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Nuclear Physics B 484 (1997) 419–434



Black hole horizon fluctuations

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Received 19 June 1996; accepted 28 October 1996

Abstract

Some authors have suggested that gravitational interactions become large at an invariant distance of order 1 from the black hole horizon. We show that due to the “atmosphere” of high angular particles near the horizon strong gravitational interactions already occur at an invariant distance of the order of $\sqrt[3]{M}$. The implications of these results for the origin of black hole radiation, the meaning of black hole entropy and the information puzzle are discussed.

PACS: 04.70.Dy

Keywords: Black hole; Hawking radiation

1. Introduction

There are at least three related unsolved problems concerning quantum black hole physics: (i) the origin of the Hawking radiation [1]; (ii) the meaning of the black hole entropy [2–4]; (iii) the information loss puzzle [5].

All of these issues are connected to the large red shift near the horizon which entails the appearance in the free field theory of transplanckian frequencies [6–8].

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On the one hand it has been argued that the transplanckian frequencies are only a coordinate artefact and that the field state is regular at the horizon in a global coordinate system. This is the basis of Hawking's conclusion that black hole evaporation is a non-unitary process [5].

On the other hand, the appearance of transplanckian frequencies has led several authors to suggest that even though the Riemann tensor near the horizon is small, $R_{\text{hor}} \simeq \frac{1}{M^2} \ll 1$,⁶ there are strong interactions near the horizon [9]. As originally stressed by 't Hooft this puts into question Hawking's original derivation of black hole radiance and suggests the possibility that the spectrum of emitted particles could differ from the exact thermal spectrum. This in turn could imply that the information is encoded in the correlations between Hawking quanta [9]. Another possibility is that the information is encoded in the correlations between Hawking radiation and a remnant [10]. Furthermore, the transplanckian frequencies are also related to the divergence of the field entropy near the horizon [11]. The strong interactions near the horizon suggest that there is a dynamical cutoff near the horizon [12]. A concrete realization of this idea is the concept of the stretched horizon [13] which is a very hot membrane, just outside the event horizon, that can absorb, thermalize, and emit information.

Several arguments suggest that the strong gravitational interactions occur at an invariant distance of the order of 1 (by invariant distance we mean the distance on constant t surfaces $\rho = \int_{2M}^r ds \simeq \sqrt{8M(r-2M)}$):

- (1) In various works not directly related to black hole physics, it was claimed that the minimal scale in quantum gravity is 1 [14]. If so then it is meaningless to describe the region near the horizon with accuracy larger than 1.
- (2) The local temperature is $T_{\text{loc}} = \frac{1}{\rho}$, so for $\rho = 1$ the temperature is planckian for which the conventional description of physics is probably incorrect.
- (3) The mean time between successive emissions of Hawking quanta is M and the energy of the emitted particle is $\frac{1}{M}$. This means that as a quantum mechanical system the black hole has a width $\frac{1}{M}$. The invariant distance between $R = 2(M - \frac{1}{M})$ and $R = 2M$ in the gravitational background of a black hole with mass $M - \frac{1}{M}$ is 1.
- (4) The number of high angular momentum particles between ρ and $r \simeq 3M$ is $N(\rho) \simeq M^2/\rho^2$ thus for $\rho = 1$ the entropy of these particles is of the order of the Bekenstein–Hawking entropy [11].

However, several authors have suggested, using different approaches, that the gravitational interactions become strong at invariant distances ρ much larger than 1: York considered quantum fluctuations of the gravitational normal modes [15], Jacobson's arguments were based on thermodynamic analogies [8], Sorkin considered the vacuum fluctuations of a scalar field [16], and Englert argued that gravitational effects which would tame the transplanckian fluctuations had to occur at a distance much larger than $\rho = 1$: they would simultaneously invalidate the conventional mechanism of Hawking emission and prevent, at least in the reconstructed history available to the external ob-

⁶ In units where $G = c = \hbar = 1$.

server, the collapse of the star [17]. However, all of these arguments are based on some questionable assumptions which have not been widely accepted.

The aim of this article is to address the above debate. Our analysis is based on the properties of the atmosphere of high angular momentum particles which surround the horizon. Let us recall that this atmosphere arises in the re-expression of the Unruh vacuum state (the state of the field after the radiation has settled into its steady state [18]) as a thermal density matrix of Schwarzschild quanta:

$$|0_U\rangle = \prod_{\omega,l,m} \exp - \left(e^{-8\pi\omega M} a_{\omega,l,m}^\dagger b_{\omega,l,m}^\dagger \right) |0_B\rangle, \tag{1}$$

where $|0_U\rangle$ is the Unruh vacuum and $|0_B\rangle$ is the Boulware vacuum. The operator $a_{\omega,l,m}^\dagger$ creates an outgoing Schwarzschild quantum of energy ω and angular momentum l, m ; $b_{\omega,l,m}^\dagger$ creates the partner of this Schwarzschild quantum and lives beyond the horizon. Upon tracing over these partners one obtains that Unruh vacuum is a thermal density matrix for the Schwarzschild quanta, with temperature $(8\pi M)^{-1}$.

