Leverage and Alpha: The Case of Funds of Hedge Funds

Benoît Dewaele

In this paper, we develop a theoretical model of fund of hedge fund net leverage and alpha where the cost of borrowing is increasing with net leverage, thereby impacting the performance. We use this model to determine the conditions under which the leverage has a negative or a positive impact on investor’s alpha. Moreover, we show that the manager of a fund of hedge fund has an incentive to take a leverage that hurts the investor’s alpha. Next, we develop a statistical method combining the Sharpe style analysis and a time-varying coefficient model; this method allows the weights of the regression to vary over time while being constrained to sum up to 1. Subsequently, we use this method to get estimates of the leverages of a sample of funds of hedge funds. The estimates of leverages are then used in predictive regression analyses to confirm the negative impact of leverage on fund of hedge fund alphas and appraisal ratios. Finally, our results being robust to various robustness checks, we argue that this effect may be an explanation for the disappointing alpha delivered by funds of hedge funds.

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JEL Classifications: C22, G11, G23.

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Abstract

In this paper, we develop a theoretical model of fund of hedge fund net leverage and alpha where the cost of borrowing is increasing with net leverage, thereby impacting the performance. We use this model to determine the conditions under which the leverage has a negative or a positive impact on investor's alpha. Moreover, we show that the manager of a fund of hedge fund has an incentive to take a leverage that hurts the investor's alpha. Next, we develop a statistical method combining the Sharpe style analysis and a time-varying coefficient model; this method allows the weights of the regression to vary over time while being constrained to sum up to 1. Subsequently, we use this method to get estimates of the leverages of a sample of funds of hedge funds. The estimates of leverages are then used in predictive regression analyses to confirm the negative impact of leverage on fund of hedge fund alphas and appraisal ratios. Finally, our results being robust to various robustness checks, we argue that this effect may be an explanation for the disappointing alpha delivered by funds of hedge funds.

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1. Introduction

One of the key features of hedge funds is the heavy and dynamic use they make of leverage. Indeed, most of them use it to enhance the expected return delivered to investors. By investing into hedge funds, funds of hedge funds are also exposed to this leverage. Furthermore, funds of hedge funds can also add an additional layer of borrowing (in the present paper, this leverage is termed the net leverage\textsuperscript{1}) to increase the return of the underlying basket of hedge funds.

Therefore, leverage is ubiquitous in the industry. Despite this, only few papers investigate its role\textsuperscript{2}, and when they do so, that is most of the time from the perspective of the financial

\textsuperscript{*} This research has benefited from the data of Thomson Reuters.
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\textsuperscript{1} In this paper, and when it is clear from the context, « leverage » is used as a substitute for « net leverage ».
\textsuperscript{2} Amongst them, we may cite Ang, Gorovyy, and van Inwegen (2011) and McGuire and Tsatsaronis (2008).
stability. This lack of literature is most certainly due to the fact that hedge funds do not report their leverage to the regulator. Even if some commercial databases provide information about it, the data reported are too basic to be used in a statistical analysis.

In this paper, we propose a method that allows measuring the dynamic leverage of funds of hedge funds and investigate theoretically and empirically the relationship between their leverages and alphas.

First, we develop a theoretical model (based on the model developed by Lan, Wang and Yang (2012), (henceforth LWY)) of fund of hedge fund (henceforth FoHF) net leverage and alpha. Using this model, we show that the after-fee alpha earned by an investor depends on the leverage taken by the fund manager. The effect of leverage on the investor’s after-fee alpha is determined by the ratio of the cost of leverage to the before-fee alpha. If this ratio is superior to one, the effect is always negative. Additionally, we show that the fund manager may have an incentive to take a leverage that harms the investor’s alpha.

More specifically, the fund manager can invest in a risk-free asset and in an index of hedge funds (henceforth HFs) (which consists of an excess return and an alpha), and can use leverage to increase the expected return of the fund. Contrary to the literature, where the cost of leverage is equal to the risk-free rate, we let the spread increase with leverage to incorporate credit risk.

The risk-neutral manager is compensated by incentive and management fees. The management fee is calculated as a fraction of the assets-under-management. To better align the incentives of investors and managers, they also earn an incentive fee, paid if the value of the assets goes over the high-water mark (henceforth HWM). The incentive fee corresponds to a fraction of the difference between the assets-under-management (henceforth AUM) and the high-water mark, and can be regarded as an option.

The fund is liquidated if the value of the assets falls below a liquidation threshold. When the liquidation occurs, the manager loses the right to future payments. As the value of the compensation contract is highly optional, the risk-neutral manager endogenously behaves in a risk-averse manner and chooses leverage accordingly. Indeed, he has conflicting views about the amount of leverage he should take; a higher leverage increasing both the potential value of the fees and the probability of receiving nothing in the case of liquidation.

We solve the manager’s problem by using dynamic programming. As suggested by Lan, Wang and Yang, we use the endogeneity property of the Hamilton-Jacobi-Bellman equation, and define a new state variable that turns the problem from a two-state-variable (the high-water mark and AUM) to a one-state-variable problem. This variable is the ratio of the value of assets-under-management to the high-water mark, and can be interpreted as the moneyness of the fund. We then solve numerically the differential equation governing the manager’s value and use it to solve for the leverage.

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3 In this paper, the alpha refers to the after-fee alpha of the fund.
4 Hedge funds can borrow through prime broker agreements, repo-market, and rely on the implicit leverage embedded in derivative products.
5 The HWM is commonly the highest peak in assets reached by the fund at the end of previous fiscal years.
It turns out that the leverage of the fund depends on the state variable $w$ and is highly dynamic. When the liquidation threshold is far (moneyness is close to one), the manager takes more leverage as it increases the volatility and the value of the incentive fee option. On the contrary, afraid of the costly liquidation, the manager decreases leverage when the moneyness is low. Subsequently, we assume that FoHF investors are interested in alpha and use the theoretical model to investigate the relation between the investor's alpha and the leverage.

At first sight, an alpha-seeking investor should benefit from leverage, since it dilutes the burden of the management fee. With an increasing cost of leverage, the investor’s after-fee alpha crucially depends on the amount of leverage taken by the managers. If the spread is superior to the underlying investment’s alpha, the alpha decreases when leverage is added to the fund.

Unfortunately, the maximization problem of the manager is not coupled with the maximization of the alpha; the manager may thus disregard the alpha it delivers to investors. Therefore, overuse of costly leverage can turn alpha-delivering funds of hedge funds to raw-return providers. An investor seeking to maximize the alpha should thus limit the amount of leverage the fund is entitled to take.

We test the implication of our theoretical model on a bias-free sample of 458 funds of hedge funds over the period January 1999 to August 2009. To get an estimate of leverage, we do not rely on the data related to leverage in the TASS database. Indeed, as explained below, information is basic and the data reported show some inconsistencies. Therefore, we develop a modified version of the style analysis to obtain this estimate. This method, called the time-varying style analysis (henceforth TVSA), is constructed in the spirit of the Sharpe analysis, but allows the weights allocated to underlying asset classes to vary with time.

Consequently, we use the estimate of leverage obtained for each fund in predictive panel and Fama-McBeth regressions of alpha on fund-specific characteristics. The results of the regression confirm the predictions of the theoretical model as the coefficient associated with leverage turns out to be negative and highly significant for various specifications. Therefore, it seems that the additional alpha brought by leverage does not offset the cost of leverage of FoHFs.

We conclude that an alpha-seeking investor should limit the leverage the fund manager is entitled to take, or invest in unlevered funds of hedge funds. Additionally, we argue that the disappointing alpha of the funds of hedge funds industry may not be due to a lack of before-fee alpha-generating skills, but the by-product of an overuse of leverage.

To the best of our knowledge this paper is the first to investigate the link between leverage and alpha for funds of hedge funds by using both a theoretical and empirical framework. Furthermore, this paper proposes a constrained time-varying style analysis that can be used to get a better understanding of the exposures of various investment vehicles.

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*6 The extent to which the investor benefits from leverage is given by the conditions developed in Section 3.2.*
2. Literature Review

The academic literature on FoHF's was born from the literature on HFs; in their seminal paper, Fung and Hsieh (1997) argue that factors used to explain HF returns have to take the location and the trading strategy into account. Building on this conclusion Fung and Hsieh (2001) construct dynamic “trend following” factors using lookback straddles. They show that those factors greatly improve explanatory power as they embed option-like features exhibited by HF returns. Later, Agarwal and Naik (2004) show that equity-oriented HF strategies exhibit payoffs resembling a short position and use buy-and-hold and option-like strategies to model those hedge fund returns. To control those option-like features, Jagannathan, Malakhov and Novikov (2010) use hedge fund style benchmarks as factors, arguing that this should also control for common factors that affect all managers, resulting in a more precise estimation of the alpha.

The biases caused by the voluntary reporting to HF databases were vastly documented in early literature. Excellent reviews of the survivorship, selection and backfilling biases can be found in Fung and Hsieh (2000), Liang (2000), Ackermann McEnally and Ravenscraft (1999) amongst others. In this literature, the autocorrelation featured by hedge funds returns is also mentioned; Getmansky, Lo and Makarov (2004) suggest that this autocorrelation could be provoked by illiquidity exposure and voluntary return smoothing by funds managers and propose a method to correct for it.

Many studies on HF returns tend to show that at least some hedge funds deliver alpha (including Ackermann, McEnally, and Ravenscraft (1999); Brown, Goetzmann, and Ibbotson (1999); Agarwal and Naik (2000); Kosowski, Naik, and Teo (2006) and Criton and Scaillet (2011)). However, most of them document low (and not significantly different from 0) or even negative alpha on average for the funds of hedge funds.

Concerning the characteristics that may influence HF performance, Liang (1999) shows that funds with high-water marks significantly outperform those without and that average hedge fund returns are positively related to incentive fees and fund assets. This positive relationship between size and performance is also demonstrated by Fung, Xu and Yau (2002). The positive influence of HWM is also documented by Agarwal, Daniel and Naik (2009), whereas the positive relation between incentive fee and performance is corroborated by the findings of Ackermann, McEnally and Ravenscraft (1999) Liang (1999) Edwards and Caglayan (2001), and Fung, Xu and Yau (2002). However, it seems that the opposite relation holds for management fees (Edwards and Caglayan (2001)).

For their part, Fung, Xu and Yau (2002) find that a young hedge fund is able to time the market better than an old fund. This negative relationship between age and performance is also demonstrated by Aggarwal and Jorion (2010) who argue that potential reasons for this outperformance are the need to build a track record and the fact that young funds may pursue more innovative strategies. Fung, Xu and Yau (2002) also find that the lockup period does not have a persistent positive impact on a hedge fund manager's stock-selection performance. Unlike the aforementioned, Aragon (2007) shows that hedge funds with higher lock-up period consistently deliver higher performance (similar conclusions are reached by Amvella, Meier and Papageorgiou (2010) develop another method to correct for this autocorrelation bias.)
Agarwal, Daniel and Naik (2009) and Liang (1999)). Aragon suggests that the reason for this performance lies in the fact that those funds invest in less liquid strategies and capture an illiquidity premium. Agarwal, Daniel and Naik (2009) and Feng, Getmansky, Kapadia (2011), find that hedge funds with greater managerial incentives perform better. The former also document that higher levels of managerial ownership and high-water mark are associated with superior performance even if the incentive fee by itself does not explain performance. They also find that funds with a higher degree of managerial discretion (proxied by longer lockup, notice, and redemption periods) deliver superior performance.

Amongst the most recent literature, Titman and Tiu (2011) suggest that the best hedge funds choose to have less exposure to factor risk, and find that hedge funds with low R-squared (a proxy for the “low-exposure” to factor risk) deliver higher alpha. Similarly, Sun, Wang and Zheng (2010) argue that skilled managers will pursue unique investment strategies that will result in outperformance. They develop a “Strategy Distinctiveness Index” to show that funds using unique strategies deliver higher performance.

