Frequency Domain Equalization for Dispersive Birefringent Nonlinear Optical Fibers

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Abstract

High order linear modulations, combined with coherent detection and signal processing at the receiver, are nowadays considered to reach the new capacity targets for the communications over the optical fibers. In order to reduce the computational complexity of the algorithms at the receiver, it has recently been proposed to compensate for the chromatic dispersion (CD) and polarization mode dispersion (PMD) in the frequency domain. Singlecarrier with frequency domain equalization (SC-FDE) is an attractive method to compensate linear channel distortions as its very low peak to average power ratio (PAPR) is likely to limit the nonlinear distortions of the transmitted signal. The goal of this paper is to show that SC-FDE can successfully equalize CD and PMD distorsion for PDM-WDM (wavelength-division multiplexing) optical point-to-point links even in the presence of nonlinear effects without the assumption of channel knownledge at the receiver.

1 Introduction

During the past decade, both capacity and reach of optical fiber communications have known substantial evolution. Whereas the current deployed long-haul systems make use of On Off Keying, reaching a transmission rate of 10 Gbit/s per channel, the current research aims at transmission rates of 40 or even 100 Gbit/s per channel, combined with the motivation of achieving higher spectral efficiency. At high bit rates, optical communications increasingly suffer from the inherent fiber impairments which can be categorised as [1]:

- Absorbtion losses compensated by in-line amplifiers. The optical amplification through stimulated emission always comes with spontaneous emission which manifests itself in white gaussian noise.
- Chromatic dispersion (CD) whose origin is the frequency-dependant refraction index of the fiber. Different spectral components travel at different speeds, causing dispersion-induced pulse broadening.
- Polarisation mode dispersion (PMD) which is a consequence of the birefringence of the fiber. The distribution of birefringence along the fiber being random, PMD is a linear but stochastic process.

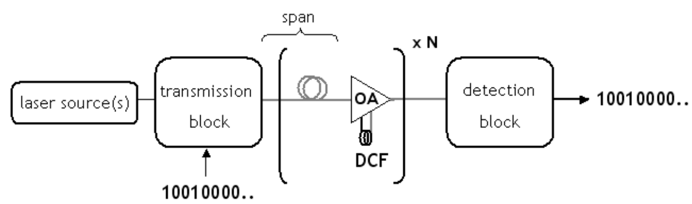

Figure 1: Simplified transmission line. Succession of fiber spans, optical amplifiers (OA) and dispersion compensating fibers (DCF). The pattern is repeated N times (typically the span length is in the order of 100 km and N in in the order of 10 for long-haul terrestrial systems).

- Nonlinear effects originate in the field-dependence of the refraction index. The nonlinear effects can be categorised as self-phase modulation (SPM), through which the intensity of a pulse modulates its own propagation, cross-phase modulation (XPM) where the intensity of one pulse affects the propagation of another pulse (XPM is thus an inter-channel effect) and four-wave mixing. The latter being constrained by phase matching conditions, it can usually be ignored.

Nonlinear effects are in many cases limiting factors for optical transmission systems. In fact, the effect increasing with optical power, nonlinear effects indirectly limit the achievable OSNR (Optical Signal to Noise Ratio) at the receiver.

The typical optical transmission scheme can be depicted as in 1: several fiber sections alternate with in-line amplifiers and sections of dispersion-compensating fibers (DCF). The role of the DCF is to annihilate the effect of chromatic dispersion of the fiber spans. However, the so-defined dispersion maps always keep a small residual dispersion. In fact, it has been both analytically and experimentally proven [2] that a small residual dispersion decreases the outcome of nonlinear effects as a consequence of averaging due to walk-off between adjacent channels.

In order to respond to higher capacity requirements, high order modulation formats like QPSK or QAM modulations are used. As these formats modulate the phase of the signal, they require coherent detection. The combination of coherent detection and digital signal processing is a promising alternative to optical equalization [3, 4]. In addition to wavelength division multiplexing (WDM), polarization division multiplexing (PDM), exploiting the two polarization axes of the optical field, is another method that can be used to double the communication capacity [4]. Orthogonal frequency-division multiplexing (OFDM) has also been recently proposed in the literature to efficiently deal with the optical fiber CD and PMD [5, 6, 7]. The principle of OFDM is to convert a time-domain convolutive channel into a frequency-domain multiplicative channel that can thus be compensated at a low complexity by scalar coefficient multiplications. In practice, inverse Fourier transform is performed on the frequency domain symbols, and at the receiver, a Fourier transform is carried out. The use of fast Fourier transforms (FFT) permits efficient implementation. Single-carrier with frequency domain equalization (SC-FDE) is an interesting alternative to OFDM that benefits from the same low complexity frequency domain channel compensation technique [8, 9]. The principle of SC-FDE is however slightly different: the signal is sent at the transmitter in the time domain. After propagation through the channel, the receiver performs first a Fourier transform to translate the signal to the frequency domain where the channel is compensated at a low complexity and second an inverse Fourier transform to translate the signal back to the time domain. Compared to OFDM, SC-FDE offers two important advantages in the case of optical fibers:

- While OFDM is known to suffer from a high peak to average power ratio (PAPR), the PAPR of SC-FDE is much lower. Since the instantaneous power of SC-FDE waveform is more stable, the robustness of the system to non-linearities which is a severe limiting factor in optical systems will be improved.

- While a complex waveform is transmitted in the case of OFDM, simple phase shift keying (PSK) or quadrature amplitude modulation (QAM) symbols are transmitted in the case of SC-FDE [10]. Therefore, the implementation of the transmitter can be significantly simplified (for example, no complex DACs are required).

A few recent papers have proposed to apply SC-FDE to optical communications to compensate for the CD [11] and PMD [12]. In [13], we firstly exploited the potential capacity doubling of the fiber link due to the use of polarization division multiplexing. However, none of the previous SC-FDE contributions take into account the nonlinearities of the fiber link, as interchannel interference has been completely ignored. The nonlinear system can no longer be implemented as a convolutive channel but is implemented by the well-known SSFM (Split-Step Fourier Transform) numerical propagation method. Furthermore, none of the previous SC-FDE contributions took into account dispersion management. The goal of the present paper is to show that a single carrier transmission scheme can efficiently achieve chromatic dispersion and polarisation mode dispersion equalization even in presence of nonlinear distorsions in the optical fiber. In order to derive the SC-FDE receiver a linear system model including CD and PMD and the transmitter and receiver effects is derived in Section 2 for a point-to-point PDM-WDM link. Section 3 explicits the realistic model of the channel transmission. Nonlinear WDM propagation with PDM, CD and attenuation and a simply-periodic dispersion map based upon a simple dispersion management criterion are presented. In Section 5, numerical simulations are carried out using the realistic channel model and the linear MMSE receiver.

2 System model

Figure 2 shows a block diagram of the optical communication system. We assume that the initial bit sequence $B_x(n)$ is mapped onto complex QPSK symbols. Furthermore, the system capacity is doubled by modulating two independent information streams $B_y(n)$ and $B_x(n)$ on two orthogonal polarizations. To enable SC-FDE equalization, a cyclic prefix at least as long as the linear channel impulse response is inserted, repeating the N_{cp} last symbols at the beginning of each symbol block.

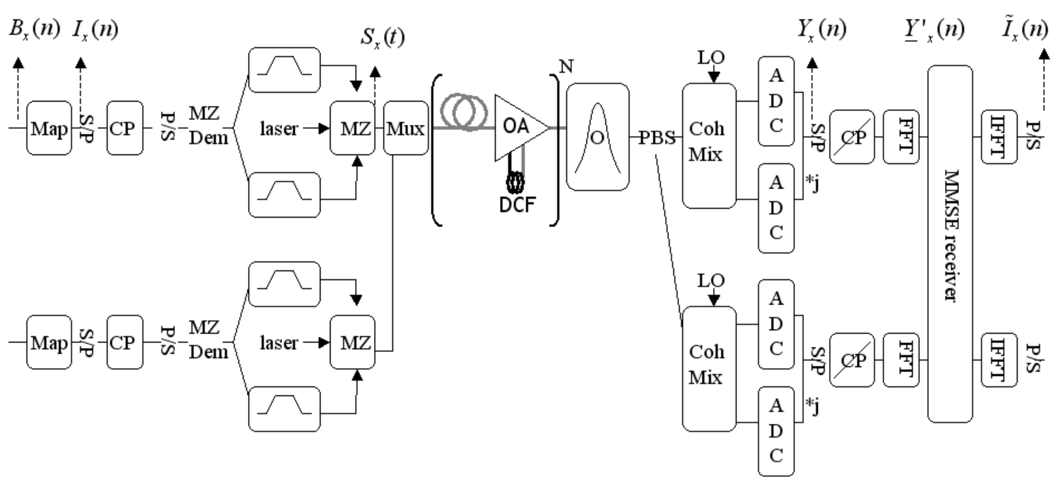

Figure 2: System model.