We present three complementary arguments which show that due to the presence of this atmosphere gravitational interactions become large at $\rho \simeq \sqrt[3]{M}$. Therefore, at this scale Hawking’s assumptions of a free field propagating on a given classical background breaks down. In particular the decomposition (1) which results from these hypothesis will no longer be valid for $\rho < \sqrt[3]{M}$.

Our first argument is thermodynamic in character and relies only on the thermal energy fluctuations in the atmosphere and on the gravitational analogue of Gauss’s law. The second argument is based on the gravitational interactions between an incoming particle and the atmosphere, and shows that these interactions can no longer be neglected for $\rho < \sqrt[3]{M}$. The third argument is concerned with the unitarity problem. We show that the information carried by an incoming particle gets encoded in the state of the atmosphere at $\rho \simeq \sqrt[3]{M}$. These arguments all lead to the same minimal distance $\rho = \sqrt[3]{M}$.

The appearance of strong fluctuations at scales much larger than $\rho = 1$ has important implications for the origin of the Hawking radiation, the interpretation of the black hole entropy and the unitary problem. These aspects are discussed in the last section of the paper.

2. Horizon fluctuations

We first review the properties of the high angular momentum particles which make up the atmosphere. To this end we recall the wave equation for a scalar field in the Schwarzschild metric,

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r_*^2} \right) \phi + \left(1 - \frac{2M}{r} \right) \left(\frac{2M}{r^3} + \frac{l(l+1)}{r^2} \right) \phi = 0, \tag{2}$$

where $r^* = r + 2M \ln(\frac{r-2M}{2M})$. The centrifugal barrier is attractive for $r < 3M$. This means that particles with high angular momentum can be trapped in the region between

the horizon and $r < 3M$. The tunneling through the angular momentum barrier may be neglected for all but the lowest angular momentum modes [19]. From Eq. (2) we see that a particle can reach a radius r only if

$$\omega^2 > \left(1 - \frac{2M}{r}\right) \left(\frac{2M}{r^3} + \frac{l(l+1)}{r^2}\right), \quad (3)$$

where ω is the Schwarzschild energy, i.e. the eigenvalue of $i\partial_t$. For Hawking radiation the typical energy is $\omega \simeq \frac{1}{M}$. This implies

$$l^2 \lesssim \frac{M^2}{\rho^2}, \quad (4)$$

where ρ is the invariant distance from the horizon in Schwarzschild coordinate, namely,

$$\rho = \int_{2M}^R ds = \int_{2M}^R \frac{dr}{\sqrt{1 - \frac{2M}{r}}} \simeq \sqrt{8M(R - 2M)}. \quad (5)$$

Since the degeneracy for each l is $2l + 1$, the number of modes that can reach ρ is

$$N(\rho) \simeq \frac{M^2}{\rho^2}. \quad (6)$$

As mentioned in Section 1, all of these particles are in a thermal distribution at the global Hawking temperature $1/8\pi M$. The average number of particles in the thermal atmosphere is estimated by noting that the emission rate for each mode is $1/M$ and the time it takes for a photon which passes ρ to fall back to ρ after being reflected by the centrifugal barrier is $O(M \ln M)$. Thus the average number of particles above a given ρ is, up to a logarithmic factor which we neglect, $N(\rho)$.

Using the above estimate, the average energy and entropy of the thermal atmosphere situated above a given ρ are

$$\begin{aligned} \langle E \rangle &\simeq N(\rho) \frac{1}{M} \simeq \frac{M}{\rho^2}, \\ S &\simeq N(\rho) \simeq \frac{M^2}{\rho^2}. \end{aligned} \quad (7)$$

These qualitative results as well as the fluctuation of the energy estimated below have been obtained in quantitative detail using the brick wall model, i.e. evaluating the partition function of the Schwarzschild modes in the WKB approximation [11].

The entropy of the atmosphere diverges as $\rho \rightarrow 0$. This led 't Hooft to suggest that there is a cutoff at $\rho = 1$ so that the entropy of the atmosphere would coincide with the Bekenstein–Hawking entropy.

The average energy $\langle E \rangle$ also diverges as ρ tends to zero. This is intimately related to the renormalization of the energy momentum in Schwarzschild background. Indeed one can show [20] that the renormalized energy density, as seen in the frame of an infalling observer, is finite in the Unruh vacuum. This is because the divergence of

$\langle E \rangle$ in Eq. (7) is compensated by the negative and divergent mean energy density in the Boulware vacuum (the state containing no Schwarzschild particles). Thus, after renormalization, $\langle E^{\text{ren}} \rangle$ is finite and of the order of the Hawking flux $1/M$. Note that the finiteness of $\langle E^{\text{ren}} \rangle$ depends on the fact that each particle in the thermal bath is correlated to a partner as in Eq. (1), i.e. it depends on the state of the field on both sides of the horizon. We shall assume in this paper that, as predicted by the semiclassical theory, $\langle E^{\text{ren}} \rangle$ is indeed finite.