Focusing on funds of hedge funds, a recent paper by Aiken, Clifford, and Ellis (2012) uses a panel of 78 FoHF holdings from SEC’s filings to examine the managerial skills of FoHFs. They find that FoHFs hire larger, younger, illiquid hedge funds with complementary contract features to their own but do not detect skilful fund selection by FoHFs. Precisely, they find that HF's hired by FoHFs outperform before being hired by FoHFs but do not perform better after being hired. However, they suggest that FoHFs add value via skilful monitoring of their underlying HF's as they find that funds fired by FoHFs subsequently perform significantly worse than other hedge funds. Therefore, it seems that once good hedge funds are spotted by the market, offer and demand typically correct the expected excess return, reducing the advantage.

The theoretical literature on hedge fund managers’ compensation, valuation and hedge fund leverage decisions is scarcer. In this paper, we use a model based on Lan, Wang and Yang (2012). They develop a model in which the risk neutral fund manager maximizes the present value of management and incentive fees taking the high-water mark into account. The manager can invest in an alpha-generating strategy and in the risk-free asset, and increase the return by using leverage. Using the endogeneity property of the Hamilton-Jacobi-Bellman equation, they define a new state variable, the moneyness (the ratio between the fund’s AUM and the HWM) and solve numerically to get the behavior of leverage, management and incentive fee values as a function of the state variable. Moreover, in their setting, the fund is endogenously liquidated if the AUM falls below the liquidation threshold; in that case, the value of the claim held by managers is null. Since the manager is afraid of costly liquidation, and as the incentive fee contract is optional in itself, the managers endogenously behave in a risk-averse manner. Additionally, they show the impact on investor’s value when various features such as restart options, managerial ownership and new money flows are added to the model.

Prior to Lan, Wang and Yang (2012), Goetzmann, Ingersoll and Ross (2003) give closed-form valuation formulas for management and incentive fees in an intertemporal valuation framework when high-water mark is present. However, they do not allow managers to use leverage. In Panageas and Westerfield (2009), the manager can use leverage to increase the performance of the fund, but the manager is only compensated by HWM contracts and the
fund cannot be endogenously liquidated. They show that despite the optionality of the compensation contract, the manager does not invest an unbounded amount in the risky asset and behaves like a CRRA (constant relative risk-aversion) investor. They reach the conclusion that the leverage of the fund should be constant and that the manager is worse off if the level of incentive fees increases. As noted by Lan, Wang and Yang, the conclusions of Panageas and Westerfield (2009) contradict theirs.

Regarding varying exposures, Liang (1999) is the first to note that by construction, factor loadings in OLS regressions are assumed to be constant, conflicting with the dynamic nature of hedge funds. Bollen and Whaley (2009) underline that if coefficients are assumed constant when, in fact, they are time-varying, parameter estimates will be unreliable and will shed no light on managers’ skill. They add that evidence shows that over 40% of living hedge funds experience a statistically significant shift in their risk exposures. They use an optimal changepoint regression and a stochastic beta model (also used by Darolles and Vaissié (2011)) and conclude that the optimal changepoint regressions give better results. In the same vein, Criton and Scaillet (2011) conclude that HF exposures are changing and use a time-varying coefficient model to capture those variations. To condition on high-frequency data, Patton and Ramadorai (2011) use a model based on Ferson and Schadt and find substantial evidence that hedge fund risk exposures vary across and within months, and that hedge funds have a tendency to abruptly cut positions in response to significant market events.

However, as noted by Criton and Scaillet (2011), using a stochastic beta model on HF returns is not adapted, as HF returns are known to be non-normal. To deal with this problem, they suggest using the time-varying model introduced by Stone (1977) and Cleveland (1979). Moreover, they argue that the time-varying model significantly reduces the modeling bias and allows avoiding the curse of dimensionality.

The style analysis was developed by Sharpe to determine the underlying exposures of mutual funds and form portfolios of underlying tracking their performance. The properties of style analysis have been extensively studied by de Roon, Nijman, and ter Horst (2003), but the first paper that applies style analysis to HFS and FoHFs is Lhabitant (2001). He uses a mixture of the style analysis, the factor push approach, and historical simulation to analyze the investment style of individual hedge funds and funds of funds. He suggests that his approach is useful and may constitute a valuable tool for assessing the investment style and risk of hedge funds. The style analysis is also used by Hasanhodzic and Lo (2006) to estimate the proportion of the fund expected returns and volatility attributable to a small set of factors. They show that for some hedge fund styles, a significant fraction of both can be captured by common factors corresponding to liquid exchange-traded instruments. They add that the clones perform well enough to warrant serious consideration as passive, transparent, scalable, and lower-cost alternatives to hedge funds. Recently, Wallerstein, Tuchschild, and Zaker (2009) used a relaxed version of the style analysis to unveil the beta exposures of fund of funds, and show that the clones constructed can be good enough for portfolio management.
3. The Model

3.1 Model Setup

Based on the model of Lan, Wang and Yang (2012) we propose a model of dynamic leverage where the cost of borrowing is increasing. Subsequently, we use the model to show the impact of leverage on the investor's alpha. In our setting, the fund manager may invest in two assets, the risk-free asset which pays a constant interest rate \( r \), and an index of hedge funds\(^8\). The latter mimics the return of the hedge fund industry; paying an excess return of \( \gamma \) with a standard deviation equal to \( \sigma \) and an alpha equal to \( \alpha \) (the before-fee unlevered alpha).

In continuous time, the incremental return of the fund \( dR_t \) can thus be written as:

\[
dR_t = (r + \gamma + \alpha)dt + \sigma dB_t
\]

where \( B \) is a standard Brownian motion. However, the fund manager can use leverage to increase the expected return. Let \( W \) denote the fund unlevered assets-under-management and \( D \) the amount invested in the risk-free asset, if \( D \) is negative the manager uses leverage. Similar to Lan, Wang and Yang (2012), we define the leverage ratio as:

\[
\pi = \left( \frac{W - D}{W} \right)
\]

This ratio can be interpreted as the amount of risky assets-under-management over the net-asset-value. The net leverage\(^9\) (in percentage) is thus equal to \( \pi - 1 \). Contrary to the literature, but in line with business practices, we assume that the cost of leverage is an increasing function of the leverage ratio\(^10\). In our setup, the total cost of leverage per unit of time is represented by the following function:

\[
(\pi - 1)(r + i)dt + (\pi - 1)^2 i dt
\]

where \( i \) is a measure of the credit spread. The total cost of leverage per unit of time is thus a quadratic function of the leverage (the cost of leverage is equal to 0 when the leverage ratio is equal to 1; for a net leverage of 0.1, the total cost of leverage is equal to 0.1\((r + i) + 0.01i\)). The spread over the risk-free rate is equal to \((\pi - 1)(i) + (\pi - 1)^2 i\) and increases linearly per unit of leverage as illustrated in equation (4).

\[
\frac{(\pi - 1)i + (\pi - 1)^2 i}{(\pi - 1)} = i + (\pi - 1)i
\]

The spread per unit of leverage thus consists of a constant part and a part that is linearly increasing with leverage. If the quadratic part was not included in equation (3), the spread would be constant for any level of leverage, inconsistent with usual practices. One may argue that we could find a better model for the cost of leverage than the quadratic form (and the linearly increasing cost per unit it entails); however, as we want to keep an analytically tractable model, we consider that this polynomial expression is a convenient approximation.

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\(^8\) For the sake of clarity, the notations are similar to Lan, Wang and Yang (2012).

\(^9\) When no risk of confusion exists, “net leverage” will be termed “leverage”.

\(^10\) The cost of leverage is thus not influenced by the amount borrowed (in $) per se, but by the leverage ratio (in %).
The following constraints are imposed on the leverage ratio:

\[ 1 < \pi_t \leq \bar{\pi} \]  

(5)

where \( \bar{\pi} \geq 1 \) represents the exogeneous maximally allowed leverage. Since \( 1 < \pi_t \), the fund is prohibited from hoarding cash.

As is common in the industry, the fund manager is paid both by incentive and management fees. Management fees are equal to a constant fraction \( c \) of the assets-under-management \( W_t \). Even if management fees are paid whatever the performance of the fund, the fund manager still have an incentive to perform as this ensures he will collect management fees on higher assets-under-management. Furthermore, the fund manager has an incentive to keep the fund alive as he has no restart option.

The incentive fees are paid when the assets-under-management exceed the high-water mark. Similar to LWY (2012), when \( W_t < H_t \), i.e. when the assets-under-management are inferior to the high-water mark, the HWM grows, without money outflow, at a rate \( g \). However this HWM is adjusted downward by money outflow to investors. This outflow occurs at a rate \( \delta \).

\[ dH_t = (g - \delta)H_t dt, \quad \text{if } W_t < H_t \]  

(6)

This means that when the HWM is higher than the AUM, the HWM will grow exponentially at the rate \( (g - \delta) \). When the AUM exceeds the HWM \( (W_t \geq H_t) \), the incremental gain is equal to \( dH_t - (g - \delta)H_t dt > 0 \) and the manager earns a fraction \( k \) of this amount, the HWM is then re-set. If \( g \) is equal to \( \delta \), as explained in LWY (2012), \( H_t \) is thus equal to the maximum level attained by AUM.

The fund can be liquidated exogenously and endogenously. When the value of assets-under-management falls below a fraction \( b \) of the HWM, the fund is endogenously liquidated and the stopping time is denoted \( \tau_1 \). The threshold \( W_{\tau_1} \) corresponds to:

\[ W_{\tau_1} = bH_{\tau_1} \]  

(7)

Additionally, the fund is exogenously liquidated with a probability of \( \lambda \), if we denote \( \tau_2 \) the exogenous stochastic liquidation time, the stopping time is thus \( \tau = \min(\tau_1, \tau_2) \). In the case of liquidation, the manager collects nothing and investors collect the total value of the AUM at that time \( (W_t) \).

Prior to liquidation, the fund’s AUM incremental change follows the following process:

\[ dW_t = rW_t dt + W_t[\pi_t(\gamma + \alpha)dt - (\pi - 1)idt - (\pi - 1)^2idt + \pi_t \sigma dB_t] \]

\[ - \delta W_t dt - cW_t dt - k[dH_t - (g - \delta)H_t dt] - W_t dJ_t \]  

(8)

where \( rW_t dt + W_t[\pi_t(\gamma + \alpha)dt - (\pi - 1)idt - (\pi - 1)^2idt + \pi_t \sigma dB_t] \) gives the incremental change due to the investment policy. \( \delta W_t dt, cW_t dt, k[dH_t - (g - \delta)H_t dt], W_t dJ_t \) represent respectively the constant repayment to investor, management and incentive fees, and the stochastic jump process with a mean arrival rate of \( \lambda \). As can be seen in the previous equation, the amplitude of the jump is equal to \( W_t \); hence, in the case of a jump the value of the funds is equal to 0.
We follow LWY (2012) and suppose the managers are risk-neutral (the risk-averse behavior of the managers is endogenously generated by the optionality of the compensation contract). Hence, managers maximize the sum of the total expected present value of management fees $M(W, H; \pi)$ and the expected present value of the incentive fees $N(W, H; \pi)$. For a given leverage policy, this sum is denoted $F(W, H; \pi)$:

$$F(W, H; \pi) = M(W, H; \pi) + N(W, H; \pi) \quad (9)$$

$$M(W, H; \pi) = \mathbb{E}_t \left[ \int_t^T e^{-r(s-t)} cW_s ds \right] \quad (10)$$

$$N(W, H; \pi) = \mathbb{E}_t \left[ \int_t^T e^{-r(s-t)} k [dH_s - (g - \delta)H_s ds] \right] \quad (11)$$

