Developing Children’s Understanding of Fractions: An Intervention Study

Florence Gabriel1,2, Frédéric Coché3, Dénes Szucs2, Vincent Carette3, Bernard Rey3, and Alain Content1

ABSTRACT—Fractions constitute a stumbling block in mathematics education. To improve children’s understanding of fractions, we designed an intervention based on learning-by-doing activities, which focused on the representation of the magnitude of fractions. Participants were 292 Grade 4 and 5 children. Half of the classes received experimental instruction, while the other half pursued their usual lessons. For 10 weeks, they played five different games using cards representing fractions (e.g., Memory and Blackjack). Wooden disks helped them represent and manipulate fractions while playing games. Our results showed an improvement in the conceptual understanding of fractions. The findings confirmed that the usual practice in teaching fractions is largely based on procedural knowledge and provides only minimal opportunities for children to conceptualize the meaning and magnitude of fractional notations. Furthermore, our results demonstrate that a short intervention inducing children to manipulate, compare, and evaluate fractions improves their ability to associate fractional notations with numerical magnitude.

1Laboratoire Cognition, Langage et Développement, Faculté des Sciences Psychologiques et de l’Education, Université Libre de Bruxelles
2Department of Experimental Psychology, Centre for Neuroscience in Education, University of Cambridge
3Service des Sciences de l’Education, Faculté des Sciences Psychologiques et de l’Education, Université Libre de Bruxelles

Address correspondence to Florence Gabriel, Laboratoire Cognition, Langage et Développement, Faculté des Sciences Psychologiques et de l’Education, Université Libre de Bruxelles, Ixelles, Belgium; e-mail: fcg25@cam.ac.uk, Dénes Szucs, Department of Experimental Psychology, Centre for Neuroscience in Education, University of Cambridge, Cambridge, UK; e-mail: ds377@cam.ac.uk, or Alain Content, Laboratoire Cognition, Langage et Développement, Faculté des Sciences Psychologiques et de l’Education, Université Libre de Bruxelles, Ixelles, Belgium; e-mail: alain.content@ulb.ac.be

Common fractions are quantities that can be represented by a ratio of integers (e.g., 3/4) and are one of the most difficult mathematical concepts to learn for primary school pupils. Indeed, most Grade 5 children cannot place a fraction on a graduated number line (Bright, Behr, Post, & Wachsmuth, 1988) and struggle when asked to order fractions (Mamede, Nunes, & Bryant, 2005). Here, we report an intervention programme designed with the objective to train the conceptual understanding of fractions in a playful setting at the beginning of fraction acquisition (Grades 4 and 5 in the French Community of Belgium).

One crucial feature of numerical conceptual knowledge is the ability to represent number magnitudes (Stigler, Thompson, & Schneider, 2011). One of the major current questions concerning fraction processing is whether the global numerical magnitude of fractions (i.e., a fraction’s real value) is directly available. That is, are educated adults able to process a fraction as a whole? To date, behavioral and imaging studies offer contradictory views. Some studies suggest that only the individual integer components of a fraction are processed (Bonato, Fabbri, Umlita & Zorzi, 2007; Kallai & Tzelgov, 2009), whereas others suggest that the global magnitude of fractions is also represented (Jacob & Nieder, 2009; Schneider & Siegler, 2010). A conciliatory view is that adults and children use a combination of both componential and global fraction processing strategies, depending on task demands and the type of fractions used (Ischebeck, Schocke, & Delazer, 2009; Meert, Gregoire, & Noel, 2009, 2010a, 2010b; Schneider & Siegler, 2010).

Another important issue to consider about children’s difficulties in learning fractions is the distinction between conceptual and procedural knowledge. Conceptual knowledge can be defined as the explicit or implicit understanding of the principles ruling a domain and the interrelations between the different parts of knowledge in a domain (Rittle-Johnson & Alibali, 1999). It can also be considered as the knowledge of central concepts and principles and their interrelations in a particular domain (Schneider & Stern, 2010). In the domain of
fractions, conceptual knowledge refers to a combination of the general properties of rational numbers (such as the principle of equivalent fractions), the understanding of the roles of the numerator and the denominator, and the understanding of global fraction magnitudes.

Procedural knowledge can be defined as sequences of actions that can be put to play to solve specific problems (Rittle-Johnson & Alibali, 1999). Some authors consider procedural knowledge as the knowledge of symbolic representations, algorithms, and rules (Byrnes & Wasik, 1991). The precision of these core numerical representations could provide the necessary foundations to build more complex mathematical abilities, such as arithmetic skills. Moreover, Siegler et al. (2011) showed that accurate representations of fraction magnitude play a key role in both fraction arithmetic and general mathematical abilities. Hence, we investigated whether a training of the better understanding of the magnitude of fractions is beneficial for more complex fraction skills. Similar training programmes have been implemented for integers but never for fractions (Kucian et al., 2011; Siegler & Ramani, 2009; Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006).