On the other hand, the thermal energy fluctuations of the atmosphere are not affected by the renormalization since they are associated with the decomposition of the Unruh vacuum as a thermal density matrix of Schwarzschild quanta and each term in this decomposition has physical significance. These thermal fluctuations are proportional to the square root of the number of particles in the thermal atmosphere as for any thermodynamic system. Therefore, the uncertainty of the Schwarzschild energy in the region between ρ and $R \simeq 3M$ is

$$\Delta M \simeq \frac{1}{M} \sqrt{N(\rho)} \simeq \frac{1}{\rho}. \quad (8)$$

Since the total energy of the black hole is fixed to be M , the uncertainty of the Schwarzschild energy between $r = 0$ and ρ is also $\frac{1}{\rho}$. Note that this uncertainty is dynamical. Indeed since $N(\rho)$ particles cross the surface ρ in a time $\Delta t = M$, the time scale over which the mass fluctuates is also $\Delta t = M$.

This uncertainty is much larger than $\langle E^{\text{ren}} \rangle$ and is much larger than the uncertainty due to the emission of s-waves (see point (3) in Section 1). It implies the existence of strong gravitational interactions at $\rho = \sqrt[3]{M}$. To see this let us first estimate how the fluctuating mass gives rise to uncertainty in the location of the horizon. A point r_0 is outside the horizon if

$$\delta(r_0) = r_0 - 2M(r_0) > 0, \quad (9)$$

where $M(r_0)$ is the Schwarzschild energy between $r = 0$ and $r = r_0$. From Eq. (8) we find that

$$\Delta \delta(r_0) = 2\Delta M \simeq \frac{2}{\rho} = \frac{2}{\sqrt{8M\delta}}. \quad (10)$$

Clearly, if $\Delta \delta(r_0) > \delta(r_0)$ then the point r_0 is in a superposition of being inside and outside the horizon. From Eqs. (10), (5), (9) this implies that the minimal ρ for which it is certain that the point is outside the horizon is

$$\rho_{\text{min}} \simeq \sqrt[3]{M}. \quad (11)$$

The quantum fluctuations smear the horizon on an invariant distance of the order of $\sqrt[3]{M}$ which is much larger than 1.

This suggests that the gravitational interactions with the atmosphere become large at $\rho = \sqrt[3]{M}$ and that the assumptions of a free field propagating on a given classical background breaks down at this scale. We illustrate this by inserting the mass fluctuation

ΔM into the Schwarzschild metric. Near the horizon one then obtains an equation of the form

$$\partial_t^2 - \left(1 - \frac{4\Delta M}{r-2M}\right) \partial_{r_*}^2 + \left(1 - \frac{2\Delta M}{r-2M}\right) \left(\frac{r-2M}{2M}\right) \left(\frac{l(l+1)+1}{M^2}\right) = 0. \quad (12)$$

The perturbation is negligible as long as

$$\frac{\Delta M}{r-2M} = \frac{M}{\rho^3} > 1, \quad (13)$$

which yield $\rho > \sqrt[3]{M}$ as above. For smaller ρ the perturbation cannot be neglected. Furthermore, since ΔM varies over time scales of the order of the inverse particle energy, the solution of Eq. (12) will contain both positive and negative frequencies below $\rho = \sqrt[3]{M}$ so the number operator $a^\dagger a$ has uncertainty of order one.

The analysis in this section was based on the thermodynamics of the fluctuating atmosphere. Gauss's law then implied that gravitational interactions occur on scales $\rho = \sqrt[3]{M}$. The main drawback of these arguments is that they treat ΔM as a classical source in Einstein's equation rather than quantum fluctuation. However, in the next section we shall recover this characteristic length $\rho = \sqrt[3]{M}$ using a completely different approach.

3. The gravitational interactions

In this section we shall show that strong gravitational interaction occurs at $\rho = \sqrt[3]{M}$ by studying the gravitational interaction between an infalling particle and the thermal atmosphere. Although the interaction between each particle is small, there are approximately $N(\rho)$ such interactions which sum up incoherently. Therefore, the total probability of scattering is proportional to $\sqrt{N(\rho)}$. The final result is that at $\rho = \sqrt[3]{M}$ the total scattering probability is of order 1. However, due to technical difficulties, at the present time we have only investigated in detail the interaction between an infalling s-wave and the high angular momentum particles of the atmosphere. Similar results may hold for the interaction among the high angular momentum particles themselves.

The detailed calculation is carried out in Appendix A. Here we summarize the results. As shown in Refs. [21,22], the semi-classical gravitational effects of a massless particle can be obtained using the gravitational shock wave (the corresponding scattering amplitude coincides up to a phase with one graviton exchange [23]). We have considered the shock wave of an infalling particle with energy E and its effect on the high angular momentum particles which constitute the atmosphere. The probability that one particle of the atmosphere be in the same state after crossing the shock wave is

$$P_1 \simeq 1 - \frac{M^2 \lambda^2}{\rho^4}, \quad (14)$$

where λ is the energy of the infalling particle. Note that this result is only valid when the gravitational interaction is weak, otherwise the shock wave interaction breaks down. From Eq. (14), the interactions are weak when $\lambda < \rho^2/M$. This condition can also be understood by noting that an infalling particle of energy λ causes a shift in the horizon radius $\delta r_H = \lambda$, which corresponds to an invariant distance $\rho = \sqrt{\lambda M}$.