Therefore, the manager dynamically chooses the leverage policy that maximizes the present value of total fees:

$$\max_{\pi} F(W, H; \pi) \quad (12)$$

We solve the optimization problem using dynamic programming. In the interior region (when the AUM is inferior to the HWM), the Hamilton-Jacobi-Bellman is11:

$$(\beta + \lambda) F(W, H) = \max_{\pi \in \Pi} cW$$

$$+ \left[ \pi (\gamma + \alpha) - \pi^2 i + \pi i + r - g - c \right] WF_W(W, H)$$

$$+ \frac{1}{2} \pi (w)^2 \sigma^2 w^2 F_{WW}(W, H) + (g - \delta) HF_H(W, H) \quad (13)$$

We use the results of LWY (2012) and conjecture that the value function is homogeneous in $W_t$ and $H_t$. Hence, the value function can be rewritten as a function of a new state variable, the moneyness of the fund manager claim $w_t$ (14). This variable is defined as the ratio of the assets-under-management to the high-water mark ($w_t = W_t / H_t$).

$$F(W, H) = f(w)H \quad (14)$$

This transformation implies that the derivatives of the value function can be rewritten as:

$$F_W(W, H) = f'(w)$$

$$F_{WW}(W, H) = f''(w)/H$$

$$F_H(W, H) = f(w) - wf'(w) \quad (15)$$

Using (13), (14) and (15) and the definition of $w_t$, the HJB equation can be restated as:

$$(\beta - g + \delta + \lambda) f(w)$$

$$= cw$$

$$+ \left[\pi(w_t)(\gamma + \alpha) - \pi^2(w_t)i + \pi(w_t)i + r - g - c \right] w f'(w)$$

$$+ \frac{1}{2} \pi (w)^2 \sigma^2 w^2 f''(w) \quad (16)$$

The manager value $f(w)$ in terms of the state variable must then solve (16), subject to the following boundary conditions:

$$f(b) = 0,$$

$$f(1) = (k + 1)f'(1) - k \quad (17)$$

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11 For additional details concerning the computation of the HJB equation we refer to LWY (2012).
The first boundary condition ensures that the manager value is equal to 0 at the liquidation boundary; the second gives the behavior of the manager value at the upper boundary (reflecting the payment of incentive fees)\(^\text{12}\). At the upper boundary, the manager earns a fraction \(k\) of the difference between the AUM and the high-water mark, and the high-water mark is reset.

Taking the first derivative of (17) with respect to the leverage and equating this first derivative to 0, we get the following first order condition (henceforth FOC) for the leverage ratio:

\[
(y + \alpha + i)wf'(w) - 2\pi if'(w) + \pi \sigma^2 w f'''(w) = 0
\]

\[
\pi(w) = \frac{-(y + \alpha + i)f(w)}{(\pi f''(w) + \sigma^2 w f'''(w))}
\]

The second order condition (henceforth SOC) for an optimum being:

\[-2if'(w) + \sigma^2 w^2 f''(w) < 0\]

If this second order condition is verified, the optimal leverage ratio, taking constraints imposed on leverage into account, can be written as:

\[
\pi^*(w) = \min \left\{ \max \left\{ \frac{-(y + \alpha + i)f'(w)}{(-2if'(w) + \sigma^2 w f''(w))}, 1 \right\}, \bar{\pi} \right\}
\]

Substituting for the optimal leverage ratio (19) into (16), we get a quadratic equation in \(f'(w)\). In Appendix, we show that the positive root of this equation never satisfies the SOC condition. Hence, the leverage obtained is not optimal, and substituting into equation (16) makes no sense. On the other hand, the negative root satisfies the SOC. Hence, we can solve the differential equation numerically to obtain \(f(w), f''(w)\) and \(f'(w)\). Using those values in equation (19), we obtain the optimal leverage.

### 3.2 Calibration and Implications

Similar to Lan, Wang and Yang (2012), we set \(g\), the growth rate of the HWM, equal to \(r\), the net growth rate of \(w\) is thus equal to 0. \(\lambda\), the exogenous liquidation intensity, is equal to 0.1, \(\delta\), the payout rate, is 0; the total expected withdrawal rate is thus equal to \(\delta + \lambda = 10\%\). We set the maximum drawdown to 31.5\% implying that the liquidation threshold \(b\) is equal to 0.685. \(\alpha\) is equal to 0.008; which corresponds to the annualized continuously compounded alpha (all rates are annualized and continuously compounded rates) of the regression of excess returns (over the LIBOR) of the DJCS hedge fund index on the 8 factors of Fung and Hsieh (over the period January 1999 to August 2009). We set \(y\) equal to 0.0307, the average excess return over the LIBOR (minus the alpha) of the DJCS hedge funds index. The management fee (\(c\)) is equal to 0.0143, the average management fee of our sample of funds of hedge funds; the incentive fee (\(k\)) is equal to 0.0841, the average incentive fee. \(r\) is equal to 0.0342, the average value of the LIBOR over the period. Finally, we set \(\sigma\) to 0.0759, the annual standard deviation of the excess return of the DJCS hedge fund index.

\(^{12}\) For more details about those boundary conditions, we refer to Lan, Wang and Yang (2012) and Goetzmann, Ingersoll and Ross (2003).
We set the spread $i$ to 0.01 using information collected in expert reports, on Internet and following discussions with an industry expert. This may be a raw estimation, and the cost of borrowing may differ from fund to fund, but unreported tests we have conducted suggest that on average the spread must be superior to the average alpha (0.008) (see Section 4.3.1 for more details). This backs up the value chosen for the spread. Moreover, the value of the spread does not threaten the validity of our conclusions as the theoretical model provides with the behavior of investor's alpha and leverage whatever the level of $i$.

Using the parameters defined above, Figure 1 plots the evolution of the optimal leverage ratio as a function of $w$. The optimal leverage ratio is equal to 1 for $w$ inferior to 0.86, above this value the fund leverage increases until it reaches approximately 1.375 at $w = 1$.

As can be seen in the figure, despite the risk-neutrality of the manager and the constant investment opportunity set, the leverage ratio depends on the moneyness of the fund. Indeed, the compensation contract of the manager is optional for two reasons. Firstly, the sum of the present values of incentive and management fees is equal to 0 if the moneyness is lower than the liquidation threshold. Secondly, the incentive fee is equivalent to an option paying a fraction $k$ of the difference between the AUM and the high-water (the strike of the option).

This optionality entails an endogenous risk-averse behavior from the manager. The amount of leverage chosen is thus a trade-off between the larger value of fees the manager could collect by using more leverage and the higher probability of receiving nothing due to the higher liquidation probability.

As can be seen in Figure 1, endogenous leverage is an increasing function of the moneyness $w$. When the liquidation threshold is far, the manager increases leverage as it increases the volatility and thus the value of the incentive fees option. When the moneyness is low, the manager decreases leverage since he is afraid of losing the present value of future fees.

The theoretical model also gives insights about the relationships between leverage, incentive fees, and moneyness (however those insights are not specific to our model, and can be deduced from the model of LWY alone). Figure 2 plots the leverage ratios for different levels of management fees (all other things being equal). As illustrated in the figure, the effect of management fees on leverage is ambiguous; when management fees are included in the range 0.5% to 2% the management fee increases the leverage, however going from 2% to 2.5% the leverage decreases. The effect of the incentive fees is illustrated in Figure 3, contrary to management fees, the effect of incentive fees on leverage is non equivocal, as the leverage ratio always increases in parallel to incentive fees. This effect was expected, as the higher the level of incentive fee, the larger the expected value of the incentive fees compared to management fees. Thus, the greater the incentive to increase the value of the incentive fees through the volatility brought by leverage.
Figure 1  Optimal leverage ratio as a function of moneyness ($\omega$)

Figure 2  Leverage ratios as a function of moneyness ($\omega$) for different levels of management fees ($c$)
Figure 3  Leverage ratios as a function of moneyness ($w$) for different levels of incentive fees ($k$)

Assuming that the investor is only interested in alpha, we now take a closer look at the relationship between leverage and investor’s alpha. At first sight, and if the unlevered alpha of the underlying investment is superior to 0; the investor should benefit from leverage. Indeed, by using leverage the fund manager dilutes the burden of management fees, making the investor better off.

However, if the leverage is more costly than the risk-free rate, this result does not hold true. Thus, let $a - c$ denote the instantaneous investor’s after fee alpha for an unlevered fund in the interior region. For a levered fund, in the interior region, the investor’s instantaneous after fee alpha is equal to the levered alpha minus the management fee and the cost of borrowing: $\pi \alpha - c - (\pi - 1)i - (\pi - 1)^2 i$. Hence, the leverage hurts the alpha in when the cost of additional leverage is higher than the additional alpha brought by leverage.

More specifically, the leverage has a negative impact on the investor’s alpha when:

$$\pi \alpha - c - (\pi - 1)i - (\pi - 1)^2 i < a - c$$  \hspace{1cm} (22)  

This can be re-written as:

$$-\pi^2 i + \pi(i + a) - a < 0$$  \hspace{1cm} (23)  

Therefore, the leverage has a positive impact on the investor's alpha as long as the leverage is included in the interval formed by the roots of the following quadratic equation:

$$-\pi^2 i + \pi(i + a) - a = 0$$  \hspace{1cm} (24)  

---

13 This after fee alpha does not take incentive fees into account; however this does not modify the conclusions of our analysis. Hence, to keep things simple, we use the after-management-fee alpha.
The roots of this equation are 1 and \( \alpha/i \), hence, when the leverage is included in the range \([1, \alpha/i]\), the effect of leverage is positive; outside those bounds the effect is negative. The behavior of the investor's after-fee alpha thus crucially depends on the ratio of \( i \) to the before-fee alpha.

In our baseline set of parameters, \( i \) is higher than \( \alpha \). Since the spread paid per unit of leverage is always higher than the unlevered alpha, the investor's alpha is decreasing with leverage as illustrated in Figure 4. If \( i \) is superior to \( \alpha \), the investors should thus expect a lower alpha as the managers disregard the alpha to maximize the value of their claim.

**Figure 4  Investor's alpha as a function of moneyness (\( w \))**

However if \( i \) is lower than \( \alpha \), the manager can improve on alpha when leverage is included in the range \([1, \alpha/i]\). Therefore, by increasing leverage, the manager can increase the investor's alpha up to a certain point. Unfortunately, behaving in self-interest, and even if \( \alpha \) is higher than \( i \), there is no guarantee that fund managers will take a leverage that maximizes the investor's alpha.

To illustrate the behavior of leverage when \( i \) is varying, Figure 5 shows the evolution of leverage for different levels of \( i \). As can be seen in the graph, the lower the \( i \), the higher the leverage used by the manager. Complementing Figure 5, Figure 6 illustrates the behavior of investor's alpha for different levels of \( i \). When \( i \) is null, (when leverage is risk-free), the alpha increases linearly with the leverage. As illustrated in the graph, when \( i \) is higher than 0, the leverage has a positive effect on leverage only if the leverage is included in the range \([1, \alpha/i]\).

In practice, as the average alpha generated by the world of hedge funds is equal to 0.8%, the unlevered alpha is unlikely to compensate for the additional cost of leverage. Therefore, the leverage should induce a lower alpha for fund of hedge fund investors. This effect should also be true for the appraisal ratio. Indeed, if the cost of leverage is equal to the risk free rate, the effect of leverage on the appraisal ratio is null (see Aragon (2007), amongst others). However, as the leverage has a negative impact on the alpha, the appraisal ratio should also
be negatively impacted. Hence, an investor seeking to maximize the alpha or the appraisal ratio should limit the amount of leverage the fund is entitled to use.

Figure 5  Leverage ratios as a function of moneyness ($w$) for different levels of $i$

Figure 6  Investor’s alphas as a function of moneyness ($w$) for different levels of $i$
4. Empirical Validation

This section presents the results of the empirical validation of the theoretical model. We start by detailing our sample of funds of hedge funds. The second subsection introduces a method that allows obtaining a dynamic estimate of leverage. This second step is made necessary by the fact that leverage-related data reported to databases are static and most of the time unreliable. In the next subsection, we show the results of this estimation and how it compares to the rough data reported. Finally, using this dynamically estimated leverage in predictive multivariate analyses, we show that leverage has indeed a negative and significant impact on alpha and appraisal ratio.