We designed an intervention aiming at developing and consolidating the representation of fraction magnitude. The intervention was based on several educational principles: learning-by-doing, starting with a concrete support to progressively build more abstract representations, playfulness, and collaboration. First, it was based on a learning-by-doing programme and followed a constructivist approach, focusing on the meaning of the magnitude of a fraction and how it is related to fractional symbols, challenging pupils' preexisting knowledge about natural numbers (Ni & Zhou, 2005; Stafylidou & Vosniadou, 2004). That is, children were confronted with obstacles that could only be overcome by conceptual change. Many authors agree that difficulties in learning fractions are linked to the whole number knowledge. This has been referred to as the whole-number bias (Ni & Zhou, 2005). Some conceptual change theories insist that the whole number bias will interfere with learning about fractions. But, as Siegler et al. (2011) noted, one of the difficulties in learning fractions originates more from drawing inaccurate analogies between fractions and whole numbers than from drawing parallels between these two types of numbers. Therefore, this intervention fostered correct analogies between fractions and whole numbers. Indeed, we focused on the fact that fractions have magnitudes that can be ordered and compared, just like whole numbers.

Second, our intervention was called “from pies to numbers” as children were invited to use concrete support at first and then build more abstract representations. A common concrete reference support (wooden pieces), based on what pupils know best (disk representation and their parts) was provided. This reference support allowed them to manipulate concrete materials and helped them find answers to problems included in the different games and solve conflicts occurring during the games. The wooden pieces could be superposed so that children could also get a concrete representation of fractions larger than 1. Children usually feel more comfortable when
using concrete aids when dealing with fractions (Arnon, Nesher, & Nirenburg, 2001), and concrete aids, such as the wooden reference support, should help them develop the corresponding abstract concepts (Arnon et al., 2001). In addition, constructivist theories suggest that development goes from the concrete operational stage to a more abstract stage, and therefore, learning should follow this trajectory (Santrock, 2009). Moreover, concrete material may be more engaging and appealing to pupils and thus engage them more in the learning process (Ball, 1992).

A circular support can be seen as a controversial choice as children already have stereotyped representations of fractions as parts of a pie (Nunes & Bryant, 1996). On the contrary, we considered that it would be a good starting point as it corresponds to something that pupils know very well and thus provides a very natural reference. Other representations of fractions such as various symbolic representations, collections of items as well as other continuous shapes were progressively introduced along with the game progression. Children should be using the reference less and less as far as the games progress as more and more diverse representations were proposed throughout the intervention. They should become less dependent of that circular representation and develop a more abstract representation of the magnitude of fractions.

Third, we developed a playful intervention, associating fraction learning with fun. Playfulness is known to have positive effects on the learning process (Sawyer, 2006) and contributes to advances in verbalization, concentration, curiosity, problem-solving strategies, cooperation, and group participation (Kangas, 2010). It can increase children’s motivation and their involvement in the learning process.

Finally, pupils were placed in groups of 3–5 and each group was given a game to play collectively. Several authors conclude that collaborative work provides a deeper understanding of taught concepts as pupils exchange views and work toward identifying the right solution (Cobb, 1993; Forman & Cazden, 1985; Palincsar, 1998).

**METHOD**

**Participants**
We chose to work with Grades 4 and 5 as this corresponds to the first fraction lessons in the French Community of Belgium. In Grade 4, pupils learn how to read and represent the value of a fraction. They start placing fractions on a graduated segment; they learn about equivalent fractions and are trained in the addition and subtraction of fractions with small common denominators. In Grade 5, children learn that fraction represents quantity, are trained to convert fractions into decimal numbers and vice versa, continue to use addition and subtraction of fractions with different denominators, and learn how to simplify fractions. Improper fractions are also introduced in Grade 5.

Eight Grade 4 and 5 classes took part in the intervention programme. These classes came from four schools of the French Community of Belgium. Every school included two classes of Grade 4 and 5. For each grade, one of the classes was the intervention group and the other the control group. Control groups received their usual lessons about fractions. The teachers of control classes were asked to give a 1-hr lesson on fractions per week to match with the time spent by the intervention group on learning fractions. The intervention group did not receive their regular lessons during the intervention and only played the games we designed. We allocated classes to intervention conditions at random.

