Considering Laminated Cores and Eddy Currents in 2D and 3D Finite Element Simulation of Electrical Machines

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***Abstract*—This paper deals with a time-domain homogenization method for laminated cores and its application to the 2D finite element simulation of rotating electrical machines. The number of additional degrees of freedom of the model, for considering the variation of the flux density along the lamination thickness, can be tuned so as to reach a good compromise between accuracy and computation time. The results obtained for a switched reluctance motor agree very well with those produced by a precise full 3D model in which eddy currents are explicitly modeled.**

I. Introduction

The behavior of an electrical machine may be considerably altered by the eddy currents in its laminated iron core, apart from the obvious effect on the losses, heating and efficiency [1]. The usual design and analysis approach which consists in an a-posteriori eddy current (and iron loss) calculation by means a 2D or 3D finite element (FE) model may then be insufficient. For real-life applications, the brute-force 3D FE modeling of each separate lamination is still far beyond practical computational capabilities [2]. As a more pragmatic alternative, homogenisation methods may be adopted in 2D FE modeling so as to include the eddy current effects at a reasonable computational cost. The time-domain method proposed in [3] is applicable to saturable magnetic cores and was validated on a real 3D test case (laminated ring core and toroidal coil, exploiting axisymmetry).

In this paper the homogenisation method is applied to the 2D modeling of an electrical machine, and in particular of a switched reluctance motor (SRM). See Fig. 1 for its geometry and main dimensions. After a brief and general discussion of the 2D and 3D modeling, 2D results will be compared directly to those obtained with a 3D FE model.

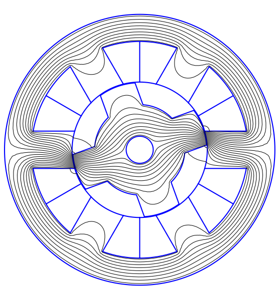

Fig. 1. Cross-section of 6/4 SRM (airgap radius: 30 mm; airgap width: 0.29 mm; stack length: 60 mm; lamination thickness d: 0.5mm; stator and rotor pole width: 16 mm; outer radius: 60 mm; 226 turns per coil) – flux lines with phase 1 excited ($\theta = 20°$)

II. 2D FE modeling with eddy currents

A. 1D time-domain lamination model

We consider a single lamination of thickness d ($-d/2 \leq z \leq d/2$), carrying a time varying flux density $b(z,t)$ along e.g. the x-axis. The governing 1D differential equation in terms of $b(z,t) = b(-z,t)$ and the magnetic field $h(z,t) = h(-z,t) = \nu b(z,t)$ reads:

$$\frac{\partial^2 h}{\partial z^2} = \sigma \frac{\partial b}{\partial t}, \qquad (1)$$

where the permeability μ, the reluctivity $\nu = \mu^{-1}$, and the electrical conductivity σ are assumed constant.

The average induction $b_a(t) = \frac{1}{d}\int_{-d/2}^{d/2} b(z,t)\,dz$ and the magnetic field $h_s(t) = h(z = \pm d/2, t)$ on the surface of the lamination are of particular interest for homogenization.

An approximate time domain solution of (1) can be obtained by considering a polynominal expansion of $b(z,t)$ comprising even basis functions $\alpha_0(z)$ =1, $\alpha_2(z) = -\frac{1}{2} + 6(z/d)^2, \ldots$ of order 0, 2, ..., which are orthogonal, $\frac{1}{d}\int_{-d/2}^{d/2} \alpha_i(z)\alpha_j(z)\,dz = 0$ if $i \neq j$, and have unit value on the lamination surface [3]:

$$b(z,t) = \alpha_0(z)\,b_0(t) + \alpha_2(z)\,b_2(t) + \ldots, \qquad (2)$$

with $b_0(t) = b_a(t)$ since $\frac{1}{d}\int_{-d/2}^{d/2} \alpha_i(z)\,dz = 0$ if $i \geq 2$.

Then, on account of (1), the magnetic field $h(z,t)$ is expanded as

$$h(z,t) = h_s(t) - \sigma d^2 \beta_2(z)\frac{d\,b_a}{d\,t} - \sigma d^2 \beta_4(z)\frac{d\,b_2}{d\,t} - \ldots, \qquad (3)$$

with $\beta_2(z) = \frac{1}{8} - \frac{1}{2}(z/d)^2$, $\beta_4(z) = -\frac{1}{32} + \frac{1}{4}(z/d)^2 - \frac{1}{2}(z/d)^4$.