4.1 Data

The data used in this section were collected from the Lipper TASS database and include fund of hedge fund monthly net-of-fee return histories for period January 1999 to August 2009. Our database contains both active and defunct funds, eliminating a priori problems due to survivorship bias in our selection\(^{14}\). Similar to Liang (1999), we delete funds reporting returns quarterly. We also delete funds with outliers\(^{15}\) and missing values. As suggested by Fung and Hsieh (1997) and many others, we delete duplicated funds, as they are essentially similar series of the same funds offered as different share classes for regulatory and accounting reasons. As suggested by Wallerstein, Tuchschmid, and Zaker (2009) for funds with the same share class, the one with the longest history or domiciled in the US is selected. Since the database still contains some gross return histories, we delete the small number of funds for which the performance is recorded gross, as we do not have any means to recover a neat net return.

Backfilling, or instant history bias, is immediately corrected by deleting the returns between the inception date and the date the fund was added to TASS (Aggarwal and Jorion (2009) and Sun, Wang and Chen (2011) and Jagannathan, Malakhov and Novikov (2010) apply a similar correction). This correction drastically reduces return histories for some funds, but knowing that the backfilling bias\(^{16}\) can have a huge impact on the fund returns, this is the only way to get rid of this bias definitely. To avoid introducing an additional bias to our database, we delete the funds for which information about the inception or registration dates are missing. Additionally, we treat only funds whose returns are reported in USD (similar correction is used by Sun, Wang and Chen (2011) and Aragon (2007)).

As the autocorrelation present in hedge fund returns entails a downward bias for the standard deviations, we remove this autocorrelation using the Getmansky, Lo and Makarov (2004) (henceforth GLM) methodology. Instead of imposing ex-ante an optimal lag of 2 we fit an AR model on the de-meaned FoHF returns with lags going from 1 to 4, and use Bayesian information criteria to determine the best lag\(^{17}\). The need for such a relaxation is corroborated by Gallais-Hammonno, Hoang and Nguyen-Thi-Thanh (2008), who show that a

\(^{14}\) For excellent reviews of data biases in HF databases, see Fung and Hsieh (2000), Liang (2000) or Ackerman, McEnally and Ravenscraft (1999) amongst others.

\(^{15}\) By outliers, we mean returns which more than likely come from coding errors.

\(^{16}\) Malkiel and Saha (2005) show that on average, the backfilled returns are more than 500 bps higher than the contemporaneously reported returns.

\(^{17}\) For the vast majority of funds (99%) lags 1 or 2 are selected.
lag of 2 is not always optimal. Similar to Getmansky, Lo and Makarov, we do not impose the constraint that the coefficients must be in the [0,1] interval, but use this as a specification check. If for the best lag, the procedure fails to give coefficients that satisfy the constraints, we use the second best lag, and if no lag gives coefficients satisfying the constraints we use the raw series. For all the results, unsmoothed returns are used. As the returns of indices of HF's also exhibit autocorrelation, the same procedure is applied to the returns of HF indices used in the DJCS model.

The final sample consists of 458 FoHFs with more than 36 months\(^{18}\) of corrected return histories for period January 1999 to August 2009 (with the backfilled returns deleted). As will be explained in the next section, we use returns of hedge fund strategies provided by DjCjS to obtain an estimation of leverage. The HF strategy indices selected are the following: Convertible Arbitrage, Dedicated Short Bias, Equity Market Neutral, Event Driven, Fixed Income Arbitrage, Managed Futures, Emerging Markets, Global Macro, Long-Short Equity Hedge and Multi-Strategy. We thus have a set of 10 factors\(^ {19}\) that will be referred to as the DJCS model.

However, to determine the alpha and the appraisal ratio that will be used in multivariate predictive analyses (Section 4.3), we use the 8-factor model of Fung and Hsieh. The factors of Fung and Hsieh are the following:

- the excess return on the S&P 500 Index (S&P-LIB),
- the "small-minus-big" factor computed as the difference between the Russell 2000 index monthly total return and the S&P 500 monthly total return (RUL-SNP),
- the monthly change in the difference between the 10-year Treasury constant maturity yield and the 1-month LIBOR (month end-to-month end) (D10Y),
- the change in the credit spread of Moody's BAA bond over the 10-year Treasury bond (DBAA-10Y),
- the excess returns on a portfolio of lookback options on bonds (PTFSBD), currencies (PTFSFX), and commodities (PTFS.COM)\(^ {20}\),
- and, as suggested by David Hsieh on his webpage, we add the excess return on the MSCI Emerging Markets Index (MSCIEM-LIB).

### 4.2 Estimation of Leverage

#### 4.2.1 Method

Since funds of hedge funds invest in other hedge funds, they are easier to model than hedge funds. Firstly, the underlying asset classes are known; secondly (and contrary to hedge funds) they cannot short sell the underlying funds. Moreover, funds of hedge funds have most of the time diversified portfolios of HF's concentrated in few HF strategies. FoHFs can thus be considered as portfolios of HF strategies. Still, FoHF's can rely on credit lines to

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\(^{18}\) As regressions will be performed on those series, we need a sufficient number of observations.  
\(^{19}\) We use the unsmoothed indices returns obtained by the aforementioned methodology.  
\(^{20}\) These non-linear factors are directly retrieved from David Hsieh's website: http://faculty.fuqua.duke.edu/dah7/HFRFD ata.htm
enhance their returns. The return of a diversified fund of hedge funds can thus be seen as a positively-weighted levered combination of HF strategies.

Using returns of HF strategies as factors, the aforementioned arguments make the style analysis developed by Sharpe readily applicable to FoHFs. Adding the LIBOR as a proxy for the cost of interest, and allowing the weight associated with it to be negative should give an estimate of FoHF exposure to interest rates and a measure of their net leverage. The net leverage is the leverage that funds of hedge funds use on top of the leverage already used by the underlying basket of hedge funds. Indeed, one may argue that the FoHF exposure to the LIBOR could represent the leverage embedded in the underlying HFs. However, as we consider that investments of FoHFs are diversified, the leverage of their underlying hedge funds should be close to the average leverage of the HF strategy, which is already reflected in the index returns.

However, the exposures and leverages of FoHFs may be less stable than in the mutual fund universe. As the style analysis is a constrained regression, it condemns us to suppose that weights are constant over the period. Therefore, to model the weights allocated by FoHFs to HF strategies, we ground our analysis on the time-varying coefficient model and adapt it to incorporate the constraints of the style analysis. Combining both approaches; we get a methodology that allows the weights to vary over the estimation period while still being constrained to sum up to 1.

The choice of the time-varying coefficient method over state-space model is explained by the following considerations. Firstly, the number of parameters to be estimated for state-space model grows rapidly with the number of regressors; hence the model will quickly become intractable in small samples. Secondly, as noted by Criton and Scaillet (2011) stochastic beta models suppose the returns are normal; hypothesis which is highly questionable for FoHFs. Thirdly, the constraints are difficult to handle in state-space models.

We thus use a one-step local linear model on FoHF gross returns to estimate the varying weights:

$$ R_{i,t} = \sum_{j=1}^{11} w^j_i(t)F^j_t + \epsilon_{i,t} $$

(25)

where $R_{i,t}$ is the gross return of fund $i$ for period $t$, $w^j_i(t)$ denotes the varying weight of the strategy $j$ for fund $i$, $F^j_t$ is the gross return of the factor $j$ for period $t$ (index 1 being the LIBOR 1-month), $\epsilon_{i,t}$ being the corresponding residual. For each $t$, we approximate the function locally as:

$$ w^j_i(t) \approx d^j_i + b^j_i(t - t_0) $$

(26)

for $t$ in a neighborhood of $t_0$. To remain consistent with the style analysis, we add the following constraints to the weights. Firstly, the estimates of function $w^j_i(t)$ must sum up to 1. Secondly, since we allow leverage and suppose that FoHFs do not hoard cash, the weight

---

21 To avoid confusion with the previous section, $R$, $w$ and $F$ are in Roman instead of italic.
associated with the LIBOR\textsuperscript{22} (the first factor) is constrained to be negative (or 0) and superior\textsuperscript{23} to MaxLeverage. Finally, as the underlying HFs cannot be shorted, estimates of the weights of HF strategies (factors 2 to 11) must be positive and sum up to 1 minus the weight associated with the LIBOR. This leads to the following least-squares problem:

\[
\text{Minimize : } \sum_{t=T_i^*}^{T_i} \left[ R_{i,t} - \sum_{j=1}^{11} \{ d^j_i + b^j_i(t - t_0) \} F^j_t \right]^2 L_{h_i}(t - t_0) \text{ for } t_0 = T_i^*, \ldots, T_i
\]

\[\text{Subject to : } \sum_{j=2}^{11} d^j_i = 1 - d^1_i(t) \]

\[
\text{MaxLeverage} \leq d^1_i(t) \leq 0 \\
0 \leq d^j_i(t) \forall i = 2, \ldots, 11
\]

for a given kernel function\textsuperscript{24} $L$ with bandwidth $h_i$ (where $L_{h_i}(\cdot) = L(\cdot/h_i)/h_i$ for the fund $i$), $T_i$ and $T_i^*$ being the last and the first reporting months of the fund $i$ in our database. Developing the previous equation we get:

\[
\text{Minimize : } \sum_{t=T_i^*}^{T_i} \left[ (R_{i,t})^2 - 2 \sum_{j=1}^{11} \{ d^j_i + b^j_i(t - t_0) \} F^j_t R_{i,t} \right. \\
\left. + \left( \sum_{j=1}^{11} \{ d^j_i + b^j_i(t - t_0) \} F^j_t \right)^2 \right] L_{h_i}(t - t_0) \text{ for } t_0 = T_i^*, \ldots, T_i
\]

As $(R_{i,t})^2$ is independent from factor weights we can omit it to get:

\[
\text{Minimize : } \sum_{t=T_i^*}^{T_i} \left[ -2 \sum_{j=1}^{11} \{ d^j_i + b^j_i(t - t_0) \} F^j_t R_{i,t} \\
+ \left( \sum_{j=1}^{11} \{ d^j_i + b^j_i(t - t_0) \} F^j_t \right)^2 \right] L_{h_i}(t - t_0) \text{ for } t_0 = T_i^*, \ldots, T_i
\]

Developing further leads to the following minimization problem:

\[
22 \text{ The use of LIBOR as a proxy for leverage is similar to the method of McGuire and Tsatsaronis (2008).} \\
23 \text{ Inferior in absolute value.} \\
24 \text{ We use the Epanechnikov kernel. For the bandwidth selection, we use an automated cross-validation algorithm based on the minimization of a one-step forecast error. See Criton and Scaillet (2011) for more details.}
\]
Using the following notations:

\[
\begin{align*}
\text{Minimize} : \quad & \sum_{t=T_i}^{T_{i+1}} \left[ -2 \sum_{j=1}^{11} \{d_j^t + b_j^t(t-t_0)\} \left( F_t^j \sqrt{L_{h_i}(t-t_0)} \right) \left( R_{i,t} \sqrt{L_{h_i}(t-t_0)} \right) \\
& + \left( \sum_{j=1}^{11} \{d_j^t + b_j^t(t-t_0)\} \left( F_t^j \sqrt{L_{h_i}(t-t_0)} \right) \right)^2 \right] \\
& \text{for } t_0 = T_i^*, ..., T_i
\end{align*}
\]

Doing this for \( t \) going from \( T_i^* \) to \( T_i \), we obtain the varying exposures to hedge fund strategies. As we suppose that the exposure to the LIBOR is a proxy for the leverage of the fund, we use it as the estimate of fund leverage for every time period. At this point, the \( \text{MaxLeverage} \) variable deserves further discussion. The TASS database gives three numbers related to leverage. Firstly, an indicator variable that is equal to 1 if the fund reports to use leverage and 0 otherwise; secondly, the maximum leverage that the fund reports to use and finally, the average leverage reported by the fund.