A total of 292 children (133 Grade 4 and 136 Grade 5 children) took part in the activities. Data for 23 pupils had to be discarded because they did not take part in either the pre- or the posttest.

**Intervention**
We created and adapted five card games involving fractions: Memory, War, Old Maid, Treasure Hunt, and Blackjack. The activities were organized twice a week for 30 min during the usual classes for a period of 10 weeks. Pupils played games in small groups of 3 up to 5 children. Every week a new game was introduced. The second session of the week implied playing the same game at the next level of difficulty (Table 1).

Wooden disks cut into pieces from halves to twelfths were manufactured to help children manipulate fractions and represent fractional magnitudes. That teaching aid allowed children to manipulate concrete equipment and progressively construct a representation of the magnitude of fractions. It also allowed them to compare different fractions during the games. One set of wooden disks (two for the larger class) was available in each classroom and served as reference during the games. At the beginning of the intervention, children were encouraged to use the teaching aid by the experimenter or the teacher every time they were not sure about the outcome of a game. Even

<table>
<thead>
<tr>
<th>Week 1</th>
<th>Presentation of wooden disks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 2</td>
<td>Memory—Level 1</td>
</tr>
<tr>
<td>Week 3</td>
<td>War—Level 1</td>
</tr>
<tr>
<td>Week 4</td>
<td>Old Maid—Level 1</td>
</tr>
<tr>
<td>Week 5</td>
<td>Treasure Hunting—Level 1</td>
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<tr>
<td>Week 6</td>
<td>Blackjack—Level 1</td>
</tr>
<tr>
<td>Week 7</td>
<td>Memory—Level 3</td>
</tr>
<tr>
<td>Week 8</td>
<td>War—Level 3</td>
</tr>
<tr>
<td>Week 9</td>
<td>Old Maid—Level 3</td>
</tr>
<tr>
<td>Week 10</td>
<td>Treasure Hunting—Level 3</td>
</tr>
</tbody>
</table>

**Table 1**

Organization of the Intervention

<table>
<thead>
<tr>
<th>1st session (30 min)</th>
<th>2nd session (30 min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week 1</td>
<td>Presentation of wooden disks</td>
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<tr>
<td>Week 2</td>
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<td>Old Maid—Level 3</td>
</tr>
<tr>
<td>Week 10</td>
<td>Treasure Hunting—Level 3</td>
</tr>
</tbody>
</table>
if there was one set per classroom, it never was a problem as groups of pupils progressed at different rhythms. Moreover, they did not have to compare the same sets of fractions as children from another group, so the reference support could be used by different small groups at the same time. The teacher and the experimenter were always supervising the games to make sure that pupils got to the correct outcome at the end of each game.

Games

Memory
The goal of the Memory game was to associate pairs of fractions that represented the same quantity. This activity implied to match two fractions, transcode symbols and numbers, and memorize the representations of fractions. Cards were laid down in a grid face down. Players took turns flipping pairs of cards over. If the cards matched, the player said “bingo” and showed the pair to the other pupils to make sure they agreed. In case of disagreement between players, they could use the wooden disks to compare both fractions. If the pair was correctly matched, the player kept both cards. The aim of the game was to match more pairs of fractions than the other players.

War
This activity involved comparison of fractions, which led pupils to get a representation of the magnitude of fractions. A total of 40 cards were equally distributed between players. Cards were placed faced down on the table. In unison, players turned over their top card and played it face up. The player who turned the card representing the largest fraction could collect all the cards in the middle of the table and move them to the bottom of his stack. When cards were of equal value, each player laid down another card and the higher valued fraction won. In case of disagreement between players, they were allowed to use the wooden disks to compare fractions. When a player did not have any cards left, he was eliminated. When there were only two players left, they were both declared winners and a new game could start again.

Old Maid
The goal of this game was to form pairs of fractions representing the same magnitude. This activity involved matching two fractions. Children had to compare symbolic as well as figural representations of fractions and associate them with their corresponding magnitudes. Unlike Memory, Old Maid did not involve memorization. In this game, children compared more than just two cards to match fractions. Indeed, they had to compare all the cards they had in their hand. The first step of the game was to remove one card, resulting in one unmatched card. This card became the “Old Maid.” Players had to discard any pairs they had faced up. Each player took turns offering his hand face down to the pupil next to him. That pupil selected a card and added it to his hand. This player then saw whether the selected card made a pair with their original cards. If so, the pair was discarded face up as well. The player who just took a card then offered his or her hand to the person sitting next to him and so on. In case of disagreement between players, they were encouraged to use the wooden disks to compare fractions. The game went on until all players except one had no cards left. The player who was left with the one unmatchable card lost the game.

