Comment on the proposed CRD amendment on significant risk transfer

M. Peters and H. Pirotte

This paper is an opinion on the public consultation on possible changes to the Capital Requirements Directive1 and more particularly on the Annex IX Part 2 (item 10 on the Technical Amendments to Directive 2006/48/Ec), related to the significance of the risk transfer in the context of securitisation operations. It will demonstrate that the onus should be put on an economic approach to significant risk transfer rather than a mechanical approach, the latter being less capable of apprehending the various risk profiles a securitisation transaction might take.

JEL Classifications: G28, G21
Keywords: Securitisation, significant credit risk transfer.

CEB Working Paper N° 08/021
June 2008
Comment on the proposed CRD amendment on significant risk transfer

Marc Peters* and Hugues Pirotte*

Abstract

This paper is an opinion on the public consultation on possible changes to the Capital Requirements Directive and more particularly on the Annex IX Part 2 (item 10 on the Technical Amendments to Directive 2006/48/Ec), related to the significance of the risk transfer in the context of securitisation operations. It will demonstrate that the onus should be put on an economic approach to significant risk transfer rather than a mechanical approach, the latter being less capable of apprehending the various risk profiles a securitisation transaction might take.

This version: June 16th, 2008

Keywords: Securitisation, significant credit risk transfer.

JEL classification: G28, G21

---

* Banking, Finance and Insurance Commission, Brussels, Belgium. The views and findings of this paper are entirely those of the author. They do not necessarily reflect the views of the Banking, Finance and Insurance Commission (CBFA).

* Center E. Bernheim, Solvay Business School, Université Libre de Bruxelles and Luxembourg School of Finance (LSF), University of Luxembourg. (corresponding author). Phone number: +32.2.650.65.21 / Fax number: +32.2.650.41.88 / e-mail address: hpirotte@ulb.ac.be

1 CRD, consisting of Directives 2006/48/EC and 2006/49/EC.
1 Introduction

In the light of the recent market crisis, securitisation products have been largely under the microscope. What is being questioned is not only the “originate-to-distribute” model but also the re-securitisation and warehousing activities. Banks have been facing commercial papers (from asset-backed commercial paper (ABCP) conduits) or senior securitisation tranches (from warehoused assets) that they were unable to sell on the market. In that situation, they are forced to rely on their balance sheet structure and to book impairments when they become unavoidable.

In the present subprime crisis case, macroeconomic conditions could not be denied as being the major driver of the “originate-to-distribute” model, but the increasing use of this kind of products was also partly due to the existing gap in the Basle I framework and to some extent, the potential it creates for regulatory arbitrage.

At the time Basel I developed, securitisation products were less used. They developed rapidly as a useful tool for credit risk management but also delivered their potential for funding while claiming capital requirements relieves (at last but not least). Some countries then elaborated some local principles framing the possible use of securitisation products (IFC [2004]). Nevertheless, regulatory bodies had to wait until Basel II to see a fully-fledged dedicated regulatory framework for securitisation operations.

In the current consultation on the Capital Requirements Directive (CRD), Annex IX Part 2 (item 10 on the Technical Amendments to Directive 2006/48/EC), the conditions under which “significant credit risk shall be considered to have been transferred” are revisited and the following proposal is made:

“(i) the risk-weighted exposure amounts of the mezzanine securitisation positions that are held by the originator credit institution in this securitisation do not exceed 50% of the risk weighted exposure amounts of all mezzanine securitisation positions that exist in this securitisation.”

An additional condition (ii) is also provided when no mezzanine positions are present. Mezzanine securitisation positions are defined indirectly, according to the risk weights which would be assigned, i.e. based on external ratings assessments.

Our desire of improvement stands on two main arguments:

1) First, the definition of what “mezzanine” represents is unclear and wide, too indirect and finally relies upon the external ratings assessment. The proposed definition of “mezzanine” would include all rating classes between A and BB-(considering aa an example the Fitch Ratings scale). Our comment is that the character of “mezzanine” is a relative one, given the other existing classes such as the “equity” and the “senior” tranches.

