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A DUAL DECISION APPROACH TO DISEQUILIBRIUM GROWTH

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1. Introduction

ITO'S (1978, 1980) basic model of disequilibrium growth retains Solow's (1956) neoclassical assumption of a constant exogenous saving rate. Recently, in a critical survey, Ito himself (1982) questions the assumption and recognizes that endogenizing the saving rate would be more consistent with optimizing behavior.

To achieve this, one has to consider the optimal consumption and saving program of households which may happen to be constrained on the labor or the goods market. The generalization of Ito's model requires thus a dynamic counterpart to the static Dual Decision Hypothesis (DDH for short) introduced by Clower (1965) and Barro and Grossman (1976).

Henin and Michel (1982a and b) have considered the disequilibrium growth path of an economy with price and wage rigidities, given by the optimal accumulation program of firms. A dual decision extension of this work is provided here, in which it is assumed that households also are intertemporal utility maximisers. Both households and firms have a fully rational (perfect foresight) behavior: their expectations are always equal to the actual solution of the intertemporal model.

Section 2 discusses a simple model, in which consumers decide on the growth of capital, and firms simply adapt employment to the given capital stock. Section 3 is concerned with an extension which takes into account dual decisions of consumers and producers, in a growth framework; it is shown that four temporary equilibrium regimes will emerge, while in the long run, full employment growth is achieved. Conclusions are offered in Section 4.

2. A simple model

In the usual formulation of the Dual Decision Hypothesis, agents perceive constraints limiting their transactions in the current period. Neary and Stiglitz (1983) generalized the idea to two period models. We consider that agents optimize over an infinite horizon, and perceive not only to-day's but also all future constraints.

We shall cope at first with the case where the dynamics results only from

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households behavior: this is the symetric of the Ito's or, more specifically, of Hénin and Michel (1982a and b) optimizing model. Next section will be devoted to the general case where both agents are optimizing over time.

The economy produces a single commodity at a rate $Y(t)$, using two factors, labor $L(t)$ and capital $K(t)$. The production function F is concave, differentiable and homogenous of degree one:

$$Y(t) = F(K(t), L(t)).$$

Assuming that firms are carrying no inventories, output is either consumed $(C(t))$ or invested $(I(t))$

$$Y(t) = C(t) + I(t).$$

The evolution of the capital stock is given by

$$\dot{K}(t) = I(t) - \delta K(t)$$

where $\dot{K}(t)$ is the rate of change of the capital stock, which decays at a rate δ. Labor supply $L^s(t)$ is exogenous and increases at a given natural rate n

$$L^s(t) = L_0 e^{nt}.$$

Households choose the path of the saving rate $s(t)$ (and hence of investment, and the capital stock) so as to maximize[1] discounted utility streams:

$$\int_0^\infty e^{-(r-n)t} U(c(t))\, dt \tag{2.1}$$

subject to

$$\dot{k}(t) = s(t)F(k(t), l(t)) - (\delta + n)k(t) \tag{2.2}$$

$$0 \leq s(t) \leq 1; \quad l(t) \text{ is exogenous} \tag{2.3}$$

where $c(t) = (1 - s(t))Y(t)/L^s(t)$, $k(t) = K(t)/L^s(t)$ and $l(t) = L(t)/L^s(t)$; $U(\cdot)$ is concave, differentiable and satisfies $U'(\cdot) > 0$, $U'(0) = +\infty$. This is the usual optimal growth model, in which labor demand $l(t)$ is exogenous to the decisions of households, and full employment does not necessarily prevail.

To solve the model, consider the following Hamiltonian function:

$$H^H = U[(1-s)F(k, l)] + q^H[sF(k, l) - (\delta + n)k] \tag{2.4}$$

where q^H is the costate variable associated with the evolution of the capital stock, with:

$$\dot{q}^H = (r + \delta)q^H - F_k'(k, l)[(1-s)U'(\cdot) + q^H s]. \tag{2.5}$$

Households decide on their saving rate $s(t)$ by maximizing H^H over $s(t)$, $0 \leq s(t) \leq 1$, leading to:

$$s(t) = 0 \quad \text{and} \quad U'(\cdot) \geq q^H(t) \tag{2.6a}$$

or

$$0 < s(t) < 1 \quad \text{and} \quad U'(\cdot) = q^H(t). \tag{2.6b}$$

[1] See Arrow and Kurz (1970) for a justification of discounting the utility of per capita consumption at the rate $(r-n)$.