Using the following notations:

\[
\begin{align*}
F_{lt} &= \left( F_{h_i}^{11} \sqrt{L_{h_i}(T_i^*-t_0)} \right) \\
& \vdots \\
& F_{h_i}^{11} \sqrt{L_{h_i}(T_i - t_0)} \\
N_{lt} &= \left( \begin{array}{c}
R_{i,T_i} \sqrt{L_{h_i}(T_i^*-t_0)} \\
\vdots \\
R_{i,T_i} \sqrt{L_{h_i}(T_i - t_0)}
\end{array} \right)
\end{align*}
\]

\[
\begin{align*}
w_{lt} &= \left( \begin{array}{c}
d_1^t \\
b_1^t \\
\vdots \\
d_{11}^t \\
b_{11}^t
\end{array} \right)
\end{align*}
\]

the previous quadratic minimization problem can be re-written as:

\[
\arg\min_{w_{lt}} 0.5w_{lt} \text{VAR}(F_{lt})w_{lt} - w_{lt} \text{COV}(N_{lt}, F_{lt}) \quad \text{for } t = T_i^*, ..., T_i
\]

Subject to:

\[
\sum_{j=2}^{11} d_j^t = 1 - d_1^t(t)
\]

\[
\text{MaxLeverage} \leq d_1^t(t) \leq 0
\]

\[
0 \leq d_i^t(t) \quad \forall i = 2, ..., 11
\]

Doing this for \( t \) going from \( T_i^* \) to \( T_i \), we obtain the varying exposures to hedge fund strategies. As we suppose that the exposure to the LIBOR is a proxy for the leverage of the fund, we use it as the estimate of fund leverage for every time period.

At this point, the \( \text{MaxLeverage} \) variable deserves further discussion. The TASS database gives three numbers related to leverage. Firstly, an indicator variable that is equal to 1 if the fund reports to use leverage and 0 otherwise; secondly, the maximum leverage that the fund reports to use and finally, the average leverage reported by the fund.

\[\text{In view of the construction of our model, our leverage estimate will thus be expressed as a fraction of the NAV of the FoHF.}\]
However, discrepancies between average leverage values, maximum leverage values and the leverage indicator variable cast some doubts on data reported. As an example, 49.04% of the funds report using leverage, yet, the percentage of funds that report a maximum leverage of 0 is 76.22% and the percentage of funds reporting an average leverage of 0 is 80.68%. Moreover, amongst the funds that report using leverage, 62.34% report a mean leverage of 0 and 54.55% report a maximum leverage of 0, which contradicts their use of leverage. To get a sense of the data reported about leverage, Figures 7 and 8 give, respectively, the maximum and average leverage distributions reported by FoHFs to the TASS database.

Since “N/A” values are never reported, some “0” reported may simply mean that the fund does not disclose this information. Therefore, instead of using the maximum leverage reported to the database in the predictive regressions, we choose to follow a “let the data talk” approach and set the maximum leverage to 10\(^26\) and maximum investment in a strategy to 11. However, it should be noted that the maximum weight is never attained and the estimates we get for leverage are close to the values reported (when they are reported)\(^27\).

Additionally, in Appendix, we show how the time-varying style analysis compares to OLS and standard style analysis in terms of adjusted r-squared.

**Figure 7** Distribution of maximum leverage (as reported to the TASS database) for funds which report using leverage

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\(^{26}\) This value corresponds to the highest maximum leverage reported by a FoHF to the TASS database.

\(^{27}\) We should also mention that McGuire and Tsatsaronis (2008) acknowledge that leverage reporting of HF reports an incomplete description of their leverage and use style analysis on aggregated data to determine leverage.
4.2.2 Estimated leverages

Figure 9 illustrates the distribution of FoHF average leverages as estimated by the aforementioned method. As can be seen in the bar chart, the percentage number of funds is decreasing with leverage. The percentage number of funds with an estimated leverage of 0 is equal to 8.74%, which is much lower than the percentage of funds that report no use of leverage to the TASS database. Furthermore, the percentages of funds with estimated leverages lower than 0.25, 0.5 and 1 are respectively equal to 47.98%, 68.83% and 87.67%.

In order to check that our method gives consistent results, we compare the average estimated leverages to the average leverages of funds that report an average leverage to the database. Figure 10 depicts this comparison; as can be seen in the graph, the distributions are quite similar.

As another anecdotal evidence of the validity of our methodology, the median value of the ratio of the average estimated leverages to the mean reported leverages is equal to 86.72%, suggesting that the estimated leverages is quite close to the reported values.

As we use our method for every fund, even those which report no use of leverage, we plot the distributions of the estimated leverages for levered and unlevered funds (as reported to the TASS database) in Figure 11. As can be seen in the graph, the distribution shapes are quite similar for the two samples but the number of funds with high leverages is larger for funds that report use of leverage. Conversely, the number of funds with low leverages is more important in the sample of funds which report no use of leverage.
Figure 9  Distribution of estimated leverage

Figure 10  Comparison of average estimated leverages (blue) and average leverages (red) for funds which report an average leverage
4.3 Results

Using the estimates of FoHF leverages, we are now in a position to confirm the predictions of our theoretical model. We use predictive panel and Fama-McBeth regressions with the future alpha as dependent variable and regress it on estimated leverages and fund-specific characteristics that may impact future performance.

Traditionally, one considers that the alpha linearly increases with leverage; however, this is true when the cost of additional leverage is constant and inferior to the underlying alpha. If the cost of an additional unit of leverage is lower than the alpha brought by this additional unit of leverage, the effect of leverage on alpha should be positive. If the opposite is true, the leverage should have, as Subsection 3.2 indicates, a negative impact.

We also use the appraisal ratio as dependent variable. This ratio is computed by dividing the alpha by the standard deviation of the residuals of the regression. As the appraisal ratio corrects the alpha for the volatility of the residuals, we use this performance measure along with the alpha.

It should be noted that without any additional cost of leverage, the leverage should leave the appraisal ratio unaffected, ceteris paribus (as explained by Aragon (2007)). However, if the additional leverage decreases the alpha, the appraisal ratio should thus also be negatively impacted by the leverage.

To investigate the effect of leverage on future performance, predictive multivariate analyses using both panel and Fama-McBeth regressions are performed in the next two subsections. The regressions estimated are:
where future performance is alternately the future alpha and the appraisal ratio. As we have an unbalanced panel dataset, we potentially need to correct for fund and time effect. Hence we use both panel and Fama-McBeth regressions as suggested by Petersen (2009) and similar to Sun, Wang and Chen (2011).

4.3.1 Panel data analysis

To determine the effect of leverage by controlling for fund-specific characteristics; we perform panel regressions of future alphas and appraisal ratios on mean estimated leverages and other control variables. At time $t$, the regression is performed as follows: the alpha is the constant of the regression of the fund future excess returns (for the period $t+1$ to $t+18$) on the 8 factors of Fung and Hsieh. The appraisal ratio is obtained by scaling the alpha by the standard deviation of the residuals of the regression. To estimate the alpha, as our set of factors is quite large (8 factors plus the constant term) an 18-month window is used. As the past ratio is used as a control variable, we also need at least 18 months of returns prior to the first month of analysis (the panel regression thus starts in July 2000).

The leverage corresponds to the absolute value of the average value of weights associated with LIBOR for the previous 18 months (the leverage variable is thus positive). We also control for the age of the fund (measured in months), the size of the fund (measured by the natural logarithm of the first reported size of the fund), management and incentive fees, and the presence of a high-water mark. As suggested by Aragon (2007), we use the redemption notice period (days) as a measure of the illiquidity of the fund. However, we do not use the lockup period and the minimum investment since data reported are missing for a large number of funds. As Titman and Tiu (2011), we add the past ratio to alleviate concerns due to persistence. The past ratio is obtained by following the procedure described in Titman and Tiu (2011), i.e. we regress the past ratio on past leverage and use the residuals as control variables.

Results of the panel regressions are reported in Table 1. The results are reported for four models: model 1 and 2 with the alpha as the dependent variable and model 3 and 4 with the appraisal ratio as endogenous variable. Contrary to models 1 and 3, models 2 and 4 also include the past ratio in the set of regressors. The $t$-statistics reported in the table are adjusted for fund and time clustering.

As can be seen in the table, besides leverage, the redemption notice and the size of the fund are the only significant variables. Size is positively and significantly (at the 10% level) related to the alpha in models 1 and 2. This can be explained by the economies of scale, the cost of running the fund being shared by a larger number of investors. The coefficient

\[ \text{Future Performance}_t = \beta_{t,0}^c + \beta_{t,1}^c \text{Est. Leverage}_t^l + \beta_{t,j}^c \text{Control}_t^l + e^l_t \]  

28 In the robustness checks section, we show that the results are qualitatively unaffected by a regression on the DJCS model.

29 As suggested by Titman and Tiu (2011).

30 In unreported robustness checks, we show that using the past ratios instead of the residuals of the past ratios does not impact our conclusions. Indeed, the coefficient associated with leverage is negative and significant at the 1% level.

31 Interestingly, the constant reported in the table (for the “Alpha” panel) is close to the difference between the alpha (0.008) and management fee (1.43) used in the theoretical model.
associated with redemption period is positive and significant at the 5% level in model 3 and 4 and at the 10% level in model 1. Since Aiken, Clifford, and Ellis (2012) report that FoHF s invest in HF s with similar features to their own, we conjecture that FoHFs with large redemption periods invest in the less liquid HF s. As explained by Aragon (2007) and Liang and Park (2008), those hedge funds thus capture the illiquidity premium and deliver a superior alpha.

The signs associated with the non-significant variables are largely consistent with the literature; however, coefficients associated with the HWM are surprisingly negative (and significant at the 10% for model 4). Nonetheless, we are not overly concerned about the counterintuitive sign of the HWM as the sign associated with it in Fama-McBeth regressions is positive and (sometimes) significant.

Concerning leverage, both for alpha and appraisal ratio, the coefficients are always negative and significant at the 1% level, in accordance with the prediction of the theoretical model. In unreported regressions, we show that this negative effect holds true whatever the level of leverage. To do this, instead of using mean estimated leverage as a regressor, we use two variables, the first being equal to the mean estimated leverage if the estimated leverage is lower than a certain threshold, 0 otherwise. The second being the mean estimated leverage if it is higher than this threshold. Whatever the threshold used, the effect is negative, suggesting that the average borrowing cost of funds of hedge funds is probably superior to the unlevered alpha of HF s whatever the amount borrowed.

4.3.2 Fama-McBeth regression

The results of the Fama-McBeth regressions are reported in Table 2. The t-statistics reported in the table are heteroskedasticity and autocorrelation consistent (Newey-West).

Concerning leverage, the results of the Fama-McBeth regressions are consistent with the results of the panel regressions. Indeed, the coefficients associated with leverage are significant (1% level) and negative whatever the specification.

The age variable has always a positive sign and is significant whatever the model. This result has, to the best of our knowledge, never been reported by studies on funds of hedge funds. However, we have good reasons to think that most of the standard arguments used to justify the negative sign of age in the HF universe are less true for FoHF s. The first argument is that young hedge funds are potentially more aggressive as they need to build a track record, which is also true for FoHF s; however as the value proposition of FoHF s is different from the one of HF this casts some doubts on the aggressiveness new FoHF s should exhibit. A second argument is that young hedge funds may conduct more innovative strategies, strategies which entail better performance (Sun, Wang and Zheng (2011), Aggarwal and Jorion (2010)). However, as the purpose of FoHF s is to select and combine the best HF s, innovativeness appears to be less important for them. On the contrary; for FoHF s, experience could mean more knowledge of the industry and potentially more ability to detect the best hedge funds. Age could thus be considered as an asset for FoHF s.