The number of particles which are affected by the shock wave of the ingoing particles when it reaches ρ is given by $N(\rho)$, so the probability for the “atmosphere” above ρ to be in the same state is

$$P^{\text{tot}} = P_1^{N(\rho)} = \left(1 - \frac{M^2 \lambda^2}{\rho^4}\right)^{N(\rho)} \simeq e^{-M^4 \lambda^2 / \rho^6}, \tag{15}$$

this means that for

$$\rho < \lambda^{1/3} M^{2/3} \tag{16}$$

the probability for the “atmosphere” to remain in the same state decreases exponentially.

In Appendix A we also show that the probability for the angular momentum of one particle of the atmosphere not to have changed coincides with P_1 , i.e.

$$P_{\Delta l=0} \simeq 1 - \frac{\lambda^2 M^2}{\rho^4} = P_1. \tag{17}$$

Hence proceeding as from Eq. (14) to Eq. (16), the angular momentum of the atmosphere is modified by one unit when the particle reaches $\rho \simeq \lambda^{1/3} M^{2/3}$.

Note that at $\rho = \lambda^{1/3} M^{2/3}$ (Eq. (16)), the probability that any individual high angular momentum particle be scattered is (see Eq. (14))

$$1 - P_1 \simeq (\lambda/M)^{2/3} \ll 1. \tag{18}$$

The weakness of the gravitational interaction justifies the semi-classical treatment of the gravitational interaction.

The minimal λ one can consider is $\frac{1}{M}$ since otherwise the wavelength of the ingoing particle is larger than the radius of the black hole. Therefore, at $\rho = \sqrt[3]{M}$ all ingoing particles have interacted strongly with the atmosphere and have acquired one unit of angular momentum. Although we have not been able to show it rigorously at this stage, we expect that the ingoing particle will also be scattered by the atmosphere (the principle of action and reaction) and that the high angular momentum particles that make up the atmosphere will be strongly self-interacting at $\sqrt[3]{M}$. This was indicated by the analysis of the end of Section 2 wherein we naively plugged the fluctuating mass into the Klein–Gordon equation and estimated its effect on the propagation of a mode.

4. The information problem

In this section we consider the implications of our results for the S -matrix ansatz proposed by 't Hooft. Since the interactions are strong at $\rho = \sqrt[3]{M}$ this distance should

play a crucial role in the information problem. Indeed, we shall show that the information of an ingoing massless charge-less spin-less particle is encoded in the state of the atmosphere when the particle reaches $\rho = \sqrt[3]{M}$.⁷

Such a particle is characterized by its energy and angular momentum. We first consider how the information about its energy is transmitted and then turn to the angular information. Consider an incoming particle in an s-wave Ψ whose energy is λ with uncertainty $\Delta\lambda$. This wave packet is spread out over an interval $\Delta t \simeq 1/\Delta\lambda$. From the previous section we know that the incoming particle will start interacting with the particles in the atmosphere at $\rho = \lambda^{1/3}M^{2/3}$. The particles in the atmosphere fall back towards the horizon after a time interval $\Delta t \simeq M$. Hence the interaction of the incoming particle with any individual particle in the atmosphere lasts a time $\Delta t \simeq M$. One should therefore decompose Ψ into a complete orthogonal set of wave packets whose uncertainty in energy is $\Delta\lambda = 1/M$ and that are spread out over a time $\Delta t \simeq M$. We study how the information about the energy of two such wave packets is encoded in the atmosphere.

Consider two particles whose wave packets have mean energy λ_1 and λ_2 and energy spread $\Delta\lambda_1 \simeq \Delta\lambda_2 \simeq 1/M$. Since these particles are orthogonal we also have $\lambda_2 - \lambda_1 > 1/M$. We want to determine at what ρ the state of the atmosphere when the energy of the ingoing particle is λ_1 is orthogonal to the state of the atmosphere when the energy of the ingoing particle is λ_2 . To find this ρ we recall that the effect of the shock wave of an incoming particle on a given particle in the atmosphere is a discontinuity in the v direction. Using Eqs. (A.4) and (A.15), we get

$$\Delta v \simeq \frac{\lambda M}{\rho} \log\left(\frac{\tilde{x}}{M}\right), \quad (19)$$

where \tilde{x} is the transverse distance between the ingoing particle and a particle in the atmosphere (see Appendix A). Except when treating problems that explicitly involve the angular momentum of the scattered particles, one can drop the logarithmic dependence of Eq. (19). Then the difference in the discontinuity is only due to $\lambda_1 \neq \lambda_2$:

$$\Delta v_1 - \Delta v_2 \simeq \frac{(\lambda_1 - \lambda_2)M}{\rho}. \quad (20)$$