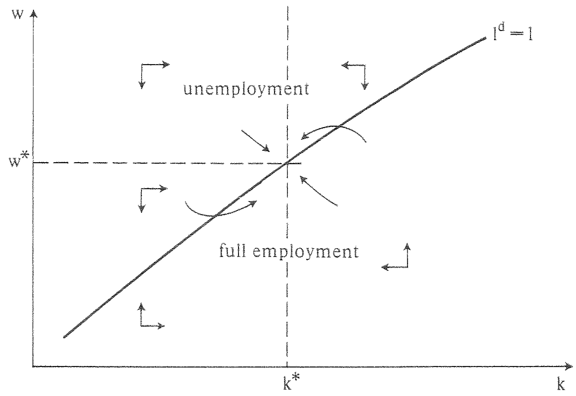

FIG. 1. Dynamics of the simple growth model.

For interior solutions, (2.5) can be written:

$$\dot{q}^H = [r + \delta - F'_k(k, \dot{l})]q^H. \tag{2.7}$$

While the saving rate is decided by households, firms simply choose at every time t the level of employment $l(t)$ so as to maximize current profits

$$F(k(t), l(t)) - w(t)l(t) \tag{2.8}$$

subject to

$$0 \leq l(t) \leq 1; \quad \text{for given } k(t) \text{ and } w(t). \tag{2.9}$$

This leads to the usual static optimality condition

$$F'_l(k, l) = w(t) \tag{2.10}$$

which defines the optimal labor demand $l^d(t)$. Effective employment is $l(t) = \min\{l^d(t), 1\}$.

The locus of equilibrium points on the labor market can be defined by the $F'_l(k, 1) = w$ curve represented in Fig. 1. In this simple model, there are only two possible disequilibrium regimes: there is unemployment above the curve, for $l^d(t) < 1$, and excess demand for labor under the curve where $l^d(t) > 1$.

If one assumes a competitive adjustment for the wage rate[2]

$$\dot{w}(t) = \lambda[l^d(t) - 1], \qquad \lambda > 0, \qquad w(0) = w_0 \tag{2.11}$$

dynamics can be studied as follows.

First, on the steady state path, $q^H(t) = \dot{w}(t) = 0$ and from (2.7), (2.10) and

[2] While the assumption of a competitive adjustment of wage is theoretically important for the long run convergence towards the market clearing value, the particular linear formulation (2.11) is inessential, and may be seen as a linear approximation of a more general adjustment process.

(2.11) one has

$$F'_k(k, l) = r + \delta; \qquad F'_l(k, l) = w; \qquad l = 1$$

which defines a point (k^*, w^*) on the curve $l = 1$ in the (k, w)-plane. The wage rate increases or decreases according to whether $l^d(t) > 1$ or $l^d(t) < 1$. The capital stock increases or decreases according to whether $k(t) < k^*$ or $k(t) > k^*$. The dynamic behavior, represented in Fig. 1, is similar to that of an economy with constant saving rate,[3] like in Ito (1978). This is not surprising, since in our model, the endogenous saving rate converges to $s^* = (\delta + n)k^*/F(k^*, 1)$.

3. A dynamic dual decision extension

The model studied in Section 2 is not symmetric, since households behave dynamically, whereas the behavior of firms is myopic. One could of course study the situation in which the behavior of firms is intertemporal, while households make short term decisions.

An approach in which both agents behave dynamically is necessary to deal with the dynamics of the dual decision hypothesis. The growth path of the economy is then the result of two optimizing agents, in which each agent has perfect foresight and takes the decisions of the other agent as given.

Therefore, the model involves basically an intertemporal Nash equilibrium because each agent does not take into account the effect of his own decisions on other's behavior.

We consider a model in which

(a) households decide on their saving rate (but not on total savings or consumption), and take as given future employment and future rates of output;

(b) firms have perfect foresight of future saving rates, and decide on the level of employment and the rate of output. As a consequence, they influence the level of investment, and capital is not given anymore, like in the simple model of Section 2.