With respect to management and incentive fees, the results are similar to the panel regressions since coefficients are respectively negative and positive but not significantly different from 0. Contrary to the panel regressions, the sign associated with personal capital
is negative (but still not significantly different from 0). For HWM, the sign is positive and significant for the first 2 models, in line with other studies on HF performance. The coefficient associated with the redemption period is still positive, but this time significant at the 1% level whatever the model (consistent with Aragon (2007)). Similar to the previous section, the coefficients of the size variable are positive but this time significant at the 1% level, consistent with return-to-scale arguments.
This table reports the results of panel regressions of alpha and appraisal ratio on leverage and other control variables (period January 1999 - August 2009). Alpha is the constant in the regression of the net-of-fees excess returns on the 8 factors of Fung and Hsieh (t+1:t+18). The Appraisal ratio variable is obtained by dividing the alpha by the standard deviation of the residuals. Leverage is the mean leverage of the last 18 months (t-17:t), ln(Age) is the age (in months) of the fund at time t, High-water mark and Personal capital are dummy variables which are equal to 1 if the fund has a high-water mark and 1 if the manager of the fund has personal capital invested in the fund, respectively. The Redemption notice control is expressed in days, ln(Size) is the first reported size of the fund. The control variable Past ratio is obtained by taking the residuals of the regression of past Appraisal ratio and Alpha (t-17:t) on Leverage. t-statistics are adjusted for fund and time clustering.

<table>
<thead>
<tr>
<th></th>
<th>Alpha</th>
<th>Appraisal ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model 1</td>
<td>Model 2</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0050**</td>
<td>-0.0044**</td>
</tr>
<tr>
<td>t-stat</td>
<td>-2.1161</td>
<td>-2.0906</td>
</tr>
<tr>
<td>Leverage</td>
<td>-0.0019***</td>
<td>-0.0020***</td>
</tr>
<tr>
<td>t-stat</td>
<td>-2.7306</td>
<td>-3.2736</td>
</tr>
<tr>
<td>ln(Age) (months)</td>
<td>0.0005</td>
<td>0.0004</td>
</tr>
<tr>
<td>t-stat</td>
<td>0.9208</td>
<td>0.7987</td>
</tr>
<tr>
<td>Management fee</td>
<td>-0.0338</td>
<td>-0.0329</td>
</tr>
<tr>
<td>t-stat</td>
<td>-0.6130</td>
<td>-0.6443</td>
</tr>
<tr>
<td>Incentive fee</td>
<td>0.0021</td>
<td>0.0017</td>
</tr>
<tr>
<td>t-stat</td>
<td>0.6634</td>
<td>0.5587</td>
</tr>
<tr>
<td>High-water mark</td>
<td>-0.0002</td>
<td>-0.0002</td>
</tr>
<tr>
<td>t-stat</td>
<td>-0.4381</td>
<td>-0.5270</td>
</tr>
<tr>
<td>Personal capital</td>
<td>0.0004</td>
<td>0.0004</td>
</tr>
<tr>
<td>t-stat</td>
<td>0.8456</td>
<td>0.7607</td>
</tr>
<tr>
<td>Redemption notice (days)</td>
<td>0.00001*</td>
<td>0.0000</td>
</tr>
<tr>
<td>t-stat</td>
<td>1.8252</td>
<td>1.6308</td>
</tr>
<tr>
<td>ln(Size) (USD)</td>
<td>0.0002*</td>
<td>0.0002*</td>
</tr>
<tr>
<td>t-stat</td>
<td>1.6690</td>
<td>1.7798</td>
</tr>
<tr>
<td>Past ratio</td>
<td>0.1375***</td>
<td>0.1342***</td>
</tr>
<tr>
<td>t-stat</td>
<td>2.0275</td>
<td>5.0306</td>
</tr>
</tbody>
</table>
This table reports the results of Fama-McBeth regressions of alpha and appraisal ratio on leverage and other control variables (period January 1999 - August 2009). Alpha is the constant in the regression of the net-of-fees excess returns on the 8 factors of Fung and Hsieh (t+1:t+18). The Appraisal ratio variable is obtained by dividing the alpha by the standard deviation of the residuals. Leverage is the mean leverage of the last 18 months (t-17:t), ln(Age) is the age (in months) of the fund at time t, High-water mark and Personal capital are dummy variables which are equal to 1 if the fund has a high-water mark and 1 if the manager of the fund has personal capital invested in the fund, respectively. The Redemption notice control is expressed in days, ln(Size) is the first reported size of the fund. The control variable Past ratio is obtained by taking the residuals of the regression of past Appraisal ratio and Alpha (t-17:t) on Leverage. t-statistics are heteroskedasticity and autocorrelation consistent (Newey-West).

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.0086***</td>
<td>-0.0084***</td>
<td>-0.6049***</td>
<td>-0.4750***</td>
</tr>
<tr>
<td>t-stat</td>
<td>-6.5013</td>
<td>-6.1121</td>
<td>-5.6752</td>
<td>-4.7996</td>
</tr>
<tr>
<td>Leverage</td>
<td>-0.0014***</td>
<td>-0.0017***</td>
<td>-0.0998***</td>
<td>-0.1044***</td>
</tr>
<tr>
<td>t-stat</td>
<td>-3.2225</td>
<td>-4.4225</td>
<td>-3.8622</td>
<td>-4.0177</td>
</tr>
<tr>
<td>ln(Age) (months)</td>
<td>0.0012***</td>
<td>0.0013***</td>
<td>0.1018***</td>
<td>0.0848***</td>
</tr>
<tr>
<td>t-stat</td>
<td>5.6150</td>
<td>5.5690</td>
<td>5.1107</td>
<td>4.4700</td>
</tr>
<tr>
<td>Management fee</td>
<td>-0.0216</td>
<td>-0.0218</td>
<td>-1.0991</td>
<td>-0.9645</td>
</tr>
<tr>
<td>t-stat</td>
<td>-1.4625</td>
<td>-1.4951</td>
<td>-0.9928</td>
<td>-0.9015</td>
</tr>
<tr>
<td>Incentive fee</td>
<td>0.0022</td>
<td>-0.0007</td>
<td>0.2244*</td>
<td>0.1000</td>
</tr>
<tr>
<td>t-stat</td>
<td>1.1718</td>
<td>-0.4077</td>
<td>1.9394</td>
<td>0.7817</td>
</tr>
<tr>
<td>High-water mark</td>
<td>0.0009***</td>
<td>0.0007***</td>
<td>0.0205</td>
<td>0.0010</td>
</tr>
<tr>
<td>t-stat</td>
<td>3.6652</td>
<td>3.2978</td>
<td>1.1443</td>
<td>0.0591</td>
</tr>
<tr>
<td>Personal capital</td>
<td>-0.0002</td>
<td>-0.0002</td>
<td>-0.0028</td>
<td>-0.0003</td>
</tr>
<tr>
<td>t-stat</td>
<td>-0.9909</td>
<td>-1.0198</td>
<td>-0.2312</td>
<td>-0.0260</td>
</tr>
<tr>
<td>Redemption notice (days)</td>
<td>0.0002***</td>
<td>0.0002***</td>
<td>0.0020***</td>
<td>0.0016***</td>
</tr>
<tr>
<td>t-stat</td>
<td>6.3293</td>
<td>5.9442</td>
<td>8.6359</td>
<td>7.1888</td>
</tr>
<tr>
<td>ln(Size) (USD)</td>
<td>0.0002***</td>
<td>0.0002***</td>
<td>0.0114***</td>
<td>0.0090***</td>
</tr>
<tr>
<td>t-stat</td>
<td>5.2056</td>
<td>4.7986</td>
<td>3.2913</td>
<td>2.7083</td>
</tr>
<tr>
<td>Past ratio</td>
<td>0.1494***</td>
<td>0.2039***</td>
<td>0.0376</td>
<td>0.0639</td>
</tr>
<tr>
<td>t-stat</td>
<td>4.1534</td>
<td>7.0181</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adj. R-squared</td>
<td>0.0326</td>
<td>0.0635</td>
<td>0.0376</td>
<td>0.0639</td>
</tr>
</tbody>
</table>
4.3.3 Determinants of leverage

As detailed in Subsection 3.2, the theoretical framework developed by LWY also gives some interesting insights into the behaviour of leverage. The theoretical model predicts that the leverage should be positively related to the moneyness and the incentive fees. The effect of the management fees is less obvious but should be positive overall.

Table 3 reports the results of panel regressions of the future mean estimated leverages on the fund-specific characteristics, past average leverages and moneyness of the funds. The moneyness is estimated by constructing an index tracking the value of the fund. Starting from 100 at the inception date, the value of the index is then incremented each month using the reported returns. The moneyness is then computed by dividing the value of the index at time t by the highest value reached by the index at the end of each 12-month period starting from the inception date. Since we have no information about the duration of the HWM, we run the HWM starting from the inception date. This variable allows us to check if the leverage increases consecutive to good performance as predicted by the theoretical model.

Since we are interested in characteristics that may influence leverage, we regress the estimated leverages on fund-specific characteristics in model 1. Supportive of the predictions of the theoretical model, results of model 1 show that, at the 10% significance level, the level of incentive fees is positively related to leverage. In contrast, the effect of the management fees is positive, but not significant at the 10% level.

Models 2 and 3 are more dynamic as they include the past leverage in the set of regressors; furthermore, the moneyness variable is added to model 3. Consistent with the predictions of the theoretical model, the coefficient associated with the moneyness is positive and significant at the 1% level.
Table 3  
Determinants of leverage: panel regression

This table reports the results of panel regressions of future leverage on the Past leverage and other control variables (period January 1999 - August 2009). Future leverage is defined as the average estimated leverage of funds of hedge funds \((t+1:t+18)\), the Past leverage is computed over the period \(t-17:t\). \(\ln(Age)\) is the age (in months) of the fund at time \(t\). High-water mark and Personal capital are dummy variables equal to 1 if the fund has a high-water mark and 1 if the manager of the fund has personal capital invested in the fund, respectively. Redemption notice is expressed in days. \(\ln(\text{Size})\) is the first reported size of the fund. Starting from the inception date, we build an index tracking the fund’s value, the Moneyness is then obtained by dividing the value of the index at \(t\) by the highest value attained at the end of each 12-month period starting at the inception date. \(t\)-statistics are adjusted for fund and time clustering.

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0,1530</td>
<td>-0,0474</td>
<td>-0,1927**</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>0,7890</td>
<td>-0,8021</td>
<td>-2,3909</td>
</tr>
<tr>
<td>(\ln(Age)) (months)</td>
<td>-0,0360</td>
<td>0,0058</td>
<td>0,0065</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>-1,2322</td>
<td>0,4895</td>
<td>0,5511</td>
</tr>
<tr>
<td>Management fee</td>
<td>4,7974</td>
<td>1,6088*</td>
<td>1,5315</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>1,4240</td>
<td>1,6530</td>
<td>1,5236</td>
</tr>
<tr>
<td>Incentive fee</td>
<td>0,5815*</td>
<td>0,0729</td>
<td>0,0596</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>1,6848</td>
<td>0,7632</td>
<td>0,6194</td>
</tr>
<tr>
<td>High-water mark</td>
<td>-0,0331</td>
<td>0,0103</td>
<td>0,0049</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>-0,7492</td>
<td>0,9318</td>
<td>0,4511</td>
</tr>
<tr>
<td>Personal capital</td>
<td>0,0695</td>
<td>-0,0096</td>
<td>-0,0062</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>1,5960</td>
<td>-0,8385</td>
<td>-0,5391</td>
</tr>
<tr>
<td>Redemption notice (days)</td>
<td>-0,0002</td>
<td>0,0002</td>
<td>0,0001</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>-0,2342</td>
<td>1,0675</td>
<td>0,6993</td>
</tr>
<tr>
<td>(\ln(\text{Size})) (USD)</td>
<td>0,0015</td>
<td>-0,0002</td>
<td>-0,0009</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>0,1562</td>
<td>-0,0605</td>
<td>-0,2933</td>
</tr>
<tr>
<td>Past leverage</td>
<td>0,8694***</td>
<td>0,875***</td>
<td>0,875***</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>27,0800</td>
<td>27,6919</td>
<td></td>
</tr>
<tr>
<td>Moneyness</td>
<td></td>
<td></td>
<td>0,1583***</td>
</tr>
<tr>
<td>(t)-stat</td>
<td></td>
<td></td>
<td>3,0529</td>
</tr>
<tr>
<td>Adj. R-squared</td>
<td>0,0274</td>
<td>0,8574</td>
<td>0,8590</td>
</tr>
</tbody>
</table>
4.3.4 Discussion

The negative influence of leverage on alpha and appraisal ratio suggests that investors interested in alpha should limit the leverage the manager is entitled to use or invest in less- or un-levered funds\textsuperscript{32}.