From Eq. (A.19) one obtains that the probability that the scattered state of the particle in the atmosphere is the same (i.e. does not depend on whether the energy of the ingoing particle is λ_1 or λ_2) is

$$P_1 \simeq 1 - \frac{M^2 \Delta\lambda^2}{\rho^4}, \quad (21)$$

⁷ We do not consider in this paper how the information about the internal degrees of freedom of the particle, i.e. its species, its spin state, etc., are transmitted to the atmosphere. The answer to this question is not clear at the moment since the information is transmitted via gravitational interaction (all other interactions are too small) and the internal degrees of freedom couple weakly to gravity.

where $\Delta\lambda = \lambda_1 - \lambda_2$. The number of particles which are affected by the shock wave is $N \simeq M^2/\rho^2$ (Eq. (6)) so the probability that the scattered state of the whole atmosphere is the same is

$$P = P_1^{N(\rho)} \simeq e^{-M^4 \Delta\lambda^2/\rho^6}. \tag{22}$$

Since $\Delta\lambda > 1/M$ we obtain that when the ingoing particle crosses $\rho = \sqrt[3]{M}$ the probability that the scattered state of the atmosphere is the same is exponentially small. Therefore, the information on the energy of the ingoing particle is encoded in the atmosphere at $\rho = \sqrt[3]{M}$.

Of course this is not all the information since there are orthogonal states of the ingoing particle with the same energy but different angular location/momentum. We will prove now that at $\rho = \sqrt[3]{M}$ the angular information of the ingoing particle is also encoded in the atmosphere.

Consider a particle which falls radially into a black hole along the direction Ω_1 with energy λ . Imagine now that the particle falls into the black hole with the same energy but along another direction Ω_2 with Ω_2 sufficiently different from Ω_1 so that the two initial states are orthogonal. This orthogonality condition implies that $\Delta\Omega > 1/\lambda M$, where $\Delta\Omega$ is the angle between the two directions Ω_1 and Ω_2 . We want to know at what ρ the state of the atmosphere when $\Omega = \Omega_1$ is orthogonal to the state of the atmosphere when $\Omega = \Omega_2$, i.e. at what ρ the information about the angular direction Ω gets encoded in the state of the atmosphere. Obviously, the most difficult case to distinguish is when $\Delta\Omega$ takes its minimal value

$$\Delta\Omega = \frac{1}{\lambda M}, \tag{23}$$

so we will consider that case. Since the energy in both cases is the same, λ , the difference in the shift of the i particle in the atmosphere is

$$\Delta v_1 - \Delta v_2 \simeq \frac{\lambda M}{\rho} \ln \frac{\tilde{x}_{1i}}{\tilde{x}_{2i}}, \tag{24}$$

where \tilde{x}_{1i} and \tilde{x}_{2i} is the transverse distance between the infalling particle (at Ω_1 and Ω_2) and the particle i . From Eq. (A.19) one obtains that the probability that the scattered state of particle i is the same in case 1 and 2 is

$$P_i \simeq 1 - \frac{M^2 \lambda^2}{\rho^4} \ln \frac{\tilde{x}_{1i}}{\tilde{x}_{2i}}. \tag{25}$$

The probability that the scattered state of all the atmosphere is the same is

$$P = \prod_i P_i. \tag{26}$$

To evaluate the product we need to estimate $\ln(\tilde{x}_{1i}/\tilde{x}_{2i})$. There is a lower bound on \tilde{x} , which is the size $1/\lambda$ of the wave packet of the incoming particle, and an upper bound,

which is the size of the horizon M . From Eqs. (22) it is clear that $\tilde{x}_{1i}/\tilde{x}_{2i}$ differs from 1 by an appreciable amount only if $\tilde{x}_{1i}, \tilde{x}_{2i} < \Delta\Omega M$. Thus

$$P \simeq \prod_{i \text{ such that } \tilde{x}_{1i}, \tilde{x}_{2i} < \Delta\Omega M} \left(1 - \frac{M^2 \lambda^2}{\rho^4} \right), \quad (27)$$

where we have neglected the log factor which is legitimate since \tilde{x}_i is bounded from below. Since the total number of particles in the atmosphere is $N(\rho)$, the number of particles such that $\tilde{x}_{1i}, \tilde{x}_{2i} < \Delta\Omega M$ is $\Delta\Omega^2 N(\rho)$ and one obtains

$$P \simeq \left(1 - \frac{M^2 \lambda^2}{\rho^4} \right)^{\Delta\Omega^2 N(\rho)} \simeq e^{-M^2/\rho^6}. \quad (28)$$

The condition of validity of Eq. (28) is that there are many particles which intervene in the product Eq. (27). This implies that $\lambda \leq T_{\text{loc}} = 1/\rho$. At $\rho = \sqrt[3]{M}$ this is exactly the same condition on λ as that derived after Eq. (14) ($\lambda < \rho^2/M$).