Treasure Hunt
This activity involved comparison between fractions, discovery of improper fractions, and addition of fractions by estimation or using the wooden disks. This way of adding fractions focused on the representation of the magnitude of fractions and did not require the understanding of the lowest common denominators procedure. For this game, the deck of cards was laid out face down on the table. Two cards were turned over. The first player took the card that he thought represented the larger magnitude. Another card was then flipped over and the next player chose the card representing the larger magnitude. When every player had two cards (or three for more experienced players), they were asked to count which player had the highest magnitude in total. The player with the highest total magnitude won a bonus card. The game started over again until every player had two (or three) cards and so on.

Blackjack
This game involved the ability of estimating the result of an addition of fractions and the relation between fractions and units. The rule of Blackjack was to add the value of cards to approach two (or three for more experienced players) without exceeding it. A referee was chosen for each team. He first dealt two cards by player. Afterward, each player could decide whether he wanted another card or would rather stop. When every player had stopped, they all showed their card and checked who was closer to two. The winner was the player with a total of fractions closer to two, without exceeding that quantity. In case of disagreement between players, they could use the wooden disks to compare fractions. When the first game was over, another referee was chosen and they could start over again.

Difficulty Levels
Four difficulty levels were defined. Each level was composed of 40 cards of different colors. The first level (yellow cards) was only composed of familiar fractions with denominators up to 5. Some fractions were represented by numbers (e.g., 1/5),
words (e.g., one-fifth), or drawings (e.g., circles or squares). This first level was mainly aimed at introducing simple and familiar fractions. The second level (red cards) was composed of fractions with denominators up to 12. Moreover, equivalent fractions were introduced, with the result that a given magnitude could be represented by different fractions. Thus, there were different ways to match pairs of fractions. Improper fractions (i.e., numerator > denominator) were introduced in the third level (blue cards). Pictorial representations were multiplied (e.g., circles, squares, octagons, and collections of objects). As the main focus in level three was put on improper fractions, equivalent fractions were set aside, leaving them for the last level. The fourth and last level (green cards) involved fractions with denominators up to 12, improper fractions, various pictorial representations, equivalent fractions, and less familiar fractions.

Treasure Hunt and Blackjack were not used with Level 4 cards because the difference with Level 3 consists of equivalent fractions. This parameter was not relevant in both these games. The use of equivalent fractions was not needed to perform better in these games. Nonetheless, two more sessions could have been added with Level 4 cards. But they would have been very similar to Level 3 in terms of learning processes.

Tests
Pre- and posttests were identical. They were divided into five different categories of questions assessing on one hand conceptual aspects (three categories: estimation, comparison, and number lines) and on the other hand procedural aspects (two categories: arithmetic operations and simplification). Diverse concepts relative to fractions were included in the tests, such as improper fractions, equivalent fractions, and unfamiliar fractions.

Conceptual items assessed a comparison between quantities or ordering. Other studies have used those criteria to assess conceptual understanding (Byrnes & Wasik, 1991; Hallett, Nunes, & Bryant, 2010). For the estimation category (n = 4), pupils had to place a symbolic fraction on a nongraduated number line going from 0 to 1. One point was given if they placed the mark at a point situated within 1 cm of the right location, and no point was attributed if the mark was further away. For the comparison category, children had to choose the larger symbolic fraction out of two (n = 16). Fractions used in the comparison task were fractions with the same numerator (n = 4), the same denominator (n = 4), different numerators and denominators (n = 4), or a fraction and one (n = 4). One point was attributed for each correct answer. For the graduated number line category (n = 9), children were asked to place a symbolic fraction on a graduated number line. Each line was 13 cm long with one tick mark every 1 cm. The digit 0 was always indicated below the first tick mark at the left end of the line, and one other fraction of reference was marked underneath one of the other 12 tick marks. The fraction of reference and its position changed from trial to trial. One point was given if they placed the fraction at the right graduation. The use of graduated number lines was particularly interesting to analyze different types of errors that pupils would make. Cronbach’s α indicated a high level of internal consistency for the conceptual category (α = .84).

Procedural skills were assessed by items that could be easily solved by applying a procedure or an algorithm, such as arithmetic operations or simplification. Arithmetic operations involved additions and subtractions of symbolic fractions with the same denominator (n = 4), and multiplications of symbolic fractions with different denominators (n = 6), multiplication of a symbolic fraction by a natural number (n = 4), and multiplication of a symbolic fraction by another symbolic fraction (n = 4). We were also particularly interested in analyzing the proportion of errors linked to the whole number bias, such as 1/2 + 1/4 = 2/6 (Ni & Zhou, 2005). The simplification category (n = 4) involved the denominator to be either divided by 2 or by 3. Cronbach’s α was .79 for the procedural category.