Although the sources of leverage of hedge funds differ from those of funds of hedge funds, the borrowing cost should also increase with leverage. Therefore, the overuse of leverage may also explain the disappointing alpha delivered by the hedge fund industry. Indeed, even if a hedge fund generates a high unlevered before-fee alpha, the manager will use leverage to maximize the present value of incentive and management fees. However, the maximization problem of the manager is not coupled with the maximization of alpha; the manager will thus overlook investor’s alpha. This overuse of costly leverage may turn alpha-delivering hedge funds and funds of hedge funds to raw-return providers.

Nonetheless, this does not mean that investors are worse off by investing in levered funds. If the cost of leverage of the fund is lower than the investor’s one and/or if the investor is constrained in the leverage he can take, he may still want to invest in the fund (if the fund delivers a before-fee alpha). Indeed, from the investor’s point of view, as long as the cost of the constraint on leverage and/or the differential cost of interest more than offset the fees in terms of utility, the investor is better off investing in the fund (i.e having the choice between two similar funds, levered and unlevered, a constrained investor (and/or an investor paying a higher interest rate) may prefer the levered fund as it delivers a higher expected return even if it is at the cost of alpha). An investor with a low risk-aversion may thus prefer a levered position in the underlying with no alpha to an unlevered position associated with an alpha.

5. Robustness Checks

A first potential concern regarding our empirical validation is that our conclusions may be driven by the set of factors we use to obtain alphas and appraisal ratios. Even if the model of Fung and Hsieh is commonly used in the literature, we still check if the alphas and appraisal ratios obtained by regressing the excess returns of FoHFs on excess returns of HF strategies indices (DJCS model) lead to the same conclusion. Regressing fund returns on style or strategy returns is suggested by Jagannathan, Malakhov and Novikov (2010), who argue that this captures the common factors affecting the industry and allows controlling for option like-features. We thus reconstruct the analysis using the excess returns of HF strategies indices provided by Dow Jones Credit Suisse as factors. Results of the panel regressions with alphas and appraisal ratios obtained using the DJCS model are reported in Table 4 in Appendix. As can be seen in the table, the results obtained are consistent with our previous results. Indeed, the coefficients associated with leverage are always negative and highly significant. Being entirely equivalent, results for the Fama-McBeth regressions are not reported\textsuperscript{33}.

\textsuperscript{32} Indeed, as the leverage has a negative impact on alpha whatever the level of leverage, it suggests that the cost of leverage of funds of hedge funds is superior to the alpha delivered by the hedge fund industry.

\textsuperscript{33} Those results are available upon request.
The leverage-estimation procedure could be another potential concern. To alleviate it, we use a dummy variable instead of the estimated leverage. This dummy variable is equal to 1 if the mean estimated leverage reported by our method is different from zero, 0 otherwise. Results of panel regressions under this specification are reported in the panel “Dummy Variable” of Table 6 in Appendix. Results are qualitatively equivalent to our baseline results; indeed, the coefficients associated with leverage are negative and significant at the 1% level.

One may also argue that the leverage estimated via our method is just a proxy for the volatility of the FoHFs, volatility that is the real factor which influences the future performance. Even if the use of the appraisal ratio should in itself alleviates this concern, we still redo the analysis using the residuals of the regression of the past volatilities (computed over the previous 18 months) on past estimated leverages\(^{34}\) as regressors. Results are displayed in the panel “With Past Volatility” of Table 6 in Appendix. As the table indicates, the addition of the volatility does not impact our conclusions as the \(t\)-statistics of the leverage coefficients are still negative and significant at the 1% level.

As an additional robustness check, we show that our results also hold for the Sharpe ratio. Results of predictive panel regressions using Sharpe ratios as endogenous variables are displayed in Table 5 in Appendix.

## 6. Conclusion

In this paper, we investigate the link between investor’s alpha and leverage in funds of hedge funds. Based on the model of Lan, Wang and Yang (2012), we develop a theoretical model where leverage is endogenously chosen by a manager that maximizes the present value of the fees. Since the cost of borrowing is increasing with leverage, we give the conditions under which the alpha benefits from leverage. Moreover, we show that the maximization problem of the manager is not coupled with the maximization of investor’s alpha. This means that the manager will potentially maximize the value of his claim at the cost of investor’s alpha.

Then, we develop a time-varying style analysis combining a time-varying coefficient model and a Style analysis to get an estimation of the leverages of a sample of funds of hedge funds. We argue that the unlevered before-fee alpha of funds of hedge funds do not compensate for the cost of leverage. Consequently, using estimated leverages in predictive multivariate regressions we show that leverage has indeed a negative and significant impact on alpha.

High costs of leverage may thus turn alpha-producing funds into raw-return generators, therefore an alpha-seeking investor should limit the leverage the fund manager is allowed to take, or invest in unlevered funds of hedge funds. However, it does not mean that an investor (not only interested in alpha) should stay away from the levered funds. After all, the investors seek the risk-return combination that brings the maximum utility. An investor that is constrained in the leverage that he can take or paying a high interest rate might

\(^{34}\text{This is done because the past leverage clearly influences the past volatility. However, using the raw past volatility also gives a negative and highly significant coefficient for leverage.}\)
prefer a levered position in the underlying asset and choose the levered fund even if it is at the cost of alpha.

This conclusion also has some implications for hedge funds. Indeed, the disappointing alpha of the hedge fund industry (funds of hedge funds included) may not be due to a lack of alpha-generating skills but be the by-product of an overuse of leverage. As some hedge fund strategies rely heavily on borrowing, the cost of leverage should thus be considered as important as the unlevered before-fee alpha.

Interestingly, the negative effect of leverage on alpha can also be interpreted as a potential explanation for the “capacity constraint” effect. Indeed, as new investor flows chase performance, we can suppose that the funds attracting new money flows evolve close to their high-water mark. As the moneyness of the fund has increased, the leverage and the borrowing cost should too; thereby decreasing the delivered alpha.

---

35 The fact that the alpha of funds is decreasing after new investor flows.
7. References


Aiken, Adam L., Christopher P. Clifford, and Jesse Ellis, 2012, Do funds of hedge funds add value? Evidence from their holdings, Working paper.


Fan, Jianqing, and Tao Huang, 2005, Profile Likelihood Inferences on Semiparametric Varying-Coefficient Partially Linear Models, Bernoulli 11, 1031-1057.


Philipp, Michel, Andreas Ruckstuhl, Peter Manz, Marcel Dettling, Franziskus Durr and Peter Meier, 2009, Performance rating of funds of hedge funds, Working paper.


8. Appendix to Section 3.1

Using the optimal leverage ratio (19) in (16) and denoting $\Omega$ and $\Psi'$:

$$\Omega \equiv (-2if'(w) + \sigma^2wf''(w))$$

$$\Psi \equiv (-2if'(w) + \sigma^2wf''(w))^2 = \left(4i^2f^2(w) + \sigma^4w^2f^{-2}(w) - 4i\sigma^2wf'(w)f''(w)\right)$$

Leads to the following differential equation:

$$\left(\beta - g + \delta + \lambda\right)f(w)\Psi - cw\Psi + (\gamma + \alpha + i)(\gamma + \alpha)wf^{-2}(w)\Omega$$
$$+ (\gamma + \alpha + i)^2iwf^{-3}(w) + (\gamma + \alpha + i)if^{-2}(w)\Omega$$
$$- (r - g - c)wf'(w)\Psi - \frac{1}{2}(\gamma + \alpha + i)^2\sigma^2w^2f^{-2}(w)f''(w) = 0$$

Replacing $\Omega$ and $\Psi$ by their expressions, we have:

$$\left[\sigma^4w^2[(\beta - g + \delta + \lambda)f(w) - cw - (r - g - c)wf'(w)]\right]f^{-2}(w)$$
$$+ \left[\sigma^2wf'(w)\left[-4(\beta - g + \delta + \lambda)if(w) + 4icw\right.$$$$+ (\gamma + \alpha + i)(\gamma + \alpha)wf'(w) + (\gamma + \alpha + i)iwf'(w)$$
$$+ 4(r - g - c)iwf'(w) - \frac{1}{2}(\gamma + \alpha + i)^2wf'(w)\right]\right]f''(w)$$
$$+ \left[if^{-2}(w)[4(\beta - g + \delta + \lambda)if(w) - 4icw$$
$$- 2(\gamma + \alpha + i)(\gamma + \alpha)wf'(w) + (\gamma + \alpha + i)^2wf'(w)$$
$$- 2(\gamma + \alpha + i)iwf'(w) - 4(r - g - c)iwf'(w)]\right] = 0$$

We thus obtain a quadratic equation in $f''(w)$. Collecting the terms in $f^{-2}(w)$, $f''(w)$ and the constant and denoting them $a^*$, $b^*$ and $c^*$, respectively:

$$a^* = \left[\sigma^4w^2[(\beta - g + \delta + \lambda)f(w) - cw - (r - g - c)wf'(w)]\right]$$

$$b^* = \left[\sigma^2wf'(w)\left[-4(\beta - g + \delta + \lambda)if(w) + 4icw\right.$$$$+ (\gamma + \alpha + i)(\gamma + \alpha)wf'(w) + (\gamma + \alpha + i)iwf'(w)$$
$$+ 4(r - g - c)iwf'(w) - \frac{1}{2}(\gamma + \alpha + i)^2wf'(w)\right]\right]$$

$$c^* = \left[if^{-2}(w)[4(\beta - g + \delta + \lambda)if(w) - 4icw - 2(\gamma + \alpha + i)(\gamma + \alpha)wf'(w)$$
$$+ (\gamma + \alpha + i)^2wf'(w) - 2(\gamma + \alpha + i)iwf'(w)$$
$$- 4(r - g - c)iwf'(w)]\right]$$

The roots of the quadratic differential equation can be expressed as:

$$x^-(w) = \frac{-b^* - \sqrt{b^{*2} - 4a^*c^*}}{2a^*}$$

$$x^+(w) = \frac{-b^* + \sqrt{b^{*2} - 4a^*c^*}}{2a^*}$$
In proposition 1, we show that the positive root (41) of the quadratic equation does not yield a solution satisfying the second order condition. Therefore, the expression for leverage obtained by substituting in the HJB equation is no longer optimal and substituting into equation (16) makes no sense. Hence, we use the negative root of this equation (for which the SOC is satisfied) to solve the differential equation numerically and get $f(w)$. Substituting in equation (19), we obtain the optimal leverage.

**Proposition 1:** If $f(w) = x^+(w)$ then $f''(w) = \frac{2l}{\sigma^2 w} f'(w)$ which implies that the SOC is not satisfied.