When considering a finite number of basis functions, up to order n for $b(z,t)$ and order $n+2$ for $h(z,t)$, the constitutive law $h(b) = \nu b$ cannot be fulfilled exactly. For e.g. $n = 2$, its weak formulation leads to a system of two differential equations in terms of $b_a(t)$, $b_2(t)$ and $h_s(t)$:

$$\begin{bmatrix} h_s \\ 0 \end{bmatrix} = \nu \begin{bmatrix} 1 & 0 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} b_a \\ b_2 \end{bmatrix} + K \begin{bmatrix} 1 & -1/5 \\ -1/5 & 1/17.5 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} b_a \\ b_2 \end{bmatrix}, \qquad (4)$$

with $K = \sigma d^2/12$, and where the elements of the diagonal matrix will further be denoted by s_{ij} ($s_{ij} = 0$ if $i \neq j$) and the ones of the other (symmetrical) matrix t_{ij}.

The accuracy of this 1D lamination model is easily assessed through comparison with the analytical frequency-domain solution (at frequency f and pulsation $\omega = 2\pi f$). Allowing a 1% maximum error on the equivalent complex reluctivity, the coarsest approximation, with $n = 0$, is valid up to roughly $d/\delta = 1$, where $\delta = \sqrt{2/(\omega\mu\sigma)}$ is the penetration depth. By adding one or two interpolation functions ($n = 2$ and $n = 4$ resp.) the validity range to extended up to d/δ equal to 4 and 8 respectively.

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B. Incorporation in 2D model

Adopting the classical one-component magnetic vector potential (MVP) formulation and, e.g., a first-order triangular discretisation of the 2D calculation domain $\Omega_{2\mathrm{D}}$, the flux density is approximated as $\operatorname{curl}\big(\sum_k a_k(t)\,\gamma_k(x,y)\,\underline{e}_z\big)$, where $\gamma_k(x,y)$ and $a_k(t)$ are the nodal basis function and degree of freedom associated to node k of the mesh, and $\underline{e}_z$ the unit vector along the z-axis. The current density $j_z(x,y)\,\underline{e}_z$ in the subdomain $\Omega_{2\mathrm{D,ind}}$ is either imposed directly or is governed by electrical circuit equations.

The additional $n/2$ flux density components $\underline{b}_i$, $i=2,4,\ldots$, for considering the eddy currents are introduced with MVP components defined in the laminated core subdomain $\Omega_{2\mathrm{D,lam}}$, discretised with the same basis functions $\gamma_k(x,y)$ and producing new degrees of freedom $a_{ik}(t)$. With $n=2$, e.g., the complete set of FE equations can thus be written in the following block matrix form:

$$\begin{bmatrix} J_0 \\ 0 \end{bmatrix} = \begin{bmatrix} S_{00} & S_{02} \\ S_{20} & S_{22} \end{bmatrix} \begin{bmatrix} A_0 \\ A_2 \end{bmatrix} + \begin{bmatrix} T_{00} & T_{02} \\ T_{20} & T_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} A_0 \\ A_2 \end{bmatrix}, \quad (5)$$

with

$$J_{0,k} = \int_{\Omega_{2\mathrm{D,ind}}} j_z\,\gamma_k\,d\Omega\,, \quad (6)$$

$$S_{ij,kl} = s_{ij}\int_{\Omega_{2\mathrm{D}}} \nu\,\operatorname{curl}(\gamma_k\,e_z)\cdot\operatorname{curl}(\gamma_l\,e_z)\,d\Omega\,, \quad (7)$$

$$T_{ij,kl} = K\,t_{ij}\int_{\Omega_{2\mathrm{D,lam}}} \operatorname{curl}(\gamma_k\,e_z)\cdot\operatorname{curl}(\gamma_l\,e_z)\,d\Omega\,. \quad (8)$$

This approach is straightforwardly extended for nonlinear material in $\Omega_{2\mathrm{D,lam}}$ [3].