Remarkably both Eq. (22) and (28) are independent of the initial energy λ of the infalling particle. Thus, provided that $\lambda \leq M^{-1/3}$, the information about the energy and angular position of an infalling particle gets encoded in the state of the atmosphere at $\rho = \sqrt[3]{M}$. For $\lambda > M^{-1/3}$ our analysis based on the gravitational shock wave is no longer valid and we cannot reach any definitive conclusion.

5. Conclusion

The existence of a thermal atmosphere above a black hole is well known. We have shown that this atmosphere plays an essential role in the gravitational back reaction to Hawking radiation. Indeed it implies the existence of strong gravitational interactions at $\rho = \sqrt[3]{M}$ and not at $\rho = 1$ as would be naively expected. This was shown by analyzing the gravitational effects of the atmosphere.

In the first approach we estimated the thermal energy fluctuations of the atmosphere. To estimate their effects, we then inserted these fluctuations as a classical source in Einstein's equations. This shows that the horizon seems to be fluctuating on scales $\rho = \sqrt[3]{M}$. We do not know if these effects would survive in a more careful treatment of the gravitational interaction, but in any case it indicates that the propagation of particles can no longer be described by a linear quantum field for $\rho < \sqrt[3]{M}$. In particular the decomposition of the Unruh vacuum as a thermal density matrix of non-interacting particles is incorrect for $\rho < \sqrt[3]{M}$.

In a second approach we calculated how the presence of an incoming particle modifies the state of the atmosphere due to the gravitational interaction. We find that the atmosphere gets scattered to an orthogonal state before the particle reaches $\rho = \sqrt[3]{M}$. Once more this shows that the atmosphere cannot be described as a gas of non-interacting particles. However, the full implications of this result cannot be understood at present because we have not been able to estimate how the infalling particle is scattered by the atmosphere and how the particles which constitute the atmosphere interact among themselves.

We then further investigated the interaction of an infalling particle with the atmosphere and showed that the information carried by the infalling particle can get encoded in the atmosphere at $\rho = \sqrt[3]{M}$. This confirms the critical role of $\rho = \sqrt[3]{M}$.

The main criticism that one can make of our approach is that we have treated the constituents of the atmosphere as on-shell particles rather than vacuum fluctuations. Indeed we have first traced over the partners before evaluating the gravitational response. This may not be legitimate if the horizon is a regular region of space time as advocated by Hawking. It is however expected to be a valid approximation if the S -matrix ansatz of 't Hooft is correct, a fact which appears to be corroborated by the analysis of Section 4. Thus our analysis implicitly implies a restriction to the region outside the horizon. The question then arises of whether an infalling observer can cross the horizon and fall into the singularity as predicted by the semiclassical theory. This question should be addressed by using a global coordinate system rather than the Schwarzschild coordinates used in this paper. In answering it the existence of partners beyond the horizon will play a crucial role. Indeed it can be shown that in certain physical processes the presence of the partners is essential in ensuring insensitivity to the transplanckian frequencies which occur in Hawking radiation [24].⁸

Nevertheless if one restricts oneself to the region outside the horizon our analysis strongly suggests that there is a new phase at $\rho = \sqrt[3]{M}$ where gravity becomes strongly coupled to the thermal atmosphere. Thus, whereas the Hawking radiation is ignited as in the conventional free field theory, the source of the thermal radiation progressively shifts to the new phase at $\rho = \sqrt[3]{M}$. This new phase can capture information of infalling matter and this is in line with the idea that the black hole evaporation is unitary and that the black hole entropy is stored in the thermal atmosphere outside the classical horizon. It remains however to be seen whether the information about the star that collapsed to form the black hole also gets encoded in the atmosphere. Possible consistency could be achieved if the star itself does not collapse but becomes a source of the burning atmosphere [17].

Whatever the details of the physics near the horizon, the essential result of our paper is that strong gravitational interactions already occur at $\rho = \sqrt[3]{M}$ where the local temperature $T_{\text{loc}} = 1/\sqrt[3]{M}$ is small. This may have important implications for several proposed scenarios of black hole evaporation that appeal to strong interactions at much smaller distances. Indeed the S -matrix proposed by 't Hooft neglects the high angular momentum particles and relies on gravitational interactions, which are strong only at the Planck scale [9], and Susskind's picture of stringy horizons makes appeal to non-perturbative effects which should arise at the Hagedorn temperature [25]. It is still too early to understand the connection with the recent advances in the string theoretic description of black holes [26,27].

⁸ It is also interesting to note that since the analysis of Appendix A was carried out in the Rindler approximation, Rindler horizons defined over a transverse distance $L \times L$ are probably fluctuating on distances $\sqrt[3]{L}$. But in the Rindler case it is obviously possible to cross the horizon.

Acknowledgements

The authors would like to thank Y. Aharonov for his participation in a fruitful discussion which was the original impetus for this work. S.M. would like to thank T. Jacobson for discussions on related problems.