---

2 Many banks were still under Basle I regulation at the time where the history of regulatory arbitrage in the context of securitisation starts.
3 The following information can be deduced from the text and the corresponding tables in the regulation:
   - For the standardized approach, mezzanine is such that : $1250\% > \text{risk-weight} > 20\%$
   - For the RBA:
     - Senior Positions only (First Column) : $1250\% > \text{risk-weight} > 8\%$
     - Base case (Second column - the more realistic to consider) : $1250\% > \text{risk-weight} > 15\%$
     - Concentrated Positions (Third Column - less than 6 underlying exposures) : $1250\% > \text{risk-weight} > 25\%$
2) Second, given the lack of granularity in this definition, proposing a ratio of 50% of the risk weighted exposure amounts is too vague. As we will show in Section 4, the economic exposure along this wide definition of the mezzanine tranche may vary considerably. Undoubtedly, the real economic exposure of holding the top 50% vs. the bottom 50% of the tranche will differ completely.

Finally, the text proposes to that:

“Notwithstanding the above, the competent authority may decide on a case-by-case basis that significant credit risk may be considered to have been transferred if the competent authority is satisfied that a possible reduction of capital requirements that the originator achieves by the securitisation is justified by a commensurate and material transfer of credit risk to third parties.”

The present paper will stand on this argument to then propose or recapitulate a framework that could easily be formalised to evaluate the real proportion of the credit risk exposure being transferred. Then a threshold percentage can be fixed.

Moreover, securitisation is a wide notion. They are many forms of securitisations and we will have a particular focus here on securitizations that involve the creation of securities of varying degrees of exposure (the so-called “tranches”) due to their ranking with respect to the others. These latest forms are much more difficult to evaluate as the risk appraisal will depend not only on the intrinsic qualities of the underlying pool of assets but also on the design of the tranches themselves.

By using the example of a cash collateralized debt obligation (CDO), we will show in the present paper that these two factors are the key drivers of the fundamental economic exposure of this type of instruments. We want to demonstrate that an approach based on a forfeit rate multiplier and a vague definition of the mezzanine notion, as the one proposed under the current CRD consultation (European Commission [2008]), is definitely not precise enough, nor accurate. Moreover, presenting a regulatory framework insensitive to these factors can cause secondary effects by allowing financial institutions and regulators to labour under misapprehension.

Next section will present some background on securitisation and what makes “tranchéd” securitisations more difficult to appraise. Section 3 will provide some illustrative examples of regulatory or prudential arbitrage under Basle I and Basle II methodologies. Section 4 is dedicated to a pedagogical presentation of various methodologies that can be used to evaluate and risk assess tranches. Section 5 aims at proposing a simple framework to make the forfeit approach more sensitive. Conclusions follow.

2 Some background on modern forms of securitisation

A securitisation operation generically involves the creation of securities that rely on a pool of assets. In its simplest form, all securities have exactly the same risk-return profile as they are all identical slices of a given pie. In this form of securitisation, the risk exposure of every tranche is identical to that of the original portfolio of assets.

4 Since we don’t really propose anything new but propose to use existing techniques, showing that we can still provide a reasonable way to assess the credit risk exposure and the amount transferred.
Collateralized debt obligations (CDOs), a modern form of securitisation, propose slices with a varying degree of risk-return exposure. It is the analogy of a pie being sliced horizontally, i.e. somebody gets the fruits, someone else gets the cream and the last one gets the hard cake. As there is no free-lunch, every holder of a slice should be compensated proportionally to the distaste inspired by that slice. Figure 1 compares both cases.

We would like to rely on this type of product, i.e. a CDO-type instrument, to illustrate how the economic exposure of a securitisation operation can vary depending on the type of slice being chosen given the overall design of the tranches.

It is important to stress here that the risk exposure attached to a particular tranche is not linear at all and that it depends on two main branches of properties:

1) The quality of the underlying loans or other instruments being securitized, i.e. the asset side of the SPV.

   a) The average credit worthiness of the pool of loans and its distribution among them.

   The quality of credit worthiness is mainly evaluated today through the rating systems used by the rating agencies. The huge dependence to this mechanism can be put into question after the acknowledgment by the same agencies of some calculation problems on the credit standing of synthetic vehicles. A thorough due diligence and the requirement of a proprietary backup of the credit worthiness estimation of counterparts should be the drivers of the evolving regulation in this matter.

   b) The economic link between the individual loans.