The maximum length of the test was 50 min.

EXPERIMENTAL DESIGN

We ran Grade (two levels: Grade 4 and Grade 5) × Condition (two levels: experimental group vs. control group) analyses of covariance (ANCOVAs) using the posttest scores as dependent variable, with the corresponding pretest score as a covariate. This was necessary to interpret our findings, given the differences between groups at pretest. By doing this, we could test that the experimental group improved more than the control group. We also introduced grade as a between-subject factor to test whether the intervention was more efficient in Grade 4 or 5. The ANCOVAs were run for both conceptual and procedural factors and also for each task separately. Similar analyses were also run on comparisons of fractions with the same denominators and same numerators, as well as comparison of a fraction to the unit and placing the unit on a graduated number line. Common patterns of errors are also reported for graduated number lines and arithmetic operations.

RESULTS

To account for varying pretest scores, the posttest scores were compared by running an ANCOVA using the corresponding pretest score as the covariate. The Grade × Condition ANCOVA with pretest score as the covariate was run for each measure.
Table 2
Mean Percentage of Correct Responses and Standard Errors for the Intervention and Control Groups in Grades 4 and 5 for the Conceptual Factor and Each Conceptual Task

<table>
<thead>
<tr>
<th></th>
<th>Conceptual pretest</th>
<th>Conceptual posttest</th>
<th>Estimation pretest</th>
<th>Estimation posttest</th>
<th>Comparison pretest</th>
<th>Comparison posttest</th>
<th>Number lines pretest</th>
<th>Number lines posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control G4</td>
<td>31.6 ± 1.6</td>
<td>36.7 ± 2.2</td>
<td>20.2 ± 6.5</td>
<td>27.1 ± 7.3</td>
<td>53.4 ± 3.5</td>
<td>56.7 ± 4.3</td>
<td>21.3 ± 2.2</td>
<td>26.2 ± 2.6</td>
</tr>
<tr>
<td>Experimental G4</td>
<td>39.3 ± 2.3</td>
<td>53.2 ± 3.2</td>
<td>31 ± 8.2</td>
<td>44 ± 9.3</td>
<td>58.9 ± 4.1</td>
<td>74.9 ± 4.1</td>
<td>28.9 ± 3.1</td>
<td>41 ± 3.9</td>
</tr>
<tr>
<td>Control G5</td>
<td>44.9 ± 2.2</td>
<td>49.5 ± 2.2</td>
<td>35 ± 8.1</td>
<td>38.1 ± 9.7</td>
<td>70.7 ± 4.2</td>
<td>75.1 ± 3.1</td>
<td>28.8 ± 3.1</td>
<td>35.2 ± 3.5</td>
</tr>
<tr>
<td>Experimental G5</td>
<td>44.3 ± 2.6</td>
<td>60.1 ± 2.9</td>
<td>33.1 ± 8.5</td>
<td>52.1 ± 8.5</td>
<td>65.2 ± 5.2</td>
<td>81 ± 3.1</td>
<td>34.7 ± 3.4</td>
<td>47.2 ± 3.9</td>
</tr>
</tbody>
</table>

Table 3
Mean Percentage of Correct Responses and Standard Errors for the Intervention and Control Groups in Grades 4 and 5 for the Procedural Factor and Each Procedural Task

<table>
<thead>
<tr>
<th></th>
<th>Procedural pretest</th>
<th>Procedural posttest</th>
<th>Operations pretest</th>
<th>Operations posttest</th>
<th>Simplification pretest</th>
<th>Simplification posttest</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control G4</td>
<td>15 ± 1.8</td>
<td>35 ± 2.9</td>
<td>15 ± 1.2</td>
<td>23.6 ± 2.2</td>
<td>14.8 ± 6.8</td>
<td>46.3 ± 9.4</td>
</tr>
<tr>
<td>Experimental G4</td>
<td>17 ± 1.6</td>
<td>27.8 ± 2.5</td>
<td>22.7 ± 1.7</td>
<td>27.1 ± 2</td>
<td>11 ± 6.3</td>
<td>28.3 ± 8.2</td>
</tr>
<tr>
<td>Control G5</td>
<td>31.9 ± 2.7</td>
<td>46.8 ± 3.4</td>
<td>35.7 ± 2.2</td>
<td>38.6 ± 3.1</td>
<td>28.1 ± 9.1</td>
<td>55 ± 9.9</td>
</tr>
<tr>
<td>Experimental G5</td>
<td>36.7 ± 2.7</td>
<td>36.5 ± 2.9</td>
<td>32.6 ± 2.2</td>
<td>30.5 ± 2.2</td>
<td>40.8 ± 9.9</td>
<td>42.6 ± 9.6</td>
</tr>
</tbody>
</table>