Appendix A

In this section we derive Eq. (14). First let us briefly summarize the effect of the shock wave (the full details are in Ref. [22]). The gravitational field of a massless point-like particle in Minkowski space is described by the line element [21]

$$ds^2 = -du \left[dv + 2p^v \ln \left(\frac{\tilde{x}^2}{M^2} \right) \delta(u - u_0) du \right] + dx^2 + dy^2, \quad (\text{A.1})$$

where $\tilde{x}^2 = x^2 + y^2$, $u = T + z$ and $v = T - z$. The massless particle moves in the v direction with constant u_0 and momentum p^v .

In Minkowski space there is an arbitrariness in the length scale appearing in the log which can be modified using the coordinate change $v \rightarrow v + \Theta(u - u_0)c$. Since we are using Minkowski and Rindler coordinates to approximate the Schwarzschild metric near the horizon, the curvature of the Schwarzschild metric fixes the length scale in the log to be of order M . None of our results depend on the exact value of this length scale.

The effects of such a shock wave on other particles are most easily analyzed in the action formalism. The solution of the Hamilton–Jacobi equation for a massless particle of initial momentum k_μ and propagating in the metric (A.1) is

$$S = S_0 + 2p^v \ln \left(\frac{\tilde{x}^2}{M^2} \right) k_v \theta(u - u_0) + O((u - u_0)\theta(u - u_0)), \quad (\text{A.2})$$

where S_0 is the solution in the absence of shock wave

$$S_0 = k_x x + k_y y + k_v v + \frac{k_x^2 + k_y^2}{4k_v} u. \quad (\text{A.3})$$

The effect of the shock wave is therefore a discontinuity in the v direction at $u = u_0$:

$$v_0 = \frac{\partial S}{\partial k_v} = -\frac{k_x^2 + k_y^2}{4k_v^2} u + v + 2p^v \ln \left(\frac{\tilde{x}^2}{M^2} \right) \theta(u - u_0) + O((u - u_0)\theta(u - u_0)), \quad (\text{A.4})$$

and a refraction in the transverse direction:

$$k_x(u) = \frac{\partial S}{\partial x} = k_x + \frac{4p^v}{\tilde{x}^2} x k_v \theta(u - u_0) + O((u - u_0)\theta(u - u_0)), \quad (\text{A.5})$$

and similarly for $k_y(u)$.

One further verifies that the solution of the Klein–Gordon equation in the presence of the shock wave is given by $\psi = e^{iS} [1 + O((u - u_0)\theta(u - u_0))]$. Thus the WKB approximation correctly describes the effect of the shock wave.

Let us now use these results to describe how a high angular momentum particle is affected by an incoming particle. First let us recall that near the horizon and for transverse distances smaller than M the Schwarzschild metric takes the approximate form

$$ds^2 = -\frac{\rho^2}{(4M)^2} dt^2 + d\rho^2 + dx^2 + dy^2, \tag{A.6}$$

which is simply Minkowski space

$$ds^2 = -du dv + dx^2 + dy^2 \tag{A.7}$$

in Rindler coordinates

$$\begin{aligned} u &= T + z = \rho e^{t/4M}, \\ v &= T - z = -\rho e^{-t/4M}. \end{aligned} \tag{A.8}$$

Particles in the atmosphere are massless and follow geodesics

$$X^\mu = X_0^\mu + \lambda V_0^\mu \tag{A.9}$$

with $V_0^2 = 0$. A boost in the T, z plane corresponds to a translation in Schwarzschild time. By such a translation in t and a rotation and translation in the x, y plane we can bring the trajectory to the following form (see Fig. A.1):

$$X^\mu(\lambda) = (T = -\rho_0 + \lambda, \quad y = 0, \quad x = -\rho_0 + \lambda, \quad z = \rho_0), \quad 0 < \lambda < 2\rho_0, \tag{A.10}$$

ρ_0 is the maximal ρ the particle can reach, so it is related to the angular momentum by $\rho_0 = \frac{M}{7}$ (see Eq. (4)).

The ingoing particle is moving along the line $u = u_0$. It is easy to see that the high angular Hawking quanta will cross the shock wave of the ingoing particle at

$$\rho_c^2 = 2\rho_0 u - u^2. \tag{A.11}$$

Most of the particles that reach the point ρ_0 will reach a maximal ρ of the order of ρ_0 . Therefore we are interested in

$$u \simeq \rho_0 \simeq \rho_c. \tag{A.12}$$

In order to be able to use Eqs. (A.4), (A.5) we need to relate Schwarzschild energies to Minkowski energies. Denoting Schwarzschild energy by λ we obtain