   In modern finance, correlation is the parameter at the heart of the diversification power of a portfolio. In credit risk, correlation of changes in value of the underlying assets being securitized must also have some

---

5 We imagine here for the sake of clarity that we securitise loans but this is applicable to any underlying instrument or asset.
importance. Indeed, some of the models we use today, like the Vasicek [1987,1991] are based on the existence of such a correlation parameter to justify the ranking of defaults over time inside the pool of loans. The latter is then a necessary condition to the existence of multiple and differentiated tranches. Otherwise said, would the loans be perfectly positively correlated, no tranching could be made and there would be only one type of security on the liability side of the SPV.

2) The design of the tranches of the CDO, i.e. the liability side of the SPV.

   a) The so-called attachment and detachment points of every tranche, i.e. the rank of the tranche.

   b) Specific features like credit enhancement techniques.

   We will keep these out of the framework as they transform the original exposure that we want to focus on.

The measurement of this risk exposure is crucial to understand how much risk is transferred when selling tranches to other counterparties, i.e. the goal of the present paper.

In order to illustrate the problem of the credit risk exposure of CDO tranches, we like to use the following example stemming from the IMF Global Financial Stability report (2006) that compares the notional value of the tranches vs. the notional size of the portfolio of underlying credits that would fully hedge the exposure (“the delta-adjusted exposure”).

Figure 2. Tranche Notional Value Versus Economic Risk Transfer (IMF report).

The conclusion of this figure is clear: such a securitisation exercise is a leveraged one where the notional amount of the tranche is inversely related to the intrinsic risk amount. It is also easier to understand the fear of some central bankers: what if those who buy the equity tranches are the most unregulated ones in the market, i.e. the hedge funds? We produce a huge amount of senior notional by pushing the risk into the lower end. We can also directly see that applying a static ratio to the notional amount of the mezzanine tranches being held is too simplistic. Not only the risk is exponentially decreasing with the seniority but the mezzanine tranche “3%-7%” is four times riskier than the mezzanine tranche “7%-10%”.

3  Current criteria and regulatory arbitrage

While developing the securitisation framework, regulators were particularly cautious about the circumstances under which regulatory relieves could be claimed by institutions, having in mind the avoidance of any regulatory arbitrage which would consist of an artificial transfer of the risk related to underlying credit positions.

The core principle was to ensure that the securitisation framework was used for legitimate purposes, i.e. real credit risk management. In the latter, a bank retaining securitized positions or maintaining a relation with the structure as sponsor could benefit from lower risk-weighted capital requirements than the one originally associated with the underlying pool of assets, under some qualitative requirements.

Table 1 presents the qualitative conditions that must be met for a securitization operation, whether traditional or synthetic, to be recognised as producing a significant risk transfer.

<table>
<thead>
<tr>
<th>Table 1. Qualitative conditions for a securitisation operation to be considered as creating a significant risk transfer.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Traditional Securitisation</strong></td>
</tr>
<tr>
<td>--------------------------------</td>
</tr>
<tr>
<td>Credit risk mitigants must comply with the requirements set out on the recognition of Credit Risk Mitigation Techniques within the Basel II framework. Banks may not recognize SPEs as eligible guarantors in the securitization framework.</td>
</tr>
<tr>
<td>Significant credit risk associated with the securitised exposures has been transferred to third parties.</td>
</tr>
<tr>
<td>The transferor does not maintain effective or indirect control over the transferred exposures. The assets are legally isolated from the transferor in such a way (e.g. through the sale of assets or through sub participation) that the exposures are put beyond the reach of the transferor and its creditors, even in bankruptcy or receivership. These conditions must be supported by an opinion provided by a qualified legal counsel.</td>
</tr>
<tr>
<td>Banks must transfer significant credit risk associated with the underlying exposure to third parties.</td>
</tr>
</tbody>
</table>

Source: BCBS [2002].
The transferor is deemed to have maintained effective control over the transferred credit risk exposures if it: (i) is able to repurchase from the transferee the previously transferred exposures in order to realise their benefits; or (ii) is obligated to retain the risk of the transferred exposures. The transferor’s retention of servicing rights to the exposures will not necessarily constitute indirect control of the exposures.

The securities issued are not obligations of the transferor. Thus, investors who purchase the securities only have claim to the underlying pool of exposures.

The transferee is an SPE and the holders of the beneficial interests in that entity have the right to pledge or exchange them without restriction.

An opinion must be obtained from a qualified legal counsel that confirms the enforceability of the contracts in all relevant jurisdictions.