The optimizing behavior of consumers is thus given by (2.1)–(2.3), while the problem of firms is[4]

$$\max \int_0^\infty e^{(r-n)t}\{[1 - s(t)]F(k(t), l(t)) - w(t)l(t)\}\,dt \tag{3.1}$$

[3] The only difference between the two models is that in figure one, capital per head k converge directly towards k^* in every regime while in Ito's model, k decrease in the North-West quadrant of the (w, k) plane, which precludes any direct convergence to equilibrium and requires a much larger decrease in the real wage as to restore full employment. This difference provides an interesting illustration of the cost in terms of adjustment of an exogeneous saving rate in the place of an optimally flexible one.

[4] The fact that the criterion for firms is different from the utility function of households may be justified on the grounds that firms' ownership is unequally distributed across households. Owing to the homogeneity of degree one of the profit function, the expression (3.1) where profit per head is discounted at the rate $(r - n)$ is identical to discounting the level of profits at rate r.

subject to

$$\dot{k}(t) = s(t)F(k(t), l(t)) - (\delta + n)k(t) \tag{3.2}$$

$$0 \leq l(t) \leq 1; \quad s(t) \text{ and } w(t) \text{ are exogenous.} \tag{3.3}$$

The economy follows a path defined by the simultaneous solution of problems (2.1)–(2.3) and (3.1)–(3.3). Since we assume that agents are rational, the constraints they anticipate are identical with the actual constraints obtained by solving (2.1)–(2.3) and (3.1)–(3.3).

Define the Hamiltonian functions of problems (2.1)–(2.3) and (3.1)–(3.3) as functions of the decisions of each agent, the decisions of the other being given:

$$\begin{aligned} H^H &= U[(1-s)F(k, l)] + q^H[sF(k, l) - (\delta + n)k] \\ H^F &= (1-s)F(k, l) - wl + q^F[sF(k, l) - (\delta + n)k] \end{aligned} \tag{3.4}$$

where q^H and q^F are the costate variables associated with the evolution of the capital stock in the households' and firms' problems respectively, with

$$\begin{aligned} \dot{q}^H &= (r+\delta)q^H - F'_k(k, l)[(1-s)U'(\cdot) + q^H s] \\ \dot{q}^F &= (r+\delta)q^F - F'_k(k, l)[(1-s) + q^F s]. \end{aligned} \tag{3.5}$$

Households decide on their saving rate $s(t)$, given by max H^H over $s(t)$, $0 \leq s(t) \leq 1$, leading to

$$s(t) = 0 \quad \text{and} \quad U'(\cdot) \geq q^H(t) \tag{3.6a}$$

or

$$0 < s(t) < 1 \quad \text{and} \quad U'(\cdot) = q^H(t). \tag{3.6b}$$

Likewise, firms decide on their demand for labor $l(t)$ given by max H^F over $l(t)$, $0 \leq l(t) \leq 1$, leading to

$$l(t) = 1 \quad \text{and} \quad v(t)F'_l(\cdot) \geq w(t) \tag{3.7a}$$

or

$$0 < l(t) < 1 \quad \text{and} \quad v(t)F'_l(\cdot) = w(t) \tag{3.7b}$$

where

$$v(t) = 1 - s(t) + q^F(t)s(t) \tag{3.8}$$

is the value for firms of one unit of output, obtained as a weighted average of 1, the value of one unit sold to households, and q^F the value of one unit of investment (the weights are evidently $1-s$ and s).

The notional employment rate of firms $l^d(t)$ is obtained by solving (3.7b) for $l(t)$; (3.7b) equalizes the marginal productivity of labor $F'_l(\cdot)$ valued at $v(t)$, with the real wage rate $w(t)$. The usual marginal productivity condition $F'_l(\cdot) = w(t)$, corresponding to a myopic behavior, is optimal only if $v(t) = 1$, implying $q^F(t) = 1$ or $s(t) = 0$. The effective employment rate is given by $l(t) = \min\{l^d(t), 1\}$.

It is straightforward to analyze temporary equilibria on the labor market: there is unemployment if $l^d(t) = l(t) < 1$, excess demand if $l^d(t) > 1$ and equilibrium if $l^d(t) = 1$.