Conceptual and Procedural Factors
The ANCOVA run on the conceptual factor showed a significant effect of pretest, $F(1, 264) = 134.7, p < .001, \eta^2_p = 0.34$, and a significant effect of condition, $F(1, 264) = 217, p < .001, \eta^2_p = 0.076$, indicating that the experimental group scored higher on the conceptual factor (Table 2). We ran the same analysis for the procedural factor. Results showed a significant effect of pretest, $F(1, 264) = 60.2, p < .001, \eta^2_p = 0.19$, and a significant effect of condition, $F(1, 264) = 15.6, p < .001, \eta^2_p = 0.056$, indicating that the control group scored higher on the procedural factor (Table 3).

We also analyzed results for each category, namely estimation, comparison, number lines, arithmetic operations, and simplification.

Estimation
In the estimation task, children were asked to place a fraction on a nongraduated number line going from 0 to 1. The mean percentage of correct responses was different for pretest in control and experimental groups in Grade 4. Nevertheless, experimental classes (+13%) improved more than control classes (+6.9%). In Grade 5, results for pretest were similar in both groups. Experimental classes (+19%) improve more than control classes (+3.1%). The Grade × Condition ANCOVA showed a significant effect of pretest, $F(1, 264) = 75.4, p < .001, \eta^2_p = 0.22$, and a significant effect of condition, $F(1, 264) = 13.1, p < .001, \eta^2_p = 0.05$, indicating a significant difference between experimental classes and the control group for the estimation task. There was no significant effect of grade ($p = .14$).

Comparison
In the comparison task, pupils were asked to decide which of two fractions was the largest. The Grade × Condition ANCOVA showed a significant effect of pretest, $F(1, 264) = 35.6, p < .001, \eta^2_p = 0.12$, a significant effect of condition, $F(1, 264) = 29.9, p < .001, \eta^2_p = 0.10$, indicating a significant difference between experimental classes and control groups in the comparison task and a significant effect of grade, $F(1, 264) = 14.1, p < .001, \eta^2_p = 0.05$, indicating that Grade 5 performed better than Grade 4. Pretest results in Grade 4 were similar in experimental and control groups. In Grade 5, results for experimental classes were lower than that for control classes (−5%). Experimental groups improved more at posttest (+16% in Grade 4, +15% in Grade 5) than control groups (+3% in Grade 4, +5% in Grade 5).

The same pattern of results was obtained for the comparison of fractions with the same denominators, the same numerators, and the comparison of a fraction to the unit (Table 4).

Number Lines
Children had to place a fraction on a graduated number line. The Grade × Condition ANCOVA showed a significant effect of pretest, $F(1, 264) = 31.8, p < .001, \eta^2_p = 0.11$, and a significant effect of condition, $F(1, 264) = 10, p = .002, \eta^2_p = 0.04$. There was no significant effect of grade ($p = .13$). Experimental groups (+12% in Grade 4, +13% in Grade 5) progressed more than the control groups (+5% in Grade 4, +6% in Grade 5).
Table 4
Mean Percentage of Correct Responses and Standard Errors for the Intervention and Control Groups in Grades 4 and 5 for the Comparison of Fractions With Same Denominators, Fractions With Same Numerators, and Fractions Referring to 1

<table>
<thead>
<tr>
<th></th>
<th>Control G4</th>
<th>Experimental G4</th>
<th>Control G5</th>
<th>Experimental G5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Comparison same</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>denominator pretest</td>
<td>69.3 ± 8.8</td>
<td>76.6 ± 8.9</td>
<td>85.8 ± 6.1</td>
<td>76.5 ± 9.6</td>
</tr>
<tr>
<td>denominator posttest</td>
<td>71.3 ± 9.1</td>
<td>87.2 ± 7.1</td>
<td>89.3 ± 5.5</td>
<td>91.6 ± 6.1</td>
</tr>
<tr>
<td><strong>Comparison same</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>numerator pretest</td>
<td>34.4 ± 9.6</td>
<td>41.7 ± 11.2</td>
<td>67.9 ± 9.1</td>
<td>59.1 ± 10.2</td>
</tr>
<tr>
<td>numerator posttest</td>
<td>34.3 ± 7.9</td>
<td>66.1 ± 8.9</td>
<td>71.5 ± 7.4</td>
<td>78.3 ± 8.2</td>
</tr>
<tr>
<td><strong>Comparison unit</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>pretest</td>
<td>52.2 ± 5.3</td>
<td>63.7 ± 6.4</td>
<td>73.7 ± 7.9</td>
<td>68 ± 6.4</td>
</tr>
<tr>
<td>posttest</td>
<td>54.1 ± 6.3</td>
<td>81.7 ± 5.9</td>
<td>84.4 ± 5.3</td>
<td>90.9 ± 4.3</td>
</tr>
</tbody>
</table>