Clean-up calls must satisfy the following conditions: (i) the exercise of the clean-up call must not be mandatory, in form or in substance, but rather must be at the discretion of the originating bank; (ii) the clean-up call must not be structured to avoid allocating losses to credit enhancements or positions held by investors or otherwise structured to provide credit enhancement; and (iii) the clean-up call must only be exercisable when 10% or less of the original underlying portfolio, or securities issued remain, or, for synthetic securitizations, when 10% or less of the original reference portfolio value remains.

Clean-up calls must satisfy the following conditions: (i) the exercise of the clean-up call must not be mandatory, in form or in substance, but rather must be at the discretion of the originating bank; (ii) the clean-up call must not be structured to avoid allocating losses to credit enhancements or positions held by investors or otherwise structured to provide credit enhancement; and (iii) the clean-up call must only be exercisable when 10% or less of the original underlying portfolio, or securities issued remain, or, for synthetic securitizations, when 10% or less of the original reference portfolio value remains.

The securitization does not contain clauses that (i) require the originating bank to alter systematically the underlying exposures such that the pool’s weighted average credit quality is improved unless this is achieved by selling assets to independent and unaffiliated third parties at market prices; (ii) allow for increases in a retained first loss position or credit enhancement provided by the originating bank after the transaction’s inception; or (iii) increase the yield payable to parties other than the originating bank, such as investors and third-party providers of credit enhancements, in response to a deterioration in the credit quality of the underlying pool.

The instruments used to transfer credit risk may not contain terms or conditions that limit the amount of credit risk transferred, such as those provided below: (i) Clauses that materially limit the credit protection or credit risk transference (e.g. significant materiality thresholds below which credit protection is deemed not to be triggered even if a credit event occurs or those that allow for the termination of the protection due to deterioration in the credit quality of the underlying exposures); (ii) Clauses that require the originating bank to alter the underlying exposures to improve the pool’s weighted average credit quality; (iii) Clauses that increase the banks’ cost of credit protection in response to deterioration in the pool’s quality; (iv) Clauses that increase the yield payable to parties other than the originating bank, such as investors and third-party providers of credit enhancements, in response to a deterioration in the credit quality of the reference pool; and, (v) Clauses that provide for increases in a retained first loss position or credit enhancement provided by the originating bank after the transaction’s inception.
Regulators then tried to develop a framework in which the capital requirements (or the risk-weights) associated to each tranche is commensurate to their inherent risk. For instance, as the equity piece of a securitisation product is supposed to contain all (or the vast majority) of the expected loss, the requirement would be a 1250 % risk-weight which corresponds to a requirement equivalent to the notional amount of the position (or to a deduction of this notional amount from own funds).

Securitisation has then been treated in a different part of the Basel II text. The first pillar is mainly considering the treatment of securitisation positions from the investor side thereby determining the capital required according to the nature of the position (i.e. the risk-weighted asset corresponding to the invested position). The second pillar is more concerned by the treatment of securitisation from an originator point of view as it mainly deals with issues related to the avoidance of regulatory arbitrage (i.e. the capital requirement should be aligned with the level of risk retained) and the monitoring of securitisation transaction developed by institutions.

The avoidance of capital arbitrage strategies has always deserved a specific attention from regulators. The new Basel II framework has been designed to better align the regulatory requirements with the underlying economic risk of the exposure and, thereby, reduce any incentive for regulatory arbitrage. This is especially true with the introduction of a specific framework for securitization. However, Fitch Ratings [2005] and Andreas Jobst [2004] tend to demonstrate that the effectiveness of the new framework in achieving that objective might strongly depends on the type of underlying assets subject to securitization and the structure of the securitization transaction.

By contrast, the Basel I framework was poorly granular and did not give much possibilities to align the requirements with the true economic risk of assets. Indeed, considering the case of loans, risk weights were fixed at 100 % in case of a corporate exposure and at 50 % in case of a residential mortgage exposure, whatever the economic risk of the underlying asset. This situation was at the origin of the regulatory arbitrage using securitization operations. Indeed, banks tend to securitize high-quality assets for which the regulatory requirements were too costly by comparison with their economic risk. The drawback was that banks were then keeping lower-quality assets on their balance sheet. Moreover, originating banks generally kept the riskiest first loss positions, thereby decreasing their Basel I capital requirements without reducing their economic exposure (actually, it could be considered as a “realignment” between the regulatory and the – then expanding notion of – economic capital).