On the market for goods, there can be no discrepancy between effective demand and supply.[5] If, by assumption, households are never rationed, firms may however perceive constraints: they may be willing to sell more (which corresponds to excess supply) or to invest more (excess demand) since saving (and consumption) is decided jointly by households and firms. The possibility of adjustment by varying the level of inventories is ruled out by assumption. The two regimes can be characterized as follows:

(i) excess supply: when $v(t) < 1$, it is easy to see from (3.8) that we must have $s(t) > 0$ and $q^F(t) < 1$. Since $\partial H^F / \partial s(t) = (q^F(t) - 1)F(k, l)$ is then negative, firms are willing to invest less, or, in other words, they would find profitable an increase in households' consumption. To reduce further undesired capital accumulation, firms have to reduce their demand for labor, obtained by (3.7b) as $l^d(w(t)/v(t)) < l^d(w(t))$ for given $w(t)$ and $v(t) < 1$. There is thus a negative spillover of the effective demand constraint (excessive saving rate) on labor demand;

(ii) excess demand: when $q^F(t) > 1$, $\partial H^F / \partial s(t) > 0$ and an increase in the saving rate would allow firms to increase their discounted profits by investing more. If $s(t) > 0$, $v(t) > 1$ and the labor demand $l^d(w(t)/v(t))$ is larger than in the myopic case $l^d(w(t))$; there is thus a positive spillover of the goods market (which is rationed) on the labor market.

The various temporary equilibrium regimes obtained are similar to the ones obtained in the standard case; these similarities should however not overcast the specific implications of the dynamic dual decision hypothesis.

On the goods market, we have singled out the cases of an insufficient or an excessive saving ratio. These two cases point to traditional business cycle theories, with their distinction between situations of "lack of saving" and "underconsumption with excessive saving".[6] Before the Keynesian revolution, the tradition, starting with Tugan–Baranovský, and continued by Cassel (1932) and Hayek (1931), was to explain cyclical peaks by a lack of saving with respect to the needs of the process of capital deepening; these theories are very different from those based on the accelerator, which discuss overcapitalization with respect to demand. These references suggest the following interpretations of the regimes generated by our model, and represented in Fig. 2.

(i) unemployment associated with a "too high" saving rate is the analogue of Keynesian unemployment, and will be called "underconsumption unemployment";

(ii) unemployment associated with a lack of saving appears as "Hayekian unemployment", a variety of classical unemployment;

[5] This results from the absence of money; this is also the case in Solow's (1956) and Ito's (1980) models of which ours is a straightforward generalization.

[6] See e.g. Haberler (1943).

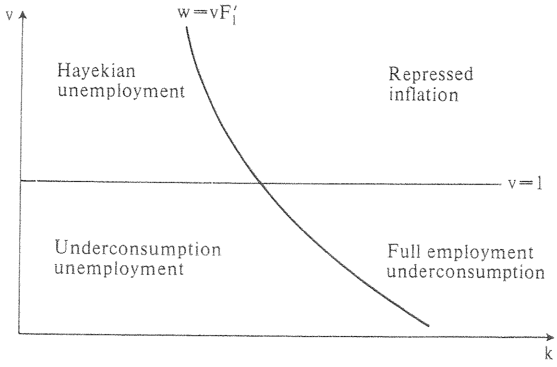

FIG. 2. Representation of temporary equilibria.

(iii) excess labor demand together with excessive saving is termed "full employment underconsumption", close to the "underconsumption" situation of the standard model;

(iv) excess demand on the labor market while firms experience rationing on investment possibilities, corresponds to the "repressed inflation" regime.

In Fig. 2, the horizontal line $v(t) = 1$ separates the "excess demand" $(v(t) > 1)$ region from the "excess supply" region $(v(t) < 1)$ on the goods market; since $F''_{kl}(\cdot) > 0$, $v(t) = w(t)F'_l(\cdot)$ is a decreasing function of $k(t)$, which separates unemployment from excess demand on the labor market.