$$\lambda = -p_t = -g_{t\mu} p^\mu = \frac{\rho^2}{(4M)^2} p^t. \tag{A.13}$$

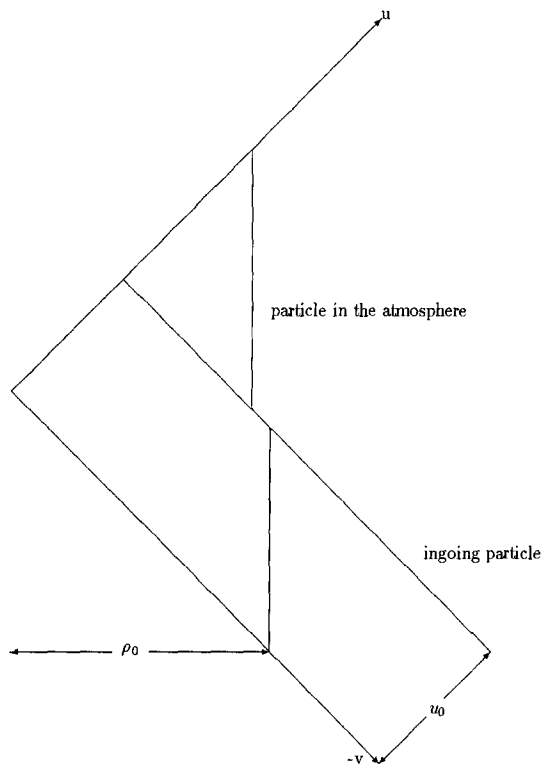


Fig. A.1. The effect of the shock wave of the ingoing particle on Hawking particle with high angular momenta is a discontinuity in v . The picture represents the trajectories projected onto the u, v plane.

Since $t = 2M \ln(u/v)$ we get

$$p_t = \frac{u}{4M} p_u - \frac{v}{4M} p_v. \tag{A.14}$$

For the incoming particle $p_v = 0$, so

$$-p_u = \frac{1}{2} p^v = \frac{4M\lambda}{u} \simeq \frac{M\lambda}{\rho_0}, \tag{A.15}$$

where we have used Eq. (A.12). For the high angular momentum particle following the trajectory Eq. (A.10), $k_v = k_u$, so Eq. (A.14) implies

$$-k_u = -k_v \simeq \frac{\omega M}{\rho_0} \simeq \frac{1}{\rho_0}, \tag{A.16}$$

where we have used the fact that the Schwarzschild energy of particles in the atmosphere is $\omega \simeq 1/M$.

The wave packet that describes such a high angular momentum Hawking particle is

$$|\phi\rangle = N \int dk f(k) e^{ik(x-T)}, \tag{A.17}$$

where $f(k)$ is a function such that $\Delta k = \bar{k} = \frac{1}{\rho_0}$ and N is a normalization factor. Eq. (A.4) implies that after the wave packet crosses the shock wave there is a discontinuity $\Delta T = p^\nu$. Neglecting logarithmic factors we find that after crossing the shock wave the state of the Hawking particle is

$$|\phi\rangle' \simeq N \int dk f(k) e^{ik(x-T+p^\nu)}. \tag{A.18}$$

Therefore, the probability to be in the same state after crossing the shock wave is

$$P = |\langle\phi'|\phi\rangle|^2 \simeq N^2 \int dk |f(k)|^2 (1 - k^2 p^{\nu 2}) = 1 - p^{\nu 2} \Delta k^2 \simeq 1 - \frac{M^2 \lambda^2}{\rho_0^4}, \tag{A.19}$$

where we have used Eq. (A.15).

In addition to the shift in the longitudinal direction, the angular momentum of the particles which constitute the atmosphere also changes. In the classical trajectories this appears as the refraction Eq. (A.5)

$$\Delta p_x = \frac{4p^\nu k_\nu \bar{x}}{\bar{x}^2} \simeq \frac{p^\nu k_\nu}{M} \simeq \frac{\lambda}{\rho_0^2}, \tag{A.20}$$

where we replaced \bar{x} and x by their typical value M and used the estimates of p^ν and k_ν obtained above. The relation between p_x and the angular momentum l is $p_x = l/M$, hence Eq. (A.20) corresponds to mean change of angular momentum $\Delta l \simeq \frac{\lambda M}{\rho^2} \ll 1$. Because of the smallness of Δl , the corresponding change in the wave function is

$$|l = l_0\rangle \rightarrow |l = l_0\rangle + i \frac{\lambda M}{\rho^2} |l = l_0 \pm 1\rangle. \tag{A.21}$$

Therefore, the probability for one particle in the atmosphere to have changed angular momentum is

$$P_{\Delta l \neq 0} = \frac{\lambda^2 M^2}{\rho_0^4}, \tag{A.22}$$

which coincides with Eq. (A.19).

Note that Eq. (A.21) can also be obtained by noting that Eq (A.18) neglects the logarithmic dependence of S and that the scattered modes are in fact $e^{ik(x-T+p^\nu \ln \bar{x}^2)}$. The \bar{x} dependence of the log can be shown to imply Eq. (A.21).

Thus we have shown how an s-wave interacts with the atmosphere. At present we cannot show how the high angular momentum particles which constitute the atmosphere interact among each other. The reason is that in order to do so one must know how to describe the gravitational interaction between two Hawking particles with high angular momentum and how to describe the gravitational effect of the Hawking particle with high angular momentum on an outgoing particle. But, unlike the ingoing particle, the high angular momenta are just vacuum fluctuations in Minkowski space. They correspond to short lines (compared to M) in Minkowski space (see Fig. A.1), and therefore their gravitational effect cannot be approximated by the shock wave.

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