Since it was not possible to develop a framework that would enable a widespread elimination of regulatory arbitrage possibilities, the Basel II framework for securitization introduced the notion of “significant risk transfer”.

As demonstrated by the two schematic following examples, the Standardized and IRB charges for securitization are generally – depending on the underlying assets and the structure of the securitization – lower than the charge required by the original portfolio. These simplistic examples are presented to clarify the potential for regulatory arbitrage and to illustrate the need for introducing the above-mentioned notion – “significant risk transfer” – in the Basel II securitization framework.

Additionally Fitch Ratings [2005] also demonstrated that depending on the type of securitization products (i.e. credit card ABS, CMBS,...), the incentive for securitization might be higher. This is important to consider as the notion of “significant risk transfer”
– especially the modifications proposed by the European Commission – is not expressly considering the type of underlying assets.

We will need the following figure presenting Basel II securitization risk weights for the development of our examples of regulatory arbitrage.

**Figure 3. Securitisation risk weights in Basel II**

Source: BCBS [2005].

### Panel A: Standardized Approach

<table>
<thead>
<tr>
<th>Long-term rating category</th>
<th>AAA to AA-</th>
<th>A+ to A-</th>
<th>BBB+ to BBB-</th>
<th>BB+ to BB-</th>
<th>B+ and below or unrated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Weight</td>
<td>20%</td>
<td>50%</td>
<td>100%</td>
<td>350%</td>
<td>Deduction</td>
</tr>
</tbody>
</table>

**Note:**
- Panel A refers to the Standardized Approach.
- Risk weights are applied to the portfolio assets.
- Deduction refers to the reduced risk weight due to securitization.

### Panel B: Rating-Based Approach

**RBA risk weights when the external assessment represents a long-term credit rating and/or an inferred rating derived from a long-term assessment**

<table>
<thead>
<tr>
<th>External Rating (Illustrative)</th>
<th>Risk weights for senior positions and eligible senior IAA exposures</th>
<th>Base risk weights</th>
<th>Risk weights for tranches backed by non-granular pools</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>7%</td>
<td>12%</td>
<td>20%</td>
</tr>
<tr>
<td>AA</td>
<td>8%</td>
<td>15%</td>
<td>25%</td>
</tr>
<tr>
<td>A+</td>
<td>10%</td>
<td>18%</td>
<td></td>
</tr>
<tr>
<td>A-</td>
<td>12%</td>
<td>20%</td>
<td>35%</td>
</tr>
<tr>
<td>A</td>
<td>20%</td>
<td>35%</td>
<td></td>
</tr>
<tr>
<td>BBB+</td>
<td>35%</td>
<td>50%</td>
<td></td>
</tr>
<tr>
<td>BBB-</td>
<td>50%</td>
<td></td>
<td>75%</td>
</tr>
<tr>
<td>BB+</td>
<td></td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>BB</td>
<td></td>
<td></td>
<td>250%</td>
</tr>
<tr>
<td>BB-</td>
<td></td>
<td></td>
<td>425%</td>
</tr>
<tr>
<td>Below BB- and unrated</td>
<td></td>
<td></td>
<td>650%</td>
</tr>
</tbody>
</table>

Example 1 here below presents the case of a commercial mortgage loan portfolio of 100 EUR held by a Standardized bank. It provides the comparison between the Basel II charges required for the original portfolio and for the securitisation assets. The
assumption is made that the bank would keep all securitised positions and that those positions are all rated.

Example 2 presents the case of a commercial mortgage loan portfolio of 100 EUR held by an IRB bank. The same assumptions are made. In addition, we assume that, for a given portfolio and all things being equals, a bank using IRB could present a regulatory capital requirement that is 20 % lower than the one for the Standardized bank.

**Example 1. Securitisation risk weights in Basel II for the Standardized Bank**

- The capital charge for the *unsecuritised* portfolio would be:
  
  \[ \text{Nominal} \times \text{Risk Weight} \times 8\% = 100 \times 100 \times 8\% = 8 \]

- The capital charge for the *securitised* portfolio would be:
  
  We consider a structure with a senior tranche representing 82 % of the entire securitized transaction, a mezzanine tranche representing 17 % and an equity piece for 1 %. We also consider the two possible risk weights for the mezzanine tranche thereby considering two different rating qualities for this position.