The symbols are then parallel to serial converted for further processing. The symbols are transformed into an optical signal using a Mach-Zehnder modulator. The output of the idealized quadrature Mach-Zehnder is [14]:

$$S_x(t) = E_{opt}(t)\frac{1}{\sqrt{2}}(e^{i\pi Re(I_x(t))} + e^{i\pi Im(I_x(t))}e^{i\pi/2}) \tag{1}$$

where $I_x(t)$ is the signal obtained by filtering the input sequence $I_x(n)$ and $E_{opt}(t)$ the laser carrier. A multiplexer assumed ideal simply transmits on the fiber the x and y optical data streams on the two orthogonal polarizations and concatenates together the different channels used in the WDM system. The optical channel will be covered in Section 3. At the receiver, a gaussian optical filter selects the WDM channel of interest. The x or y component of the received field obtained through a polarization beam splitter (PBS) enters the coherent detection chain. After sampling, we get the two polarization sequences $Y_x(n)$ and $Y_y(n)$. After another serial to parallel conversion, the cyclic prefic is removed.

3 The transmission channel

In this Section, we present the channel model used to represent the Kerr nonlinearity of optical fibers, the dispersion management and the optical amplification.

3.1 Scalar propagation (neglecting birefringence)

The scalar propagation of optical pulses in single mode fibers is described by the well-known nonlinear Schrödinger equation [15]:

$$\frac{\partial A}{\partial z} + \beta_1\frac{\partial A}{\partial t} + \frac{j\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\alpha}{2}A = j\gamma|A|^2 A \tag{2}$$

where A gives the baseband representation of the optical pulse which has been separated from the transversal pulse profile and normalized in such a way that $|A|^2$ represents the optical power, β_1 represents the group velocity of the pulses, β_2 is the second order chromatic dispersion, γ is the nonlinear coefficient whose magnitude shows the strength of nonlinear effects in the fiber medium and α is fiber attenuation. Chromatic dispersion is easily generalised by adding terms of the form $\frac{j\beta_n}{n!}\frac{\partial^n A}{\partial t^n}$. Note that if only dispersion is considered, the integration of (2) is straightforward in the frequency domain and yields the following frequency response:

$$H_{lin}^{fiber}(\omega) = e^{(-\frac{j}{2}*\beta_2\omega^2 - \frac{j}{6}*\beta_3\omega^3 + \ldots)L} \tag{3}$$

In the presence of non linear phenomena, propagation must be carried out numerically. We use the well-known Split-Step Fourier Method (SSFM), a pseudo-spectral method integrating (2) over small z steps alternating frequency domain steps for linear part of (2) and time-domain steps for the nonlinear part. Generalisation of the SSFM to the WDM case is straightforward in the absence of four wave mixing, the power $|A|^2$ in the nonlinear term simply becomes the total power of all channels.

3.2 Vector propagation with PMD

Without fiber birefringence, PDM propagation for the vector field $\mathbf{A} = [A_x \quad A_y]^T$ are trivially two independant (2) equations for A_x and A_y. In the presence of fiber birefringence, (2) becomes [16], using the approximation of complete mixing of the nonlinear polarization term [17], the Manakov-PMD equation:

$$\frac{\partial \mathbf{A}}{\partial \tau} + \frac{\alpha}{2}\mathbf{A} + j\frac{\beta_2}{2}\frac{\partial^2 \mathbf{A}}{\partial \tau^2} = j\frac{\Delta\beta_0}{2}(l(z).\vec{\sigma})\mathbf{A} + \frac{\Delta\beta_1}{2}(l(z).\vec{\sigma})\frac{\partial \mathbf{A}}{\partial \tau} + j\gamma\frac{8}{9}|\mathbf{A}|^2\mathbf{A} \tag{4}$$

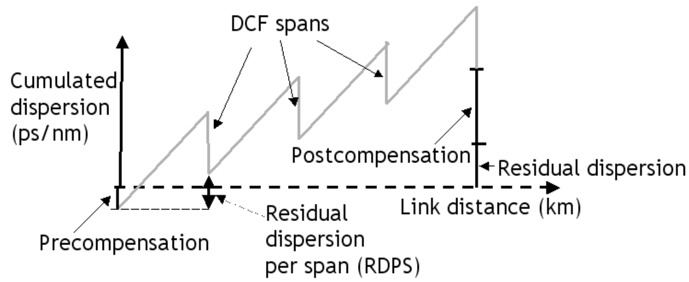

Figure 3: The dispersion map, showing the cumulative dispersion over link distance. The simplest form of the dispersion map is the singly-periodic map depicted here