Placing the Unit on the Number Line
Some items required placing 1 on the number line. The Grade × Condition ANCOVA run on posttest scores revealed a significant effect of pretest, $F(1, 264) = 27.7, p < .001$, $\eta_p^2 = 0.095$, and a significant effect of condition, $F(1, 264) = 11.6, p < .001$, $\eta_p^2 = 0.042$, indicating a significant difference between experimental classes and control classes. There were large differences in Grade 4 between control (15.7%) and experimental classes (31.5%) for the pretest. However, experimental classes improved more than control classes (+16.9% and +7.8%, respectively). In Grade 5, there were also differences between control (30.9%) and experimental classes (36.8%) for the pretest. Experimental classes improved more than control classes (+19.6% and +8.5%, respectively).

Error Analysis
We were particularly interested to see the representation that children have of the relationship between the unit and a fraction. When pupils were asked to place 1 on the number line, common errors could be pinpointed. When they made a mistake, they often placed 1 at the very beginning of the line as if it could only be representing the beginning of the counting sequence. Many pupils also placed the unit at the end of the number line, showing their poor understanding of improper fractions. For those children, a fraction can only be smaller than one. Another common error was to place the unit at the 10th grading, showing the influence of the decimal numeral system (Table 5).

Arithmetic Operations
Arithmetic operations included additions, subtractions, and multiplications of fractions. The Grade × Condition ANCOVA run on posttest scores revealed a significant effect of pretest, $F(1, 264) = 10.4, p = .001$, $\eta_p^2 = 0.038$. In general, the control groups progressed more than experimental classes both in Grades 4 and 5. However, there were no significant effect of condition ($p > .2$), nor grade ($p = .81$).

Whole Number Bias Errors
Errors linked to the whole number bias were, for example, $1/3 + 1/4 = 2/7$ (Ni & Zhou, 2005). Table 6 shows the percentage of errors linked to the whole number bias for each grade and condition. Interestingly, errors linked to the whole number bias only dropped in the experimental group in Grade 5 (−12%).

Simplification
The Grade × Condition ANCOVA revealed a significant effect of pretest, $F(1, 264) = 38.1, p < .001$, $\eta_p^2 = 0.13$, and a significant effect of condition, $F(1, 264) = 15.8, p < .001$, $\eta_p^2 = 0.06$, showing a significant difference between control and experimental classes in the simplification task. There was no significant effect of grade ($p = .54$). Grade 4 children had equivalent scores at pretest, that is, between 11% and 15% of correct responses (Table 3). However, in Grade 5, control...
groups had poorer performance than experimental classes at the pretest. Globally, control groups progressed more than experimental groups for the simplification task.

**DISCUSSION**

We designed an educational intervention to improve the learning of fractions in Grades 4 and 5. The development of this intervention was based on the premise that most of the difficulties in fraction learning stem from the fact that pupils have a poor representation of the global magnitude of fractions and that they do not grasp the conceptual meaning of fractions.

Our data showed that pupils who received the intervention improved more than children from the control groups for conceptual items of the test (estimation, comparison, and number lines). Even if these results do not allow us to conclude that pupils developed the numerical representation of fractions, we can still posit that it allows children to develop conceptual knowledge about fractions and compensate for major difficulties.

The intervention had a positive impact on pupils' motivation. In line with research showing that motivation can be fostered through the introduction of playfulness into teaching (Kangas, 2010; Smilansky & Shefatya, 1990), pupils actively took part in the learning process and showed great enthusiasm when playing the fraction games. Even if we did not explicitly measure motivation levels in the classroom, teachers were positively surprised to see their pupils enjoying learning fractions. Motivation in the classroom is an important factor as it has been shown to develop feelings of mastery and competence and is related to school achievement (Gottfried, 1982). Future studies examining the role of learning-by-doing and playful interventions should assess more objectively the impact of motivation in learning fractions. We would have liked to also control for the possibility that fraction skills improved due to an increase in motivation through the playing of games rather than any fraction-specific training but did not have the resources to do this. However, children from our control group selectively improved in procedural skills, while the experimental group selectively improved in conceptual understanding, thus suggesting that the effect of the intervention was not solely a global motivational effect.