  - **Case A :**
    
    \[
    \begin{align*}
    & \text{Nominal} \times \text{Risk Weight} \times 8\% \\
    & 82 \times 20 \% \times 8\% = 1.312 \\
    & 17 \times 100 \% \times 8\% = 1.36 \\
    & 1 \times 1250 \% \times 8\% = 1 \\
    \text{Total charge} &= 1.312 + 1.36 + 1 = 3.672
    \end{align*}
    \]
  
  - **Case B :**
    
    \[
    \begin{align*}
    & \text{Nominal} \times \text{Risk Weight} \times 8\% \\
    & 82 \times 20 \% \times 8\% = 1.312 \\
    & 17 \times 350 \% \times 8\% = 4.76 \\
    & 1 \times 1250 \% \times 8\% = 1 \\
    \text{Total charge} &= 1.312 + 4.76 + 1 = 7.072
    \end{align*}
    \]

**Example 2. Securitisation risk weights in Basel II for the IRB Bank**

- The capital charge for the *unsecuritised* portfolio would be:
  
  We assume that the standardized charge (Nominal x Risk Weight x 8 % = 100 x 100 x 8 % = 8) is reduced by 20 %, which results in a IRB charge of 6.4.

- The capital charge for the *securitised* portfolio would be:
  
  We consider a structure with a senior tranche representing 82 % of the entire securitized transaction, a mezzanine tranche representing 17 % and an equity piece for 1 %. We also consider two different risk weights for the senior and the mezzanine tranches thereby considering two different rating qualities for these positions.

  - **Case A :**
    
    \[ \text{Nominal} \times \text{Risk Weight} \times 8\% \]

---

6 For the sake of the example, we use an extreme case to compute the capital requirement under two different approaches (securitisation vs. no securitisation). In reality, the bank would have a hybrid behavior, choosing some tranches to keep and other to sell.

7 Given our experience, this can be considered as a realistic scenario.
These examples show that a bank could, under certain circumstances (depending on the structure of the securitisation and the type of underlying assets as suggested by Fitch Ratings [2005]) and with the same risk profile, benefits from a capital relief by using securitisation.

In addition, these examples also illustrate the fact that investing in securitization products might be more interesting than directly investing in the underlying assets from a regulatory capital viewpoint as the capital charge for an equivalent risk profile could be reduced.

4 Analysis of the economic exposure of a securitisation operation

Practitioners and academics are aware that the quantitative models in use nowadays are not yet optimal in terms of behaviour and sensitivity to special features and events. Yet, they are based on an economic abstract representation of the reality which provides a fundamental reasoning that we have a hard time to find out in the static ratio of 50% of a broad definition of what mezzanine financing is. Simplification assumptions are widely used in financial economics but there is always a red line linking the model to the fundamentals.

We will thus now focus on two main branches of quantitative models to identify opportunities to provide a more sensitive framework that is still sufficiently easy to implement and to understand.

A The capital structure model

For any financial economics researcher, it should be clear that the tranched securitisation operation proposed by a CDO is nothing else than a perfect replica of the hierarchical capital structure of a real firm. Then it becomes quite attractive to reuse the Mertonian contingent claims approach in the sense of Ingersoll [1977], where mezzanine financing, in the form of a convertible bond, is examined and modelled.

Of course, this model lacks many practical details since it relies on: a single maturity where the firm can only default at maturity, a constant interest rate and asset volatility measure. But yet, we can map realistically these values to implied estimations from the firm’s observables and come up with an intuitive framework.
Generically, a given layer or tranche of a CDO is mezzanine as long as there is some lower layer to rely on as a shock absorber, i.e. a more "equity-like" layer. The bigger this layer is, the closer is the mezzanine tranche to be a senior tranche.

Simplifying a little bit the context and admitting a given number of tranches, – an equity tranche, a mezzanine tranche and a senior tranche – we can easily show that the payoff profile follows the structure presented in Table 2.

Table 2. Contingent-claims payoff profile at maturity for a capital structure consisting of three tranches: senior, mezzanine and equity-like.