$\tau = t - \beta_1 z$. The new linear terms are due to fiber birefringence: the propagation constant β depends on the polarization of light, which results in the presence of $\Delta\beta$ terms. At each position z, the fiber owns two eigenmodes ($l(z)$). This feature leads to mode coupling via $l(z).\vec{\sigma}$, a 2×2 Jones matrix which acts of the field $\mathbf{A}$ [16].

The variation of birefringence is commonly described by the evolution of the birefringence vector $\vec{\beta} = |\Delta\beta(z)|\vec{l}(z)/|\vec{l}(z)|$. Its distribution of the eigenvecors is a random process. Experimental results show that the distribution of the local birefringence $|\delta\beta(z)|$ is Rayleigh-distributed. The distribution is caracterised by the beat length $L_B = \frac{2\pi}{<|\delta\beta(z)|>}$ of the fiber. Second, the mean differential group delay between optical pulses propagating on orthogonal polarizations has a maxwellian distribution. The distribution is caracterized by both the beat length and the coupling length L_C, a parameter which caracterises the autocorrelation function of the birefringence. The model used commonly to describe the distribution of $\vec{\beta}$ is the Wai and Menyuk model [18] :

$$\frac{d\beta_k}{dz} = -p\beta_k + s\eta_k(z) \; k = 1, 2 \;\; \beta_3 = 0 \tag{5}$$

η_1 and η_2 are independant white noise samples. p and s are directly related to L_C and L_B [19]. An optical fiber can be described by a concatenation of polarization maintaining fiber sections. In order to describe PMD in the SSFM algorithm, we thus compute the birefringence distribution using 5 for each section. At each SSFM step, we project the field onto the local eigenmodes.

3.3 In-line amplification and dispersion management

The optical equivalent of the noise normally encountered in wireless systems is the spontaneous emission in the in-line amplifiers. The quality of amplifier is expressed through its noise figure: $NF = \frac{SNR_{in}}{SNR_{out}}$. The value of the OSNR is triggered by the channel power and the presence of the optical amplifier [14].

During propagation, nonlinear effects and dispersion interact in a complex way. Consequently, the result of the detection strongly depends on the distribution of dispersion compensation. It is caracterised by the dispersion map, a graph showing the evolution of the accumulated dispersion with distance, as shown in figure 3; in practice the maps are singly or doubly-periodic in order to simplify the optimization problem [20]. A simplistic but convenient tool to optimize dispersion maps is the Phase to Intensity Conversion Criterion (PIC) [20]. It is an estimation of the intensity noise caused by conversion of Kerr induced phase shifts into Intensity noise by chromatic dispersion [15]: $dPIC(z, \omega) = dP(z, \omega)/ < P >$ where $dP(z, \omega)$ is the intensity noise generated at ω in the interval $[z, z + dz]$ and $< P >$ the mean power of the channel. The integration over distance yields a simple link between dispersion map parameters to fulfill in order to set the overall PIC to zero[20].

4 MIMO - MMSE receiver for optical point-to-point links

The receiver reconstructs an estimate $\tilde{I}_i(n)$ based upon the observation of the received sequence $Y'_x(n)$ and the knowledge of the pilot symbols passed through the channel. The goal of the MMSE receiver is to account for dispersion - induced distorsion and PMD-induced polarisation mixing. In Section 5 we will show that the detector is still usable in the case of nonlinear propagation. We now assume a linear system input - output behaviour such that we can write the received sequence $Y'_x(n)$ as a convolution:

$$Y'_x(n) = \sum_{i=1,2} \sum_{l=0}^{L_h} h_{ij}(l) U_i(n-l) + w_j(n) \tag{6}$$

L_h is the length of the total channel response, $w(n)$ are white gaussian noise samples. The noise is caused by spontaneous emission in the in-line erbium doped fiber amplifiers. $h_{ij}(l)$ is the total channel impulse response, and thus the convolution of the linear fiber response h_{lin}^{fiber} as in 3, the transmitter response h_{trans} and the reception response h_{rec}:$h_{ij}(l) = h_{trans}(t) * h_{opt}(t) * h_{rec}(t)|_{t=lT_s}$, where T_s is the symbol period.The block input - output relationship necessary for detector design can be found in [13] For the final channel matrix, we concatenate the receive vectors: $\underline{Y}'(n) = [\underline{Y}'_x(n)\ \underline{Y}'_y(n)]^T$, the matrix model becomes:

$$\underline{Y}'(n) = \underline{\underline{H}}.\underline{U}(n) + \underline{z}(n) \tag{7}$$