Understanding the relationship between a fraction and the unit is well known to be a major difficulty when learning fractions (Case, 1985; Smith, Solomon, & Carey, 2005). However, the performance of the experimental classes suggests that class activities focusing on comparing and manipulating fraction magnitudes allow children to grasp the link between fractions and the concept of unity. Indeed, experimental classes had better performance in understanding the link between fractions and one as a “unit.” They performed better than children of the control group when comparing fractions to the unit and when placing the unit on a number line. Typical mistakes observed in the number line subtest consisted in placing one at the beginning or end of the line or placing it at the 10th grading. At posttest, Grade 5 children from the experimental classes made fewer of these typical mistakes.

We also examined the potential effects of the intervention on procedural skills. One current hypothesis is that improvements of conceptual knowledge could facilitate the mastery of procedures (Byrnes & Wasik, 1991). Our results do not support this hypothesis as experimental classes showed less improvement than control classes for arithmetic operations and simplification tasks. Nevertheless, two interesting results were found. First, there was a drop in errors linked to the whole-number bias in arithmetic operations in Grade 5 experimental classes. Second, children from the experimental group showed some improvement for additions and subtractions with the same denominator. It is possible that conceptual knowledge (in this case, the construction of the mental representation of the magnitude of fraction) allowed children to answer questions that were considered as procedural items in the design of the tests. However, the use of conceptual knowledge as a substitute for procedural knowledge seems to be limited. More complex operations, such as additive operations with fractions of different denominators, remain inaccessible for children who have not explicitly learned the procedure. Conceptual knowledge thus appears insufficient to allow pupils to invent appropriate procedures to solve such problems. However, longer and reinforced conceptual learning might perhaps lead to improved procedural performance.

One of the surprising results of this study was that pupils from the control group improved more than pupils from the experimental groups on most of the procedural tasks. This seems related to the lessons that teachers gave children of the control group. Teachers from the control classes put more emphasis on procedural knowledge than on conceptual knowledge. Children of the control group did not show better conceptual understanding after 10 weeks. It seems that they have been learning procedures in a mechanical way and still have poor representations of the magnitude of fractions.

As mentioned earlier, another characterization of the relationship between conceptual and procedural knowledge (Rittle-Johnson & Alibali, 1999) is the suggestion of iterative development with interactive and reciprocal influences. Children would gradually improve both their conceptual and procedural knowledge, and each type of improvement would permit progress on the other in turn. If this view is correct, better conceptual understanding could be fruitful for the rest of the curriculum when pupils learn procedures with their teacher. Their increased understanding of the meaning of fractions would help them make sense of the procedures and hence gain faster mastery and better control of their application. Longitudinal studies would be necessary to further investigate this question.
Further studies should investigate long-term advantages of these types of teaching methods. As conceptual learning is considered to lead to longer and deeper understanding (Sawyer, 2006), children may benefit from learning-by-doing style interventions in the long term. Pupils could use their conceptual understanding to develop a more appropriate representation of fractions, which might then help them create further procedural knowledge. As shown in this study, experimental classes showed improvement in certain procedures, such as addition and subtraction of fractions with the same denominator. Their conceptual understanding of numerical fraction helped them deal with procedural skills. Long-term advantages should be examined in future research.

Most theories about conceptual change underline how knowledge about whole numbers interferes with learning fractions (e.g., Ni & Zhou, 2005). However, Siegler et al. (2011) offered another viewpoint on the whole number bias. They argue that difficulties in learning fractions do not come from comparisons between fractions and whole numbers as such but rather in inaccurate comparisons between the two types of numbers. Teaching methods should focus more on the common characteristics between fractions and whole numbers, such as both fractions and whole numbers have magnitudes that can be ordered, compared, and represented on number lines.

In conclusion, this study showed clear improvements in conceptual understanding of fractions in Grades 4 and 5 after an educational intervention aimed at helping children associate fractional notations with magnitudes. The conceptual improvements allowed children to develop some elementary arithmetic procedures with fractions, but further research is needed to assess the influence of conceptual understanding on procedural learning. Further research should investigate through longitudinal studies whether such interventions have long-lasting positive effects and whether, as we would hypothesize, such conceptual improvements facilitate children’s acquisition and use of procedures and algorithms and help avoid mathematical blockage at later stages of the curriculum.

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Developing Children's Understanding of Fractions