<table>
<thead>
<tr>
<th></th>
<th>$V_T &lt; S^* + M^*$</th>
<th>$V_T &gt; S^* + M^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Equity (E₀)</strong></td>
<td>0</td>
<td>$V_T - S^* + M^*$</td>
</tr>
<tr>
<td><strong>Mezzanine (M₀)</strong></td>
<td>$V_T &lt; S^*$</td>
<td>$V_T - S^*$</td>
</tr>
<tr>
<td><strong>Senior (S₀)</strong></td>
<td>$V_T$</td>
<td>$S^*$</td>
</tr>
</tbody>
</table>

It can be easily shown that the payoff at maturity of the mezzanine tranche is

$$M_T = \min(M^*, \max(0, V_T - S^*))$$

$$= M^* - \max(M^* - \max(0, S^* - V_T), 0)$$

$$= M^* - \max((S^* + M^*) - V_T, 0) + \max(S^* - V_T, 0)$$

(1)

i.e. the value of a mezzanine tranche is equivalent to

$$M_0 = M^* e^{-r_f T} - \text{Put}(V_T, (S^* + M^*)) + \text{Put}(V_T, S^*)$$

(2)

where $r_f$ is the risk-free interest rate and $(S^* + M^*)$ and $S^*$ can be viewed as attachment and detachment points respectively of the mezzanine tranche. The put values can be obtained by applying the Black & Scholes (& Merton) option pricing model.

This approach is still simplistic and suffers from two major drawbacks for its application to our current case:

1) If there is only one broad mezzanine tranche, it does not solve the granularity problem. The whole tranche is valued only with respect to its attachment and detachment points.

2) It considers the pool of assets as a homogeneous one, evolving through time, without any particular differentiation in terms of behaviour of its individual components.

These two points are solved in practice by using Monte Carlo simulation on each individual asset component of the pool of assets $V$. Each individual asset is simulated with some correlation to the others and, for a particular time-to-maturity, a function counts the number of survivors in each simulation exercise. This is of course a heavier numerical exercise.
Notwithstanding, equation (2) can still be used as a simple way to quickly assess the credit risk exposure of a given tranche. The granularity problem can be overcome by applying the same equation to smaller (and more numerous) tranches.

B Using the one-factor Gaussian copula

Vasicek (1987) argues that the Mertonian framework presented earlier, in its version where firms’ assets values are correlated to each other and they may default when that value falls below some threshold, is isomorphic with a one-factor Gaussian copula model. Other authors since then, like Hull and White [2003], have been showing various modelling applications around this framework.

Figure 4. The Gaussian Copula

Figure 4 presents the idea of the Gaussian copula model. The default of every individual loan in the portfolio can be viewed as a “stopping-time” problem, to take back some jargon of mathematicians, i.e. a distribution of the time-to-default. If the time-to-default happens to be beyond the maturity of a given instrument, then there is no default. But, since that default time has some randomness, there is some risk related to it.

Since we don’t really know what kind of distribution could be used for that “stopping-time”, we use a quantile-by-quantile mapping of that “unknown” distribution to one that we know well, the normal or Gaussian distribution. Marginally, each individual new distribution is isomorphic with the original one.

Then, we can refine the representation of $R$ by combining a common component with an idiosyncratic one:

$$ R_i = a_i M_{\text{common}} + \sqrt{1-a_i^2} Z_{\text{specific}}, \quad M, Z \sim N(0,1) $$

(3)

where $M$ is the common component to all counterparts. For two counterparts $i, j$, the correlation between them results from the following multiplication: $a_i a_j$. 
We can then compute the probability to mature after the stopping-time $T^*$ which can be mapped to $R^*$. By standardising and replacing $R^*$ by its mapping to the inverse of the $Q$ distribution on $T^*$, we obtain

$$
\Pr(R^* < R_t | M) = \Pr(T^* < T | M) \rightarrow N\left[ \frac{R_t - aM}{\sqrt{1-a^2}} \right] = N\left[ \frac{N^{-1}(Q(T)) - aM}{\sqrt{1-a^2}} \right]
$$

(4)

Assuming that all individual loans or counterparts share the same average $a$, we replace it by $\sqrt{\rho}$ and we also replace $-M$ by $N^{-1}(X)$, the inverse of the state of the market:

$$
\Pr(T^* < T | M) = Q(T | M) = N\left[ \frac{N^{-1}(Q(T)) + \sqrt{\rho}N^{-1}(X)}{\sqrt{1-\rho}} \right]
$$

(5)