$\underline{z}_n$ are the bloc noise samples. $\underline{\underline{\mathrm{H}}}$ is defined by:

$$\underline{\underline{H}} = \begin{bmatrix} \underline{\underline{H}}_{1,1} & \underline{\underline{H}}_{2,1} \\ \underline{\underline{H}}_{2,1} & \underline{\underline{H}}_{2,2} \end{bmatrix} \tag{8}$$

The receiver optimized according to the minimum mean square error (MMSE) criterion, it is thus designed to minimize the symbol estimation error variance. The linear MMSE detector is given by [21]:

$$\underline{\underline{G}} = \left(\frac{\sigma_w^2}{\sigma_U^2} \underline{\underline{I}}_Q + \underline{\underline{H}}^H \cdot \underline{\underline{H}} \right)^{-1} \cdot \underline{\underline{H}}^H . \tag{9}$$

5 Numerical results

For numerical simulations, we chose a standard single mode fiber (SSFM) for the fiber spans. Chromatic dispersion is taken into account for up to third order. Nonlinear effects and third-order dispersion are neglected in the DCF. We consider a typical terrestrial link with 10 spans of 100 km. Channel estimation is carried out using time-separated known symbols for the x and y polarization.

Traditionnaly, dispersion maps were mostly designed so that the residual dispersion at the end of the link was zero [20]. This situation changed in the past few years as coherent detection made digital dispersion compensation possible [4]. Economically speaking, this feature permits to save some DCF length. Figure 4 shows BER curves for different residual dispersions for a one-channel system where nonlinear effect were neglected. The results show that the SC-FDE detector can effectively account for fiber dispersion. For very low residual dispersions, the receiver struggles as the channel estimation is degraded in noise-dominated channel.

We carry out simulations including nonlinear effects over the defined point-to-point link. Results are compared with the linear case in 4. We conclude that nonlinear effects degrade the performance of the MMSE receiver.

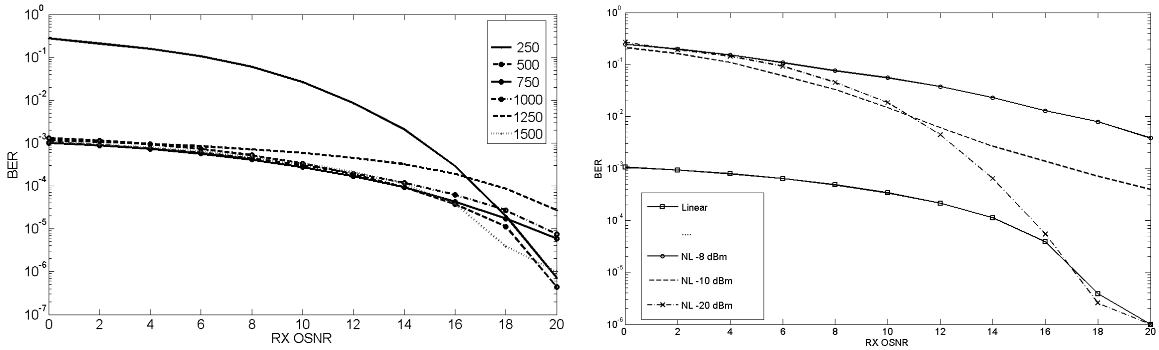

Figure 4: The left figure shows BER curves for different residual dispersions. The legend shows the residual dispersions in ps/nm. The right figure shows BER curves comparing the results at 1500 ps/nm residual dispersion, for a purely linear channel as well including nonlinear effects for different channel power levels. The channel performance is strongly affected by non linear effects.

6 Future works

Channel time-variance is to be implemented in the channel model. We will thus be able to evaluate the effects of channel estimation frequency upon system performance. Furthermore, in future works, channel power will be optimised with respect to nonlinear effects for each value of noise figure of the in-line amplifiers. BER curves will thus be shown as a function of noise figure and thus constant noise power.

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