (1997), pp. 375–382.

[75] D. Kirilova: "Overproduction of primordial helium-4 in the presence of neutrino oscillations", *Astropart. Phys.*, Vol. 19, (2003), pp. 409–417.

[76] K. Enqvist, K. Kainulainen, J. Maalampi: "Neutrino asymmetry and oscillations in the early universe", *Phys. Lett.*, Vol. B 244, (1990), pp. 186–190.

[77] A. D. Dolgov: "Restrictions on neutrino oscillations from BBN. Non-resonant case", *Phys. Lett. B* Vol. 506, (2001), pp. 7–12.

[78] D. Kirilova, M. Chizhov: "Neutrino oscillations in the early universe", *Nucl. Phys. Suppl.*, Vol. B 100, (2001), pp. 360–362.

[79] G. Fogli, E. Lisi, A. Marrone: "Four-neutrino oscillation solutions of the atmospheric neutrino anomaly", *Phys. Rev.*, Vol. D 63, (2001), pp. 053008 [14 pages].

[80] A.D. Dolgov, S.H. Hansen, S. Pastor, S.T. Petcov, G.G. Raffelt, D.V. Semikoz: "Cosmological bounds on neutrino degeneracy improved by flavor oscillation", *Nucl. Phys. B* Vol. 632, (2002), pp. 363-382.