The Museum Pass Game and its Value*

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May 2002

Abstract

We discuss a subscription game in which service providers (e.g., museums) team up in offering a limited time subscription or access pass allowing unlimited usage of their services. In this game, a natural way to allocate the subscription income among the service providers is by using the Shapley value. We show that, for the particular game considered, the Shapley value takes a very intuitive and computationally simple form.

Published *Games and Economic Behavior* 43 (2003), 322-325.

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* We are indebted to Abraham Neyman, Dov Samet, Yair Tauman and Shlomo Weber for helpful discussions and comments. Special thanks go to the Conseil Bruxellois des Musées for introducing us to the problem. Israel Zang's research was supported by a Research in Brussels 2001 grant from the Brussels capital region.

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1. Introduction and Background

Museum Passes which give visitors (tourists as well as residents) unlimited access to participating museums during a limited period of one to several days (perhaps a whole year for residents) have become very common in many European[1] or North-American cities[2], regions, or even entire countries[3]. The problem is how to share the net income from the sale of passes among the participating museums. Another application yielding exactly the same formulation is the case of an Internet service provider who charges a limited time subscription fee, allowing customers to freely download files (application software, music or video pieces) during a given period. Here, the problem is to allocate the income among the files that are downloaded.

Consider a group of $N = \{1,..., n\}$ service providers (the museums), offering differentiated and imperfectly substitutable services (museum visits) whose usage can be described in binary terms (i.e. 0 if the service is not used by a customer and 1 if it is). The providers charge possibly different prices for their services. They decide to issue a limited time subscription pass, sold to customers (visitors) at a fixed price P. This pass can be used by a customer to access services from any member(s) of N. The question is how to share the income, derived from subscription sales during a given base period, among the service providers. We assume that the detailed data, concerning the usage of each pass sold during the base period, is available.

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1 In 2000, such passes existed in Amsterdam, Barcelona, Bologna, Bonn, Budapest, Copenhagen, Helsinki, Lisbon, London, Luxemburg, Paris, Salzburg, Stockholm, Vienna.
2 Montreal, Philadelphia, among others.
3 The Netherlands (Nederlandse Museumjaarkaart), the UK (Great British Heritage Pass), Flanders (OKV-Museumkaart), Switzerland (Passeport Musées Suisses). Some passes are even international: The Pass-musées du Rhin Supérieur, gives access to museums in the Upper-Rhine region, which includes the East of France, the West of Germany and some parts of Switzerland.

2. Layout of the Problem

Let $K \subseteq N$ be a subset of size $|K| = k$ of N. Let $M = \{1,..., m\}$ denote the set of customers who have purchased the pass, and let $K_j \subseteq N$ denote the subset of providers whose services were utilized by customer $j \in M$. For any subset $K \subseteq N$ of providers, we denote by $V(K)$ the number of customers who only use services of providers belonging to *K*. Namely

(1) $$V(K) = \sum_{j \in M, K_j \subseteq K} 1 .$$

Note that $V(N) = m$ is the total number of pass buyers. We assume that each customer $j \in M$, purchasing a pass, has preferences over providers' services. These preferences are independent of the potential groupings of providers. Hence, a customer decides which group of services $K_j \subseteq N$ to use before purchasing the pass. Consequently, for any subset $K \subseteq N$ of providers, $V(K)$ is what this subset can guarantee to itself when the pass price is *P*.

3. The Problem

How does one allocate the subscription income among the participating service providers? If we can figure out $\phi(i)$, the number of subscribers who can be attributed to service provider $i \in N$, then we can let provider *i* earn $\phi(i) \times P$. Hence we need to find the real contribution $\phi(i)$ to $V(N)$ of each provider $i \in N$.

One can, of course, allocate the income in proportion to the total number of service utilizations of provider *i* in the subscribers group *M*. But this will not take into account the fact that some providers will have an externality, in the sense that customers subscribe mainly because of their service but also use others' services. So the ***real power*** of provider $i \in N$ might be different from the actual number of service usages he has experienced. For example, in a proportional allocation scheme, the income derived from an additional subscription, utilized to acquire the service from only one provider, will usually be split among all service providers. This is counterintuitive, as one expects this additional income to be allocated to the single provider who supplied the service[4].

4 It should be mentioned that the income allocation schemes presently utilized by some European museum pass programs are even worse in terms of adequacy or fairness. There are cases where an increase in the number of visits to a given museum (due, say, to a special exhibition) results in other museums being better compensated. Some mechanisms even give to participating museums wrong incentives in pricing their own entry tickets.

4. The game structure and proposed solution

We consider a *cooperative game in characteristic form* in which the *n players* are the service providers. For this game the *characteristic function V* is given by (1). Namely, for each *coalition K* of providers in *N*, $V(K)$ is the number of pass buyers who used services of providers in *K* only. The real power $\phi(i)$, of player *i*, can be expressed by the *Shapley value* (Shapley, 1953):

(2) $$\phi(i) = \sum_{K \subseteq N} \left[V(K) - V\left(K \setminus \{i\}\right)\right]\frac{(k-1)! \times (n-k)!}{n!} \ , \ i \in N \ .$$

This formula gives the number of passes sold that can be attributed to provider *i*, with $\phi(i) \times P$ as income.

In general, due to the combinatorial nature of the Shapley value, its computation could become quite cumbersome (when, say, fifty providers participate in a subscription plan). However, for the particular game structure considered, we have:

Proposition: For the museum pass game, the Shapley value implies that the income derived from each subscriber is equally distributed among the providers that this subscriber has utilized. Namely:

(3) $$\phi(i) = \sum_{\substack{j=1,\ldots,m \\ K_j \ni i}} \frac{1}{|K_j|} \ , \ i \in N \ .$$

Proof: The proof follows the observation that the game considered here is merely the sum of *unanimity games* considered in Shapley's (1953) proof. Let

(4) $$v_S(K) = \begin{cases} 1 & if \quad K \supseteq S \\ 0 & otherwise, \end{cases}$$

denote the characteristic function of the unanimity game corresponding to a carrier subset $S \subseteq N$. For this game, the symmetry and efficiency axioms imply that the value of player *i,* $\phi_S(i)$, is given by

(5) $$\phi_S(i) = \begin{cases} \dfrac{1}{|S|}, & if \quad i \in S, \\ 0 \ , & otherwise. \end{cases}$$

From (1), the characteristic function of the museum pass game is given by:

(6) $$V(K) = \sum_{j \in M} v_{K_j}(K) \, .$$

Expression (3) now follows the additivity axiom. QED

Note that for the museum pass game, the Shapley value satisfies the very intuitive property that the income generated by additional subscribers will be allocated only to those providers whose services are utilized by the new subscribers.

Some additional remarks are in order:

1) The subscription game is convex (Shapley, 1971/72), and consequently its Shapley value is contained in its core.
2) If there are only two players, the solution defined above coincides with the nucleolus.5
3) A similar allocation scheme can be devised for cases where one allows multiple uses, by a single subscriber, of the service provided by a unique provider.
4) To expand the scope for possible applications, one may also consider a weighted Shapley value that assigns a weight to each player, thus taking into account for example the size and importance of the collection of a museum, its distance from the center of the city, etc.

5 This was brought to our attention by Shlomo Weber.

5. References

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