III. 3D FE Modeling

We consider the magnetic vector potential formulation in the 3D domain $\Omega_{3\mathrm{D}}$ obtained by extruding the domain $\Omega_{2\mathrm{D}}$ over one lamination thickness d ($-d/2 \le z \le d/2$). The current density $\underline{j}$ in subdomain $\Omega_{3\mathrm{D,ind}}$ is either imposed directly or is governed by electrical circuit equations, whereas in subdomain $\Omega_{3\mathrm{D,lam}}$ the eddy currents are explicitly modeled, with $\underline{j}=\sigma\underline{e}$ and the electric field $\underline{e}$ given by $-\partial_t\underline{a}$. The vector potential is, e.g., discretised by means of edge basis functions $\underline{\gamma}_k$: $\underline{a}=\sum_k a_k\underline{\gamma}_k$. The uniqueness of $\underline{a}$ can be ensured by considering an edge co-tree in the nonconducting domain $\Omega_{3\mathrm{D}}\setminus\Omega_{3\mathrm{D,lam}}$.

IV. Comparison of 2D and 3D Results

Dynamic 2D calculations, with the eddy currents ignored ($\sigma=0$) or considered via the homogenization approach ($n=0,2,4$), are carried out exciting the first phase only and with aligned rotor poles. The corresponding 3D calculations, the results of which are taken as reference, are done with a mesh of prismatic elements obtained by extruding the triangular elements of the 2D mesh. The space discretization is illustrated by Fig. 2. A conductivity of 5 MS/m and a relative permeability of 1000 are set for the laminated core, corresponding to a penetration depth of 0.07 mm at 10 kHz.

Frequency-domain calculations are first carried out imposing a sinusoidal current of unit amplitude and with the frequency ranging from 10 Hz to 10 kHz. Fig. 3 shows the

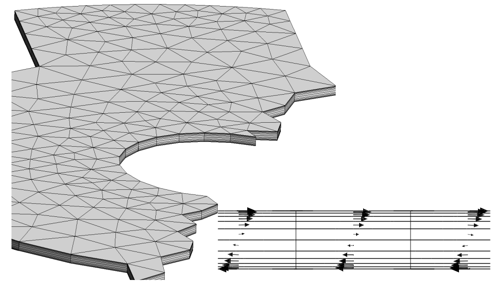

Fig. 2. Detail of the surface and thickness discretisation in the 3D model (layer thickness varying between 0.2 and 0.02 mm) and current density vectors

complex inductance obtained with the various models. With $n=2$ the 2D model produces very accurate results in the complete frequency interval.

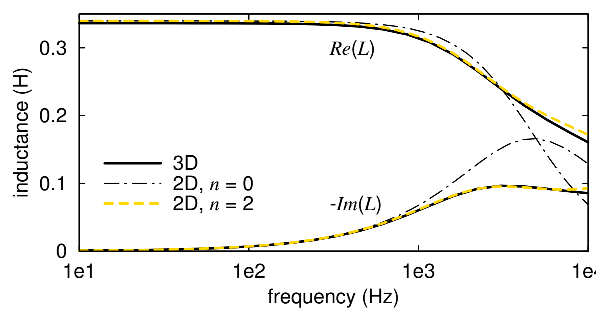

Fig. 3. Inductance (real and imaginary parts) versus frequency obtained with 2D and 3D FE models

Time-domain results with pulsed 5 kHz, ±500 V voltage supply are shown in Fig. 4. Again with $n=2$, the 2D model is very accurate.

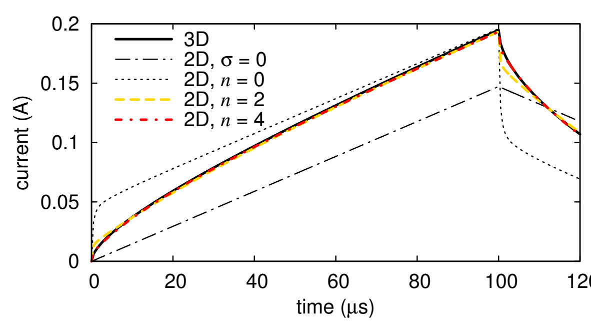

Fig. 4. 5 kHz PWM voltage: current versus time obtained with 2D and 3D FE models

The rotation of the rotor can be handled by a moving band technique in both 2D and 3D modeling. This aspect will be developed in the full paper.

References

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