**Proof:** Replace $a^*, b^*, c^*$ in $x^+(w)$ take first and second derivatives with respect to $w$. The conclusion follows.
9. Appendix to Section 3.2

To test the behaviour of the constrained time-varying style analysis, we simulate returns of artificial funds of hedge funds and apply the time-varying method to those artificial funds. The results of the time-varying style analysis applied to an artificial FoHF are reported in Figure 14. To construct the returns of this artificial FoHF, we proceed as follows: we select the most common strategies in our databases of HF's (i.e. Emerging Markets, Event Driven, Fixed Income Arbitrage, Long Short Equity Hedge and Managed Future strategies), and count the number of funds per strategy (from August 1995 to August 2009). We compute the weights associated with those strategies by dividing by the total number of hedge funds for the five strategies. We then multiply the weights obtained by the returns of the strategies and add a residual noise with a noise-to-signal ratio of 1/5. This artificial FoHF thus follows the evolution of the FoHF industry for the 5 main strategies. We then use the time-varying style analysis to get the estimated weights and compare them to the real simulated weights. The results reported in Figure 14 show that weights obtained closely match the time-varying behaviour of the real weights.

As a further validation check of the time-varying style analysis, we deem useful to report the behaviour of the time-varying style analysis compared to OLS in terms of adjusted R-squared. The cumulative distributions of the adjusted R-squared of the time-varying style analysis and the classical OLS are reported below. Figure 12 indicates that the time-varying style analysis gives lower adjusted R-squared for the funds with low adjusted R-squared, but on the other hand, performs better than OLS for the funds with high adjusted R-squared. For the whole sample, the average difference between the adjusted R-squared of the time-varying style analysis and OLS is 1.09%.

Figure 12 Cumulative distributions of adjusted R-squared – OLS vs. time-varying style analysis

Since the time-varying style analysis is constrained, the small improvement is not surprising. Hence, we also report the cumulative distribution of adjusted R-squared of the
classical style analysis compared to the distribution of the time-varying style analysis. Figure 13 gives a sense of the improvement, the average difference between the adjusted R-squared of the standard style analysis and time-varying style analysis being 11.48%.

Figure 13  Cumulative distributions of adjusted R-squared – Standard style analysis vs. time-varying style analysis
Figure 14  Comparison of simulated and estimated coefficients
10. Appendix to Section 5

Table 4
Alpha, appraisal ratio and leverage: panel regressions (DJCS Model)

This table reports the results of panel regressions of alpha and appraisal ratio on leverage and other control variables (January 1999 - August 2009). Alpha is the constant in the regression of the net-of-fees excess returns on the DJCS model \((t+1:t+18)\). The Appraisal ratio variable is obtained by dividing the alpha by the standard deviation of the residuals. Leverage is the mean leverage of the last 18 months \((t-17:t)\), \(\ln(\text{Age})\) is the age (in months) of the fund at time \(t\), High-water mark and Personal capital are dummy variables which are equal to 1 if the fund has a high-water mark and 1 if the manager of the fund has personal capital invested in the fund, respectively. The Redemption notice control is expressed in days, \(\ln(\text{Size})\) is the first reported size of the fund. The control variable Past ratio is obtained by taking the residuals of the regression of past Appraisal ratio and Alpha \((t-17:t)\) on Leverage. \(t\)-statistics are adjusted for fund and time clustering.

<table>
<thead>
<tr>
<th>Alpha</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.0080***</td>
<td>-0.0079***</td>
<td>-0.9304*</td>
<td>-0.9289*</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>-2.6442</td>
<td>-2.6959</td>
<td>-1.7427</td>
<td>-1.7999</td>
</tr>
<tr>
<td>Leverage</td>
<td>-0.0035***</td>
<td>-0.0035***</td>
<td>-0.3187**</td>
<td>-0.3182**</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>-2.9106</td>
<td>-3.0810</td>
<td>-2.3570</td>
<td>-2.4294</td>
</tr>
<tr>
<td>(\ln(\text{Age})) (months)</td>
<td>0.0009*</td>
<td>0.0009*</td>
<td>0.1466*</td>
<td>0.1395*</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>1.8320</td>
<td>1.8554</td>
<td>1.7124</td>
<td>1.6699</td>
</tr>
<tr>
<td>Management fee</td>
<td>0.0586</td>
<td>0.0582</td>
<td>3.8893</td>
<td>4.3506</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>0.5318</td>
<td>0.5576</td>
<td>0.2619</td>
<td>0.3063</td>
</tr>
<tr>
<td>Incentive fee</td>
<td>0.0035</td>
<td>0.0022</td>
<td>0.5389</td>
<td>0.4380</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>0.7632</td>
<td>0.4902</td>
<td>0.7787</td>
<td>0.6550</td>
</tr>
<tr>
<td>High-water mark</td>
<td>0.0005</td>
<td>0.0005</td>
<td>-0.0322</td>
<td>-0.0277</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>0.8141</td>
<td>0.8597</td>
<td>-0.3327</td>
<td>-0.2956</td>
</tr>
<tr>
<td>Personal capital</td>
<td>0.0004</td>
<td>0.0003</td>
<td>0.1095</td>
<td>0.1057</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>0.6633</td>
<td>0.5378</td>
<td>1.2067</td>
<td>1.2141</td>
</tr>
<tr>
<td>Redemption notice (days)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0030*</td>
<td>0.0028*</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>1.1576</td>
<td>1.1384</td>
<td>1.8379</td>
<td>1.8058</td>
</tr>
<tr>
<td>(\ln(\text{Size})) (USD)</td>
<td>0.0001</td>
<td>0.0001</td>
<td>-0.0165</td>
<td>-0.0145</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>0.5188</td>
<td>0.5486</td>
<td>-0.7108</td>
<td>-0.6496</td>
</tr>
<tr>
<td>Past ratio</td>
<td>0.0674**</td>
<td>0.0674**</td>
<td>0.0462***</td>
<td></td>
</tr>
<tr>
<td>(t)-stat</td>
<td>2.4957</td>
<td>2.9902</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Adj. R-squared | 0.0466 | 0.0627 | 0.0385 | 0.0513 |
Table 5
Sharpe ratio and leverage: panel regressions

This table reports the results of panel regressions of the Sharpe ratio on leverage and other control variables (January 1999 - August 2009). **Leverage** is the mean leverage of the last 18 months \((t-17:t)\), **ln(Age)** is the age (in months) of the fund at time \(t\), **High-water mark** and **Personal capital** are dummy variables which are equal to 1 if the fund has a high-water mark and 1 if the manager of the fund has personal capital invested in the fund, respectively. The **Redemption notice** control is expressed in days, **ln(Size)** is the first reported size of the fund. The control variable **Past ratio** is obtained by taking the residuals of the regression of past Sharpe ratio \((t-17:t)\) on **Leverage**. \(t\)-statistics are adjusted for fund and time clustering.

<table>
<thead>
<tr>
<th>Sharpe ratio</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.0933</td>
<td>0.1519</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>0.8062</td>
<td>1.4974</td>
</tr>
<tr>
<td>Leverage</td>
<td>-0.0811***</td>
<td>-0.0827***</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>-3.3969</td>
<td>-3.9999</td>
</tr>
<tr>
<td>ln(Age) (months)</td>
<td>-0.0369</td>
<td>-0.0413*</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>-1.4258</td>
<td>-1.7070</td>
</tr>
<tr>
<td>Management fee</td>
<td>0.3997</td>
<td>0.4229</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>0.2773</td>
<td>0.3477</td>
</tr>
<tr>
<td>Incentive fee</td>
<td>0.3797**</td>
<td>0.3251**</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>2.4415</td>
<td>2.4319</td>
</tr>
<tr>
<td>High-water mark</td>
<td>-0.0601**</td>
<td>-0.0658**</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>-2.1147</td>
<td>-2.5512</td>
</tr>
<tr>
<td>Personal capital</td>
<td>0.0645**</td>
<td>0.0622**</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>2.5219</td>
<td>2.6129</td>
</tr>
<tr>
<td>Redemption notice (days)</td>
<td>0.0003</td>
<td>-0.00001</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>0.8468</td>
<td>-0.0360</td>
</tr>
<tr>
<td>ln(Size) (USD)</td>
<td>0.0083*</td>
<td>0.0071*</td>
</tr>
<tr>
<td>(t)-stat</td>
<td>1.9378</td>
<td>1.8782</td>
</tr>
<tr>
<td>Past ratio</td>
<td></td>
<td>0.2211***</td>
</tr>
<tr>
<td>(t)-stat</td>
<td></td>
<td>4.4394</td>
</tr>
<tr>
<td>Adj. R-squared</td>
<td>0.0570</td>
<td>0.0924</td>
</tr>
</tbody>
</table>
Table 6
Leverage, appraisal ratio and alpha: panel regressions
- Robustness checks

This table reports the results of panel regressions of alpha and appraisal ratio on leverage and other control variables (January 1999 - August 2009) for different specifications. In the panel "Dummy variable", the leverage is expressed as a dummy variable; in the panel "With past volatility" the past volatility of excess return is included in the regressions. Alpha is the constant in the regression of the net-of-fees excess returns on the 8 factors of Fung and Hsieh (t+1:t+18). The Appraisal ratio variable is obtained by dividing the alpha by the standard deviation of the residuals. Dummy leverage is a dummy variable equal to 1 when the estimated leverage is different from 0. Leverage is the mean leverage over the last 18 months (t-17:t), ln(Age) is the age (in months) of the funds at time t, High-water mark and Personal capital are dummy variables which are equal to 1 if the fund has a high-water mark and 1 if the manager of the fund has personal capital invested in the fund, respectively. The Redemption notice control is expressed in days, ln(Size) is the first reported size of the fund. The control variable Past ratio is obtained by taking the residuals of the regression of past Appraisal ratio and Alpha (t-17:t) on Leverage, Past volatility is obtained by taking the residuals of the regression of the past volatility (t-17:t) on Leverage. t-statistics are adjusted for fund and time clustering.

<table>
<thead>
<tr>
<th>With past volatility</th>
<th>Dummy variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Alpha</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0038*</td>
</tr>
<tr>
<td>t-stat</td>
<td>-1.8222</td>
</tr>
<tr>
<td>Leverage</td>
<td>-0.0020***</td>
</tr>
<tr>
<td>t-stat</td>
<td>-3.1654</td>
</tr>
<tr>
<td>Dummy leverage</td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td></td>
</tr>
<tr>
<td>ln(Age) (months)</td>
<td>0.0004</td>
</tr>
<tr>
<td>t-stat</td>
<td>0.7676</td>
</tr>
<tr>
<td>Management fee</td>
<td>-0.0250</td>
</tr>
<tr>
<td>t-stat</td>
<td>-0.5199</td>
</tr>
<tr>
<td>Incentive fee</td>
<td>0.0024</td>
</tr>
<tr>
<td>t-stat</td>
<td>0.7126</td>
</tr>
<tr>
<td>High-water mark</td>
<td>-0.0003</td>
</tr>
<tr>
<td>t-stat</td>
<td>-0.6639</td>
</tr>
<tr>
<td>Personal capital</td>
<td>0.0004</td>
</tr>
<tr>
<td>t-stat</td>
<td>0.7415</td>
</tr>
<tr>
<td>Redemption notice (Days)</td>
<td>0.0000</td>
</tr>
<tr>
<td>t-stat</td>
<td>1.3075</td>
</tr>
<tr>
<td>ln(Size) (USD)</td>
<td>0.0001</td>
</tr>
<tr>
<td>t-stat</td>
<td>1.3308</td>
</tr>
<tr>
<td>Past ratio</td>
<td>0.1320**</td>
</tr>
<tr>
<td>t-stat</td>
<td>1.9612</td>
</tr>
<tr>
<td>Past volatility</td>
<td>-0.0509**</td>
</tr>
<tr>
<td>t-stat</td>
<td>-2.1466</td>
</tr>
<tr>
<td>Adj. R-squared</td>
<td>0.0929</td>
</tr>
</tbody>
</table>