The dynamics of the model, for a given initial capital stock $k(0)$ are fully defined by the differential equations associated with the two optimization problems of households and firms

$$\dot{k}(t) = s(t)F(k(t), l(t)) - (\delta + n)k(t) \tag{3.9}$$

$$\dot{q}^H(t) = (r+\delta)q^H(t) - F'_k(k(t), l(t))[(1-s(t)U' + s(t)q^H(t)] \tag{3.10}$$

$$\dot{q}^F(t) = (r+\delta)q^F(t) - F'_k(k(t), l(t))[1-s(t) + s(t)q^F(t)] \tag{3.11}$$

to which we add an equation explaining wage adjustments as a function of the excess demand for labor

$$\dot{w}(t) = \lambda[l^d(t) - 1], \qquad \lambda > 0, \qquad w(0) = w_0 \tag{3.12}$$

while $s(t)$ and $l(t)$ satisfy the optimality conditions (3.6)–(3.7).

We first examine the steady state solution $(\bar{k}, \bar{s}, \bar{q}^F, \bar{q}^H, \bar{l}, \bar{w}, \bar{v})$ and show that, if it exists, it is uniquely determined and satisfies all the optimality conditions. Indeed, for $\dot{w} = 0$, $l^d = \bar{l} = 1$; for $\bar{k} > 0$ and $\dot{k} = 0$, it follows from (3.9) that $\bar{s}F(\cdot) = (\delta + n)\bar{k}$ and $\bar{s} > 0$; then, from (3.6b), we have $\bar{q}^H = U'(\cdot)$; replacing this in (3.10) with $\dot{q}^H = 0$ leads to $F'_k(\cdot) = r + \delta$ (which is the usual condition on a stationary path); this in turn defines a unique value of $\bar{k}$ and $\bar{s}$ (using $\bar{s}F = (\delta + n)\bar{k}$), and of $\bar{q}^H$. Replacing $F'_k = r + \delta$ in (3.11) gives $\bar{q}^F = 1$, and hence, using (3.8), $\bar{v} = 1$; finally, from (3.7b) it follows that $\bar{w} = F'_l(\bar{k}, \bar{l})$.

Under the usual concavity assumptions on the utility and production functions, $U(\cdot)$ and $F(\cdot)$, the stationary solution is unique and is an optimal solution for the consumers and the firms.

Any trajectory of the system (k, q^F, q^H, w) which converges to the unique stationary state is an intertemporal equilibrium of the economy. Indeed, when the economy is following such a trajectory, for each agent the path of the variables under his control is optimal for a given path of uncontrolled variables: it satisfies the sufficient conditions of Arrow and Kurz.

It is not possible to give a picture of the full dynamics of such trajectories lying in a four-dimensional space. Moreover, the dynamics of the model will in general involve changes in regimes. At these points, a change in differential equations occurs. Thus, the four state variables, either predeterminated like k and w, or forward looking as q^F and q^H are continuous but their time derivatives are not continuously differentiable functions. There are no stability conditions available for such systems (Honkapojha et Ito 1983). Only for paths belonging for ever to a same regime do standard stability conditions apply.[7]

In the present case, one may hazard the following appreciation: On the one hand, the transversality conditions are stability conditions for forward looking variables. On the other hand, the dynamics of pre-determined k and w is similar to this one represented in Fig. 1. This gives some intuitive support to the idea that stability prevails.

4. Concluding comments

The model discussed provides a dynamic extension of the basic static dual decision hypothesis, and a more satisfactory formulation of a disequilibrium version of the neoclassical growth model.

Like in the Solow–Ito tradition, money is not explicitly introduced, and there is no possibility for effective (or ex post) disequilibria to appear on the goods market; however, the value of implicit prices provides information on notional (or ex ante) disequilibria, and restores the existence of four regimes of temporary equilibria, instead of two as is the case in Ito's model.

When the assumption is made that the wage rate adjusts to excess demand for labor, disequilibria will only be temporary, and it appears that full employment growth is the only sustainable steady state.

Finally, by assuming non-competitive wage adjustments, like Pitchford and Turnovsky (1978) or Picard (1982) it is possible to obtain non-Walrasian steady states, with lasting unemployment; this is clearly a decisive step away from neoclassical conclusions, using a neoclassical framework.

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[7] According to the now classical stability conditions, as two state variables are predeterminated, k and w, and two are forward looking, q^H and q^F, the convergence towards the steady state requires that, among the roots of the linearized system, two have a positive real part and two have a negative one.

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