A simple binomial model applied on this can allow us to know the probability of more than “n” defaults:

$$
\Pr(\text{#defaults} \geq n | M) = \sum_{k=n}^{N} \frac{N!}{(N-k)!k!} Q(T | M)^k \left[ 1 - Q(T | M) \right]^{N-k}
$$

(6)

or

$$
\Pr(\text{#defaults} = n | M) = \frac{N!}{(N-n)!n!} Q(T | M)^n \left[ 1 - Q(T | M) \right]^{N-n}
$$

We can use this type of expression to know the probability of a tranche to be hit and therefore to put a price on it (conditional on $M$). Using this idea and integrating over $M$ allows us to find the average price estimation of an “nth-to-default” swap or of a given tranche of a CDO.

We programmed this framework in Scilab*, putting into perspective the attachment and detachment points of every tranche.

Imagine that we want to estimate the expected loss on a particular tranche, let’s say the tranche “3%-7%” of a pool of 125 counterparts, i.e. the tranche from 4 to 9 defaults. That means that is more than 8 defaults occur, the whole tranche is consumed (100% loss). And if 4 or less defaults occur, the tranche is safe (0% loss).

Therefore, we need to assess the probability of each “nth-to-default” case with equation (6) and integrate numerically over all possibilities of $M$, for all the potential numbers of default belonging to that tranche.

**Example 3. Computation of the expected loss using the One-factor Gaussian copula approach**

We use equation (6) with the following parameters:

- A (conditional) probability of default of 2% per annum.
- An average correlation coefficient between the counterparts of 50%.
- A recovery rate of 0%.
- Attachment and detachment points of 4 to 9 defaults on a portfolio of 125 counterparts.

* See www.scilab.org
We can obtain the following results:
- The expected loss is 10.64% of the nominal amount.
- The total probability of the tranche being hit is 13.79%.

<table>
<thead>
<tr>
<th># of defaults</th>
<th>Probability</th>
<th>% of the tranche lost</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 or more</td>
<td>8.1024%</td>
<td>100%</td>
</tr>
<tr>
<td>8</td>
<td>1.0051%</td>
<td>80%</td>
</tr>
<tr>
<td>7</td>
<td>1.2208%</td>
<td>60%</td>
</tr>
<tr>
<td>6</td>
<td>1.5173%</td>
<td>40%</td>
</tr>
<tr>
<td>5</td>
<td>1.9448%</td>
<td>20%</td>
</tr>
<tr>
<td></td>
<td>Compounded average</td>
<td>10.64%</td>
</tr>
</tbody>
</table>

Example 3 shows that it is possible to obtain estimations of the credit risk exposure of a given tranche by taking into account all the factors mentioned above. Also, since it is possible to stratify the expected loss result into sub-layers per amount of defaults considered, we can obtain the desired granularity inside each tranche irrespective of its size, i.e. irrespective of the difference between the extreme attachment and detachment points.

Moreover, the estimations show a strong sensitivity to the correlation coefficient. Figure 5 shows this effect.

**Figure 5. Sensitivity of the Expected Loss measure of tranches to a change in the correlation.**

The parameters used are: a (conditional) probability of default of 2% per annum, a recovery rate of 0%, attachment and detachment points based on a standard US tranching.

---

*The result is impressively quick to obtain and clearly justifies the amount of additional sophistication.*
The comparison of these two graphs shows that the interest alignment of the equity tranche vs. other tranches is far from being obvious and that conveys governance issues.

## 5 Conclusion

The present paper aims at providing some guidelines around the assessment of the credit risk exposure of securitisation such like CDOs.

Our presentation shows that the current framework does not define the mezzanine tranche in a clear and transparent way, nor does it satisfy the granularity requirement. Moreover, it relies too much on the credit rating framework.

In such a complex securitisation operation, the quality of the underlying portfolio as well as the parameters of the design – here the attachment and detachment points – are supremely important in the valuation process.

Finally, some methods exist today and have been shown to answer most of these questions.

## References

- European Commission [2008], “CRD Potential Changes, Co-decision, Comitology”, Consultation by the DG Internal Market and Services, Financial Institutions, Banking and financial conglomerates, April 16th 2008.


Hull, John and Alan White [2003], “Valuation of a CDO and an nth to default CDS without Monte Carlo simulation”, Working paper, University of Toronto.


